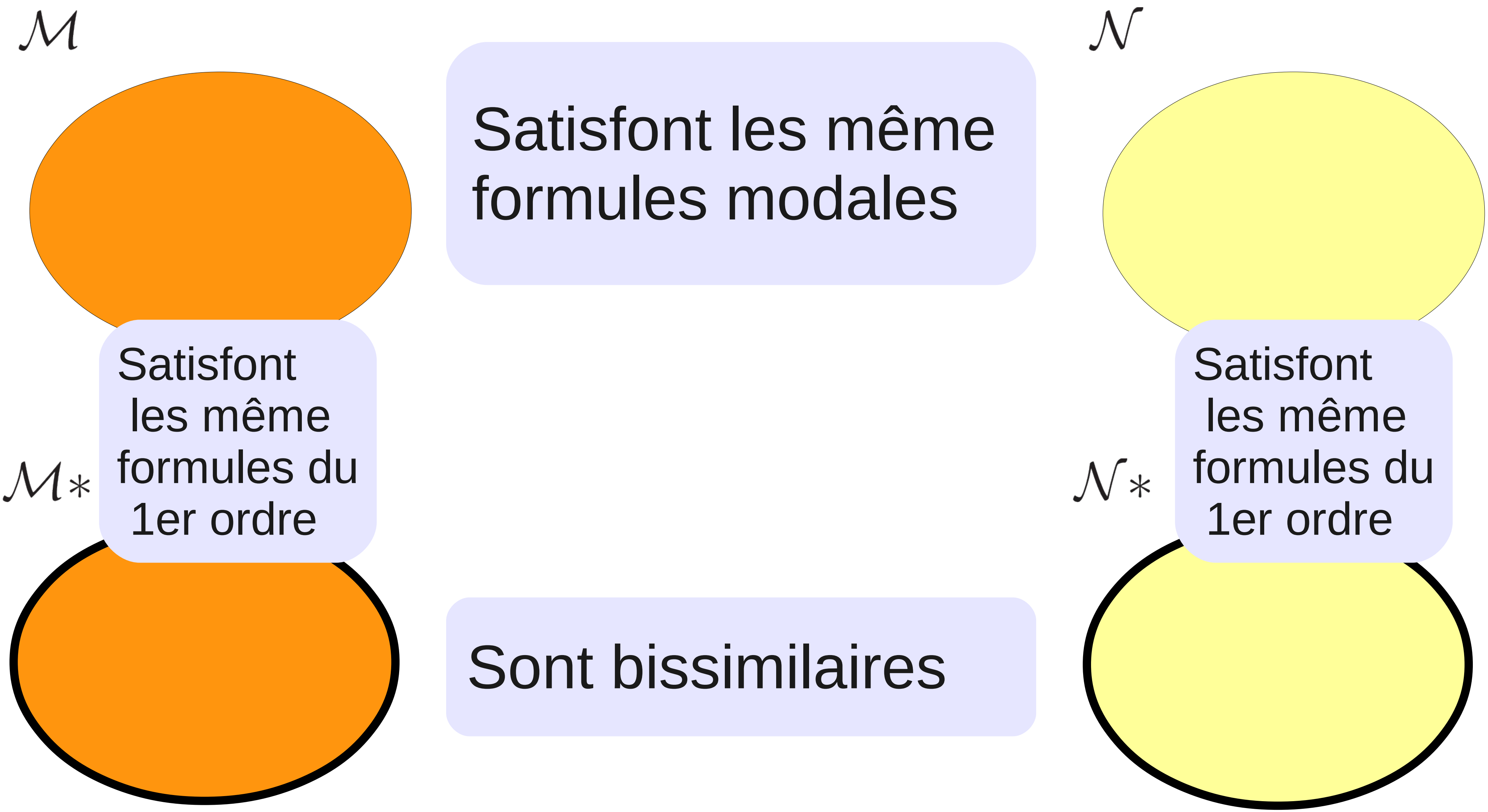
Caractérisation de Van Benthem

François Schwarzentruber
ENS Cachan – Antenne de Bretagne

Detour lemma (admis)



