Un peu d'algorithmique géométrique

François Schwarzentruber
ENS Cachan – Antenne de Bretagne

Références :

• Beauquier, Berstel, Chrétienne. Elements d'algorithmique.
• Cormen, Leiserson, Rivest, Stein : Algorithmique

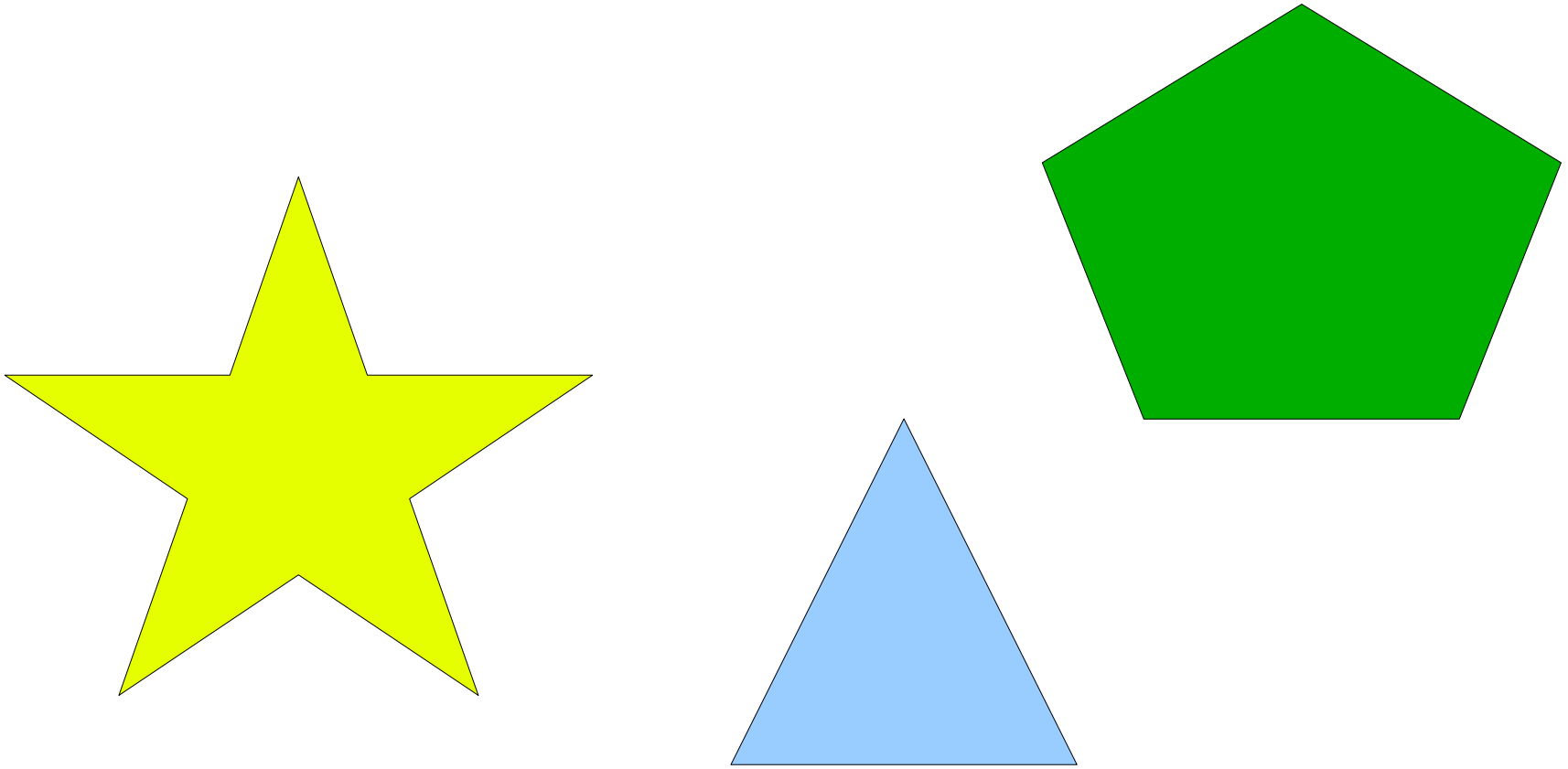
Plan

- Appartenance d'un point à un polygone
  - Algorithme « lent » mais générique
  - Algorithme « rapide » mais spécifique (polygone convexe)
- Calcul d'une enveloppe convexe de n points
  - Algorithme de Graham (qui utilise le tri et une pile)
  - Optimalité via une « réduction »

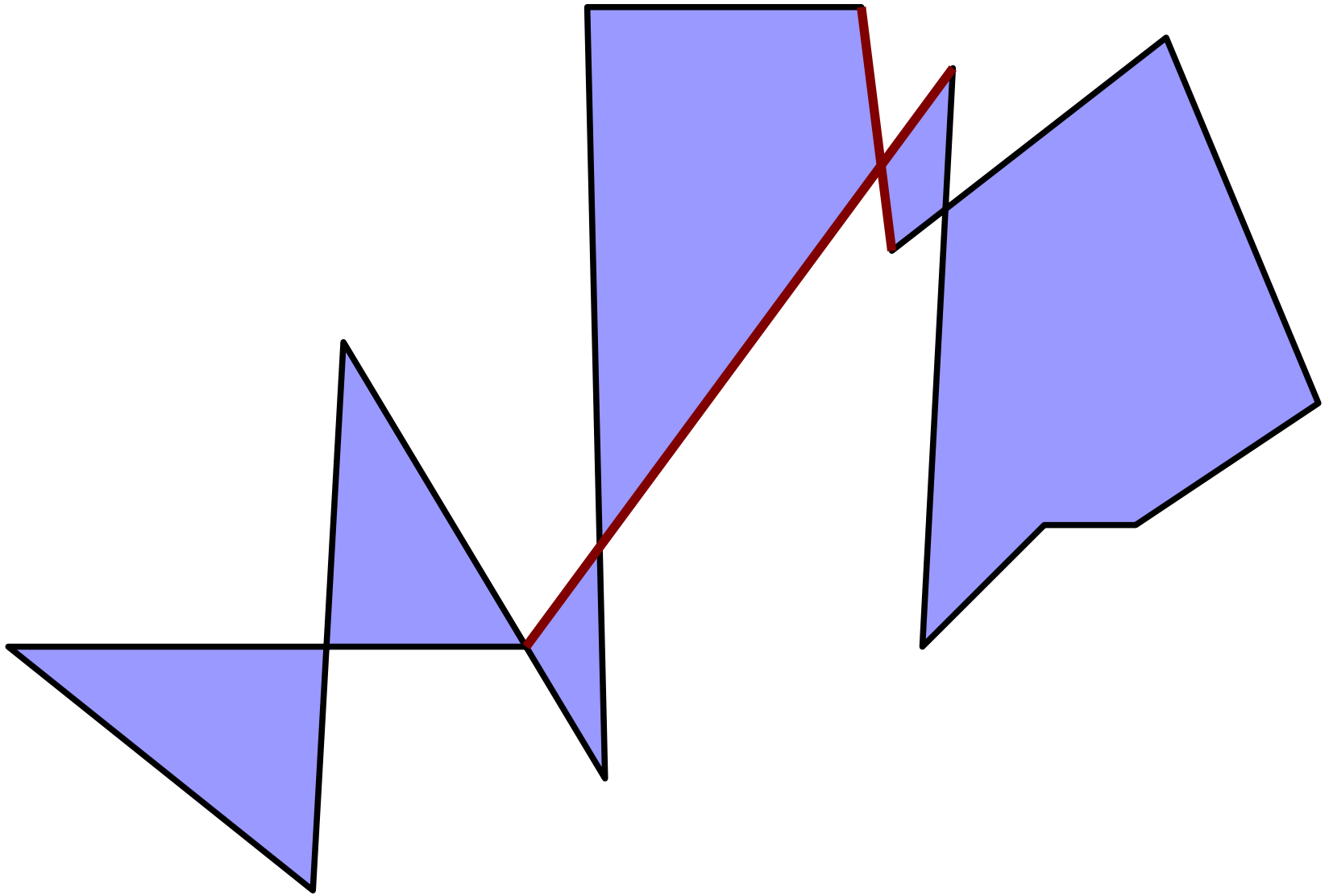
Appartenance d'un point à un polygone

Algorithme lent mais générique

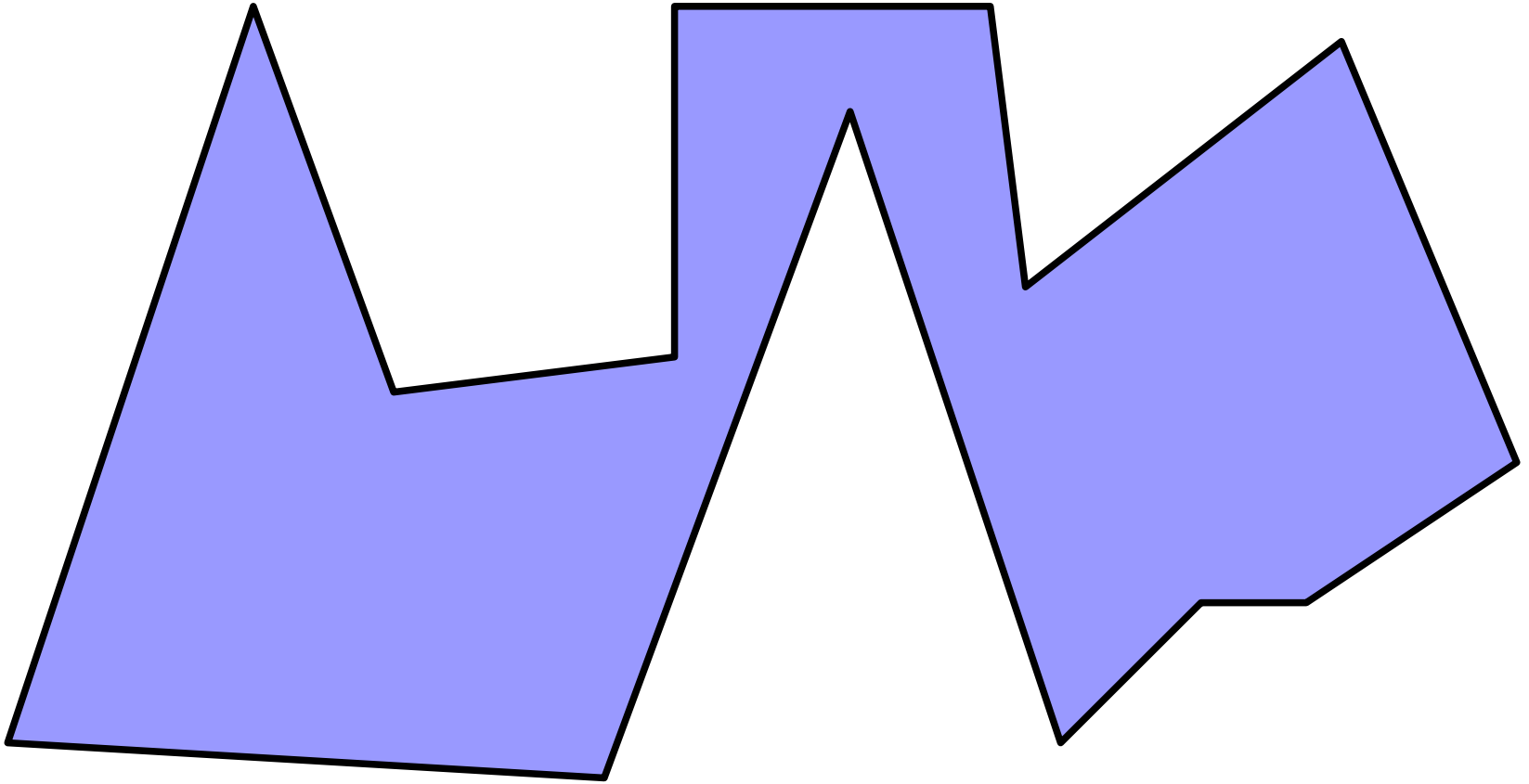
Des polygones



Un polygone pas simple

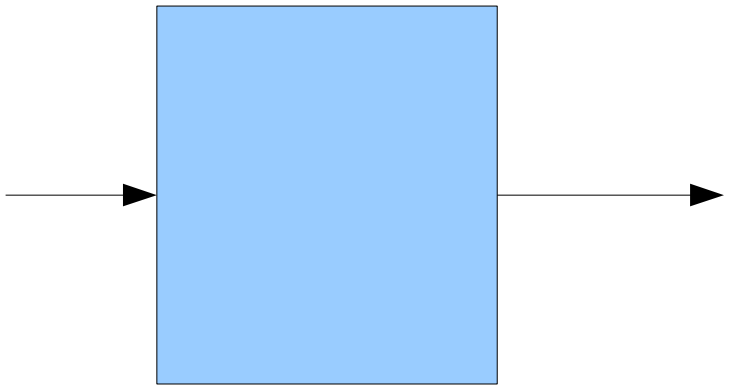


Un polygone simple



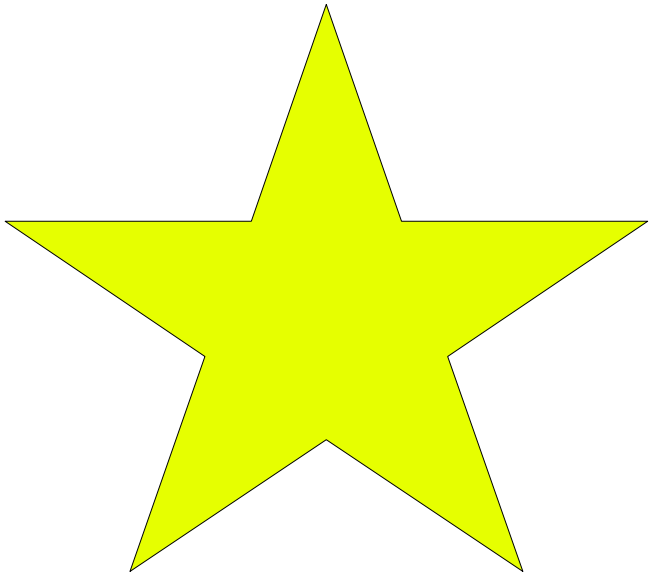
Le problème

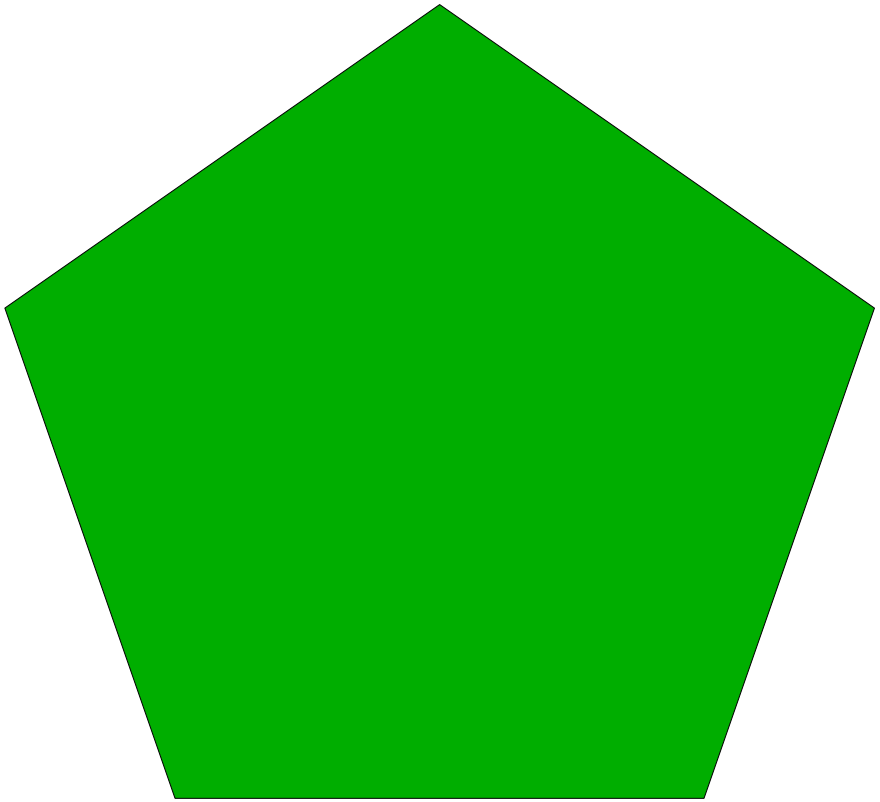
Un point x
Un polygone simple
P = (p1, …, pn)



