Knowledge and time

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May 14, 2019

Outline

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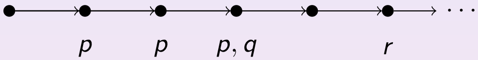
1. Linear temporal logic
   - Models
   - Syntax and semantics
   - Satisfiability problem
   - Model checking
2. Epistemic linear temporal logic
3. Interaction between knowledge and time
4. Model checking

Models



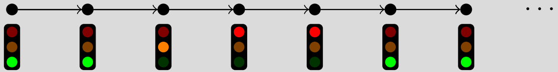
Definition

A linear temporal model is a structure $\langle \mathbb{N}, V \rangle$ such that:

- $V : \mathbb{N} \to 2^{AP}$.

Example

Example (traffic light)

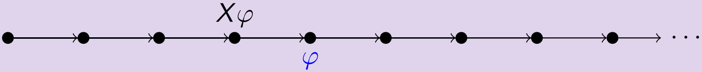


Outline

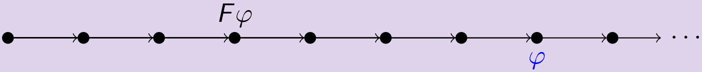
Syntax and semantics

Temporal modalities of LTL

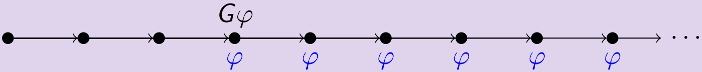
$X\varphi$ — φ is true at the next time



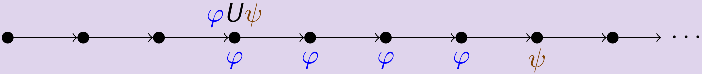
$F\varphi$ — φ is true at some point in the future



$G\varphi$ — φ is true in all points in the future



$\varphi U \psi$ — ψ holds in the future and φ is true until ψ holds



Syntax and semantics

$$
\begin{array}{lll}
\langle \mathbb{N}, V\rangle, t \models p & \text{if} & p \in V(t) \\
\langle \mathbb{N}, V\rangle, t \models \neg\varphi & \text{if} & \langle \mathbb{N}, V\rangle, t \not\models \varphi \\
\langle \mathbb{N}, V\rangle, t \models \varphi \vee \psi & \text{if} & \langle \mathbb{N}, V\rangle, t \models \varphi \text{ or } \langle \mathbb{N}, V\rangle, t \models \psi \\
\\
\langle \mathbb{N}, V\rangle, t \models X\varphi & \text{if} & \langle \mathbb{N}, V\rangle, t+1 \models \varphi \\
\langle \mathbb{N}, V\rangle, t \models F\varphi & \text{if} & \text{there is } t' \geq t \text{ such that } \langle \mathbb{N}, V\rangle, t' \models \varphi \\
\langle \mathbb{N}, V\rangle, t \models G\varphi & \text{if} & \text{for all } t' \geq t,\ \langle \mathbb{N}, V\rangle, t' \models \varphi \\
\langle \mathbb{N}, V\rangle, t \models \varphi U \psi & \text{if} & \text{there is } t' \geq t \text{ such that } \langle \mathbb{N}, V\rangle, t' \models \psi \\
 & & \text{and } \langle \mathbb{N}, V\rangle, t'' \models \varphi \text{ for all } t'' \in \{t, \ldots, t'-1\},
\end{array}
$$

Outline

Satisfiabiliy problem

Definition

The satisfiability problem is:

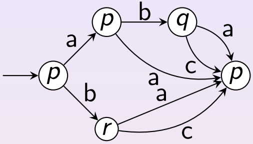
- input: a formula φ;
- output: yes if there is V such that $\langle \mathbb{N}, V \rangle, t \models \varphi$

Theorem

The satisfiability problem is PSPACE-complete.

Outline

Model checking



Definition

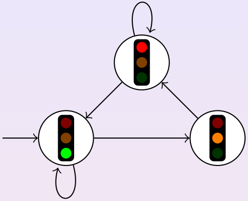
- input: a transition system $\mathcal{S}$; a formula φ of LTL;
- output: yes, if all paths of $\mathcal{S}$ starting from an initial state of $\mathcal{S}$ satisfy φ.

