Power Optimal Design of Multicast Light-Trees in WDM Networks

Fen Zhou, Miklós Molnár, Bernard Cousin, and Gwendal Simon

***Abstract*—Given a multicast session in Wavelength Division Multiplexing (WDM) networks, we try to find the multicast light-trees with the minimum power budget while taking into account the optical power loss as light splitting loss, node tapping loss and light attenuation loss. Although light splitting causes non-linear power relationship, we succeed to formulate this problem as a Mixed-Integer Linear Programming (MILP) by developing a set of equivalent linear equations to replace the non-linear ones. The distribution of power loss is analyzed by simulations, which suggests to bound the combined power loss ratio of the node tapping loss and the light attenuation in each source-destination path, and make power-symmetric light-trees by properly using light-splitters in order to minimize the overall power loss.**

***Index Terms*—All-Optical multicast routing, light-tree, power optimal, mixed-integer linear programming (MILP).**

I. INTRODUCTION

POWER-aware multicast routing is studied in WDM networks when only a fraction of nodes are equipped with light splitters, *i.e.* the so-called sparse splitting [1]. We refer to the nodes with light splitters as *Multicast-Capable Optical Cross-Connect* switches (MC-OXC). The remaining nodes are *Multicast-Incapable OXC* (MI-OXC). Both of them are also equipped with *Tap-and-Continue* (TaC) devices to support multicasting [1]. In multicast communications, a light signal generally suffers from the following power losses:

(1) **Splitting loss**: an MC-OXC is able to split an incoming light signal into several identical outgoing ones. When a light signal is split into f copies, the power for each outgoing link is reduced to $\frac{1}{f}$ of the original one [2].

(2) **Signal attenuation loss**: a power loss is proportional with the length of the optical fiber l (km). Near 1550 nm, the standard fiber attenuation factor is approximately $10^{\frac{\beta \cdot l}{10}}$ with $\beta = 0.2$ dB/km [2].

(3) **Taping loss for local usage**: when the light signal traverses a TaC device, a portion of power is consumed for measurement and management in the network control plane. The tapping loss ratio is about $10^{\frac{\gamma}{10}}$ with $\gamma = 1$ dB [3].

For a successful transmission over an optical fiber, one must ensure that the light power arriving at a sink node is above the sensitive threshold of receivers. Before establishing a multicast session, one should know at least the minimum power budget required by the source node in order to guarantee the quality of signal at each sink node. Moreover, all-optical multicasting should avoid unnecessary power loss and consume as little energy as possible for reducing the number of costly optical amplifiers in WDM networks. Due to severe optical power losses, the design of *power-optimal routing strategies* is challenging for all-optical multicasting in WDM networks.

A light-tree or a light-forest (a set of light-trees rooted at the same source) is generally used for multicasting in WDM networks. The centralized algorithm in [4] only considers the splitting loss and tries to achieve small power loss by reconstructing the light-forest computed by the Member-Only algorithm [1]. In [2], only the attenuation loss and the splitting loss are taken into account. Balanced light-tree algorithms are proposed to satisfy the source-destination power loss constraint and the inter-destination power variation constraint. These works ignore the node tapping loss and only present heuristics, which have practical interest, but do not provide any theoretical proof of optimality. Paper [5] proposes an integer linear programming (ILP) model to search the load-balanced light-trees with minimal cost so that the number of destinations is bounded in each light-tree. The problem of the power-aware routing and wavelength assignment for a set of concurrent multicast sessions is modeled by an MILP in [3], which tries to minimize the blocking probability. However, these works either focus on the cost or on the connection provision while neglecting the total power consumption of multicast sessions.

In this paper, we try to find a light-forest with the minimum power budget for a given multicast session while taking into account both the three aforementioned power losses and the wavelength channel cost. As an MC-OXC divides the power level of a light signal by its fanout, non-linear power relationship is incurred. We develop a linearization technique to transform the non-linear equations into linear ones. Therefore, we make possible the formulation of an MILP model that formulates the power optimal design of light-trees. The distribution of power loss in multicast communications is also studied by simulations in two sample WDM networks.

The rest of the paper is organized as follows. An MILP formulation is developed to search the power-optimal multicast light-trees in Section II. Then, numerical cases are studied in Section III. Finally, the paper is concluded in Section IV.

II. POWER OPTIMAL ALL-OPTICAL MULTICAST ROUTING

A. System Model

Less than half of nodes in usual sparse splitting WDM networks are MC-OXC [1]. We consider one multicast session $ms(s, D)$, which requires computing a light-forest to multicast the light signal from the source s to a set of destination nodes D. Therefore the studied problem is to search a set of light-trees with the minimum power budget while complying the following three constraints in the WDM layer:

(i) **Wavelength continuity constraint**: the same wavelength should be retained over all the links in each light-tree in the absence of wavelength converters.