Enoncé

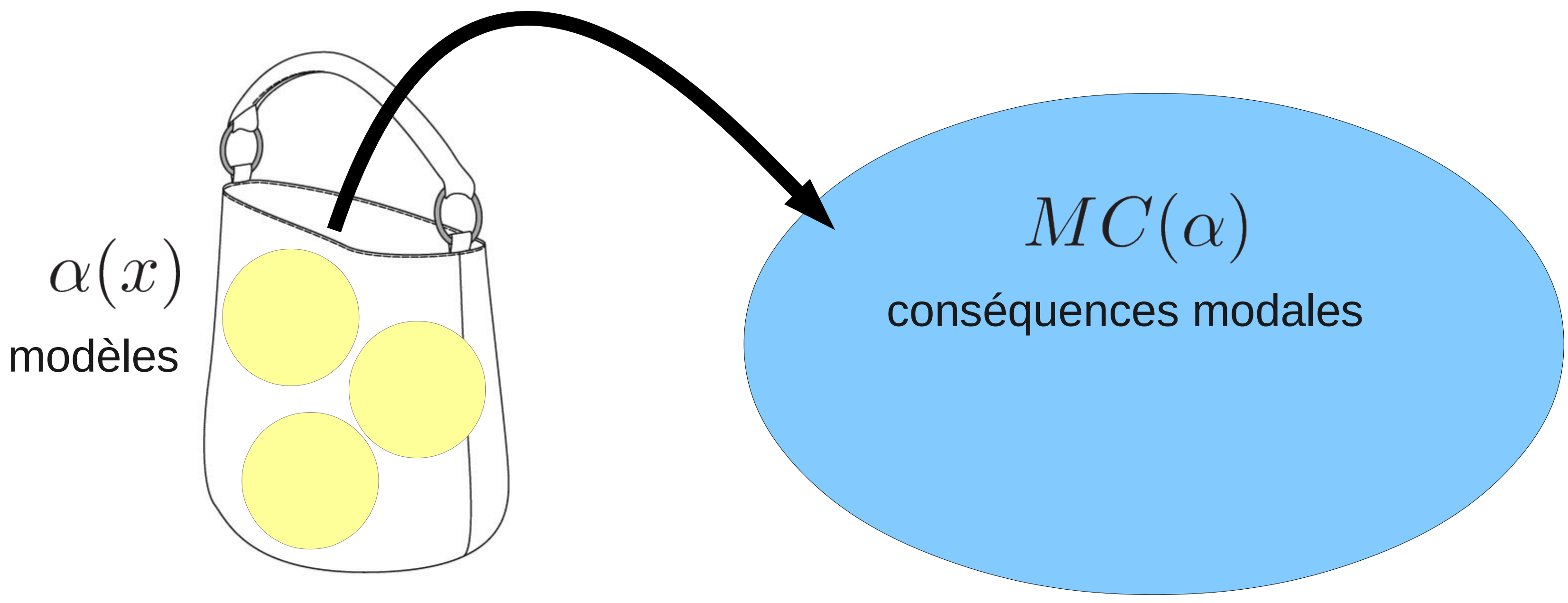
$\alpha(x)$ invariant par bissimulation

$\alpha(x)$ équivalente à la traduction d'une formule en logique modale par bissimulation