Theorem

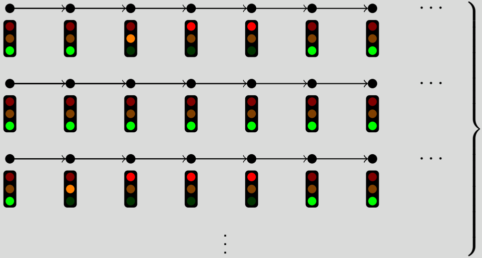
The model checking of LTL is PSPACE-complete.

Example

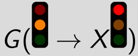
Transition system $\mathcal{S}$:



Example (paths of $\mathcal{S}$ starting from an initial state of $\mathcal{S}$)



they all satisfy



Outline

Epistemic linear temporal logic

Epistemic linear temporal logic	$=$	epistemic logic	$+$	linear temporal logic
		K_a		X, F, G, U

Outline

1. Linear temporal logic
2. Epistemic linear temporal logic
   - Models
   - Semantics
3. Interaction between knowledge and time
4. Model checking

Models

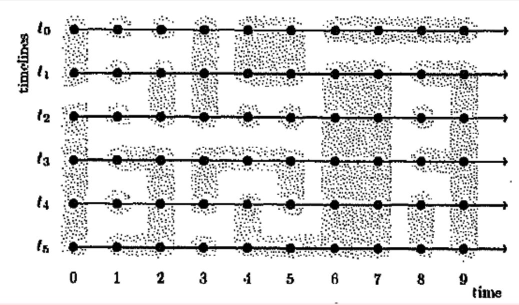
Definition

An ELTL model is a structure $\mathcal{M} = \langle TL \times \mathbb{N}, (\sim_a)_{a \in AGT}, V \rangle$ such that:

- TL is a non-empty set of *timelines*;
- for all agents a,

$$\sim_a \text{ is an equivalence relation on } TL \times \mathbb{N};$$

- $V : TL \times \mathbb{N} \rightarrow 2^{AP}$.



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Semantics

$$
\begin{array}{lll}
\mathcal{M},(\rho,t) \models p & \text{if} & p \in V(\rho,t) \\
\mathcal{M},(\rho,t) \models \neg\varphi & \text{if} & \mathcal{M},(\rho,t) \not\models \varphi \\
\mathcal{M},(\rho,t) \models \varphi \vee \psi & \text{if} & \mathcal{M},(\rho,t) \models \varphi \text{ or } \mathcal{M},(\rho,t) \models \psi
\end{array}
$$

$$
\begin{array}{lll}
\mathcal{M},(\rho,t) \models X\varphi & \text{if} & \mathcal{M},(\rho,t+1) \models \varphi \\
\mathcal{M},(\rho,t) \models F\varphi & \text{if} & \text{there is } t' \geq t \text{ such that } \mathcal{M},(\rho,t') \models \varphi \\
\mathcal{M},(\rho,t) \models G\varphi & \text{if} & \text{for all } t' \geq t,\ \mathcal{M},(\rho,t') \models \varphi \\
\mathcal{M},(\rho,t) \models \varphi U \psi & \text{if} & \text{there is } t' \geq t \text{ such that } \mathcal{M},(\rho,t') \models \psi \\
 & & \text{and } \mathcal{M},(\rho,t'') \models \varphi \text{ for all } t'' \in \{t,\ldots,t'-1\},
\end{array}
$$

$$
\mathcal{M},(\rho,t) \models K_a\varphi \quad \text{if} \quad \text{for all } (\rho',t') \sim_a (\rho,t),\ \mathcal{M},(\rho',t') \models \varphi
$$

Outline

Outline

1. Linear temporal logic
2. Epistemic linear temporal logic
3. Interaction between knowledge and time
   - When no interaction
   - Adding interaction
   - Impact on the complexity
4. Model checking