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(ii) **Distinct wavelength constraint**: two light-trees should be assigned with distinct wavelengths if they are not link disjoint. (iii) **Quality of signal constraint**: the power level arriving at each destination should be kept above a certain threshold p_{sen} to guarantee the successful recovery of multicast messages.

B. MILP Formulation

The studied WDM network is modeled as a symmetric digraph $G = (V, E, W)$. For a vertex $v \in V$, the set of neighbors of v in G is written as $N_G(v)$ and its nodal degree is denoted by d_v. V_{MC} is the set of MC-OXC while V_{MI} is the set of MI-OXC, and $V_s = V \setminus \{s\}$. An arc $e \in E$ from node v to u is noted as e_{vu} and its associated cost is written as c_e. We use $\delta^+(v)$ and $\delta^-(v)$ to denote the set of arcs leaving and entering the node v in G respectively. All optical fibers support the same set of wavelengths W. For each wavelength $\lambda \in W$, $x^\lambda \in \mathbb{R}^E$ is an edge-indexed vector defined as:

$$\forall e \in E, x_e^\lambda = \begin{cases} 1 & \text{if } e \text{ is used by a light-tree on } \lambda \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

For any given vector $x^\lambda \in \{0,1\}^E$, the subgraph of G whose set of edges is $E^\lambda(x) = \{e \in E : x_e^\lambda = 1\}$, and whose set of vertices is $V^\lambda(x) = \{v \in V : x^\lambda(\delta^-(v)) = 1\} \cup \{s\}$ will be denoted by $T^\lambda(x) = (V^\lambda(x), E^\lambda(x))$. For each $\lambda \in W$, let $h^\lambda \in \mathbb{N}^V$, and $p^\lambda, \bar{p}^\lambda \in \mathbb{R}^V$ be three vertex-indexed vectors. For each $v \in V$ and $\lambda \in W$, h_v^λ denotes the number of hop counts from v to the source s in the light-tree using wavelength λ if v is covered inside, where $h_v^\lambda \in [1, n-1]$ for $v \in V_s$ and $h_s^\lambda = 0$. The light power (in **mW** unit) arriving at a node v is noted as p_v^λ, while $\bar{p}_v^\lambda$ (in **mW** unit) is the corresponding light power after splitting if applicable ($p_v^\lambda = \bar{p}_v^\lambda$ for any MI-OXC).

The objective of our design problem is to find a set of multicast light-trees for $ms(s, D)$, so that the optical power loss produced is minimized. Besides, among the multicast light-forests with the optimal power loss, it is also preferred to choose the one consuming the least cost. Hence, the objective function of our MILP formulation can be expressed as:

$$\text{Minimize: } \alpha \cdot \sum_{\lambda \in W} p_s^\lambda + \sum_{\lambda \in W} \sum_{e \in E} c_e \cdot x_e^\lambda \quad (2)$$

The objective function is subject to constraints (3)-(19), which result in the light-forest $\bigcup_{\lambda \in W} T^\lambda(x)$.

$$x^\lambda(\delta^-(s)) = 0 \qquad \forall \lambda \in W \quad (3)$$

$$x^\lambda(\delta^-(v)) \leq 1 \qquad \forall v \in V_s, \lambda \in W \quad (4)$$

$$x^\lambda(\delta^+(v)) \leq x^\lambda(\delta^-(v)) \qquad \forall v \in V_{MI}, \lambda \in W \quad (5)$$

$$x^\lambda(\delta^+(v)) \leq d_v \cdot x^\lambda(\delta^-(v)) \qquad \forall v \in V_{MC}, \lambda \in W \quad (6)$$

$$x^\lambda(\delta^+(v)) \geq x(\delta^-(v)) \qquad \forall v \in V_s \setminus D, \lambda \in W \quad (7)$$

$$\sum_{\lambda \in W} x^\lambda(\delta^-(v)) \geq 1 \qquad \forall v \in D \quad (8)$$

$$h_v^\lambda + 1 - h_u^\lambda \leq n \cdot (1 - x^\lambda(e_{vu})) \qquad \forall v \in V, u \in N_G(v), \lambda \in W \quad (9)$$

Inequality (3) makes each light-tree rooted at the source node s and inequality(4) assures that each node has at most one input link. Inequalities (5) and (6) impose the splitting constraint on each node. Inequalities (7) and (8) guarantee that non-destination node could not be a leaf in a light-tree and each destination is spanned in at least one light-tree. Constraint (9) helps to avoid the loops in each $T^\lambda(x)$. Thus constraints (3)-(9) guarantee the tree structure, while constraints (10)-(19) are used for computing nodal optical power, where M is a big enough number. Inequality (12) makes sure each node in light-trees has a power level above the sensitivity threshold of light receivers p_{sen}. Inequality (13) shows that the splitting loss is not applicable for any MI-OXC.