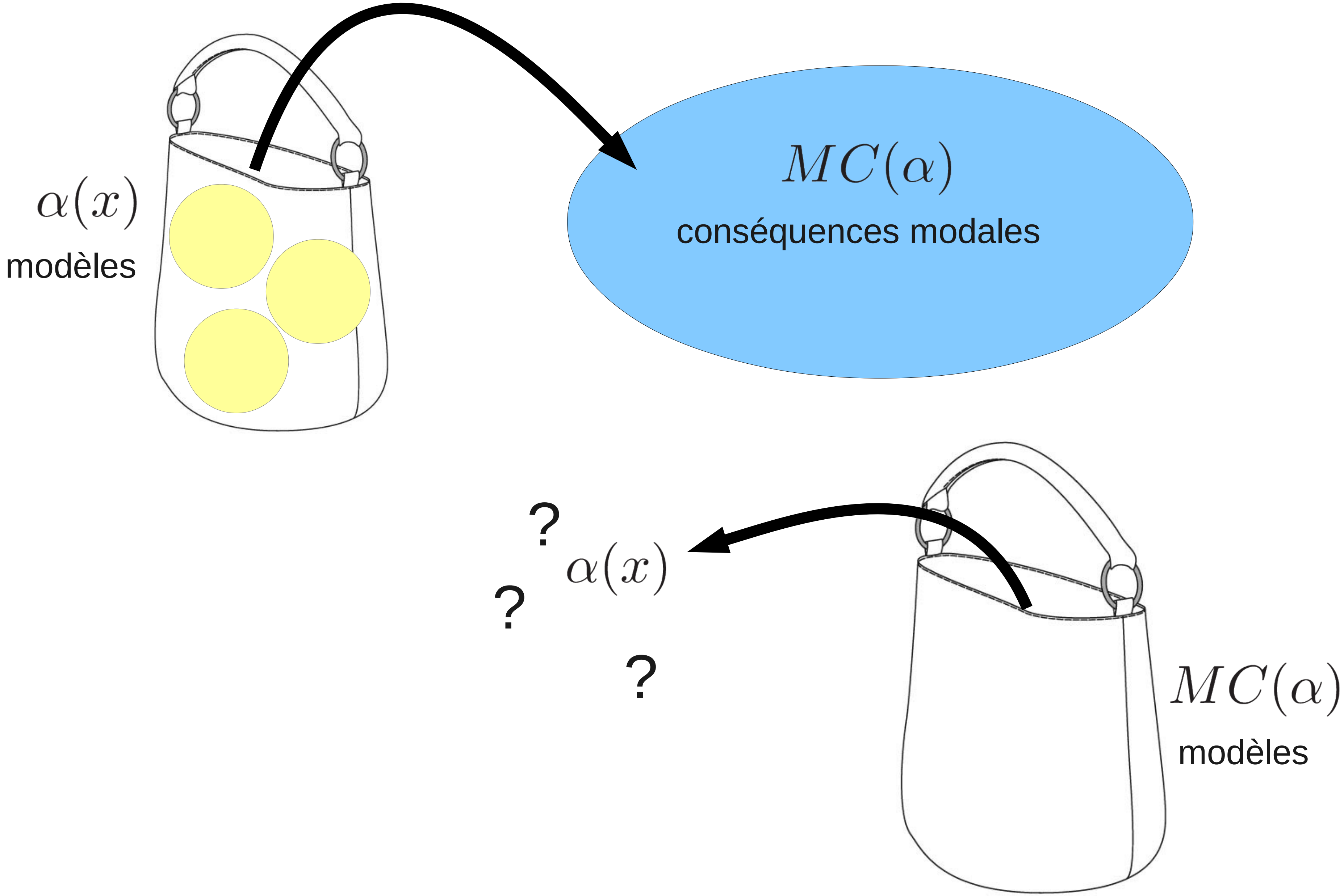
Début de la démonstration

$\alpha(x)$ invariant par bissimulation

$$MC(\alpha) = \{ST_x(\varphi) \mid \varphi \text{ modale et } \alpha(x) \models_{FO} ST_x(\varphi)\}$$



But : montrer que $MC(\alpha) \models \alpha(x)$



Pourquoi $MC(\alpha) \models \alpha(x)$ c'est bien ?