Axiomatisation: fusion of EL and LTL

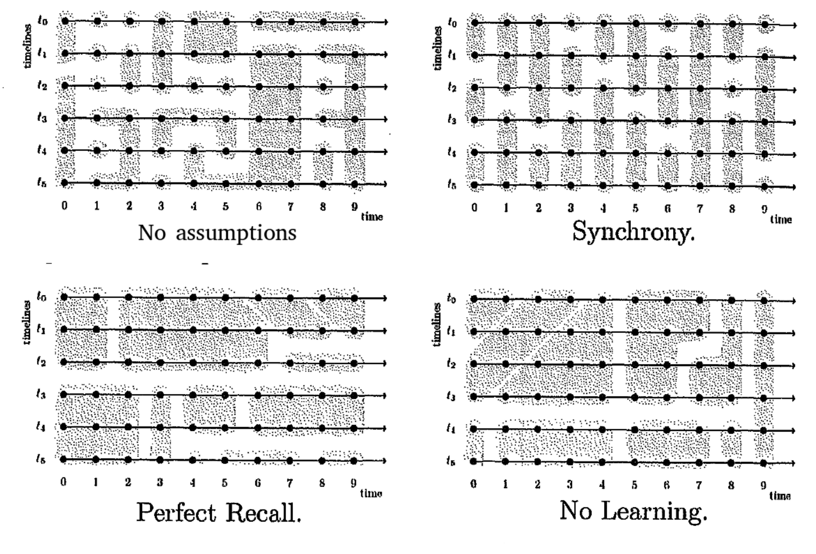
all classical tautologies

$K_a(\varphi \to \psi) \to (K_a\varphi \to K_a\psi)$
$K_a\varphi \to \varphi$
$\hat{K}_a\top$
$K_a\varphi \to K_a\ K_a\varphi$
$\neg K_a\varphi \to K_a\neg\ K_a\varphi$

$G(\varphi \to \psi) \to (G\varphi \to G\psi)$
$X(\varphi \to \psi) \to (X\varphi \to X\psi)$
$X\neg\varphi \leftrightarrow \neg X\varphi$
$G\varphi \to (\varphi \wedge XG\varphi)$
$G(\varphi \to X\varphi) \to (\varphi \to G\varphi)$
$(\varphi U\psi) \to F\psi$
$(\varphi U\psi) \leftrightarrow (\psi \vee X(\varphi U\psi))$

Outline

Corresponding properties in the models

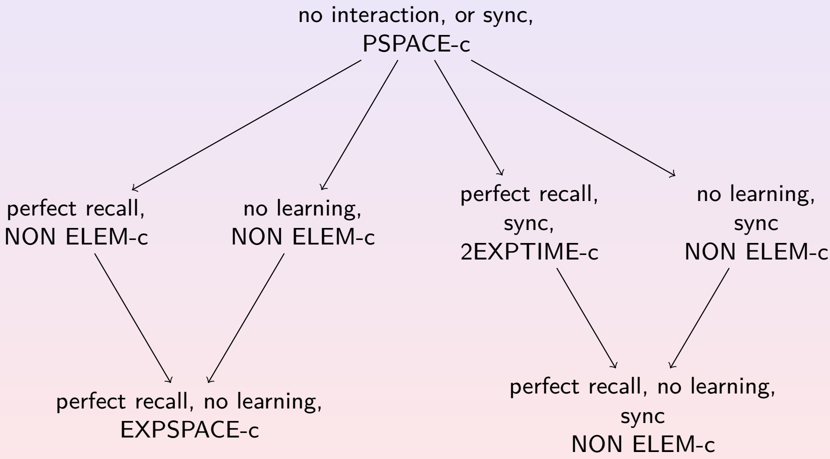


Additional axioms for interaction

synchronous	agents know the time t (not an axiom)
perfect recall, synchronous	$K_a X\varphi \rightarrow XK_a\varphi$
perfect recall	$K_a\varphi \wedge X(K_a\psi \wedge \neg K_a\chi) \rightarrow \neg K_a\neg(K_a\varphi U(K_a\psi U\neg\chi))$
no learning	$(K_a\varphi U K_a\psi) \rightarrow K_a(K_a\varphi U K_a\psi)$
no learning, synchronous	$XK_a\varphi \rightarrow K_a X\varphi$