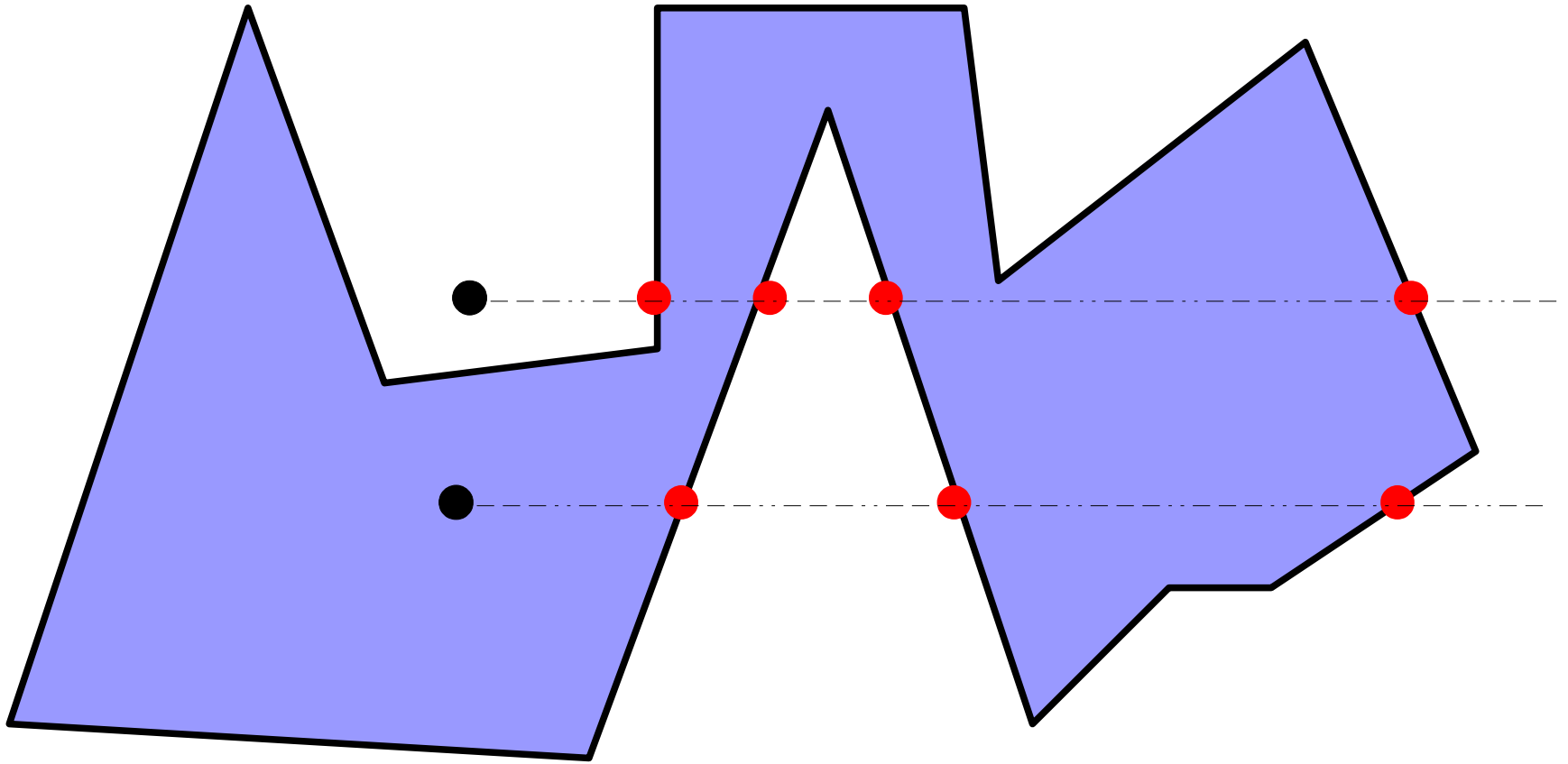
- Oui x appartient à l'intérieur de P
- Non

Applications : Logiciel de dessin. Reconnaître où est le curseur

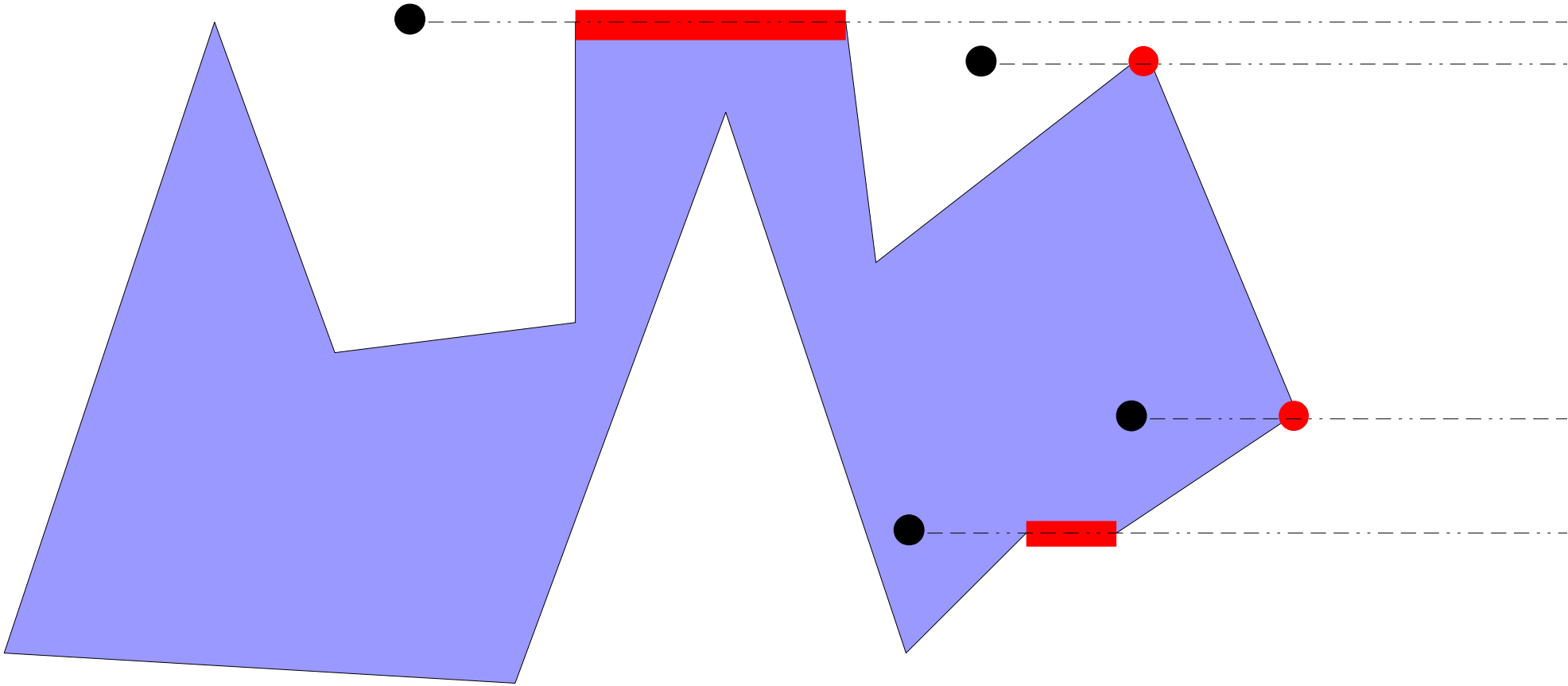




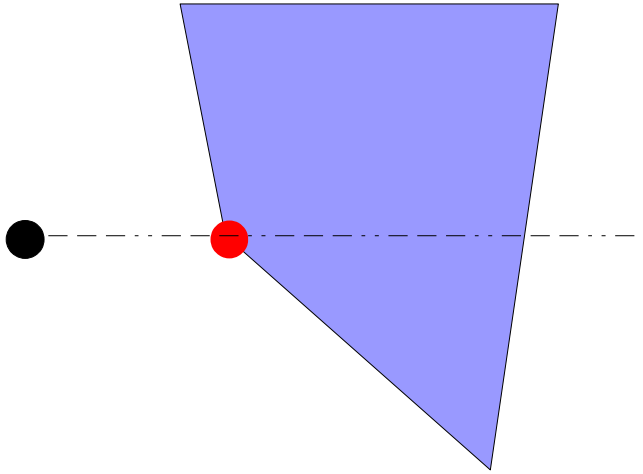
Principe de l'algorithme



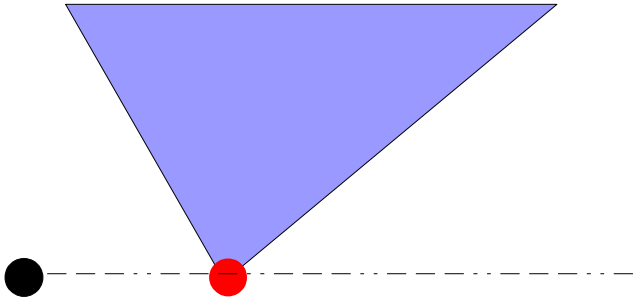
Que faire des singularités ?



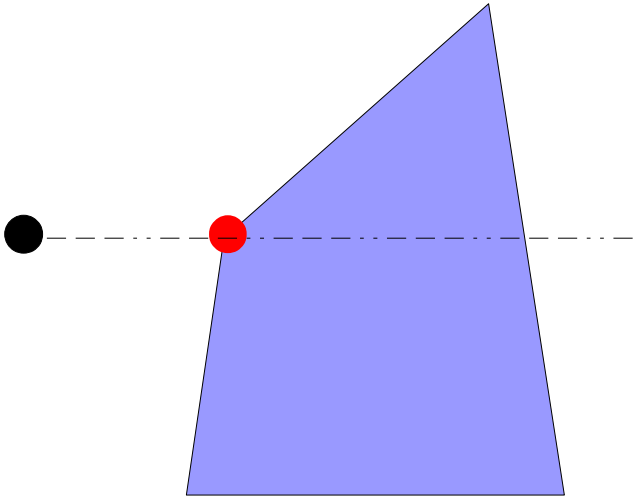
Un point...

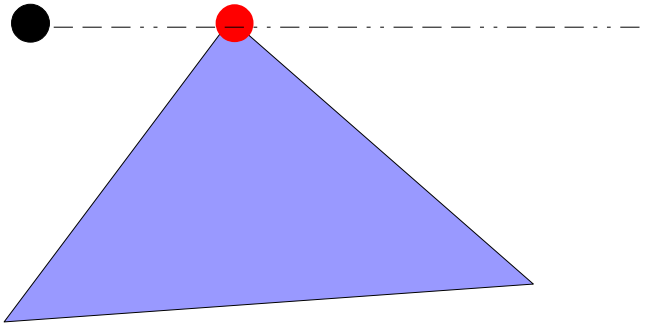


Compte pour 1

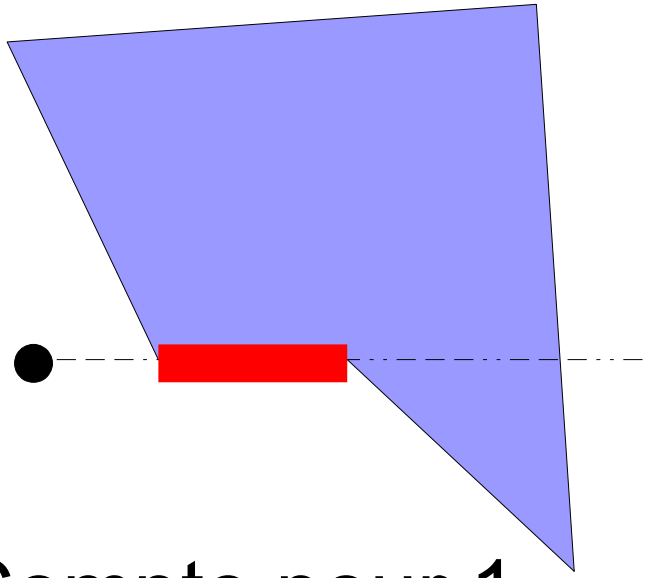


Compte pour 0

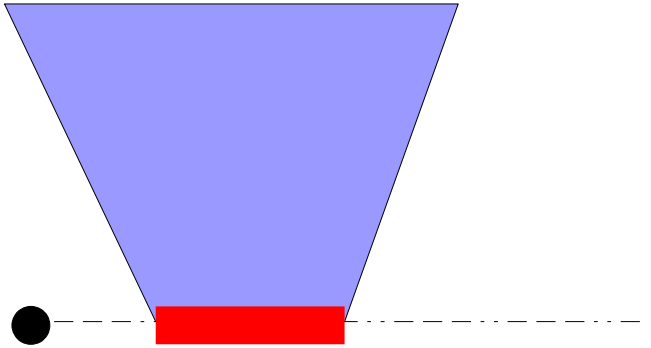




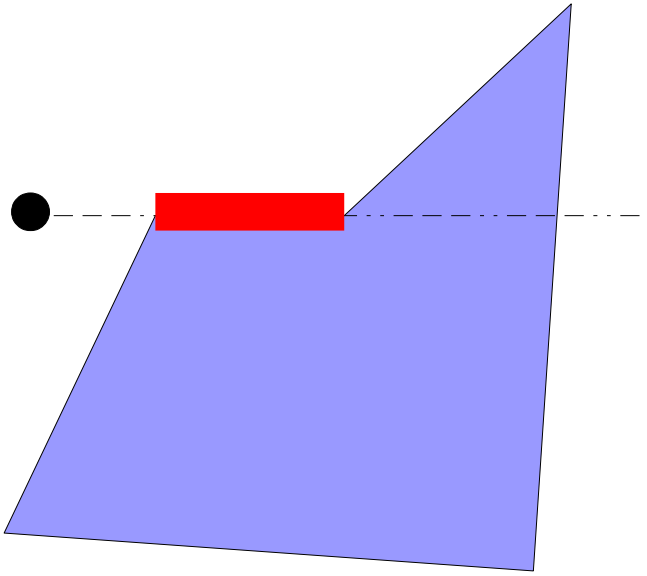
Deux points...