Compacité du premier ordre

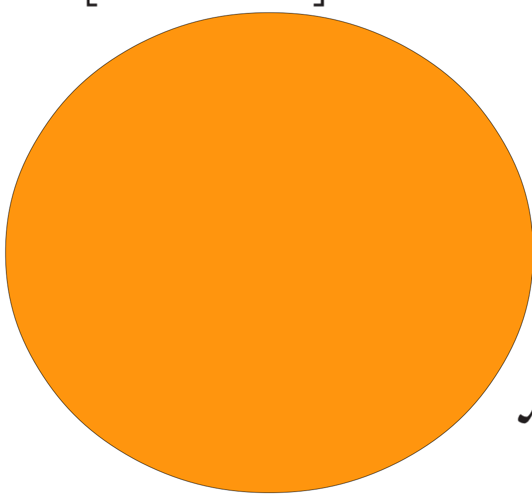
$$\bigwedge_{i \in I_{fini}} \psi_i \models \alpha(x)$$

$$\alpha(x) \models \bigwedge_{i \in I_{fini}} \psi_i$$

$\alpha(x)$ modale !

But : $MC(\alpha) \models \alpha(x)$

$\mathcal{M}[x \leftarrow w]$



$\mathcal{M} \models MC(\alpha)$

$\mathcal{M} \models \alpha(x)$?

?

?

$$MT(x) = \{ST_x(\varphi) \mid \mathcal{M} \models ST_x(\varphi)\}$$

Formules modales vraies dans $\mathcal{M}$

$MT(x) \cup \{\alpha(x)\}$ consistent

$MT(x) \cup \{\alpha(x)\}$ inconsistent

Compacité du premier ordre

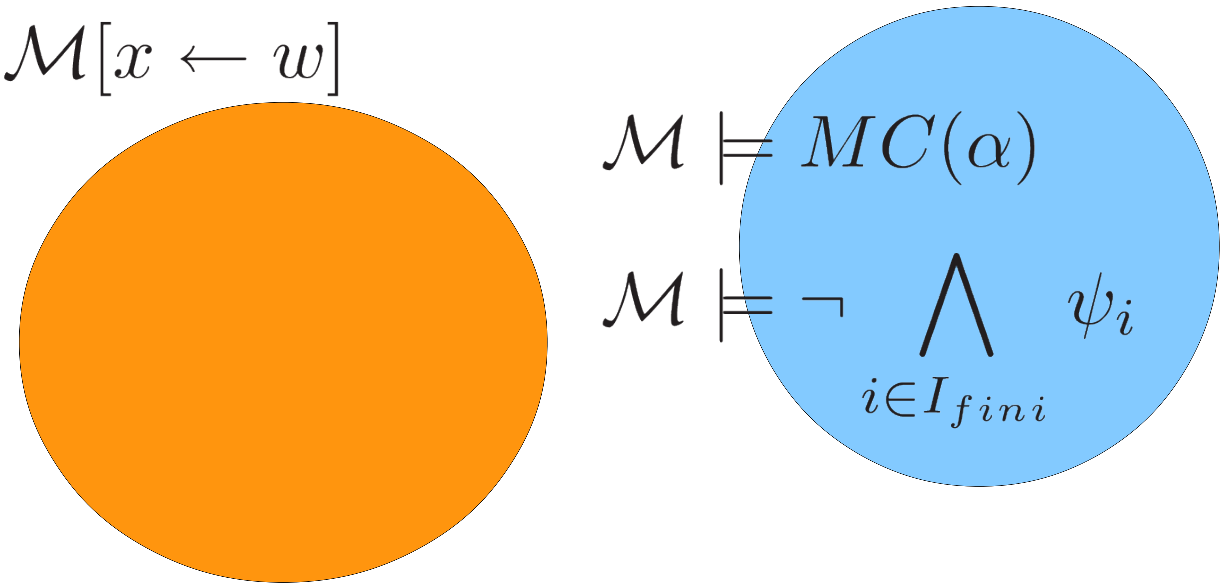
$$\models \bigwedge_{i \in I_{fini}} \psi_i \wedge \alpha(x) \rightarrow \bot$$