$$p_s^\lambda \leq M \cdot x^\lambda(\delta^+(v)) \qquad \forall \lambda \in W \quad (10)$$

$$p_v^\lambda \leq M \cdot x^\lambda(\delta^-(v)) \qquad \forall v \in V_s, \lambda \in W \quad (11)$$

$$p_v^\lambda \geq p_{sen} \cdot x^\lambda(\delta^-(v)) \qquad \forall v \in V_s, \lambda \in W \quad (12)$$

$$p_v^\lambda = \bar{p}_v^\lambda \qquad \forall v \in V_{MI}, \lambda \in W \quad (13)$$

For an MC-OXC with f-fanout in a light-tree, its power level after splitting is expressed by equation (14), which is obviously

$$\bar{p}_v^\lambda = \frac{p_v^\lambda}{f} = \frac{p_v^\lambda}{x^\lambda(\delta^+(v))}, \quad \forall v \in V_{MC}, \lambda \in W \quad (14)$$

non-linear and thus stumps the MILP model. To solve this, two sequences of binary variables $a_i^{\lambda v}$ and $b_i^{\lambda v}$ are introduced to determine the splitting loss for each MC-OXC on each wavelength. Let M be a big number, and we impose the same inequalities from (15) to (17) for $i = 1, 2, ..., d_v$ respectively:

$$a_i^{\lambda v} - 1 \leq \frac{x^\lambda(\delta^+(v)) - i + \frac{1}{2}}{d_v + 1} \leq a_i^{\lambda v} \quad \forall v \in V_{MC}, \lambda \in W \quad (15)$$

$$b_i^{\lambda v} - 1 \leq \frac{i - x^\lambda(\delta^+(v)) + \frac{1}{2}}{d_v + 1} \leq b_i^{\lambda v} \quad \forall v \in V_{MC}, \lambda \in W \quad (16)$$

$$p_v^\lambda - i \cdot \bar{p}_v^\lambda \geq (a_i^{\lambda v} + b_i^{\lambda v} - 2) \cdot M \qquad \forall v \in V_{MC}, \lambda \in W \quad (17)$$

According to the inequalities (15) and (16), it is derived that

$$\begin{cases} a_i^{\lambda v} = 1 \text{ and } b_i^{\lambda v} = 0, & \text{if } i < x^\lambda(\delta^+(v)) \\ a_i^{\lambda v} = 1 \text{ and } b_i^{\lambda v} = 1, & \text{if } i = x^\lambda(\delta^+(v)) \\ a_i^{\lambda v} = 0 \text{ and } b_i^{\lambda v} = 1, & \text{if } i > x^\lambda(\delta^+(v)) \end{cases} \quad (18)$$

Thus, for any MC-OXC v of a light-tree on wavelength λ, only when its fanout $x^\lambda(\delta^+(v)) = i$ we are able to obtain $p_v^\lambda - i \cdot \bar{p}_v^\lambda \geq 0$ from inequality (17). Thus non-linear equation (14) can be equivalently replaced by linear ones (15)-(17).

$$\bar{p}_v^\lambda - 10^{\frac{\beta \cdot l_{vu} + \gamma}{10}} \cdot p_u^\lambda \geq (x^\lambda(e_{vu}) - 1) \cdot M \quad \forall v \in V, \lambda \in W, u \in N_G(v) \quad (19)$$

The final inequality (19) reflects the power relation between tow adjacent nodes in a light-tree by taking into account the tapping loss and the attenuation loss.

III. Simulation and Case Studies

The proposed MILP is implemented by using C++ with Cplex package in a weighted 6-node mesh network (Figure 3.3 of [6]) and the 14-node NSF network [5]. The system parameters are listed in Table I. Given a group size $|D|$, 10 random multicast sessions are generated and the membership of each

TABLE I

Parameters	β	γ	p_{sen}	M
Values	0.2 dB/km	1 dB	-9 dBm=0.1259 mW	1000

TABLE II

6-node mesh network: nodes 2 and 3 are MC-OXC, T+A= $h + 0.2 \cdot l$								
	Power Optimal (PO)				Cost Optimal (CO)			
$\|D\|$	P (mW)	Cost	T+A	SP	P (mW)	Cost	T+A	SP
2	0.6495	23.5	6.52	0	0.6904	23.5	6.74	0
3	0.927	32.8	7.39	0	0.9654	32.5	7.58	0
4	1.1913	42.4	7.72	0	1.5622	34.5	9.32	3
5	1.3793	44.9	8.78	0	2.0048	41.0	10.18	2
14-node NSF network: nodes 4, 6, 8, 9 are MC-OXC, T+A= $h + 0.2 \cdot l$								
	Power Optimal (PO)				Cost Optimal (CO)			
$\|D\|$	P (mW)	Cost	T+A	SP	P (mW)	Cost	T+A	SP
2	0.9291	27	7.24	0	0.9538	22.8	7.24	2
4	1.5406	44.8	8.08	0	2.3899	37.6	9	2
6	1.7154	53.6	7.92	0	3.7608	38.6	13.24	4
8	2.3556	62.5	9.2	0	4.9370	47	11.6	13