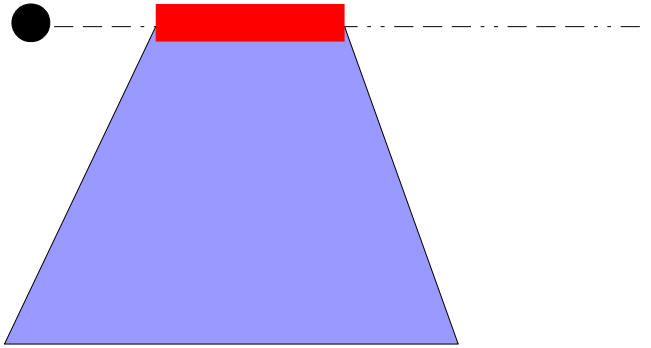


Compte pour 1

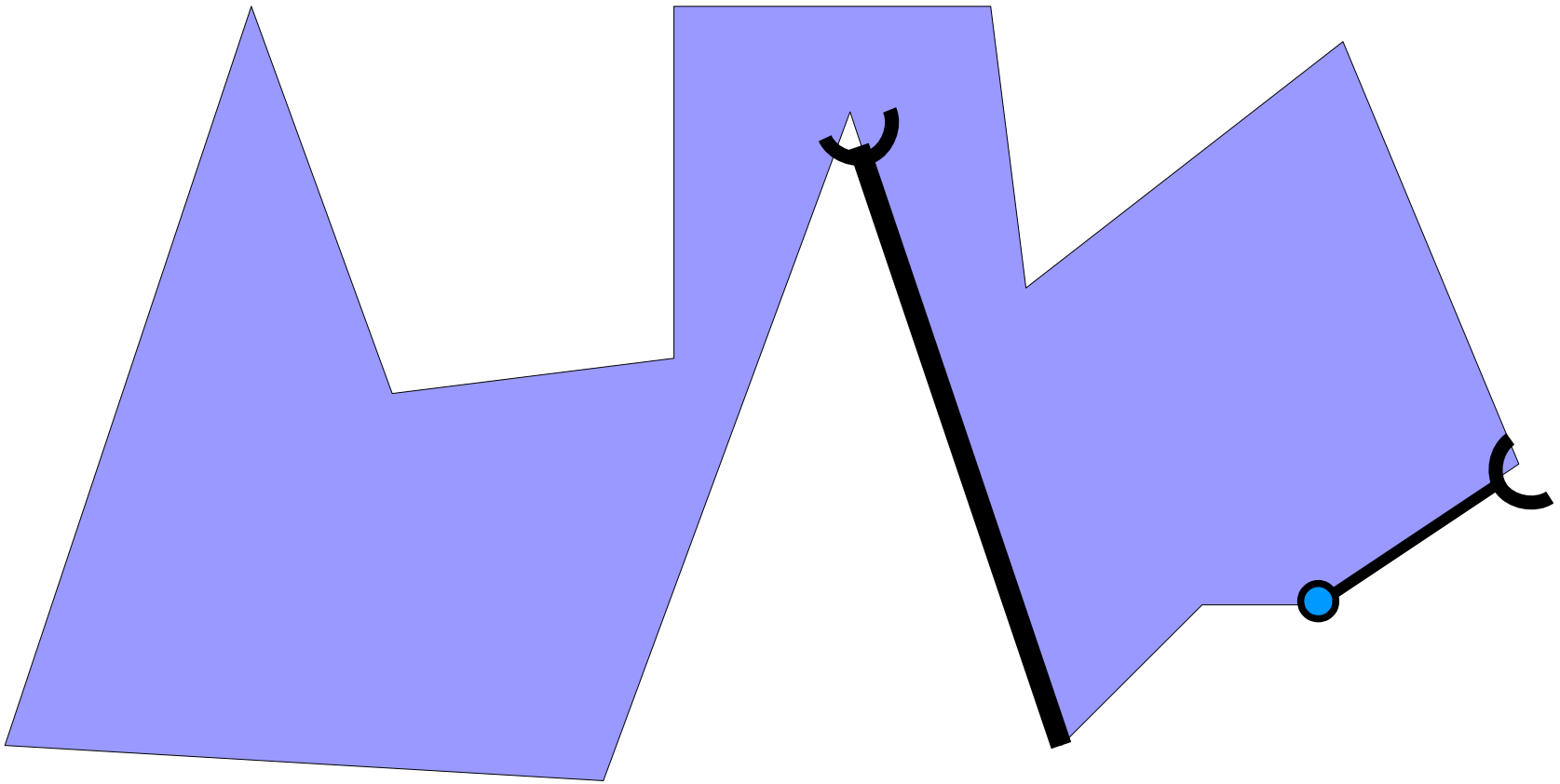


Compte pour 0

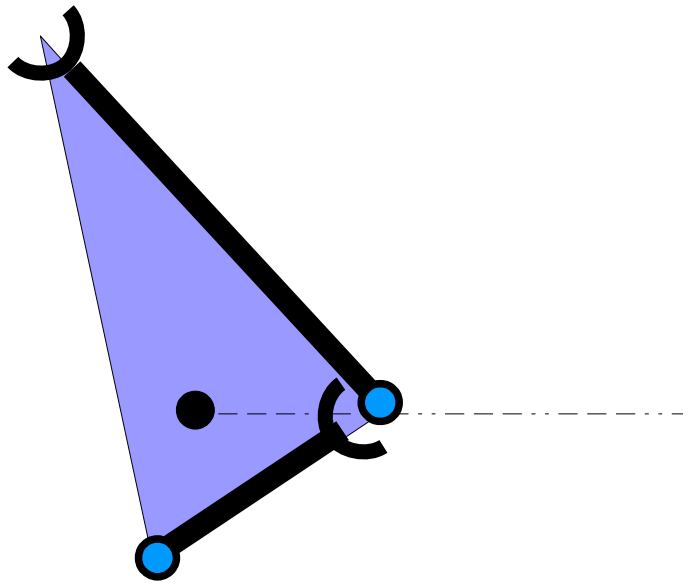




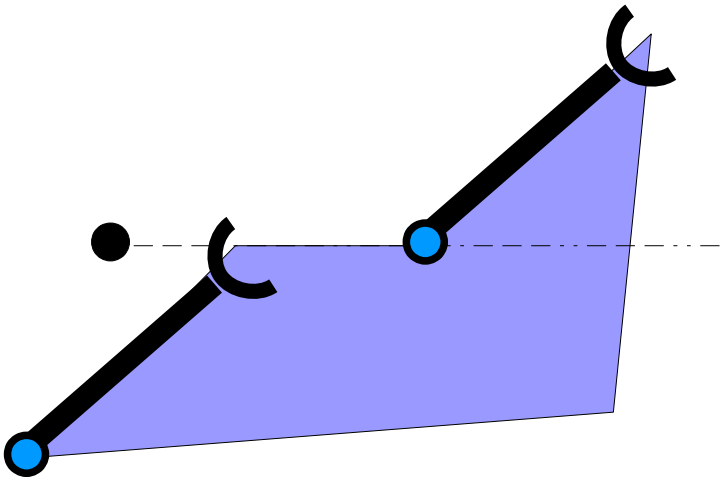
Solution



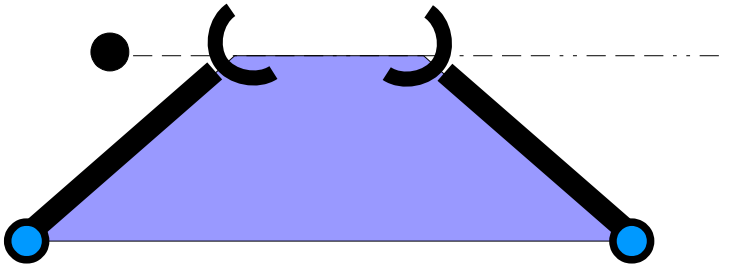
Différents cas



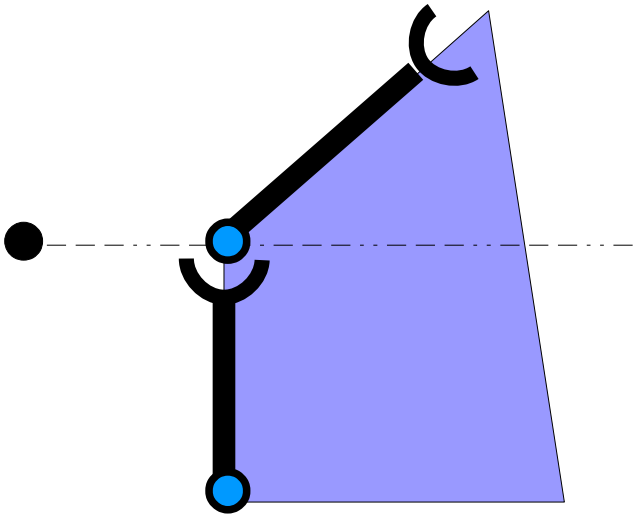
Différents cas

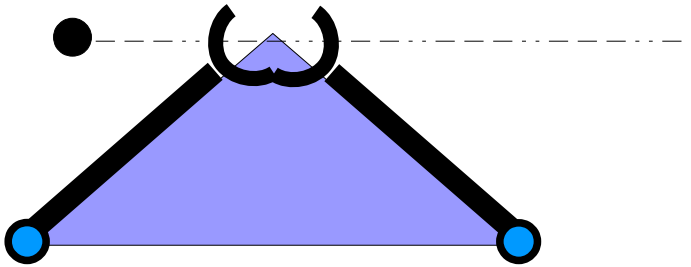


Compte pour 1

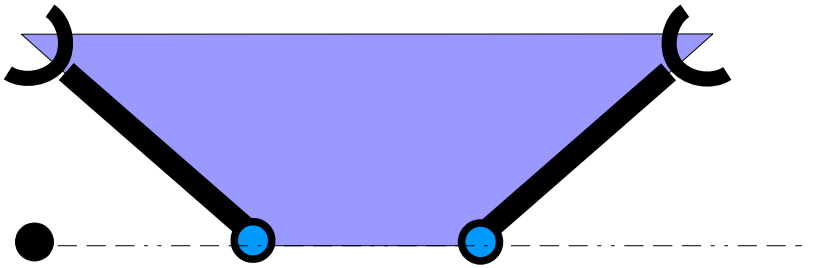


Compte pour 0

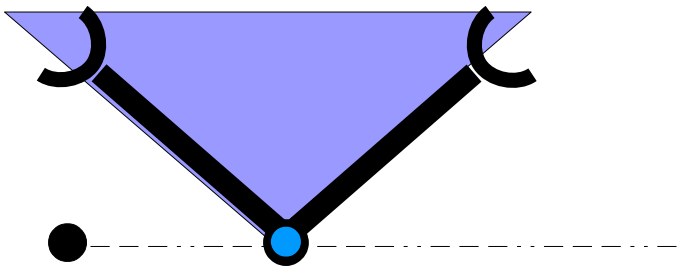




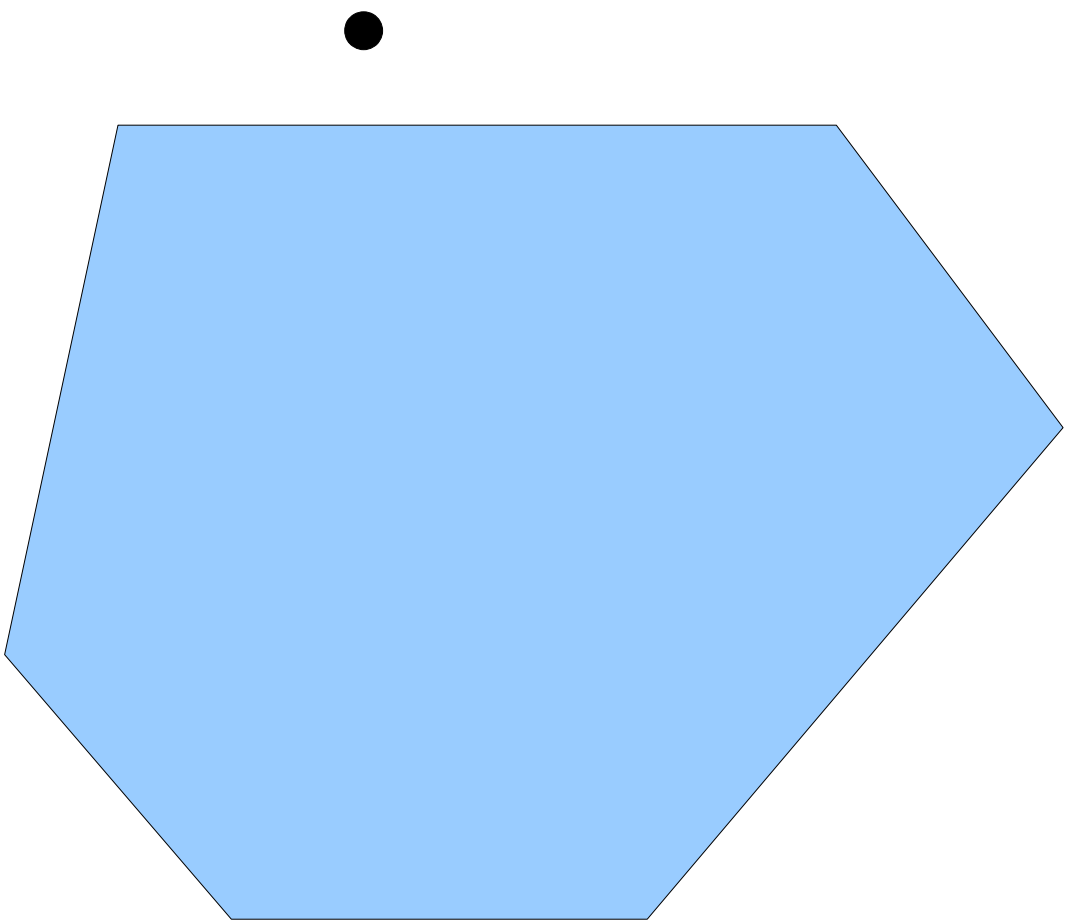
Différents cas



Compte pour 2 (modulo 2) = 0



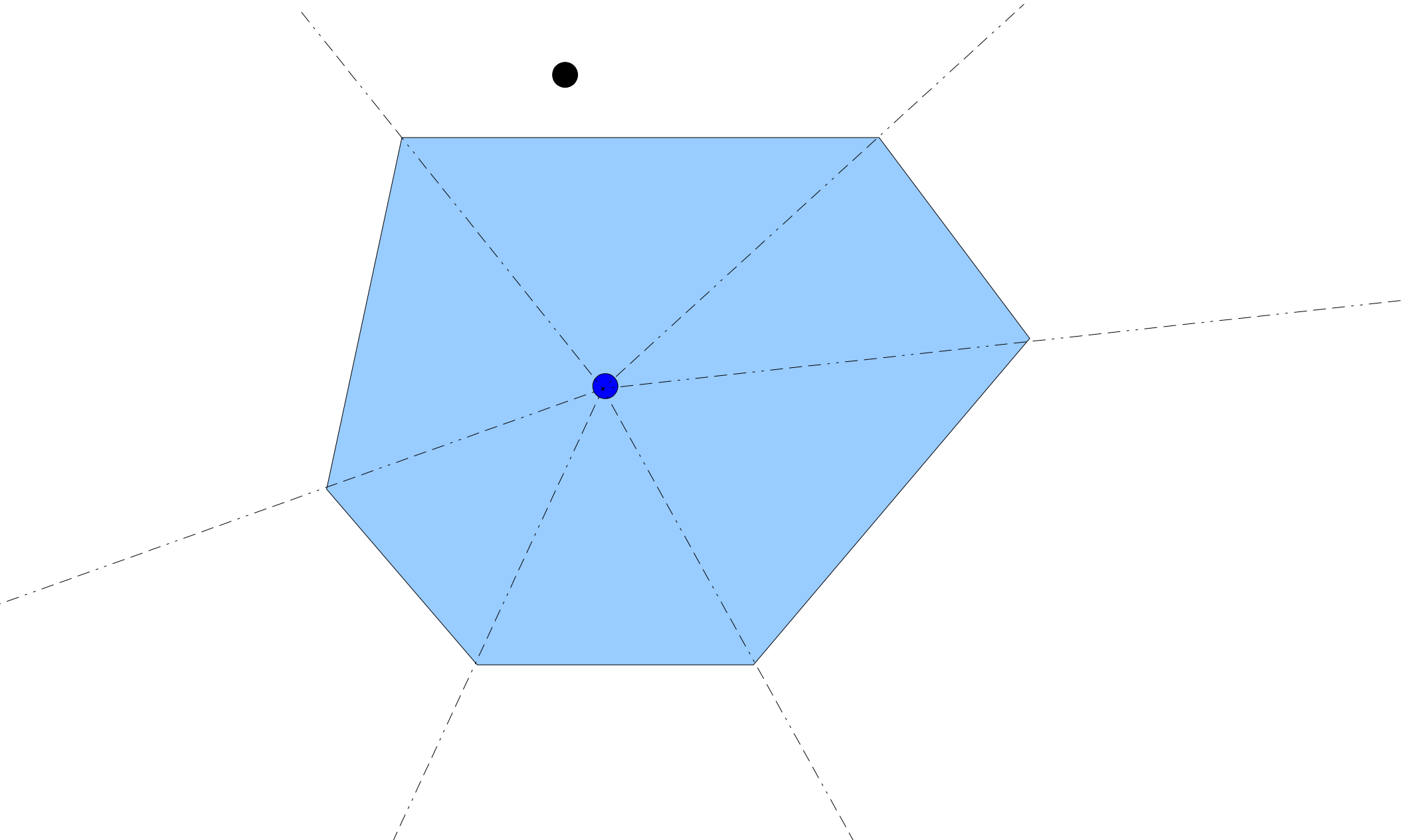
Algorithme « rapide » pour les polygones convexes



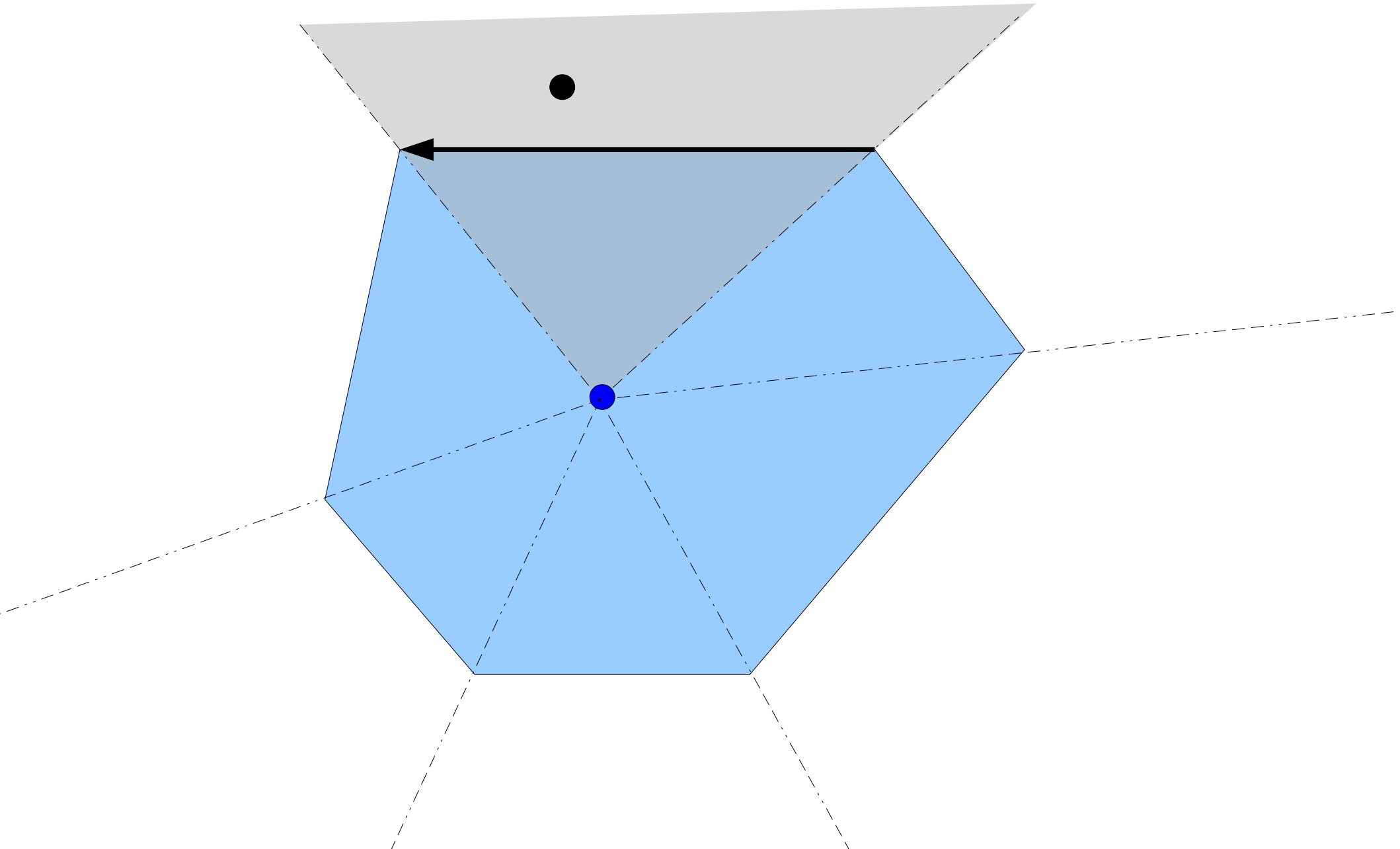
Algorithme « rapide » pour les polygones convexes