Outline

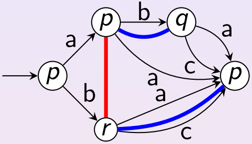
Complexity of the satisfiability problem



[Halpern and Vardi, 1989]

Outline

Model checking problem



Definition

- Input: An epistemic transition system $\mathcal{S}$, that is a transition system augmented with epistemic relations $(R_a)_{a \in AGT}$, with a set of initial states; a formula φ of epistemic linear temporal logic;
- Output: Yes, if "$\mathcal{M}_\mathcal{S}, (\rho, 0) \models \varphi$" for all paths ρ of $\mathcal{S}$ starting in an initial state of $\mathcal{S}$, no otherwise.

$\mathcal{M}_\mathcal{S}$ should be defined...

Possible definitions of $\mathcal{M}_\mathcal{S}$

Definition

Given a transition system $\mathcal{S}$,
we define $\mathcal{M}_\mathcal{S} = \langle TL \times \mathbb{N}, (\sim_a)_{a \in AGT}, V \rangle$ such that:

- TL is the set of paths of $\mathcal{S}$ starting in an initial state of $\mathcal{S}$;
- for all agents a, $(\rho, t) \sim_a (\rho', t')$ if:
  - $t = t'$; — synchrony
  - $\rho[i] R_a \rho'[i]$ for all $i \in \{0, \ldots, t\}$ — perfect recall
- $V : TL \times \mathbb{N} \to 2^{AP}$ defined by:

$$V(\rho, t) = \text{set of propositions true at } \rho[t]$$

Possible definitions of $\mathcal{M}_{\mathcal{S}}$

Definition

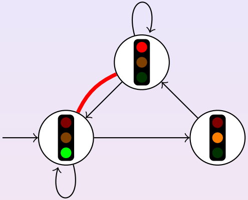
Given a transition system $\mathcal{S}$,
we define $\mathcal{M}_{\mathcal{S}} = \langle TL \times \mathbb{N}, (\sim_a)_{a \in AGT}, V \rangle$ such that:

- TL is the set of paths of $\mathcal{S}$ starting in an initial state of $\mathcal{S}$;
- for all agents a, $(\rho, t) \sim_a (\rho', t')$ if:
  - $t = t'$; synchrony
  - $\rho[t] R_a \rho'[t]$ memory less
- $V : TL \times \mathbb{N} \to 2^{AP}$ defined by:

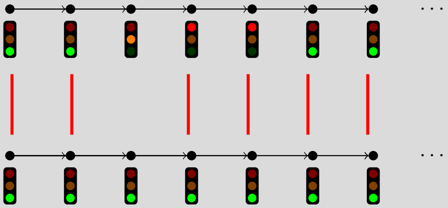
$$V(\rho, t) = \text{set of propositions true at } \rho[t]$$

Example

Transition system $\mathcal{S}$:

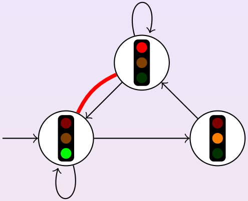


Example (Memory less)

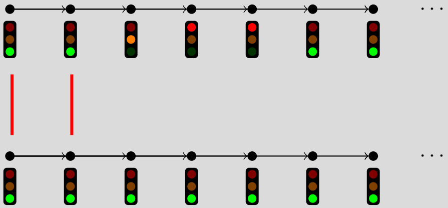


Example

Transition system $\mathcal{S}$:



Example (Perfect recall)



Results

Theorem

The model checking memoryless and synchrony is PSPACE-complete. [Engelhardt, Peter Gammie, and Ron Van Der Meyden, 2007]

Theorem

The model checking under perfect recall and synchrony is:

- *undecidable if CK and until*
- *NON ELEM-c if until but no CK*
- *PSPACE-c if CK but no until*

[van der Meyden and Shilov, 1999]

[Maubert et al. IJCAI 2019]

Thanks to Bastien Maubert for discussions about knowledge and time.