$$\models \alpha(x) \rightarrow \neg \bigwedge_{i \in I_{fini}} \psi_i$$

$MC(\alpha)$

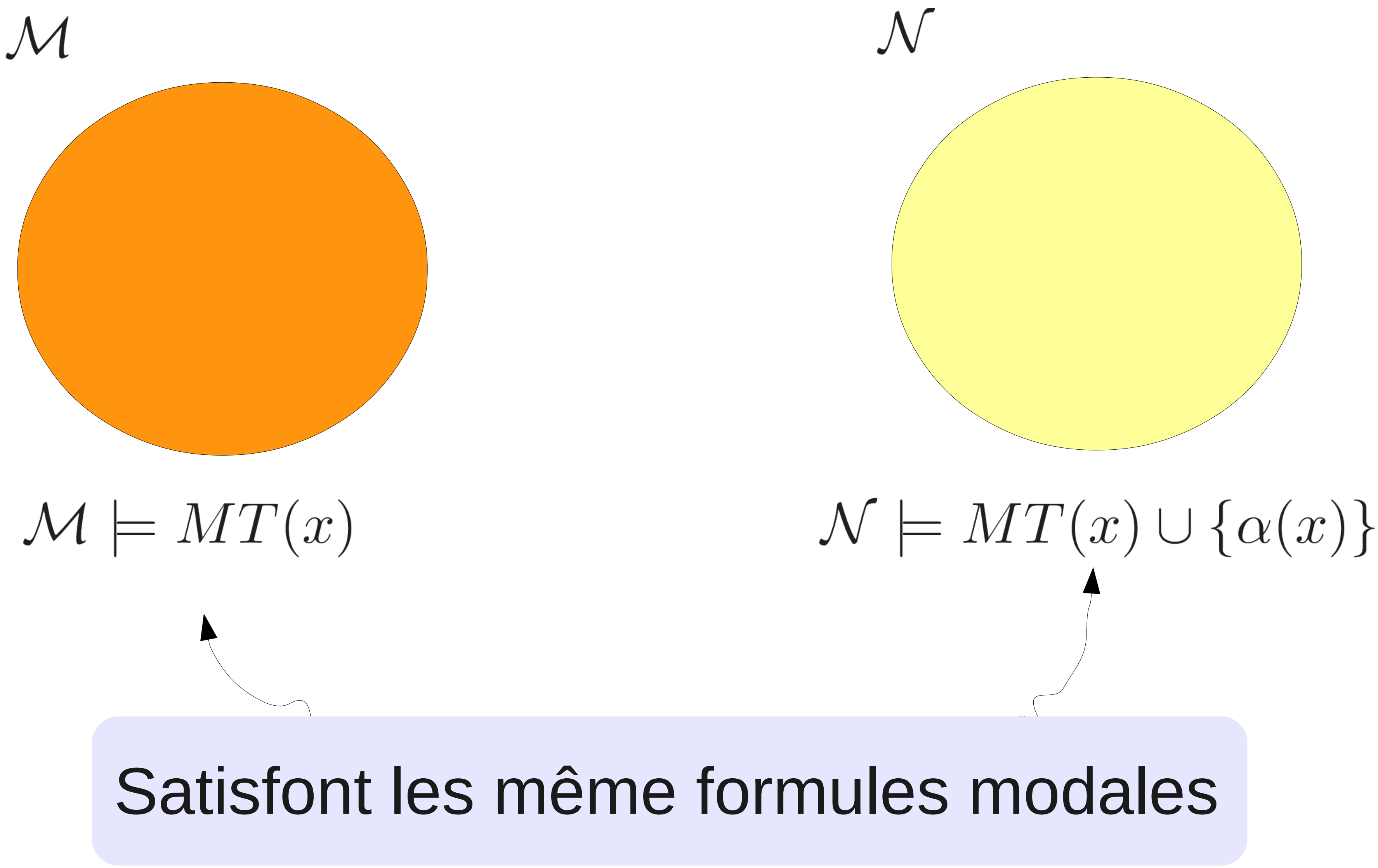
$$\neg \bigwedge_{i \in I_{fini}} \psi_i$$