- Trouver le secteur angulaire
- Tester si le point est intérieur

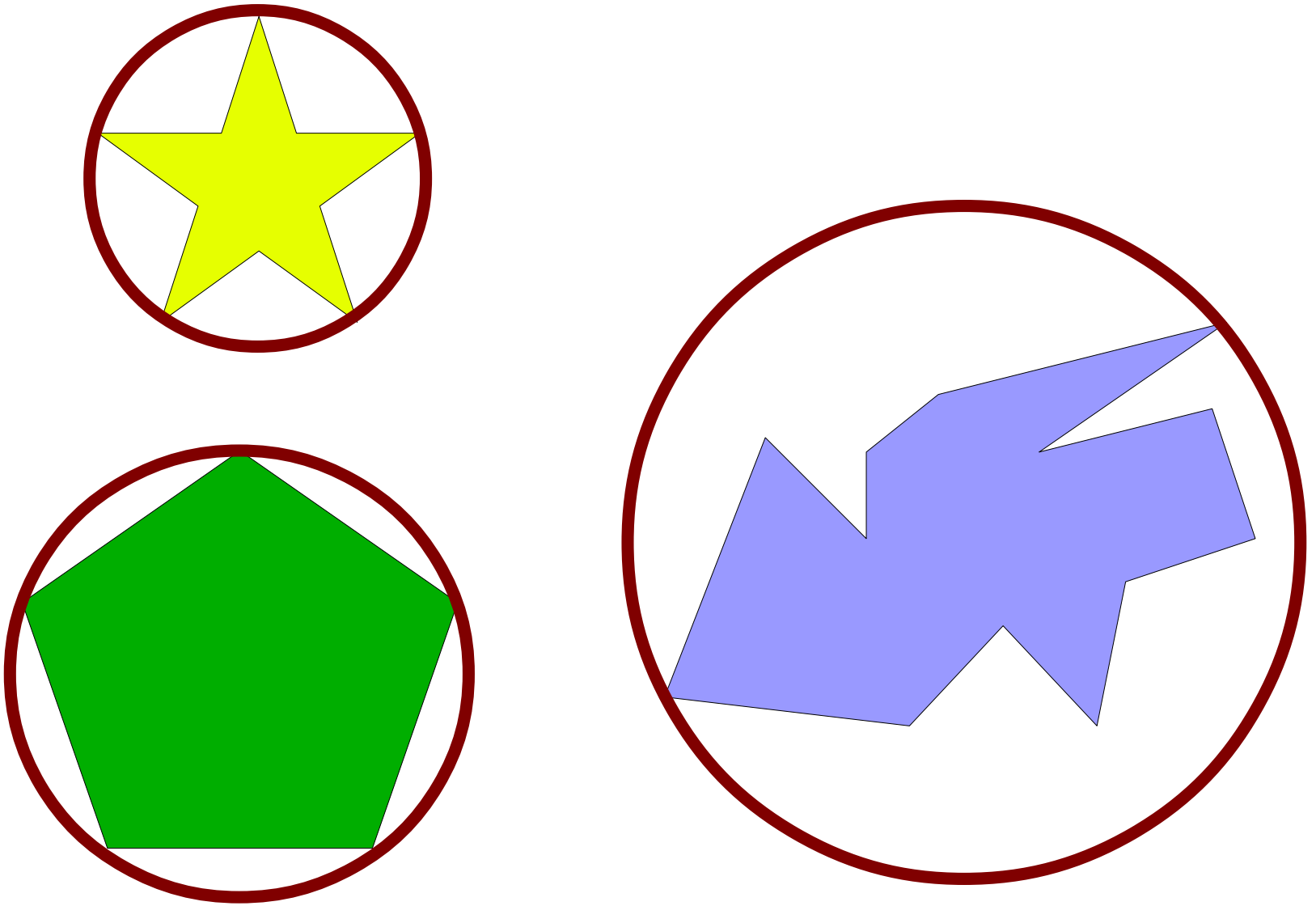
Trouver le secteur angulaire



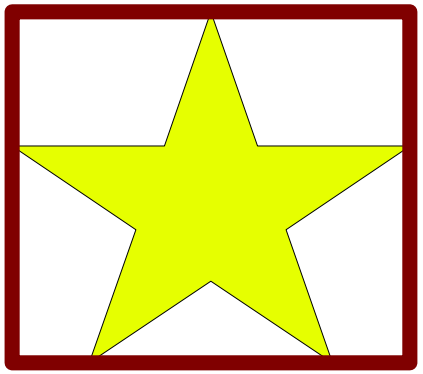
Tester si le point est à l'intérieur

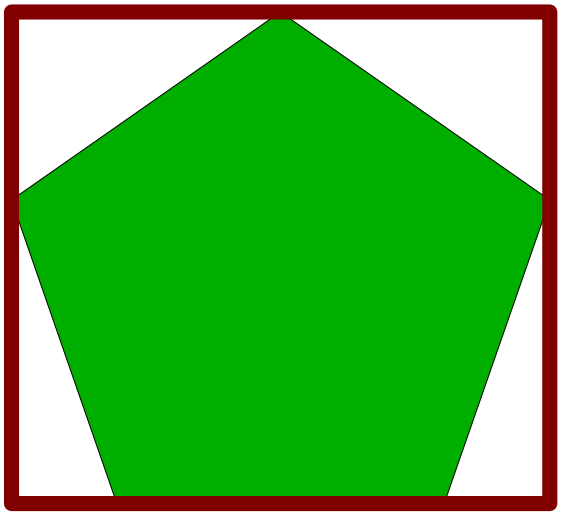


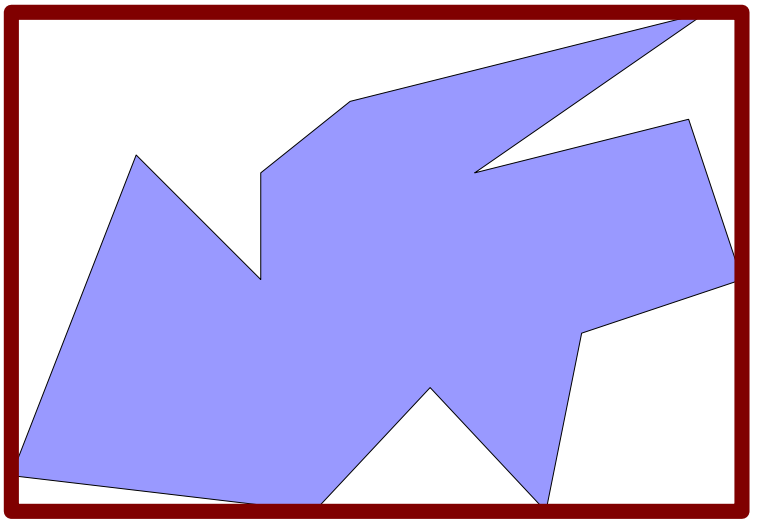
Bounding box



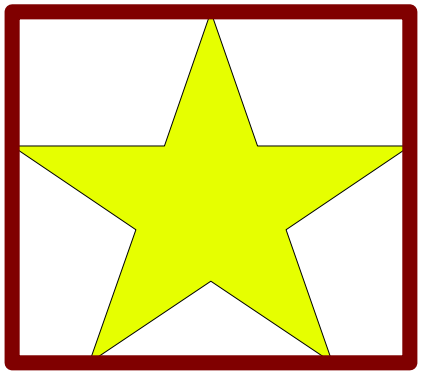
Bounding box

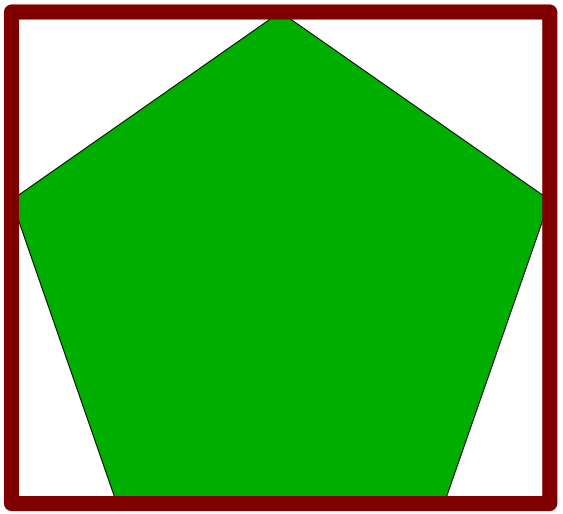


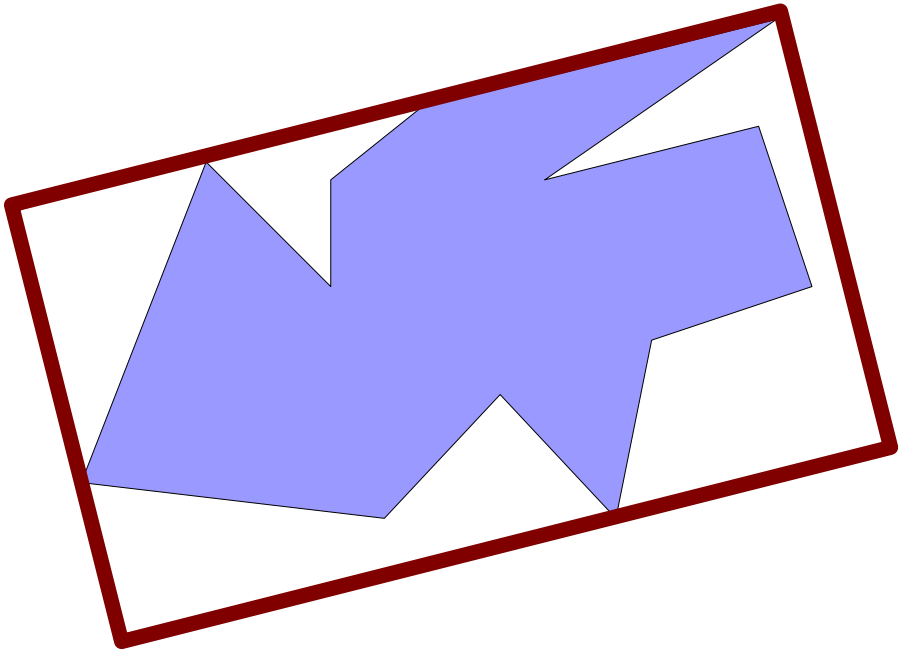




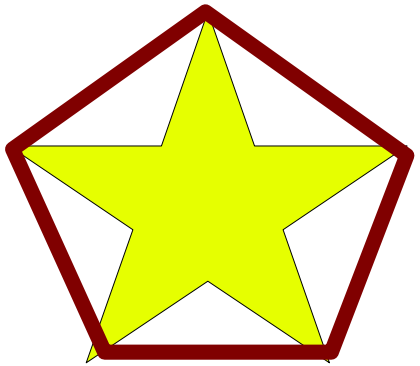
Bounding box

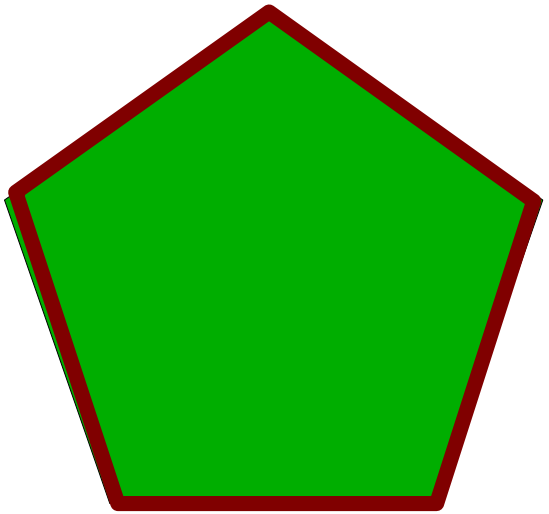


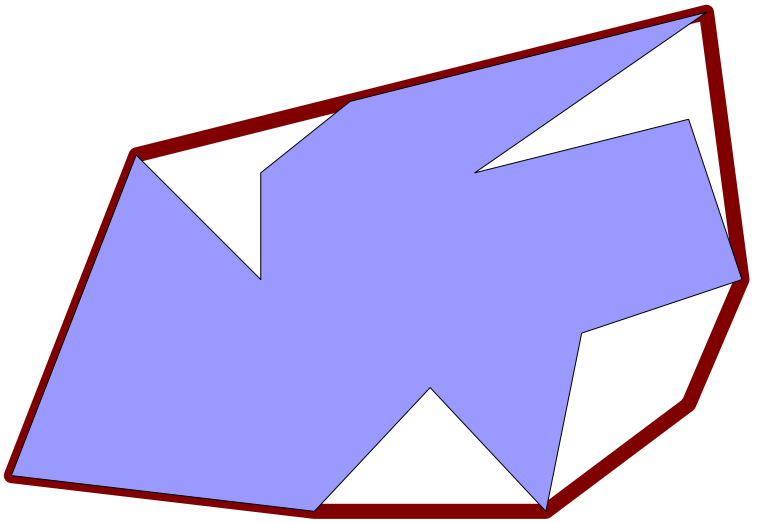




Bounding box





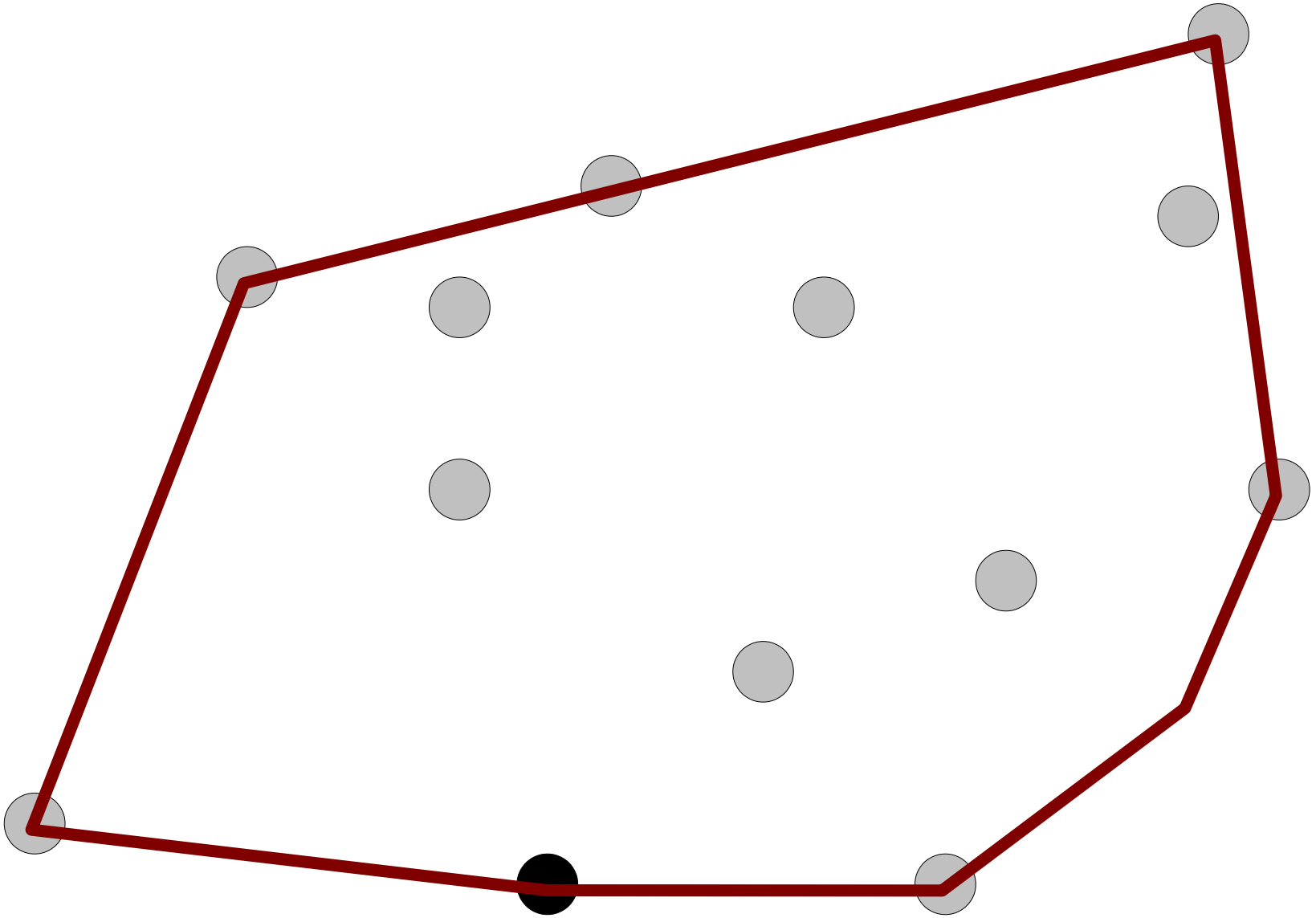


Calcul de l'enveloppe convexe de n points

n points

Un polygone convexe simple
P = (p1, …, pn)
dont l'intérieur contient les n points

