

Go is PSPACE-hard

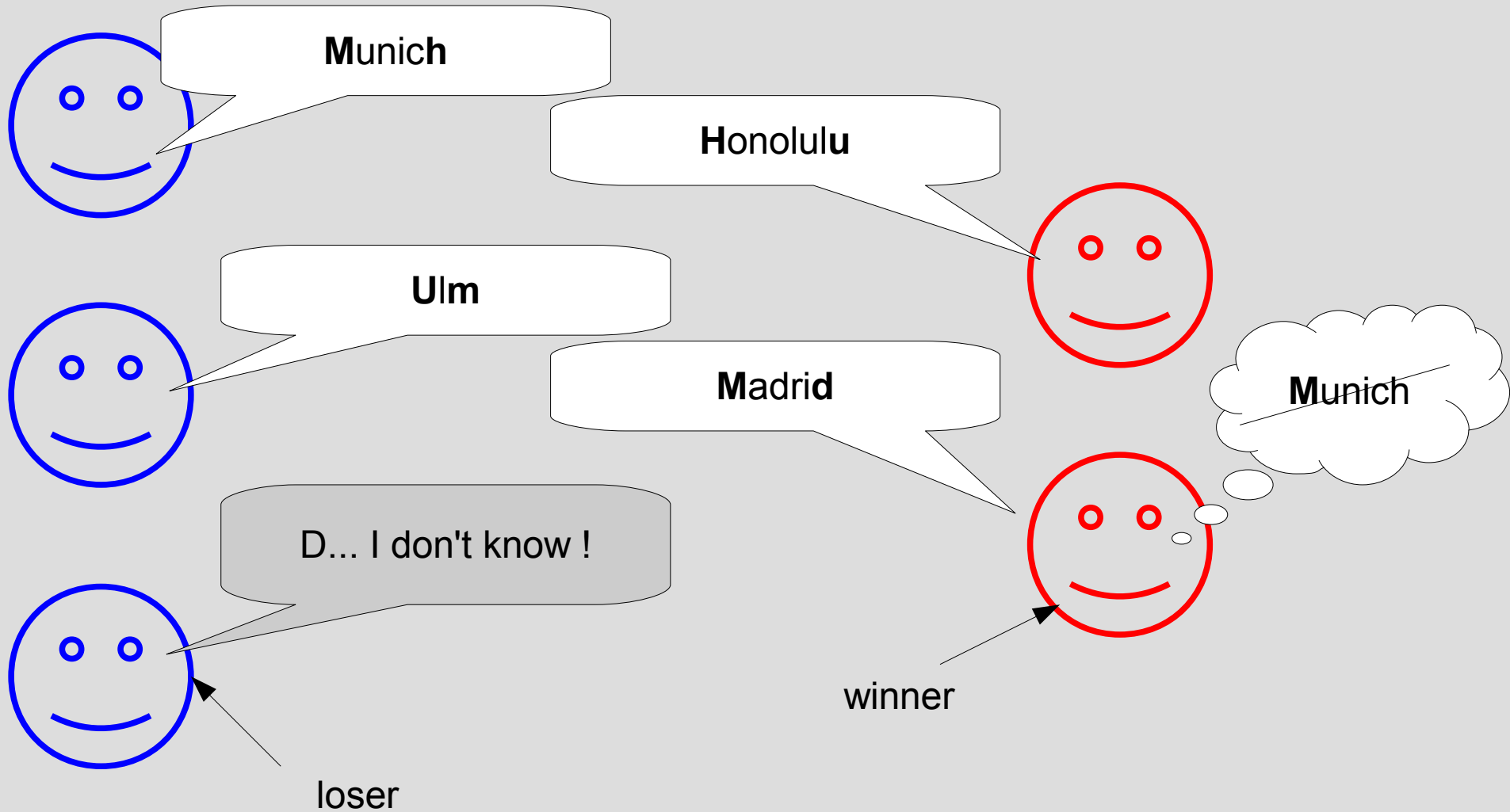
François Schwarzentruber
ENS Cachan – Antenne de Bretagne

- “Go is Polynomial-Space Hard” David Lichtenstein and Michael Sipser.
- Computational complexity. Papadimitriou.