TABLE III

Average computation time in second (s)								
	6-node mesh network				14-node NSF network			
$\|D\|$	2	3	4	5	2	4	6	8
Time (PO)	0.5	1.7	4.4	89.7	17.6	594	3927	16177
Time (CO)	0.1	0.2	0.2	0.3	0.1	0.6	1.1	1.6

session follows a uniform distribution over the topology. We set $\alpha = 10^6$ in equation (2) to compute the power-optimal (PO) light-trees while we set $\alpha = 10^{-6}$ to search the cost-optimal (CO) light-trees. In Tables II and III, we compared the following criteria between PO and CO light-forest, where each value is the average of 10 instances except **SP**.

- **P** is the total power budget in **mW** unit for a light-forest
- **Cost** is the total cost of a light-forest
- **T+A** represents the maximum non-splitting power loss ratio of destinations in D, i.e. $max_{d \in D}(h \cdot \gamma + \beta \cdot l)$, where h is the number of hop counts in the path from s to a destination d and l is the optical fiber length of the path. The first part corresponds to the total node tapping loss ratio in the path from s to d while the latter one signifies the overall attenuation loss ratio in that path.
- **SP** is the number of splitters used in 10 sessions.
- **Time** is the computation time of a light-forest

Based on the numerical results, it is observed that

(1) As multicast group size grows, more intermediate nodes may be traversed (i.e. $h \uparrow$) and more fiber links should be used (i.e. $l \uparrow$) in a source-destination path. Thus, the value of **T+A** of a light-forest increases, which results in bigger tapping loss and attenuation loss. In addition, a light-forest may contain more light-trees and light splitters may be required to span all destinations. Thus the overall power budget increases also.

(2) The **T+A** value in the CO light-forest is always bigger than that in the PO one, which thus makes the power of a CO light-forest bigger. In a light-tree without light splitters, the source power should be above $10^{\frac{\mathbf{T+A}}{10}} \cdot p_{sen}$. Thus, the augmentation of **T+A** makes the source power increase exponentially. In a CO light-forest, some light-trees consume huge power while the others use small power. Differently, the power difference between light-trees of a PO light-forest is significantly smaller.

(3) Compared to the PO light-forest, splitting loss leads a more important role in the power loss of a CO one. Among 10 multicast sessions, up to 3 light splitters are used in 6-node network and up to 13 light splitters are employed in NSF networks by the CO light-forests, which causes unnecessary power waste. To achieve the minimum cost, light splitters make it easier for a destination to join a light-tree using a path with smaller cost. However, to obtain the minimal power loss, it is required that the branches of a light splitter should be as power-symmetric as possible. For instance, a light splitter has two downstream branches in a light-tee, where the **T+A** of one branch is 5 (short branch) while the **T+A** of the other branch is 10 (long branch). To ensure the power of the leaf node of the long branch be above p_{sen}, the power of the leaf node of the short branch will be 3.16 times higher than p_{sen}. Consequently, unnecessary power will be wasted in the short branch. As a result, a PO light-forest should avoid light splitters in order not to produce power-asymmetric light-trees.

(4) It is time consuming to compute PO light-forest by using MILP even in small WDM networks. Time efficient heuristic algorithm will be practical to compute online PO light-forest.

IV. Conclusion

We addressed the power-optimal multicast routing problem in WDM networks. We transformed the non-linear equation caused by light splitters into a set of exactly equivalent linear ones and formulated the studied problem as an MILP. According to the simulation results, two approaches may be helpful for developing heuristic algorithms to cut down the power budget of a light-forest. First, the maximum combined ratio of node tapping loss and attenuation loss (i.e. **T+A**) should be bounded. This ratio impacts both the number of intermediate nodes and the total fiber length of the path from the source to each destination. On one hand, it helps to forbid a long line light-tree with big **T+A**. On the other hand, it helps to restrict the power differences among distinct branches of a light splitter and that among different light-trees. Thus, the tapping loss induced by intermediate nodes and the light attenuation loss could be diminished or limited in each source-destination path. In addition, light splitters most of the time should be avoided in order to construct power-balanced light-trees. Otherwise, unnecessary power loss will be produced if two branches of an MC-OXC are not power-symmetric.

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