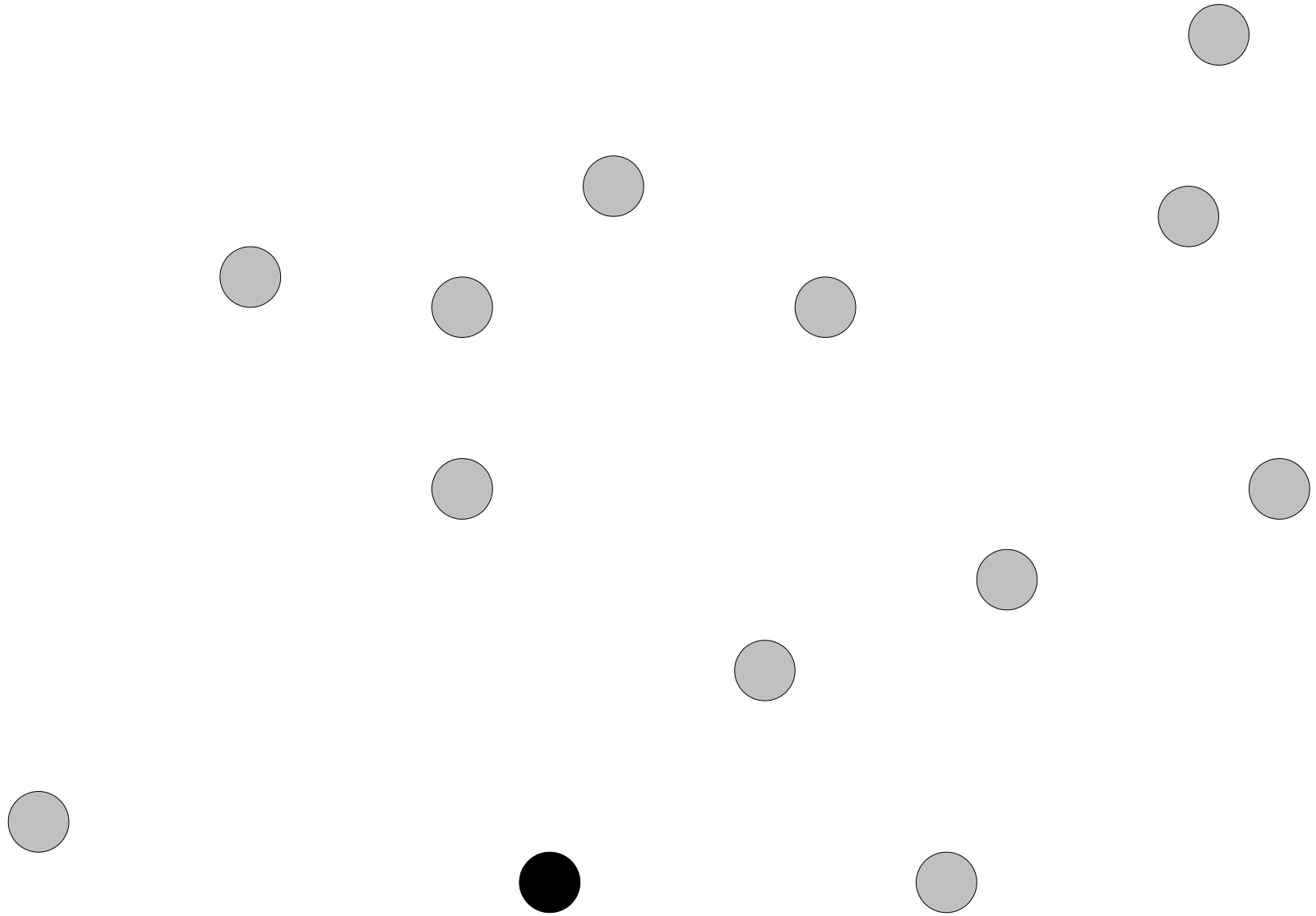
Algorithme de Graham pour calculer l'enveloppe convexe



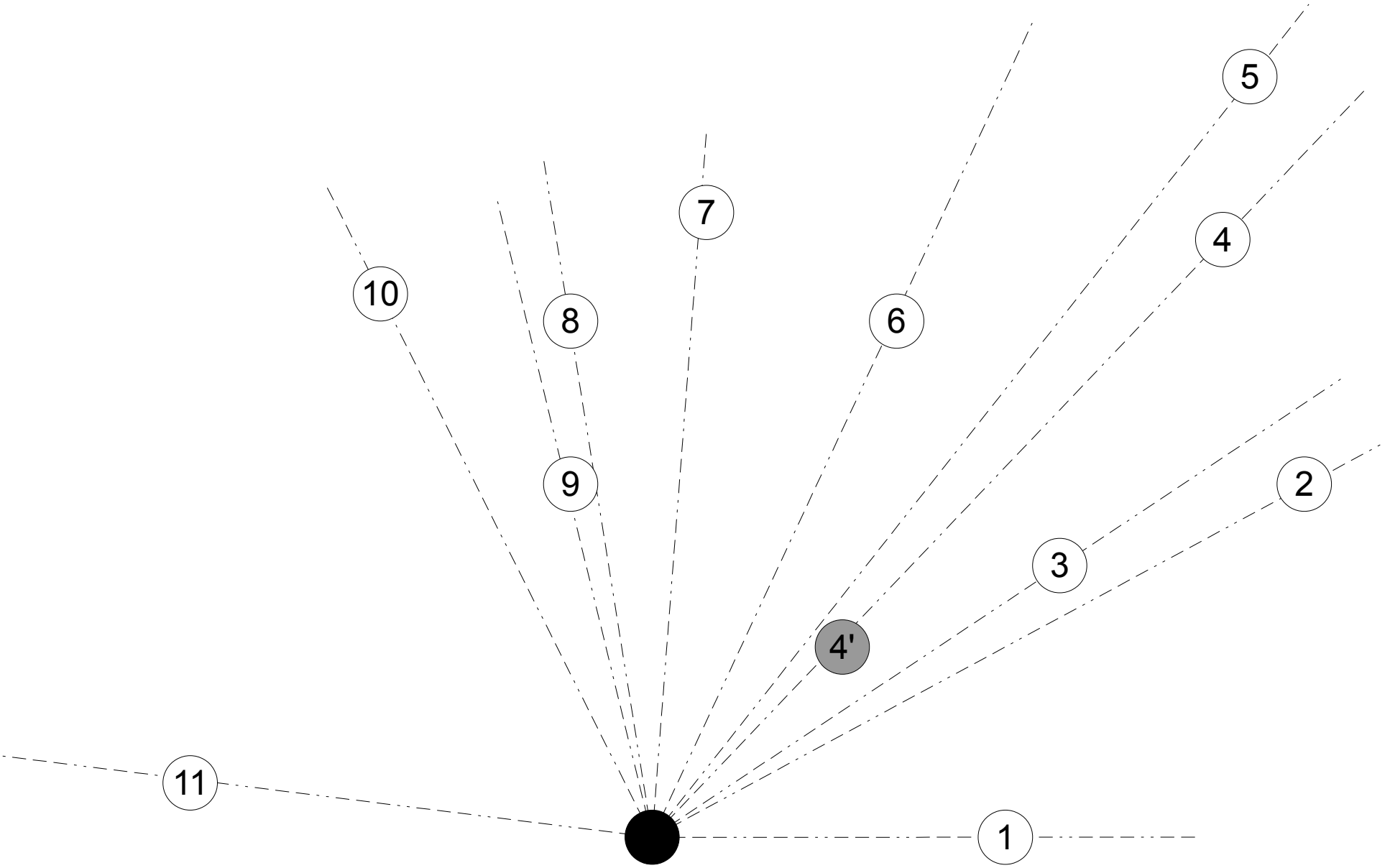
Algorithme de Graham pour calculer l'enveloppe convexe

- Trouver le point le plus en bas
- Balayage
- Construction du contour de l'enveloppe convexe

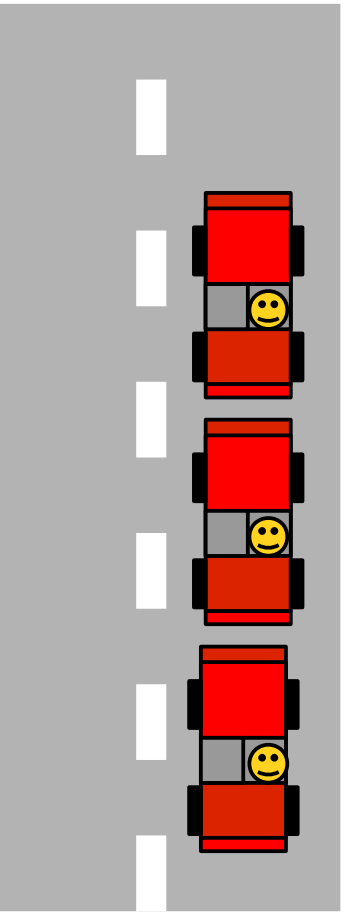
Trouver le point le plus en bas, puis si plusieurs le plus à gauche

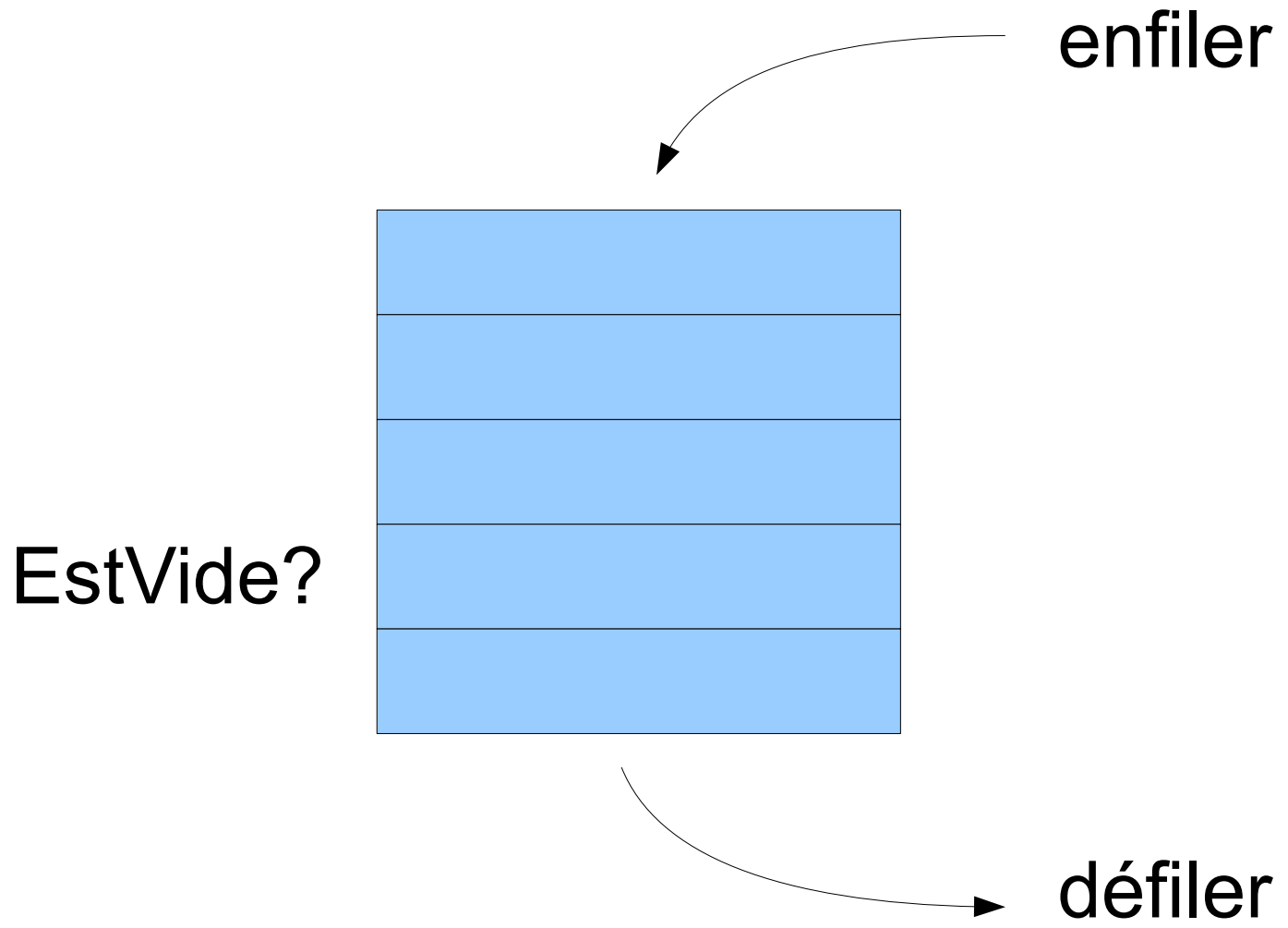


Balayage : trier les points selon leurs angles



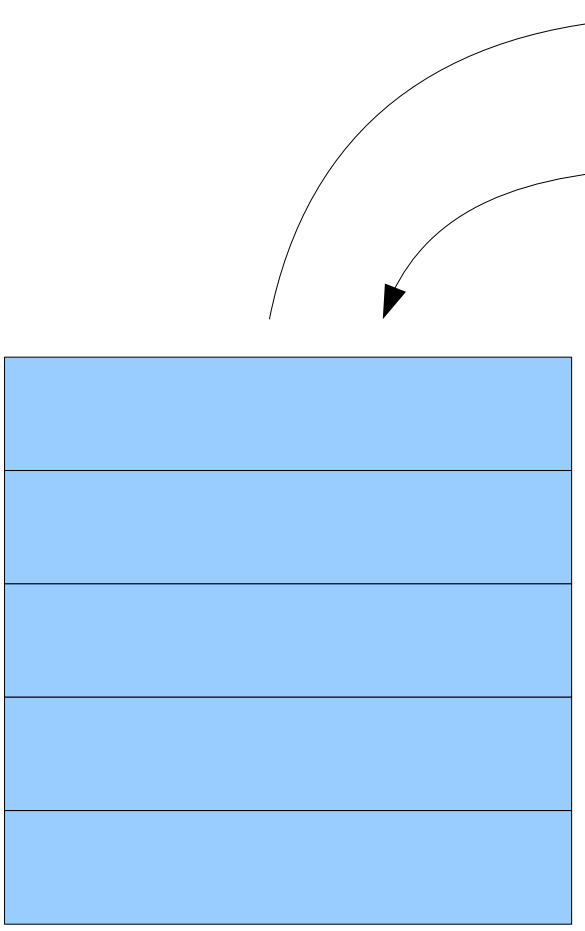
File (FIFO : first in, first out)



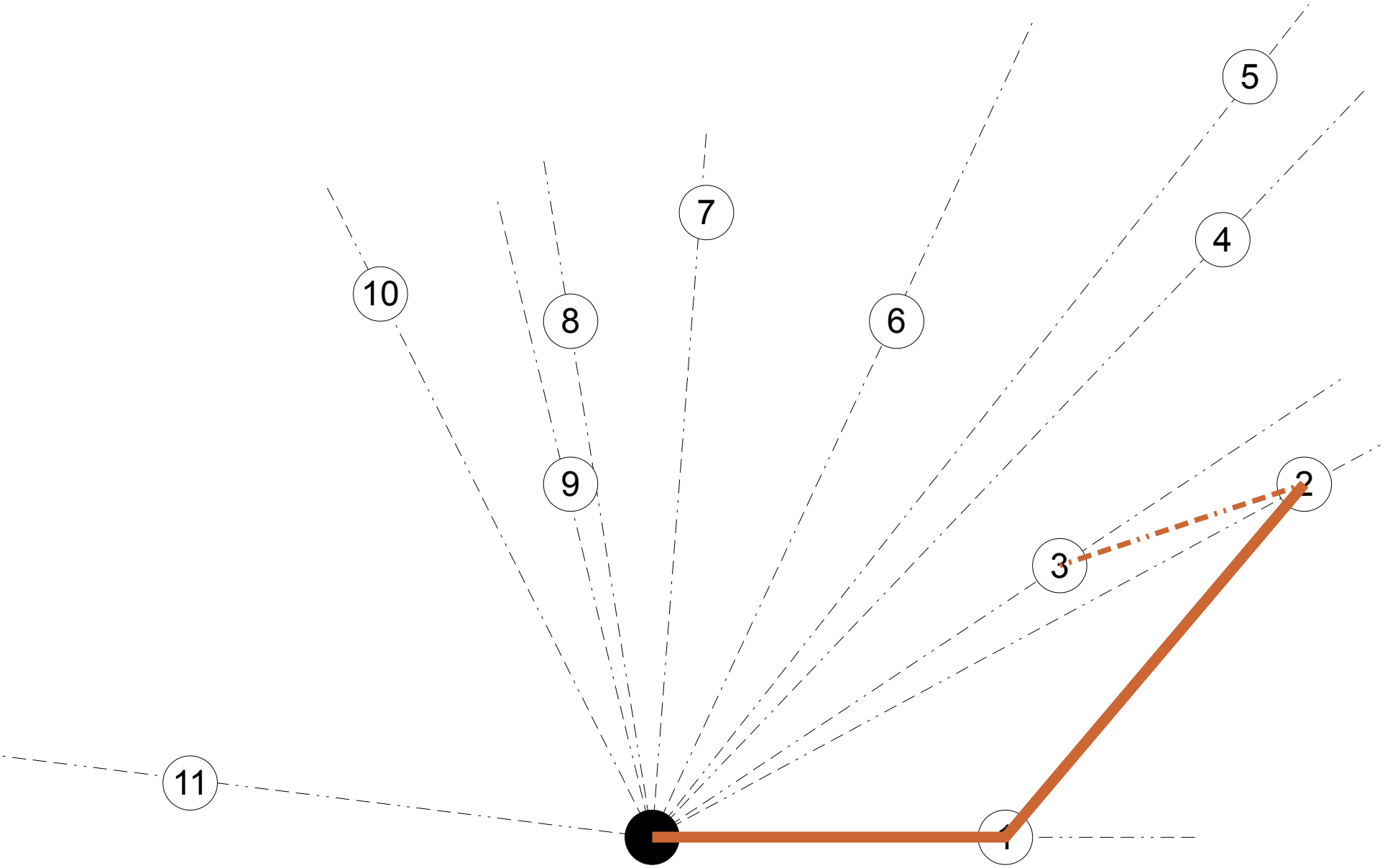


Pile (LIFO : last in, first out)

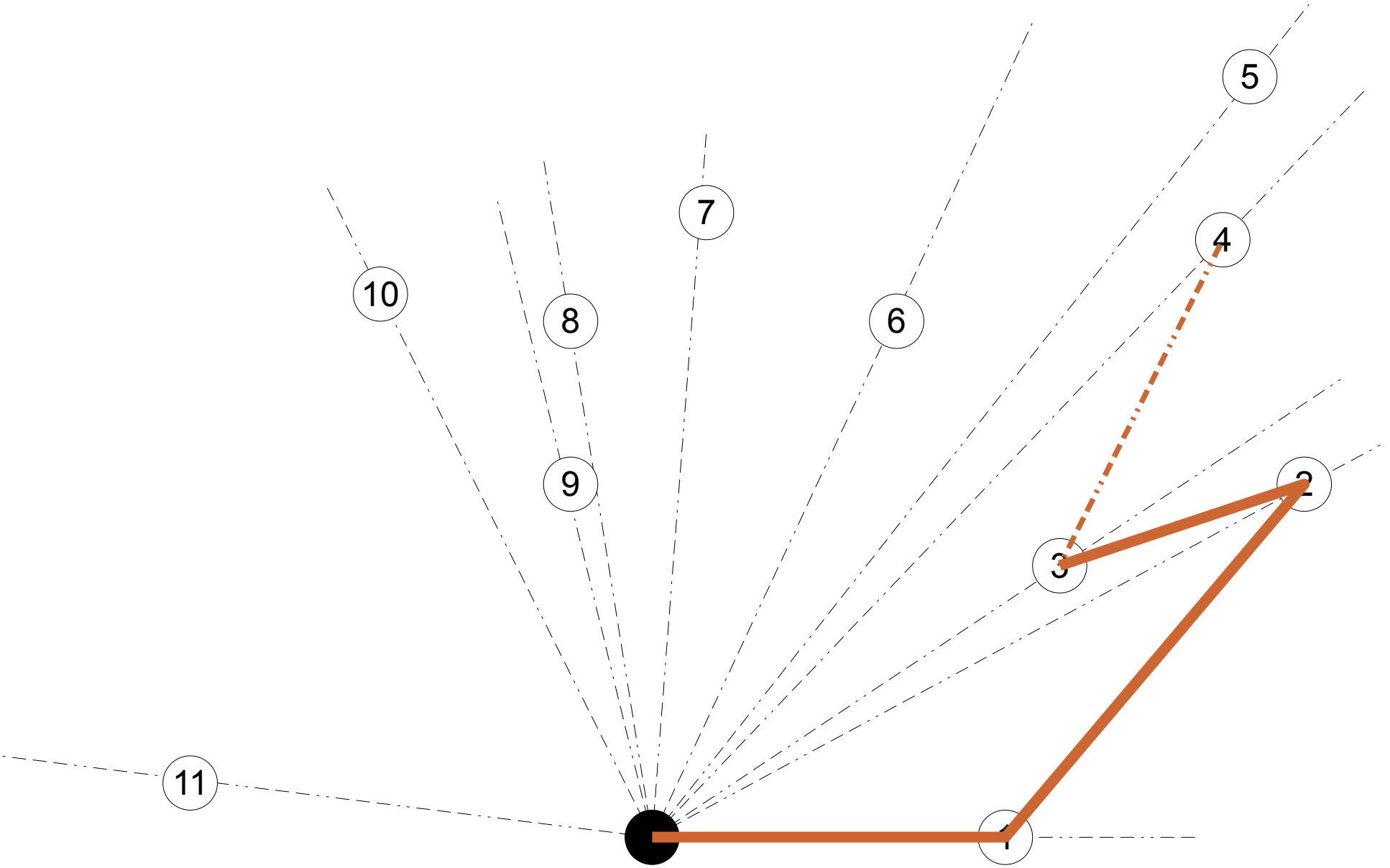
EstVide?