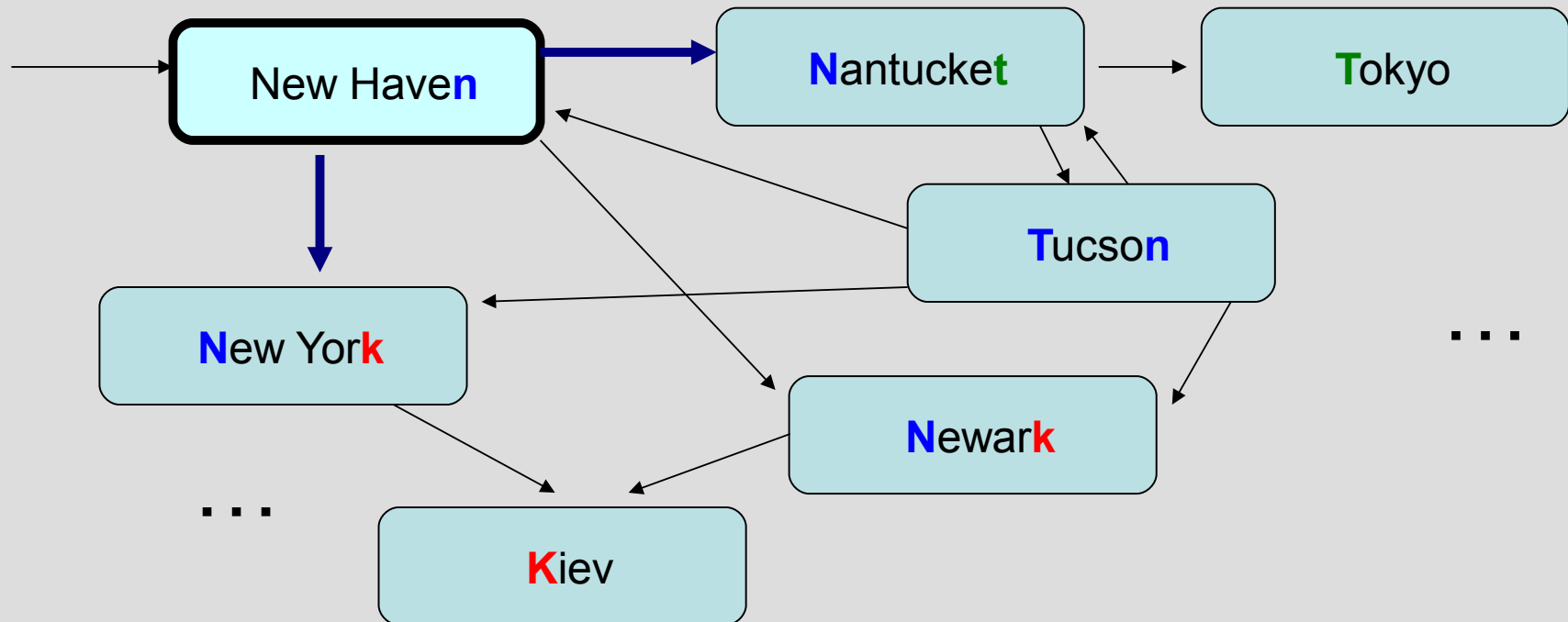
Outline

- Winning strategy in Geography game
- Winning strategy in Go

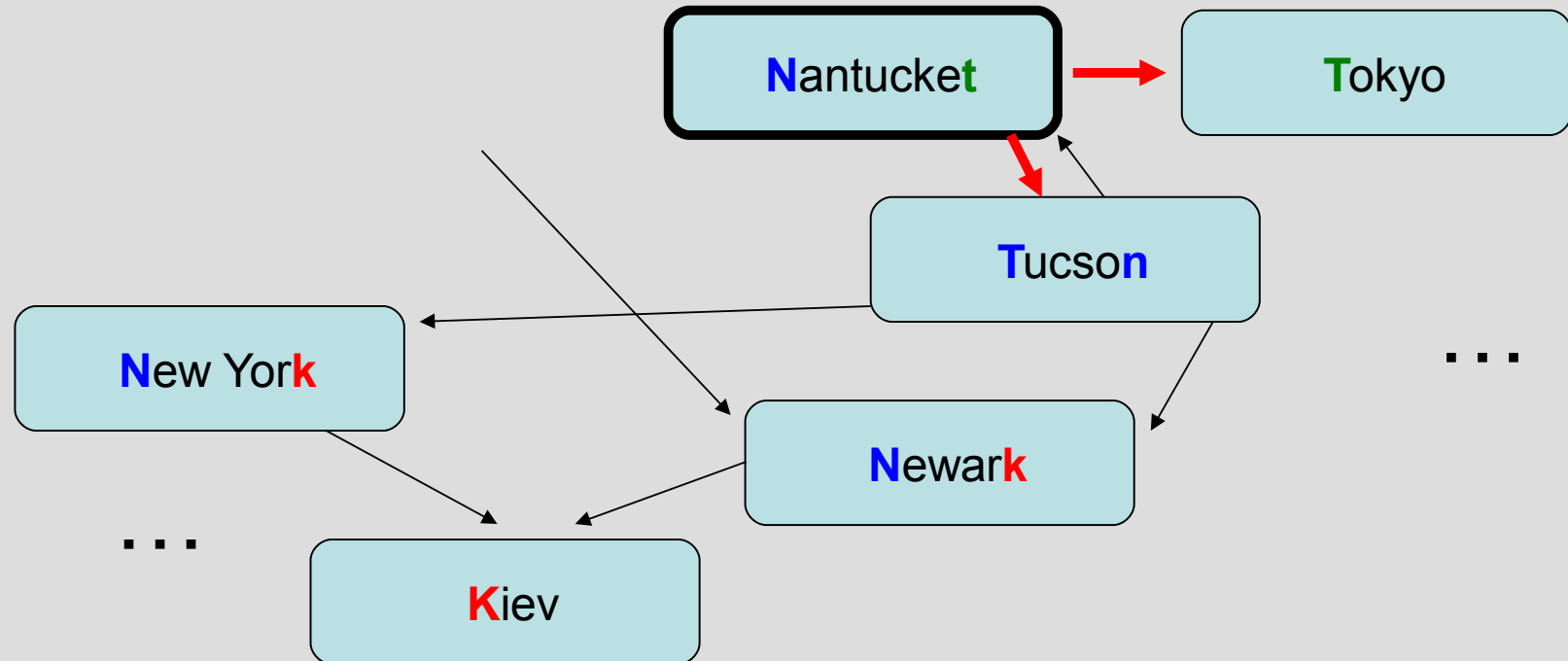
Geography game



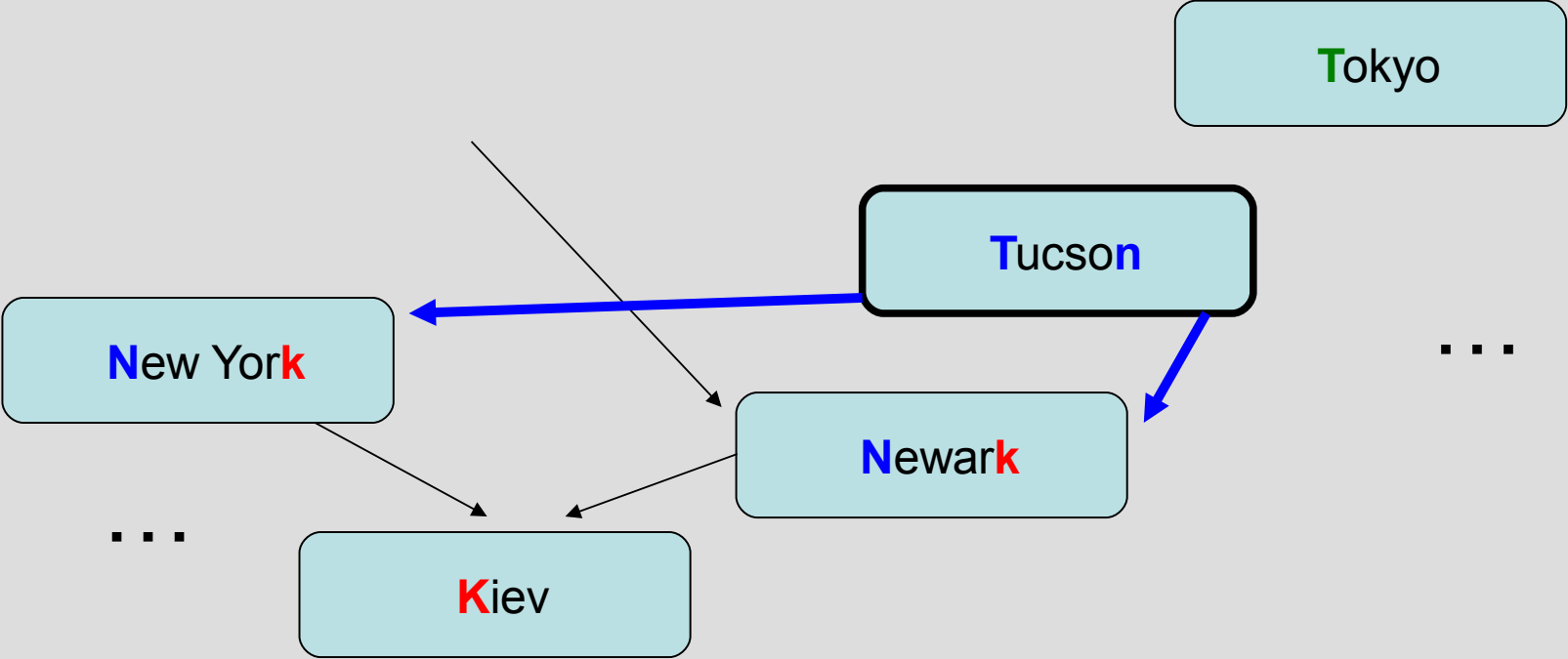
Generalized Geography game



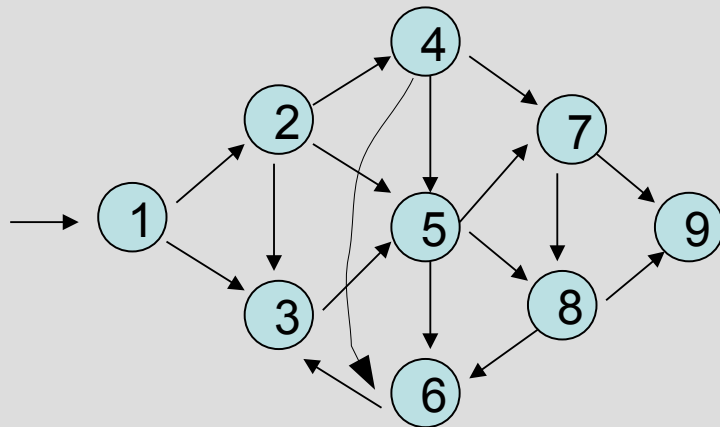
Generalized Geography game



Generalized Geography game



Generalized Geography game



Yes iff the first
Player has
a winning
strategy

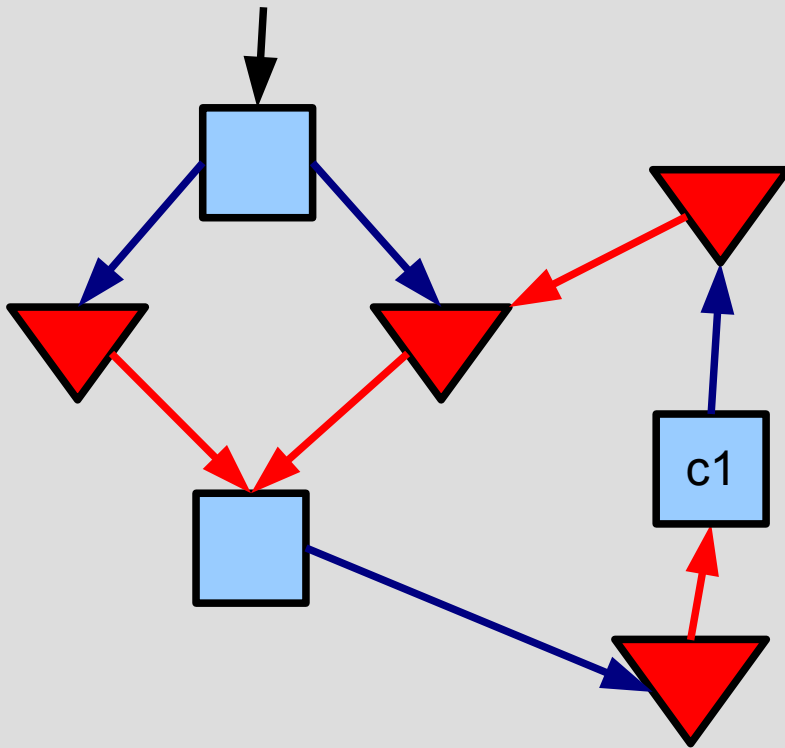
Generalized Geography game is PSPACE-hard

Theorem : Generalized Geography game is PSPACE-hard.