$MT(x) \cup \{\alpha(x)\}$ consistent

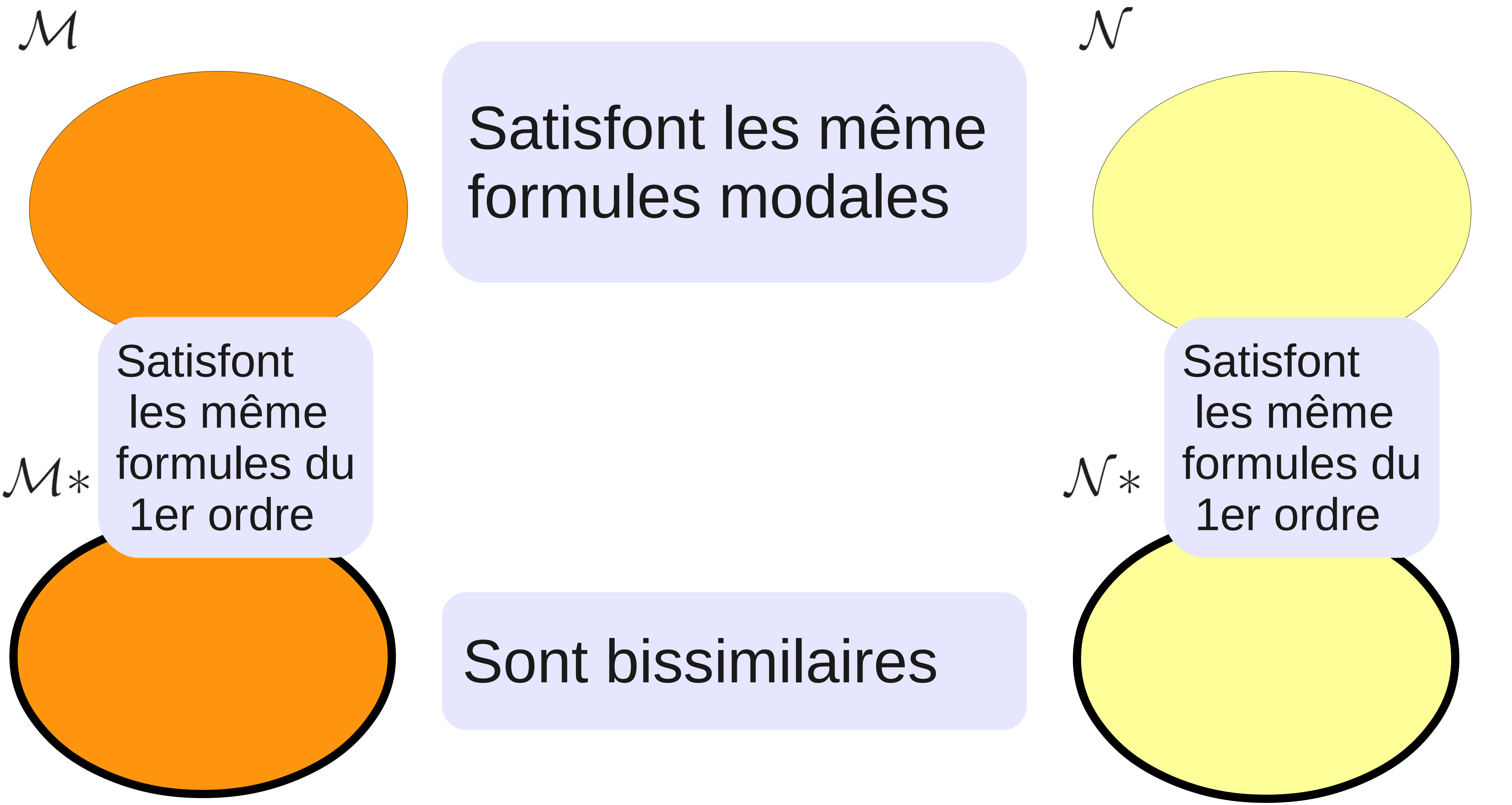


NON !

Un modèle pour $MT(x) \cup \{\alpha(x)\}$



Detour lemma



Detour lemma