

$$\begin{array}{c}
\overline{\Gamma, \phi \vdash \phi} \text{ ax} \\
\frac{\Gamma \vdash \phi_1 \quad \Gamma \vdash \phi_2}{\Gamma \vdash \phi_1 \wedge \phi_2} \wedge_I \\
\frac{\Gamma \vdash \phi_i}{\Gamma \vdash \phi_1 \vee \phi_2} \vee_I^i \\
\frac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi \Rightarrow \psi} \Rightarrow_I \\
\frac{\Gamma, \phi \vdash \perp}{\Gamma \vdash \neg \phi} \neg_I \\
\frac{\Gamma \vdash \phi[x := t]}{\Gamma \vdash \exists x. \phi} \exists_I \\
\frac{\Gamma \vdash \phi}{\Gamma \vdash \forall x. \phi} \forall_I \quad (x \notin \text{fv}(\Gamma))
\end{array}
\qquad
\begin{array}{c}
\frac{\Gamma \vdash \perp}{\Gamma \vdash \phi} \perp_E \\
\frac{\Gamma \vdash \phi_1 \wedge \phi_2}{\Gamma \vdash \phi_i} \wedge_E^i \\
\frac{\Gamma \vdash \phi_1 \vee \phi_2 \quad \Gamma, \phi_1 \vdash \psi \quad \Gamma, \phi_2 \vdash \psi}{\Gamma \vdash \psi} \vee_E \\
\frac{\Gamma \vdash \neg \neg \phi}{\Gamma \vdash \phi} \text{RAA} \\
\frac{\Gamma \vdash \exists x. \phi \quad \Gamma, \phi \vdash \psi}{\Gamma \vdash \psi} \exists_E \quad (x \notin \text{fv}(\Gamma, \psi))
\end{array}
\qquad
\begin{array}{c}
\overline{\Gamma \vdash \top} \top_I \\
\frac{\Gamma \vdash \phi \Rightarrow \psi \quad \Gamma \vdash \phi}{\Gamma \vdash \psi} \Rightarrow_E \\
\frac{\Gamma \vdash \neg \phi \quad \Gamma \vdash \phi}{\Gamma \vdash \perp} \neg_E \\
\frac{\Gamma \vdash \forall x. \phi}{\Gamma \vdash \phi[x := t]} \forall_E
\end{array}$$

Figure 1: Dédution naturelle pour la logique classique du premier ordre