Proof : reduction QBF-sat to Generalized Geography.

Example

$$\phi = \exists p. p$$



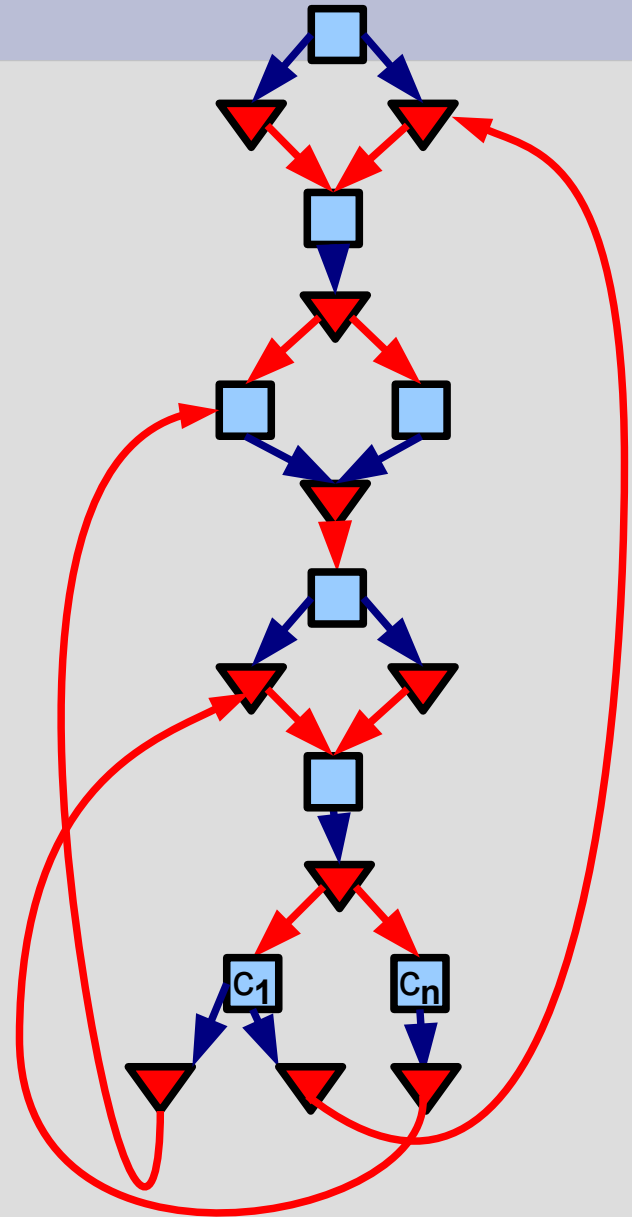
Bleu gagne

Reduction from QBF to GG

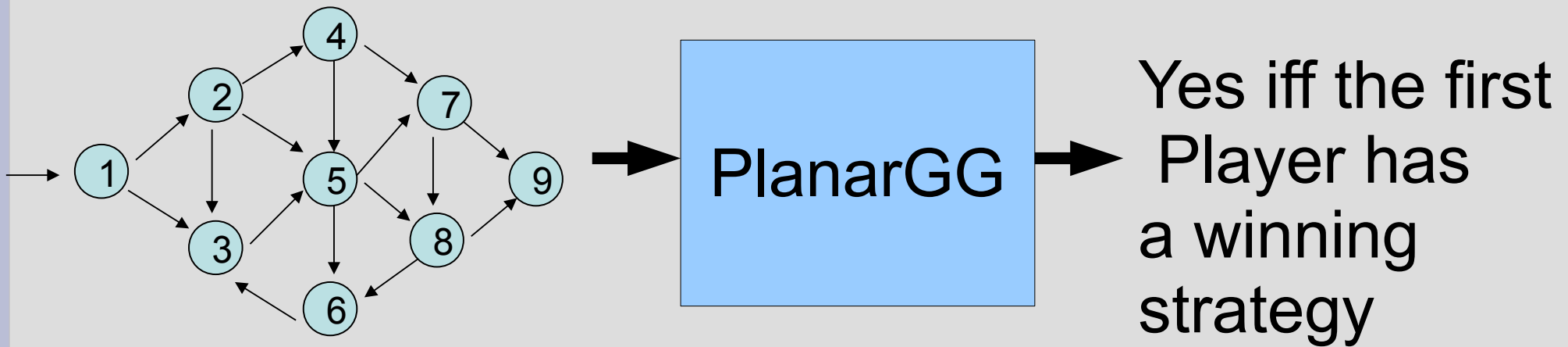
$$\phi = \exists x_1 \forall x_2 \exists x_3 \psi$$

$$c_1 \wedge \dots \wedge c_n$$

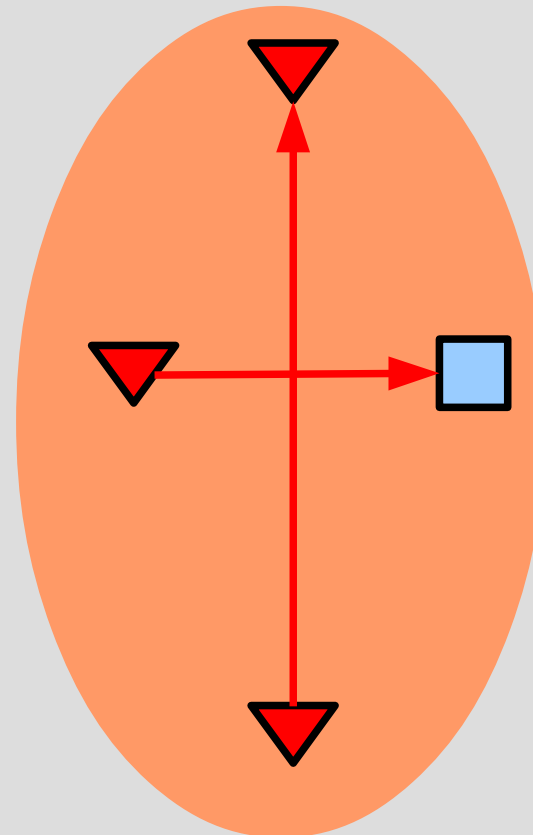
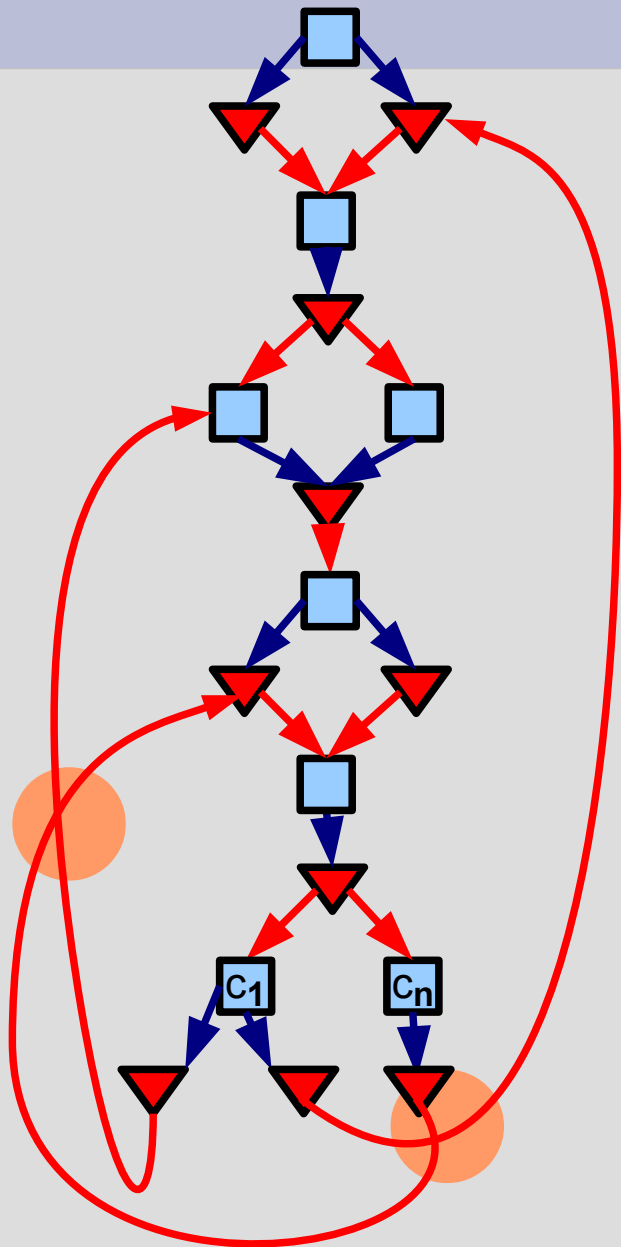
ϕ is true iff blue has winning strategy.