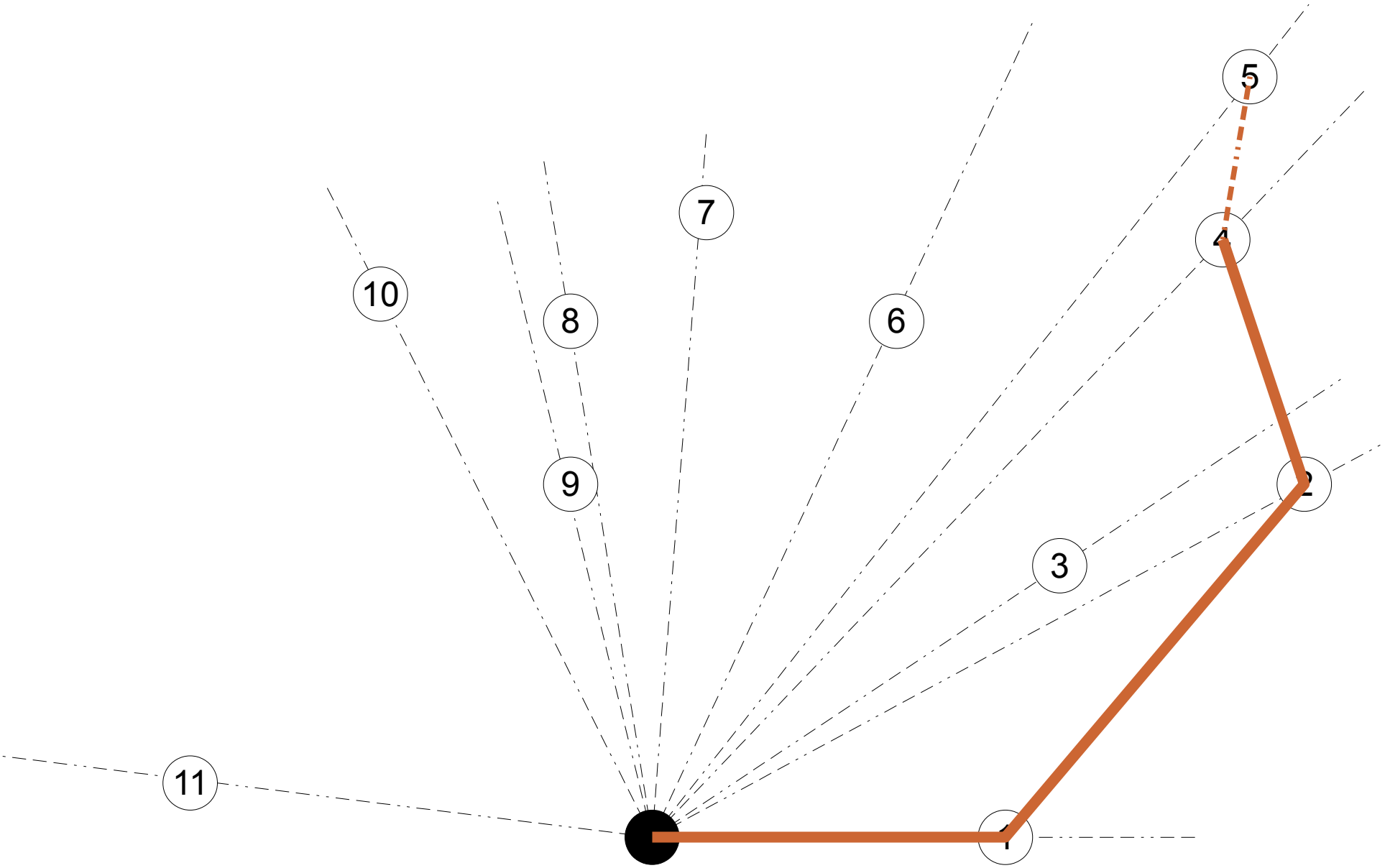
Construction du contour de l'enveloppe convexe



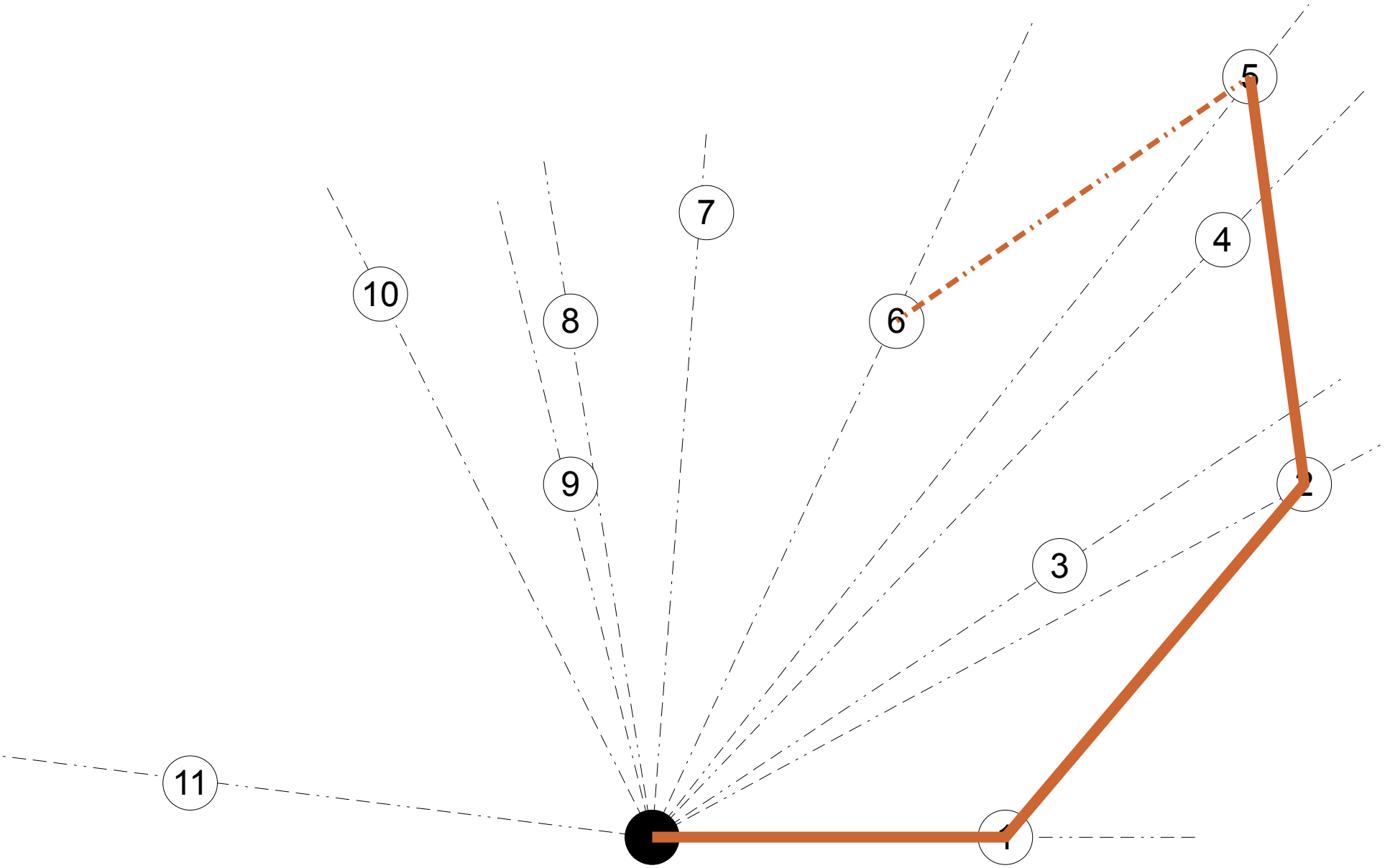
Construction du contour de l'enveloppe convexe



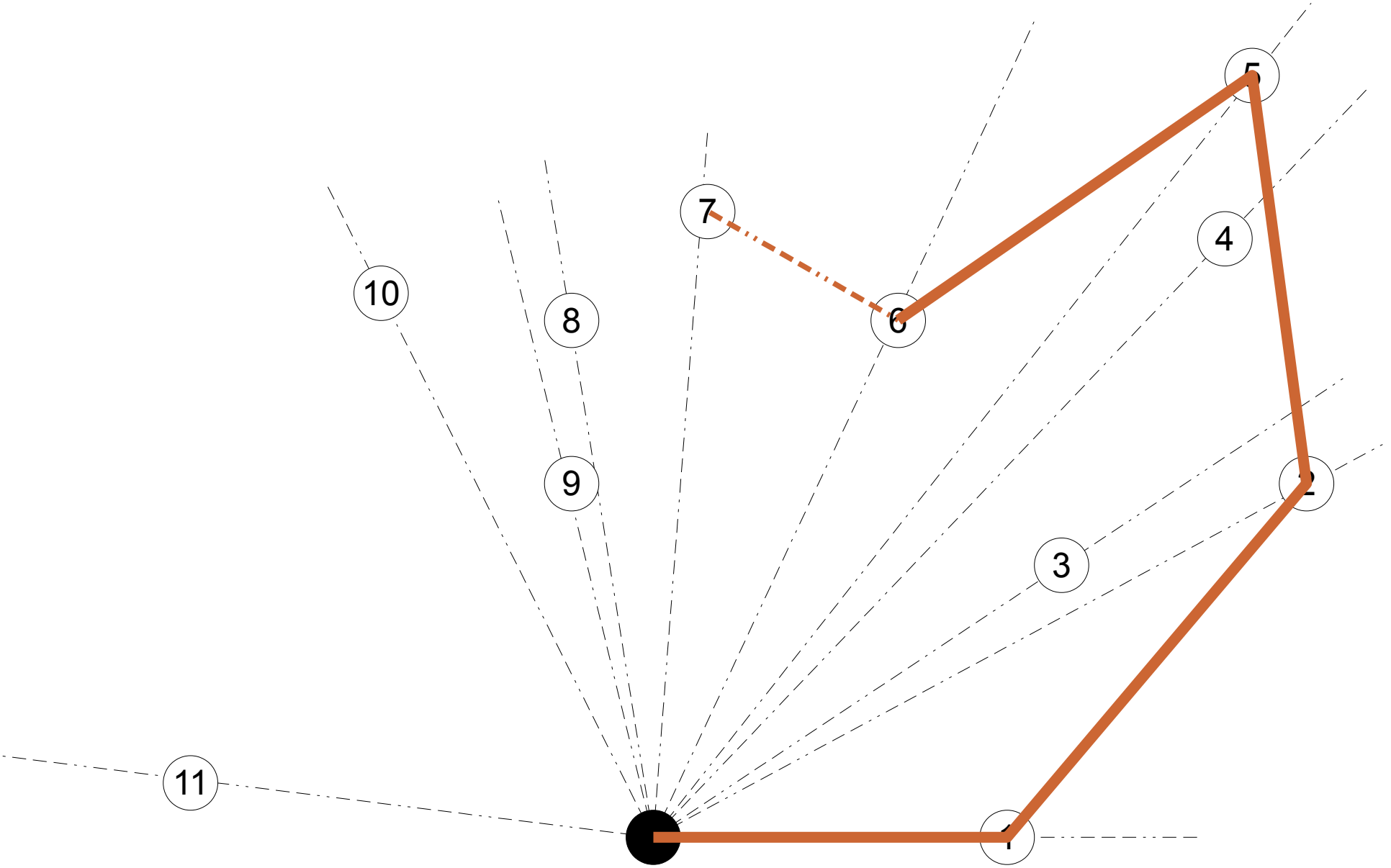
Construction du contour de l'enveloppe convexe



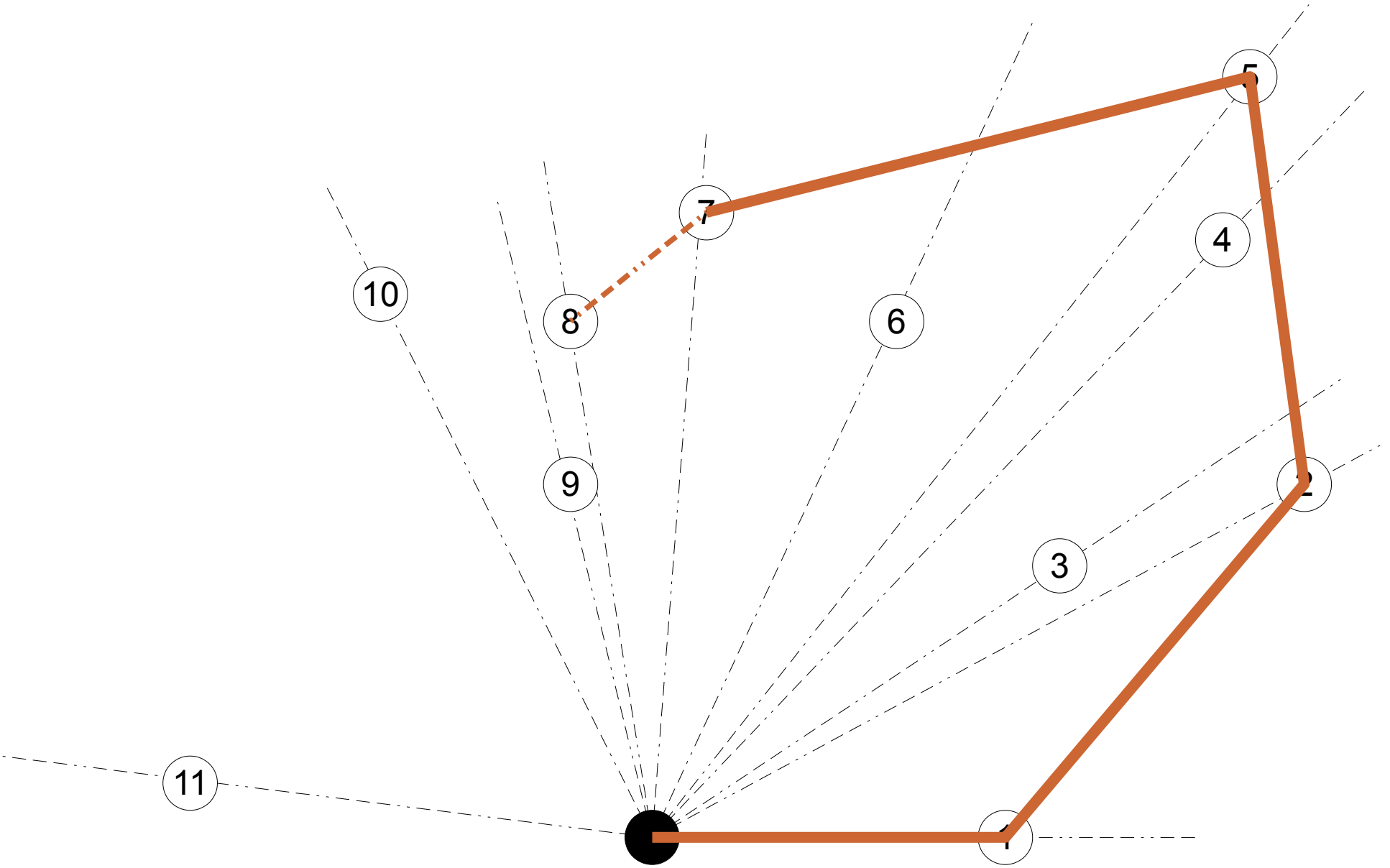
Construction du contour de l'enveloppe convexe



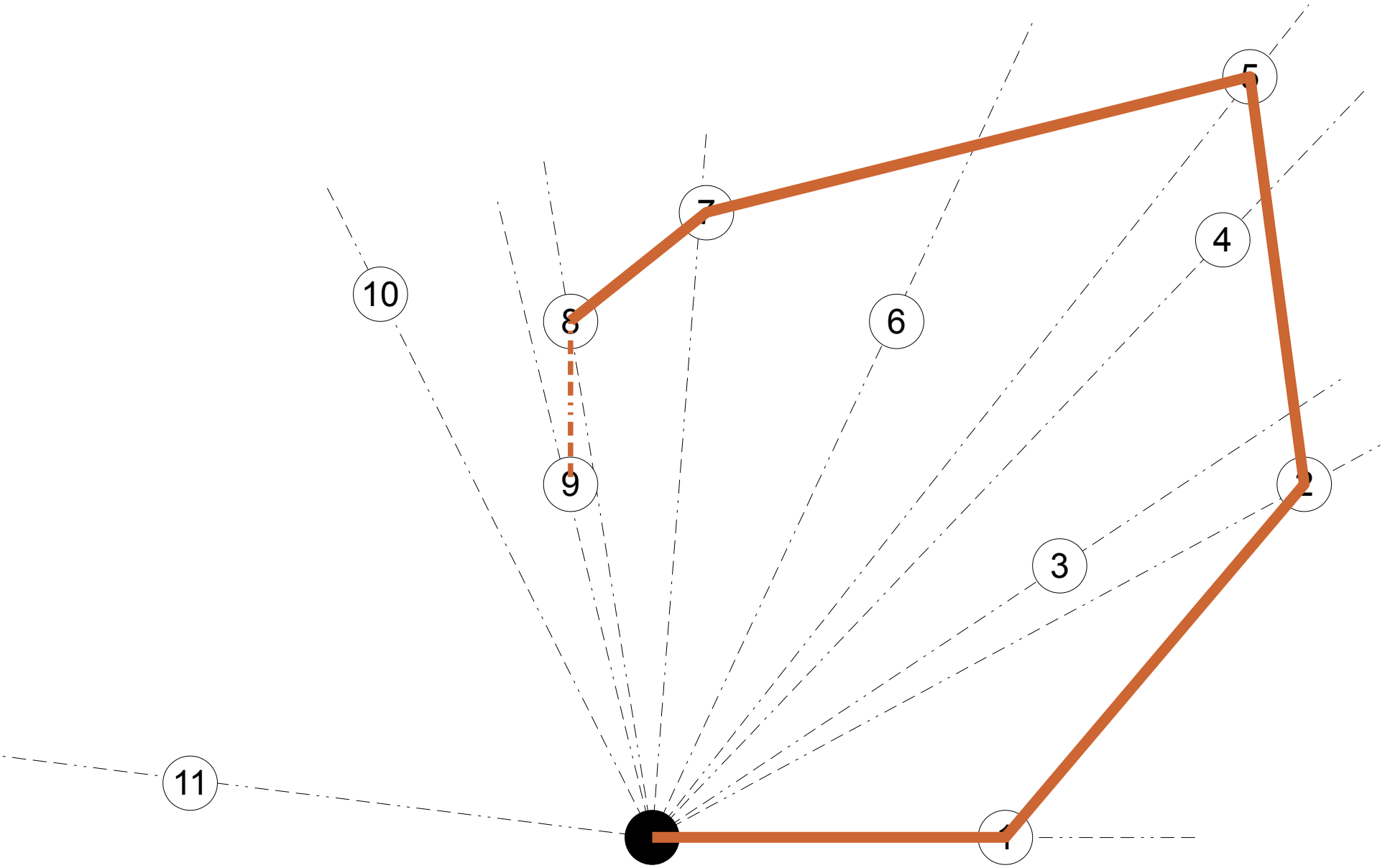
Construction du contour de l'enveloppe convexe



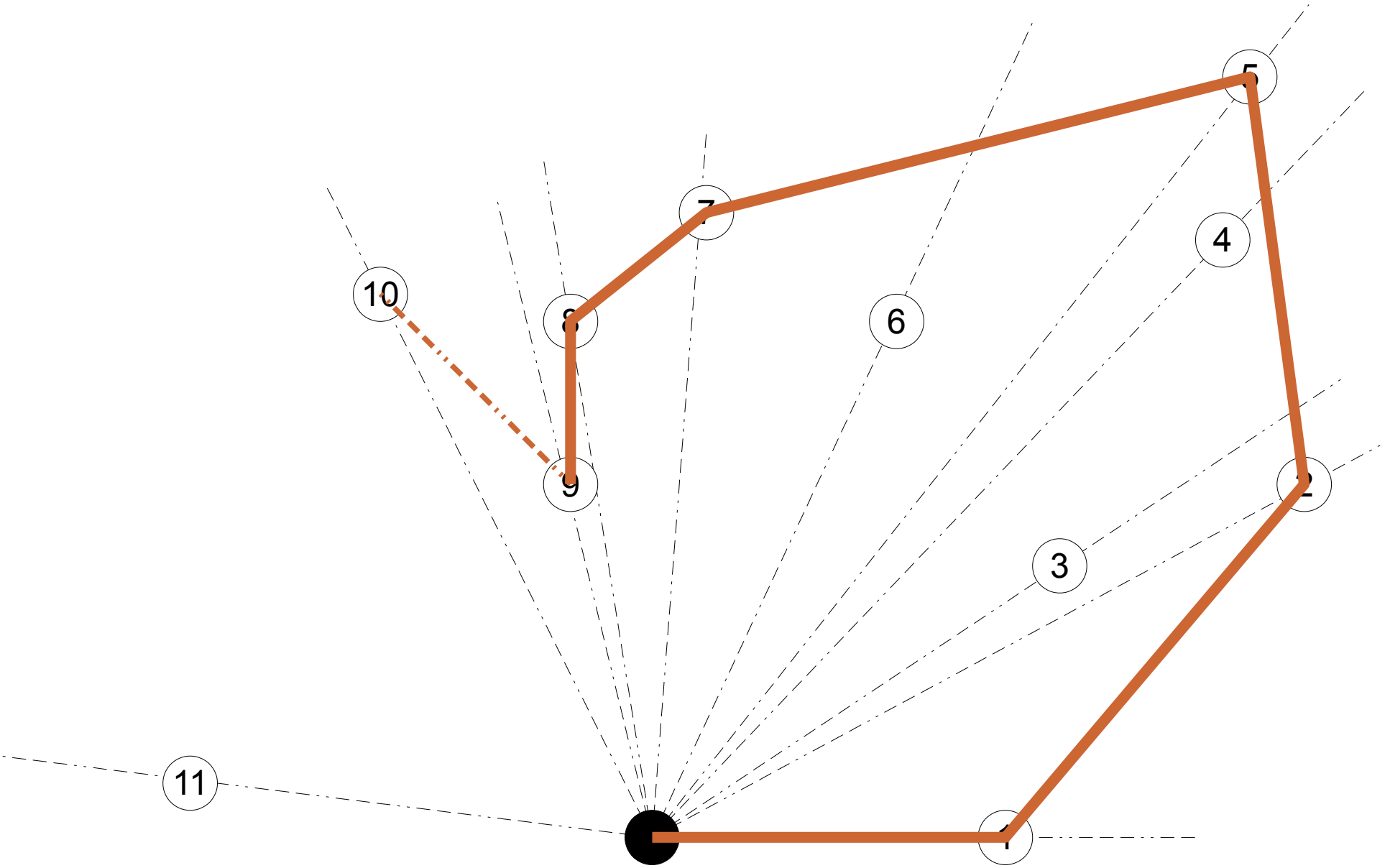
Construction du contour de l'enveloppe convexe



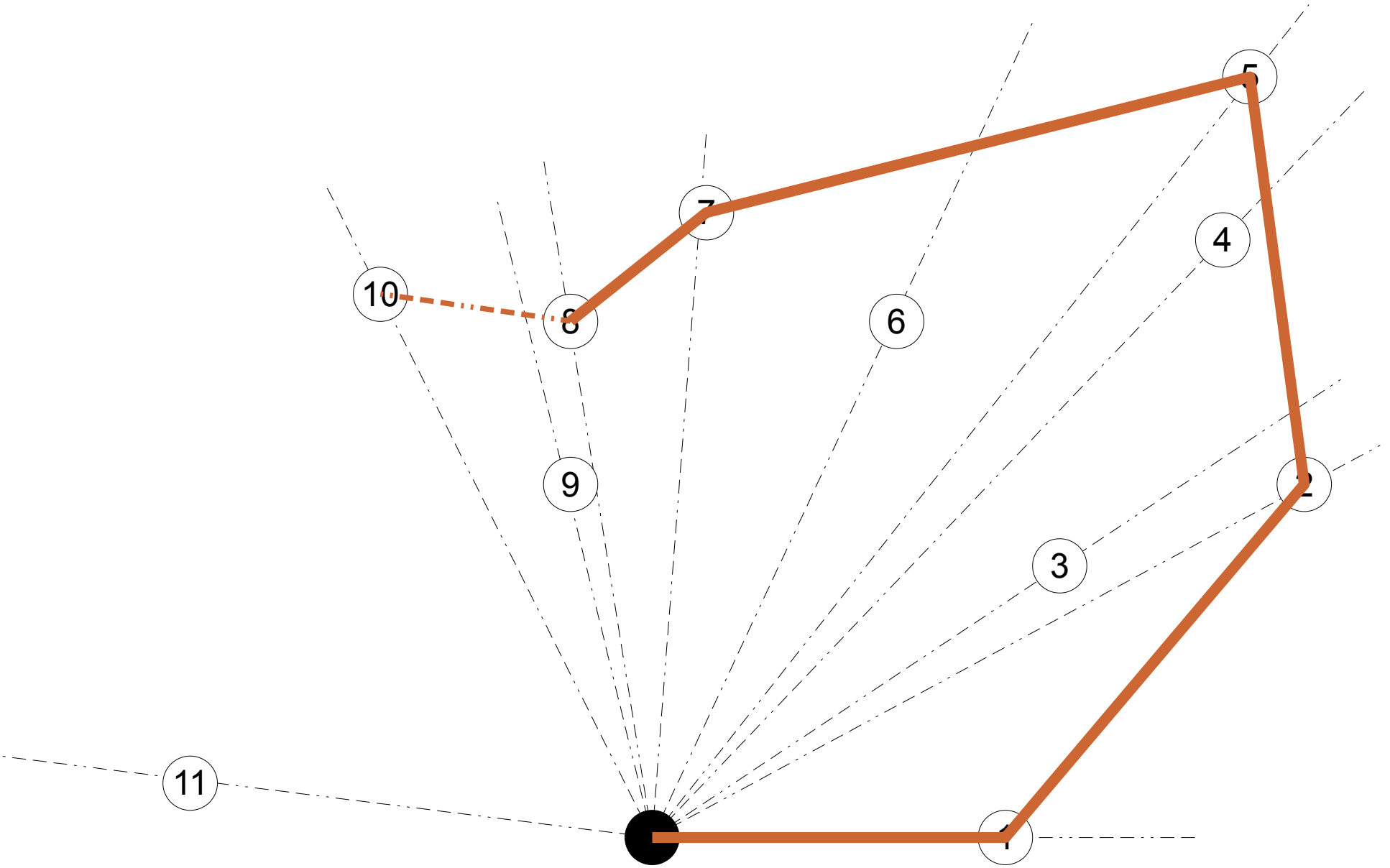
Construction du contour de l'enveloppe convexe



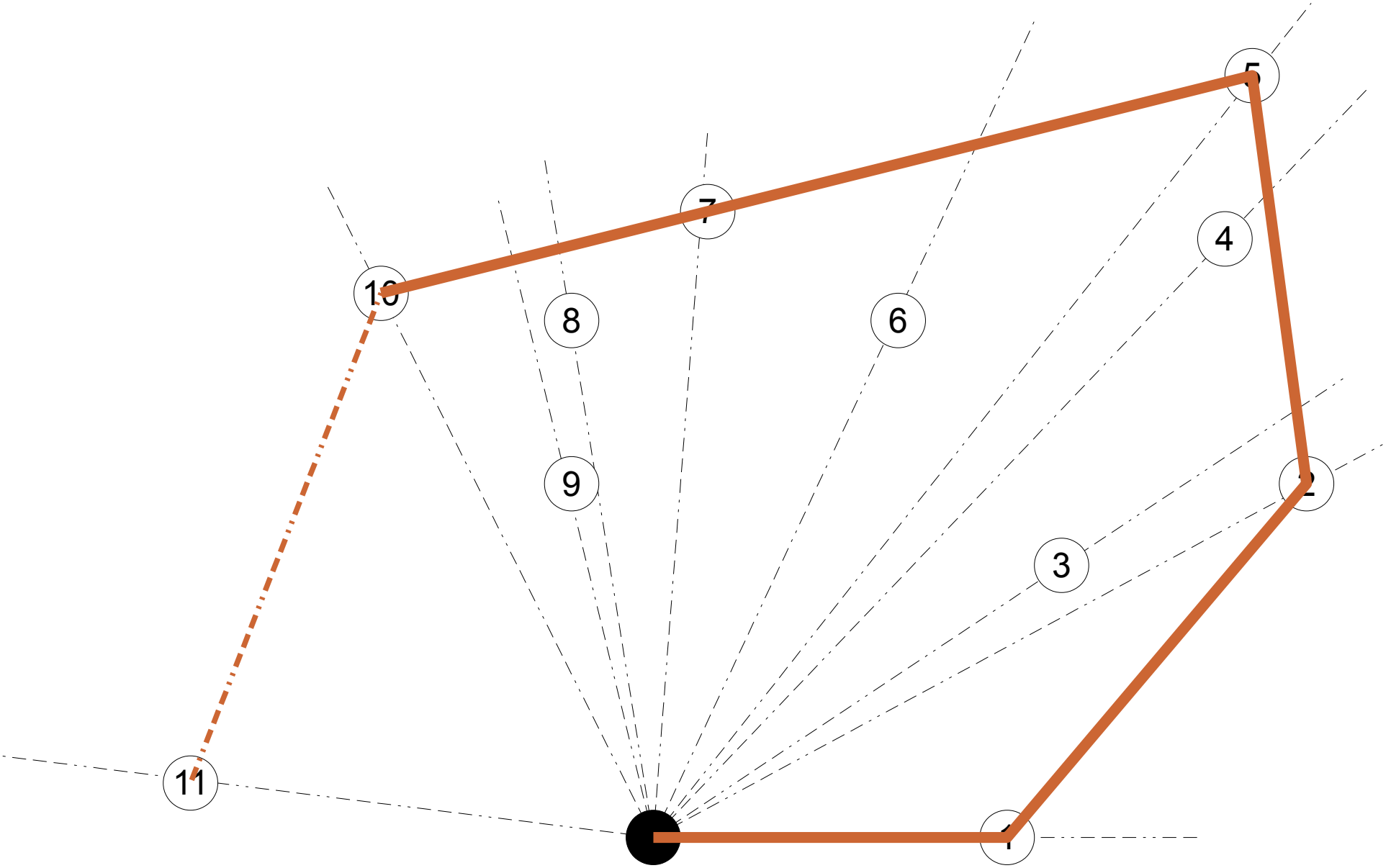
Construction du contour de l'enveloppe convexe



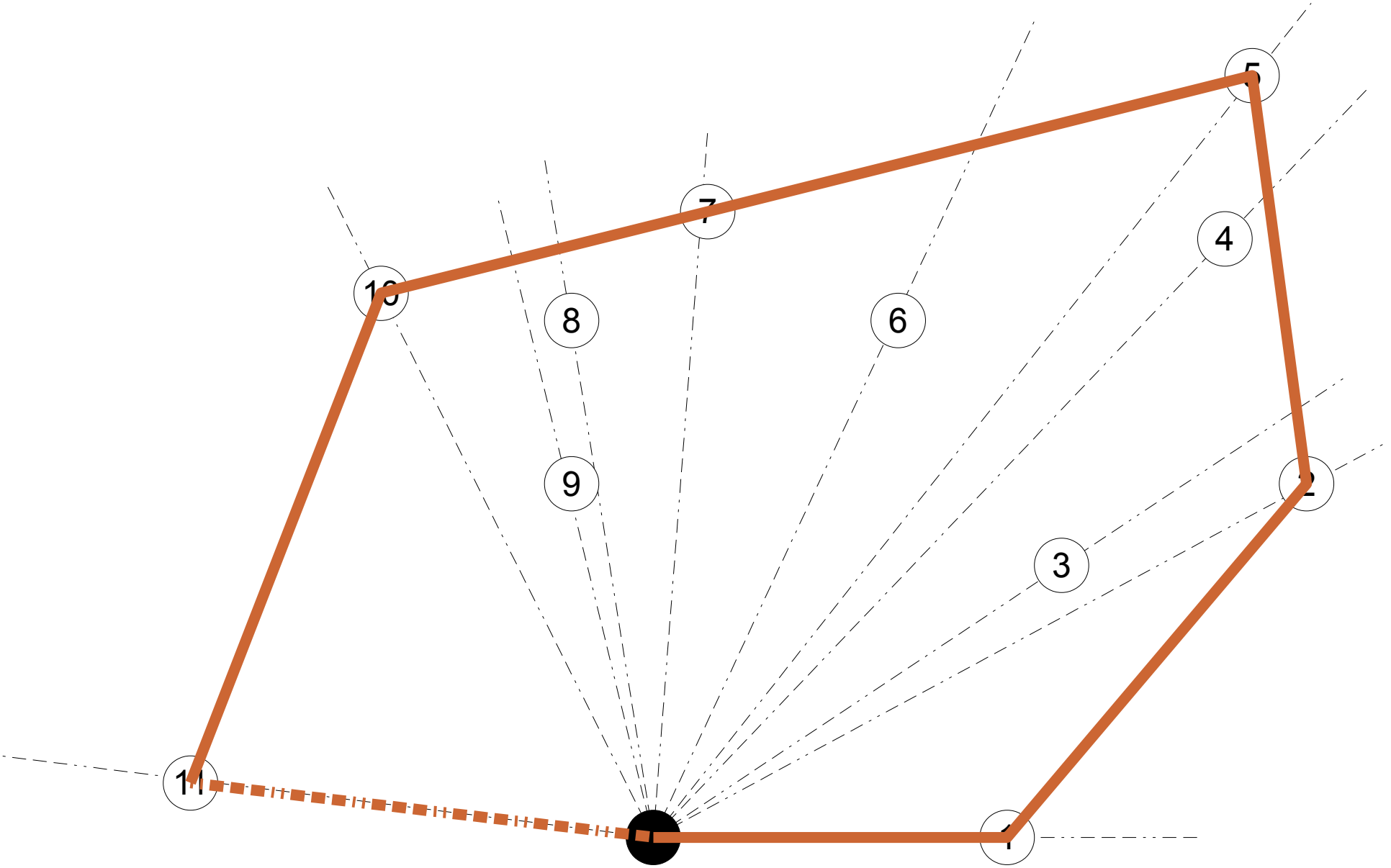
Construction du contour de l'enveloppe convexe