Planar GG is also PSPACE-hard

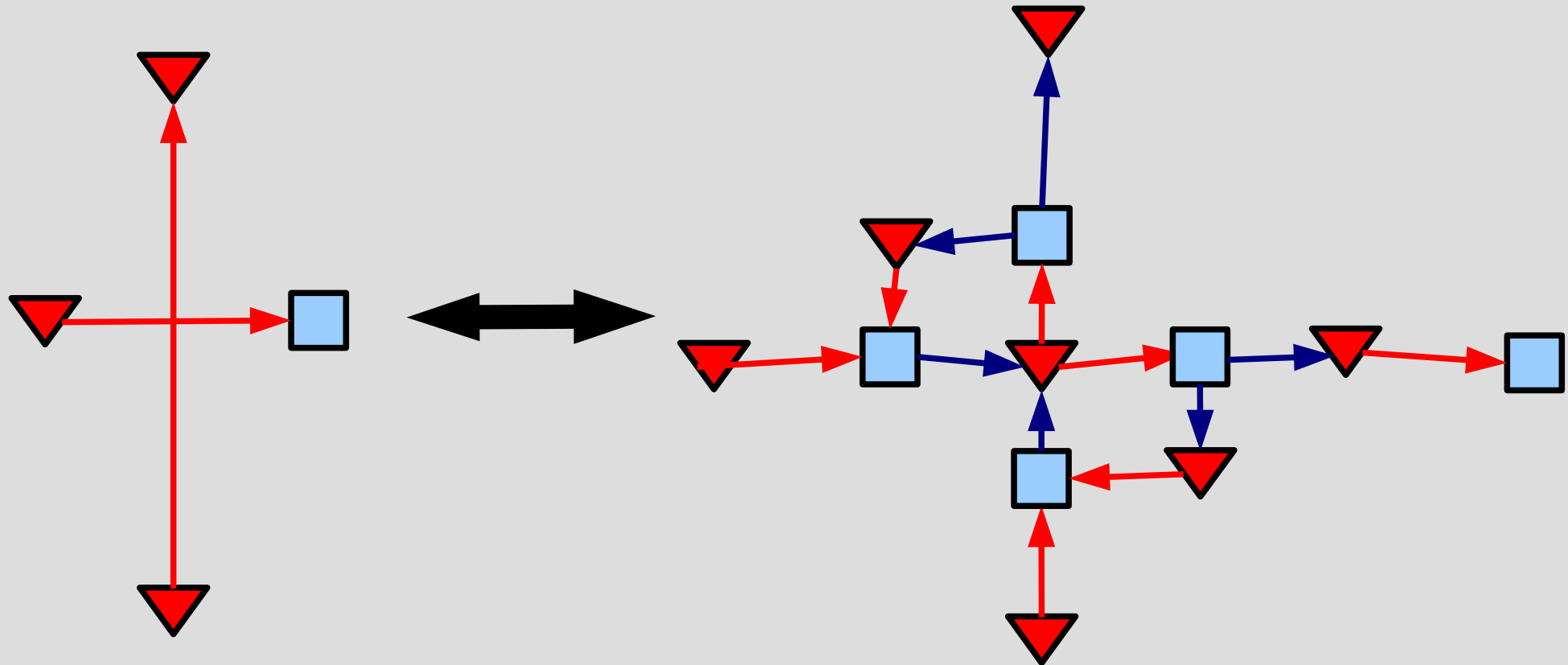


Remove crossings

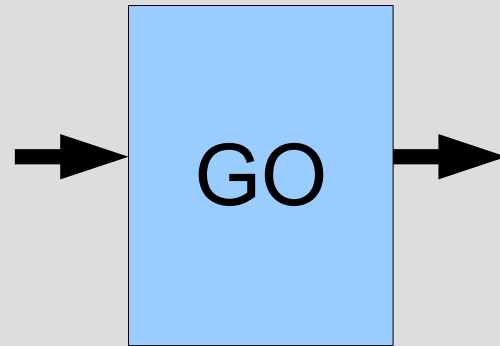
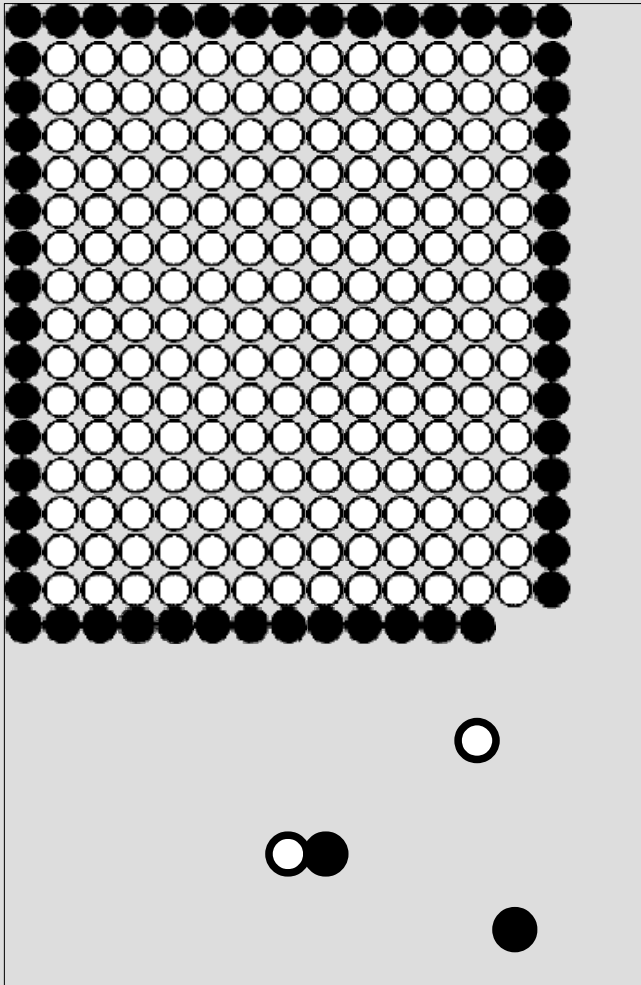


In the GG game, at most one edge will be used.

Remove crossings



Go winning strategy problem



Yes iff
black has
a winning
strategy to take
the white territory

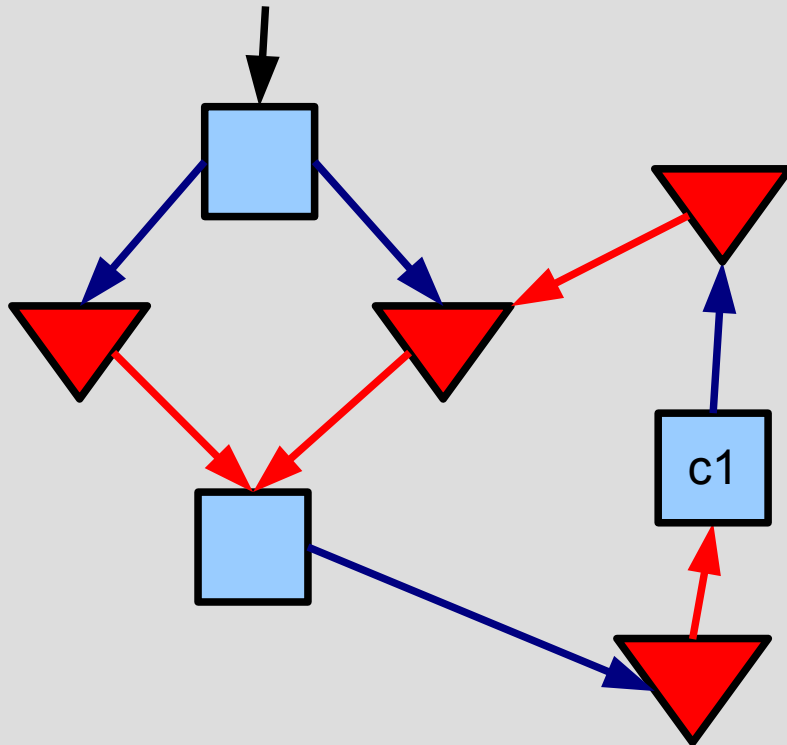
GO is PSPACE-hard

Theorem : GO is PSPACE-hard.

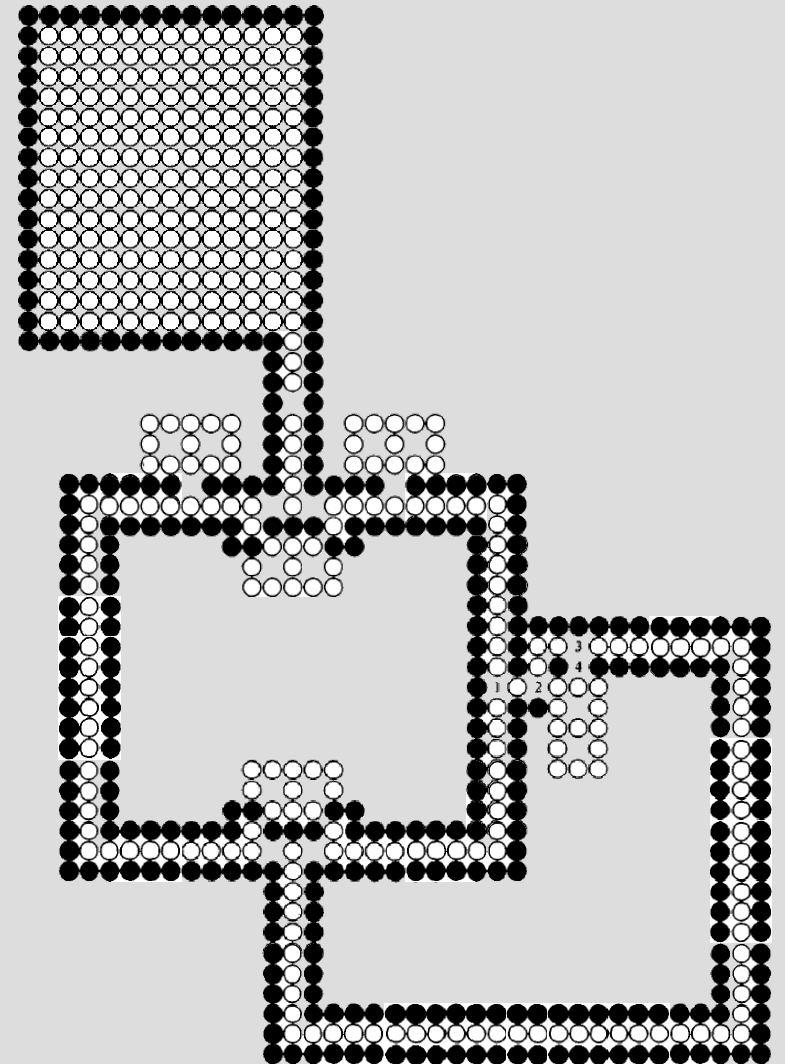
Proof : We reduce Planar GG to GO.

We encode the graph on the goban.

$$\phi = \exists p. p$$

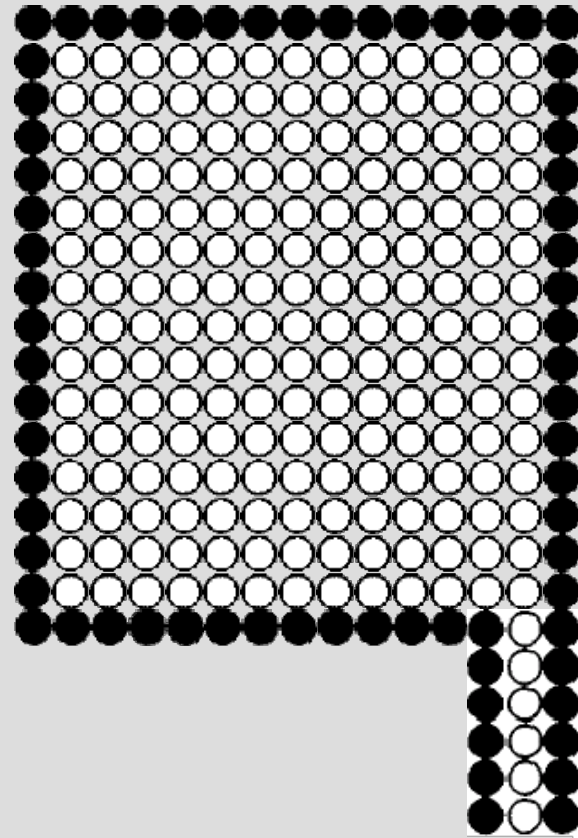
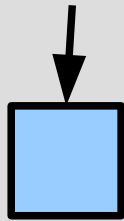


Bleu gagne

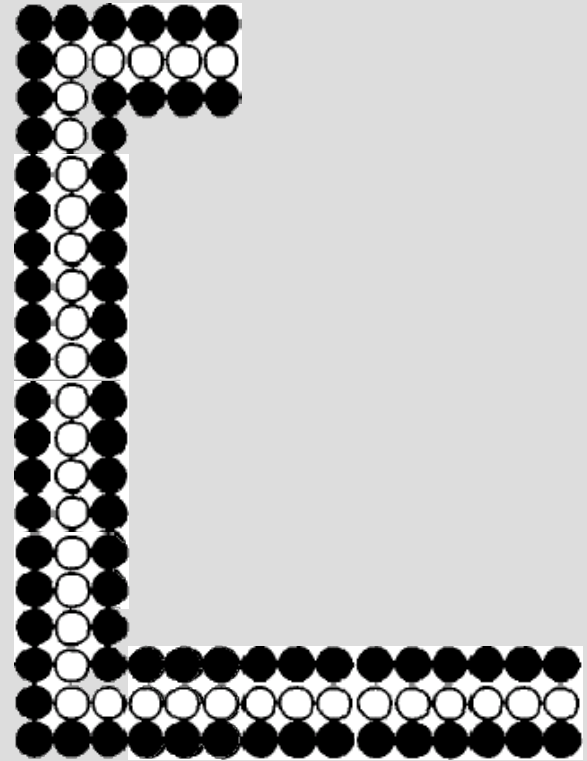
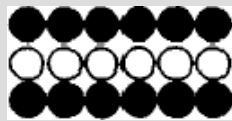
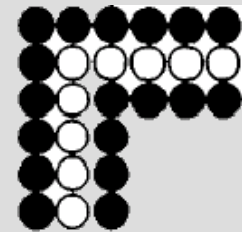
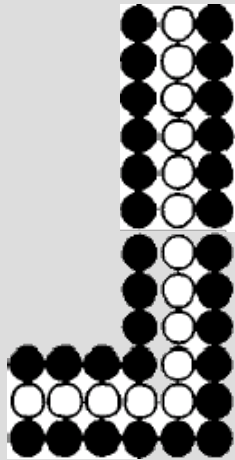
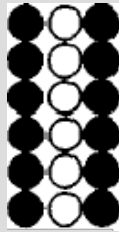
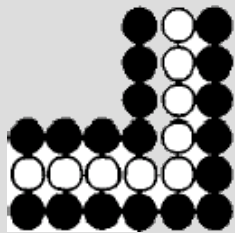