Construction du contour de l'enveloppe convexe

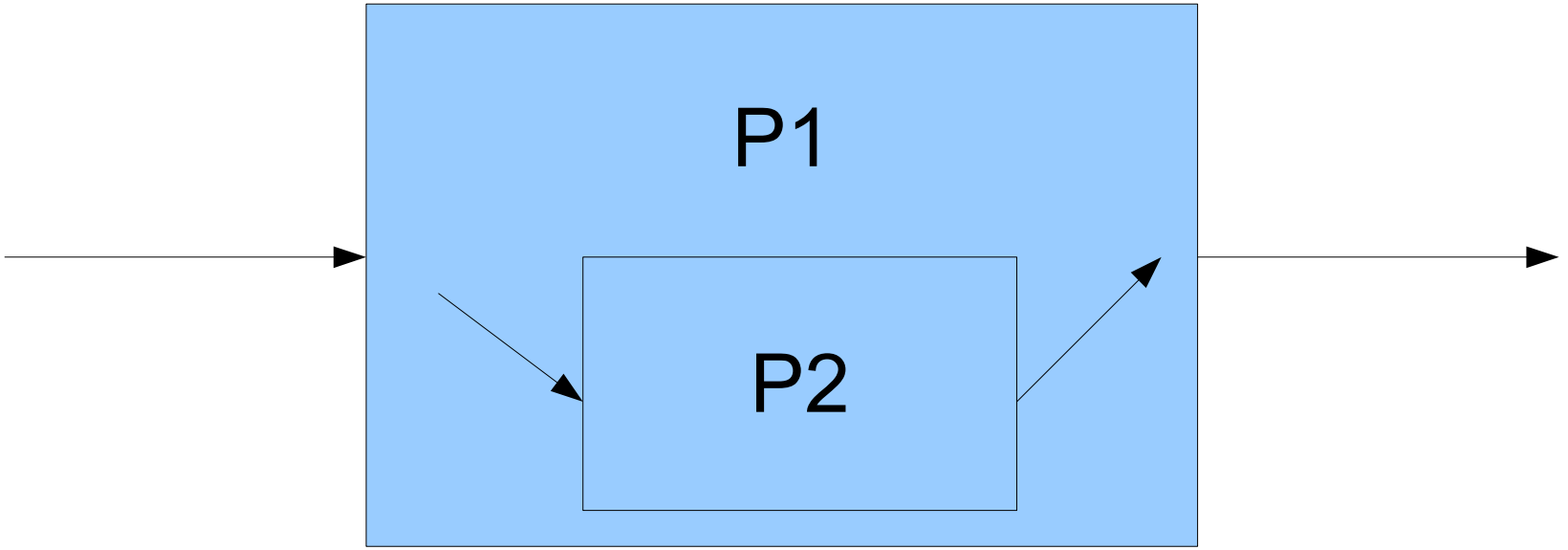


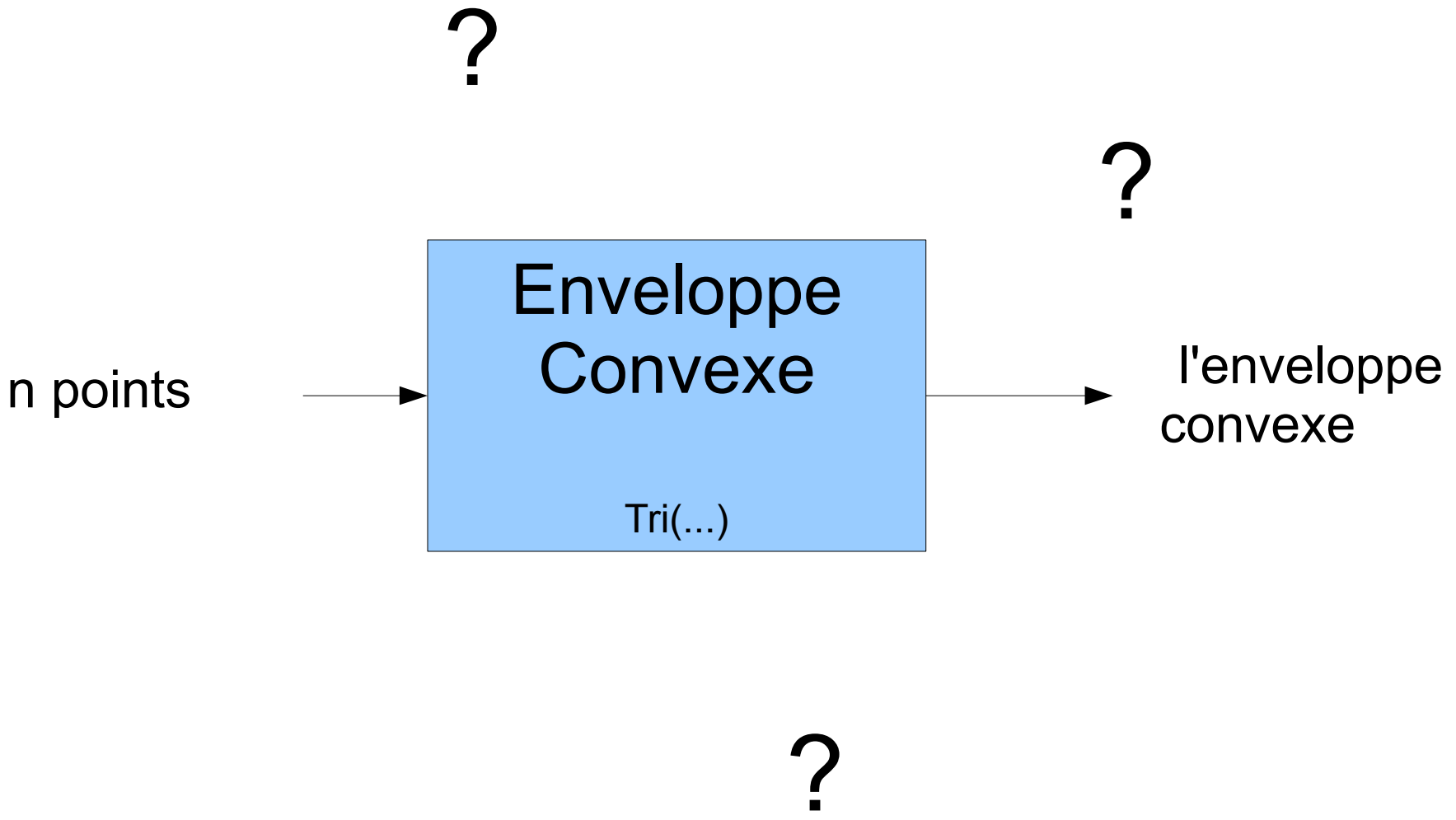
Construction du contour de l'enveloppe convexe



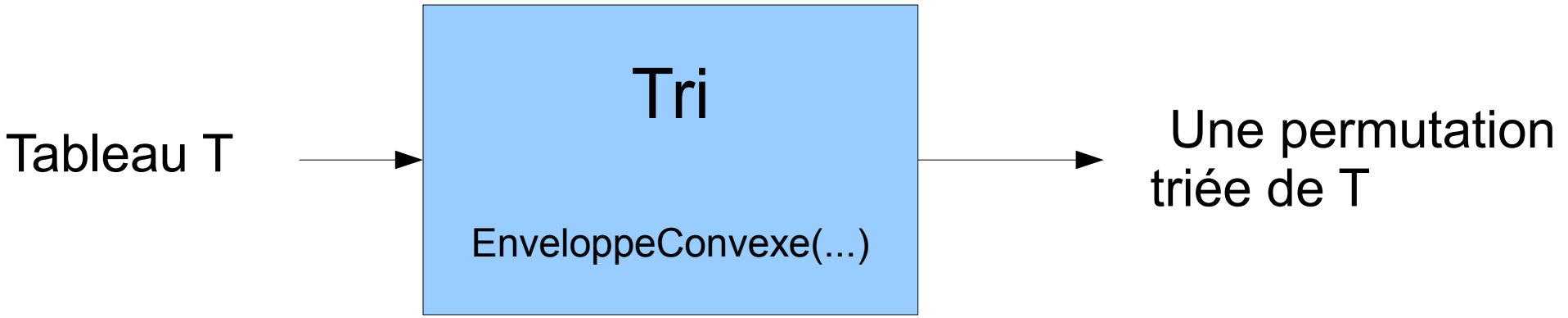
Optimalité de la complexité du calcul de l'enveloppe convexe

Réduction




?
?
Enveloppe
Convexe
Tri(...)
n points
l'enveloppe
convexe
?

Optimalité de la complexité du calcul de l'enveloppe convexe



Optimalité de la complexité du calcul de l'enveloppe convexe

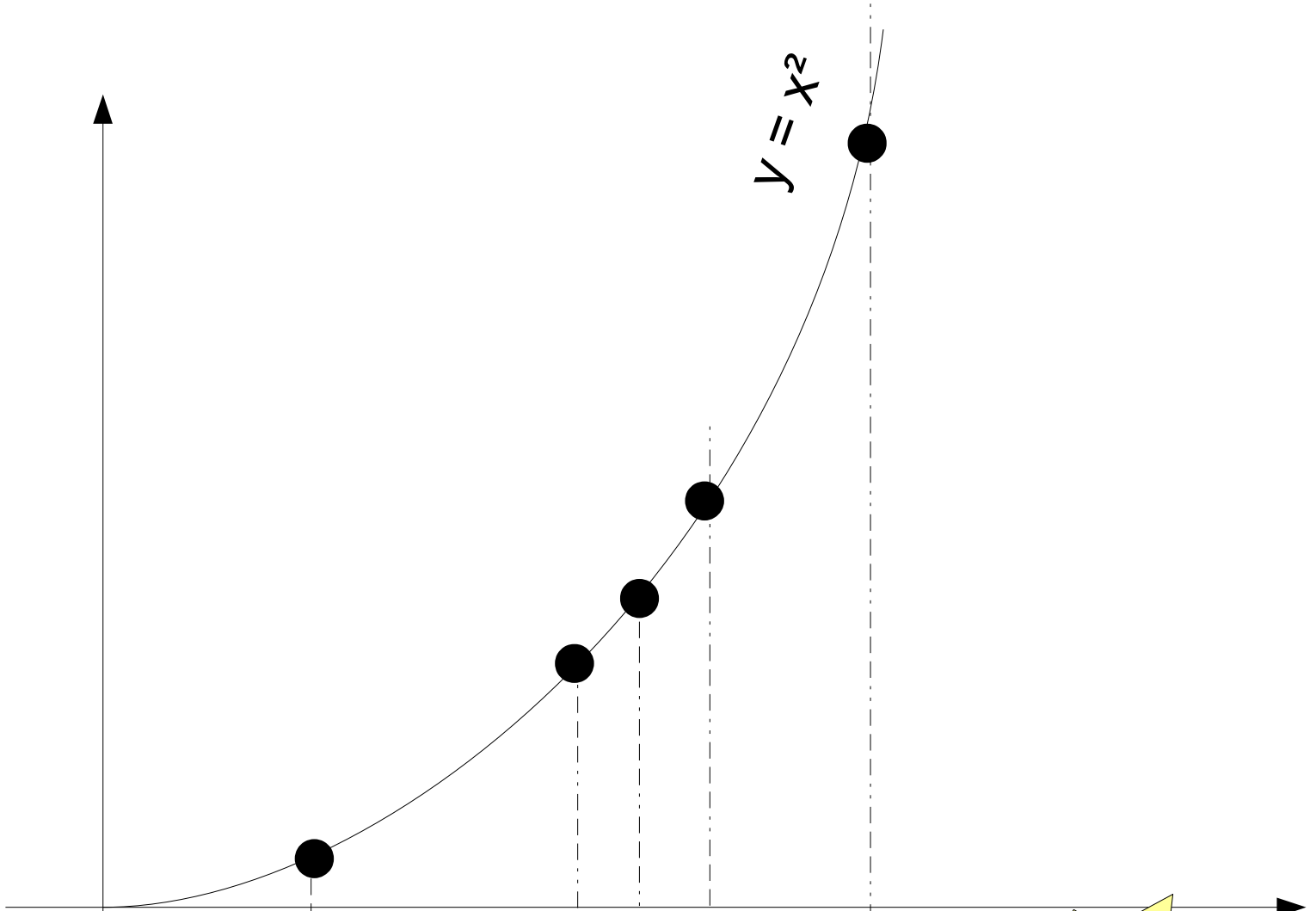
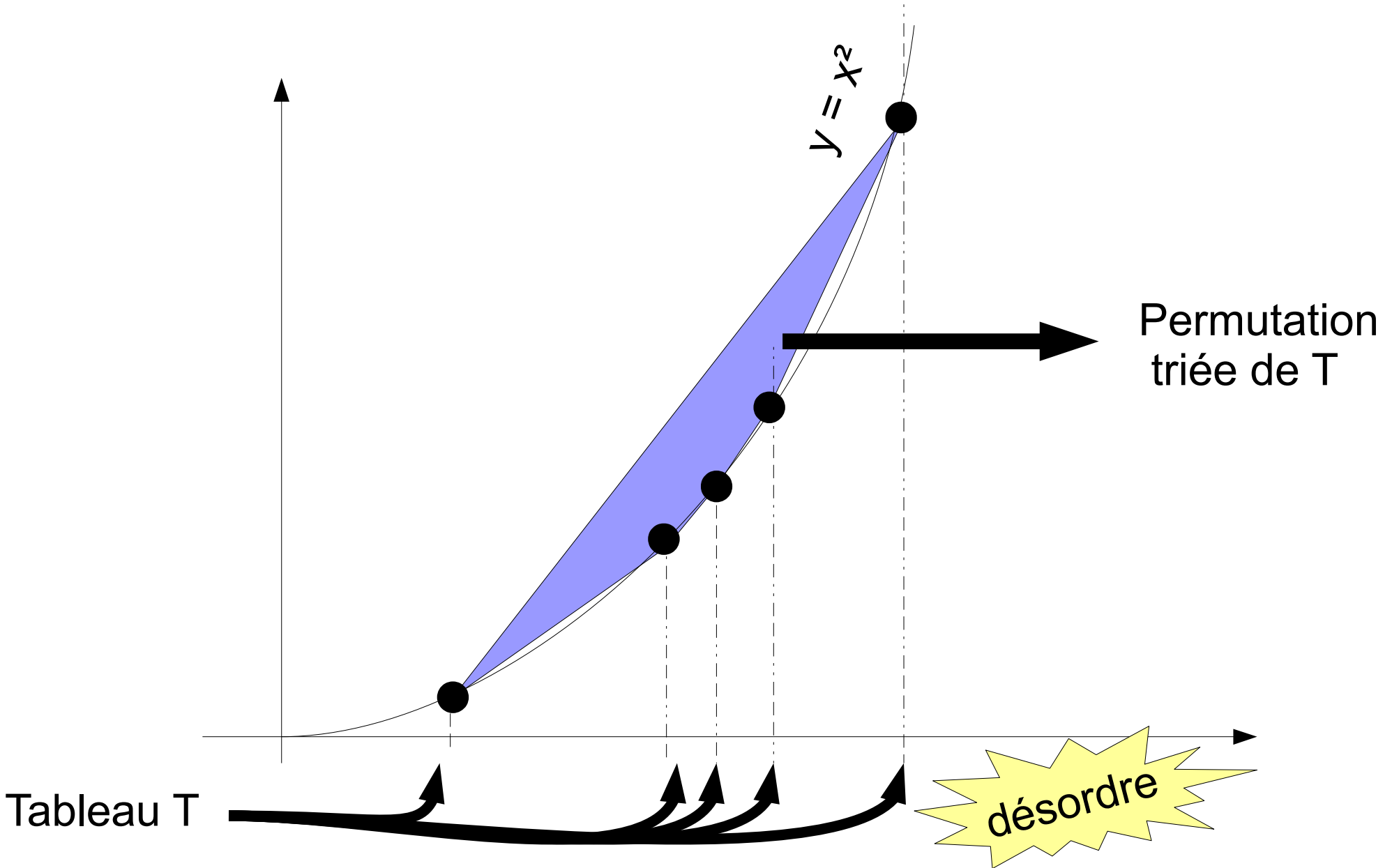


Tableau T

Optimalité de la complexité du calcul de l'enveloppe convexe



Conclusion

- Importance du tri même dans un domaine « lointain »
- Réduction