Noir gagne

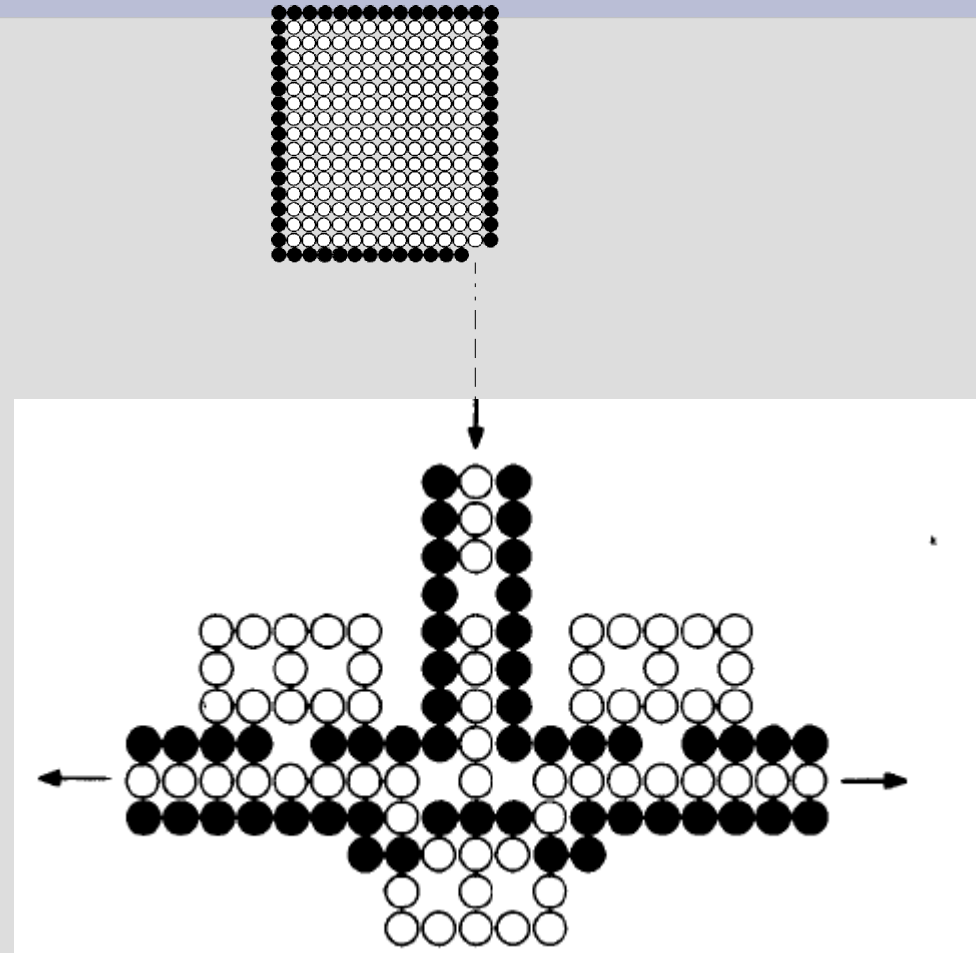
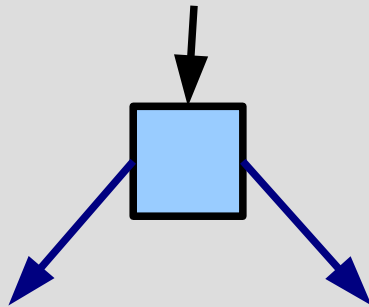
Starting point



Edges

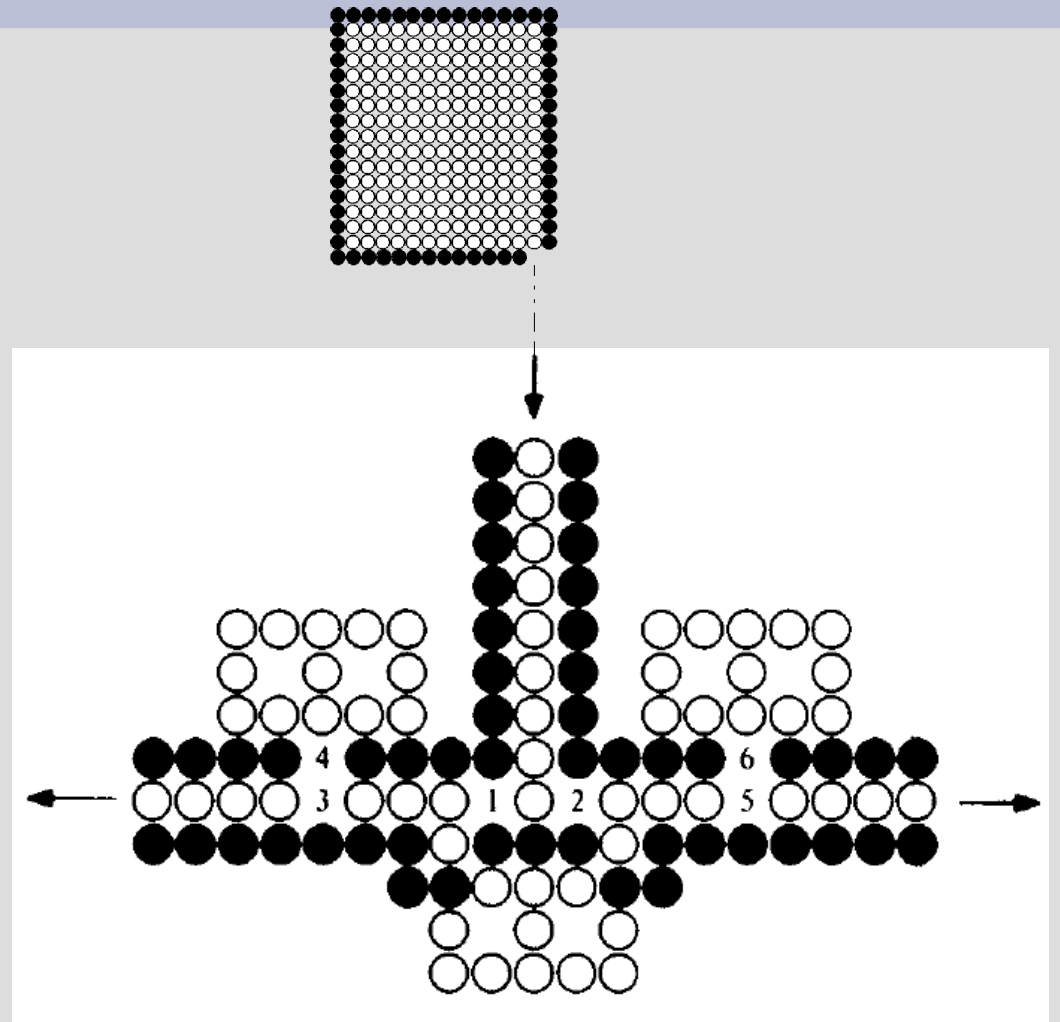
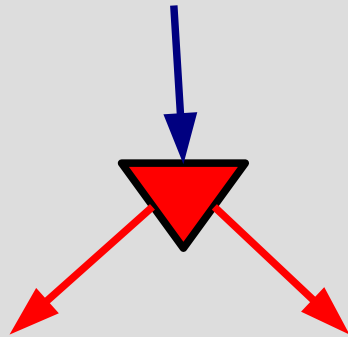


Player E's choice



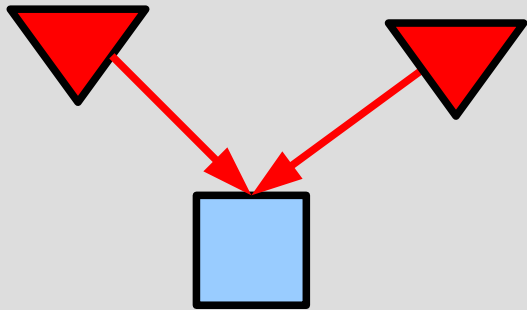
Noir décide le côté où on va

Player A's choice

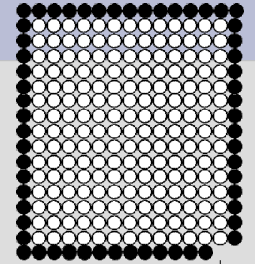
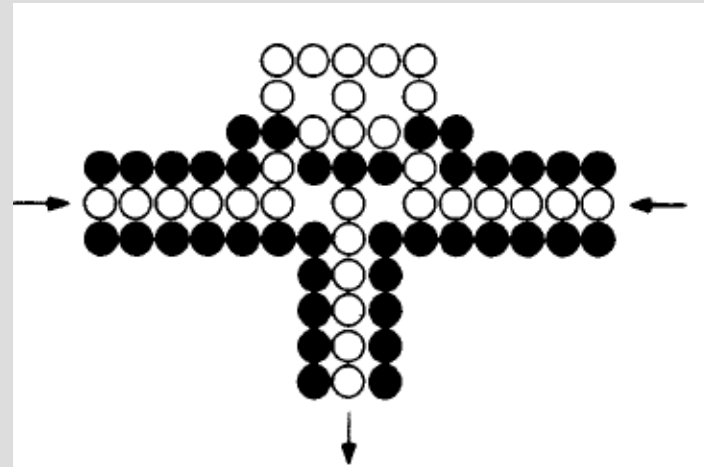
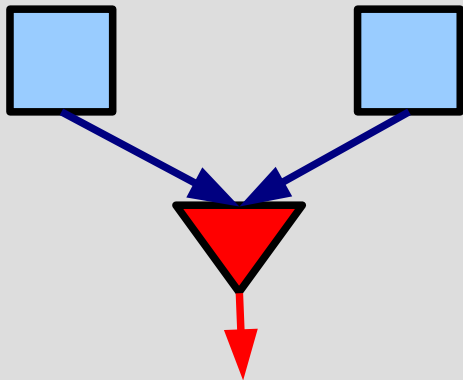


Blanc décide le côté où on va

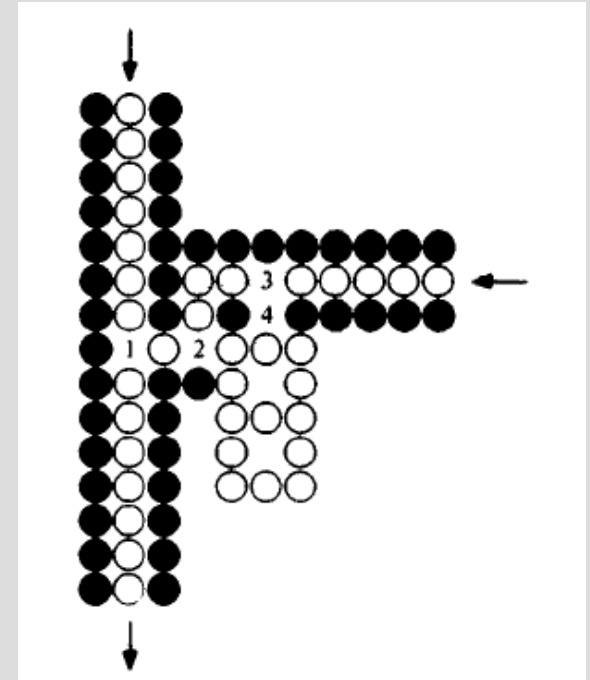
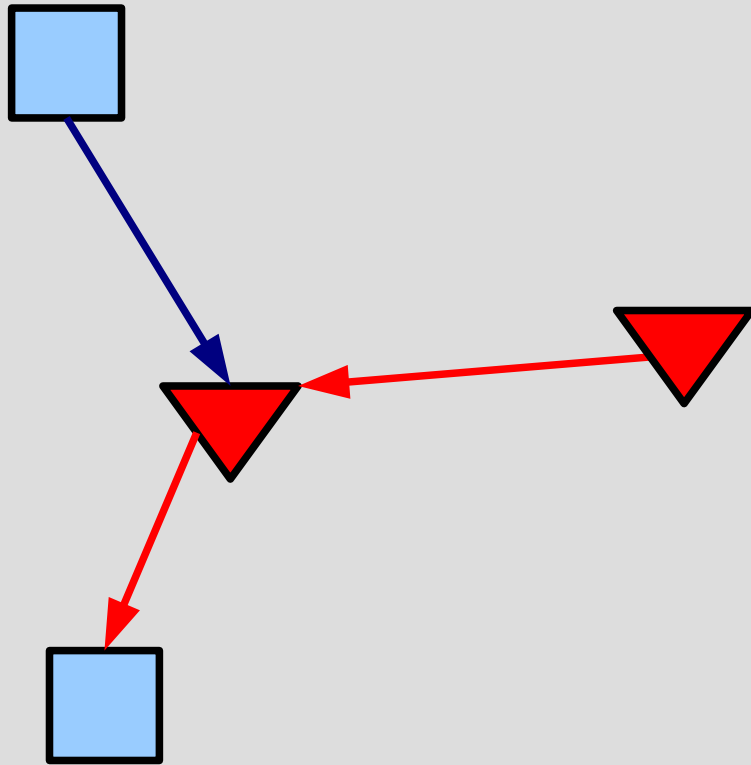
Join



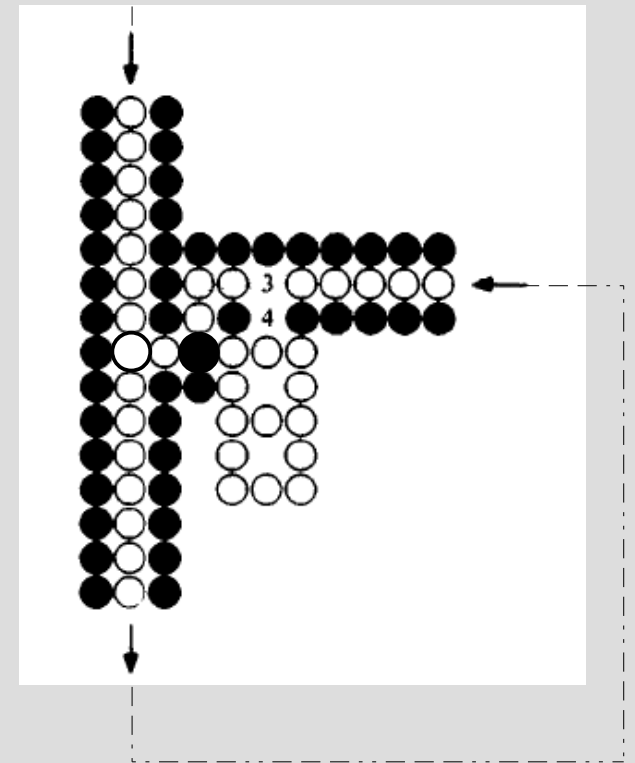
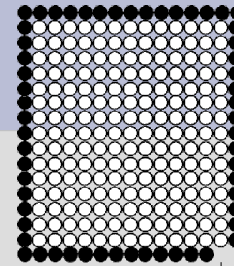
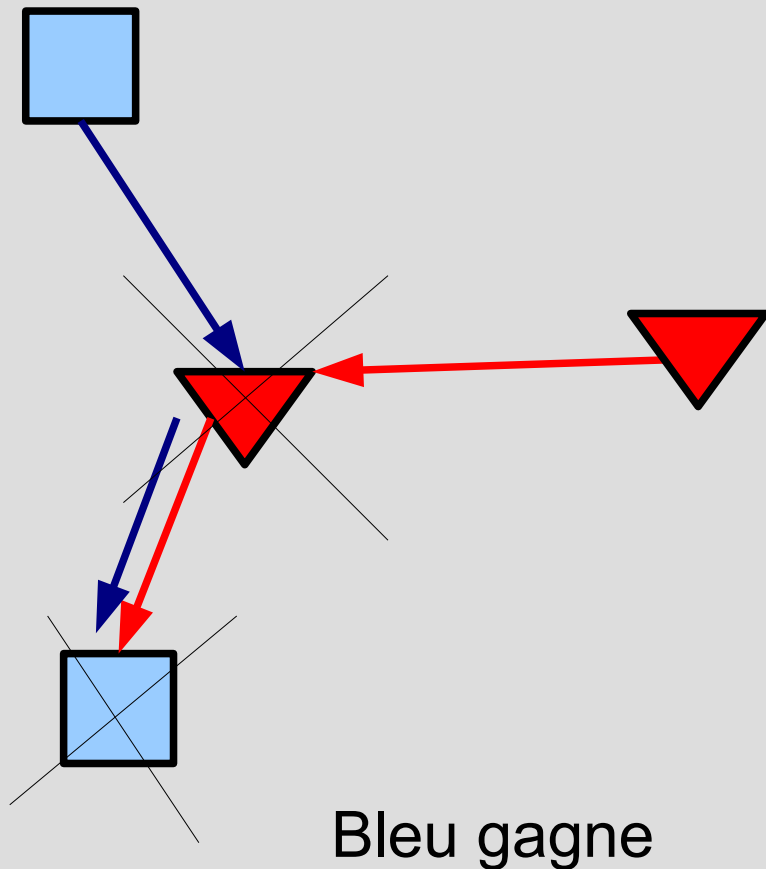
or



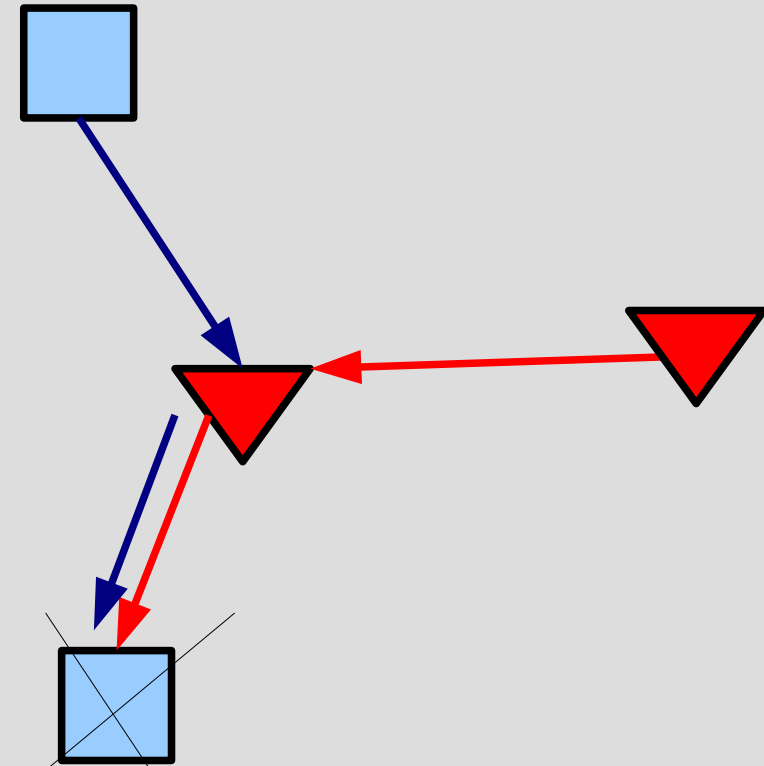
Test



Test : cas où connecté au territoire

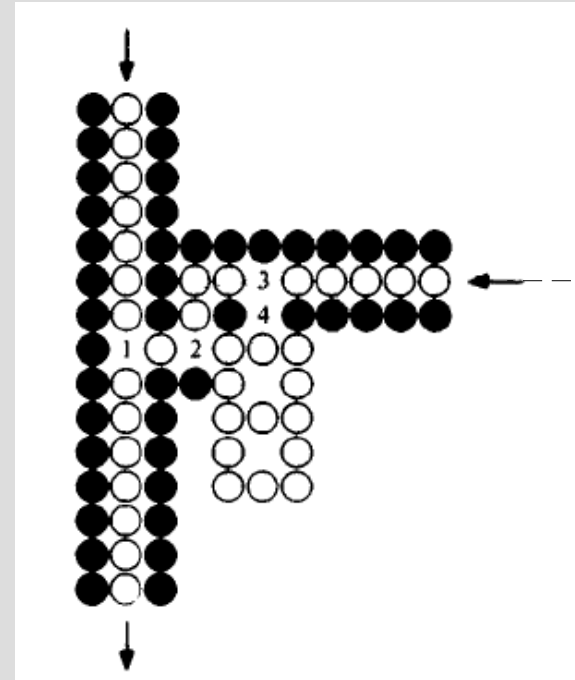


Cas où non connecté au territoire

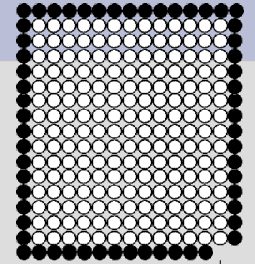


Rouge gagne

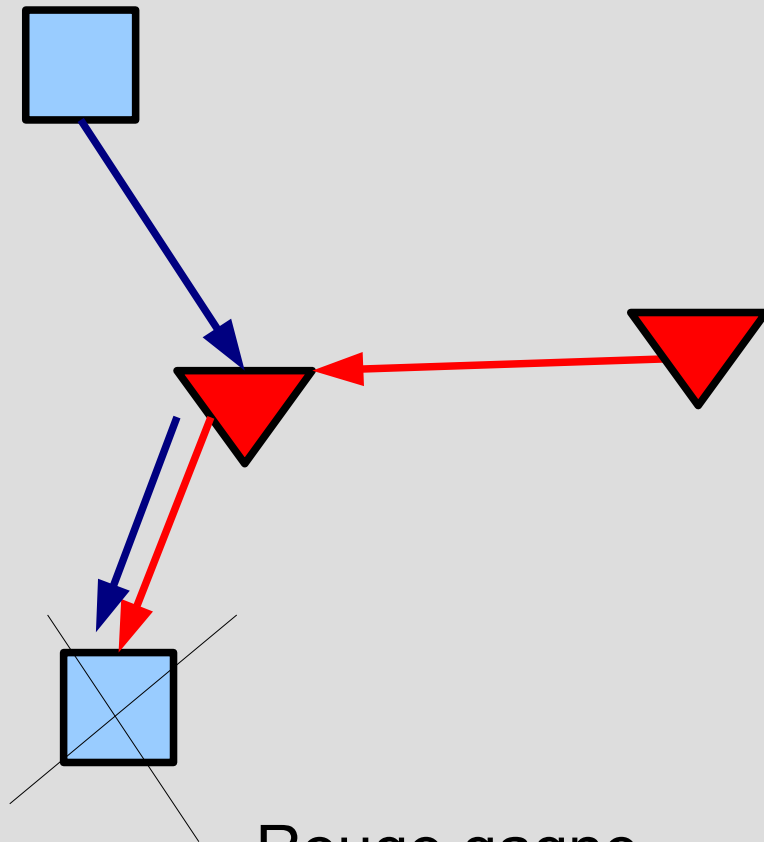
Pas connecté
au territoire



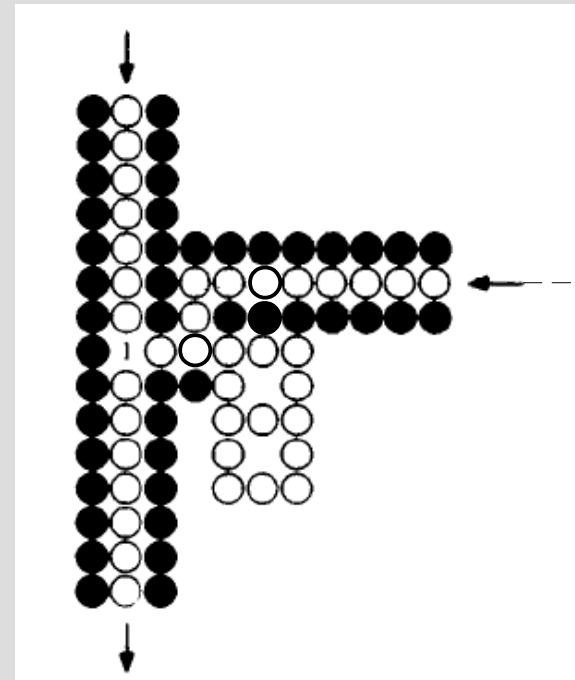
Blanc gagne



Cas où non connecté au territoire



Pas connecté
au territoire



Blanc gagne

Conclusion

- Is GO in PSPACE ?
- Write a program which converts a QBF formula into a GO configuration (or look if it exists)!