History-Preserving Bisimilarity for Higher-Dimensional Automata via Open Maps

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(cf. MFPS)
History-preserving bisimilarity is, “morally”, a relation on paths.

But we can show that for higher-dimensional automata, it is equivalent to a relation on states and (higher-dimensional) transitions.

This adds weight to the claim that HDA are a natural and useful (and beautiful!) formalism for concurrency.
Higher-Dimensional Automata

- Formalism for concurrency
- Generalizes Petri nets and most others
- Invented by V. Pratt and R. van Glabbeek
- Like automata, but with higher-dimensional objects which signify independence / concurrency
- States, transitions, squares, cubes, etc.
Simple Bisimilarity

Def.: HDA \( X \), \( Y \) *om-bisimilar* if exists sub-HDA \( R \subseteq X \times Y \) s.t. for all reachable \( x \in X \), \( y \in Y \) with \( (x, y) \in R \):

- for all \( x = \delta_k^0 x' \), there is \( y = \delta_k^0 y' \) with \( (x', y') \in R \)
- for all \( y = \delta_k^0 y' \), there is \( x = \delta_k^0 x' \) with \( (x', y') \in R \)

- Easy generalization of standard bisimilarity for transition systems
- Comes from a natural notion of open maps
- But how does it relate to standard notions of concurrent bisimilarity?
History-Preserving Bisimilarity

- Hp-bisimilarity: relation on computations which respects extensions and independence
- For HDA:
  - computations $= \text{cube paths}$
- independence $= \text{homotopy}$ (simple combinatorial notion)
Main Result

- Def.: HDA $X$, $Y$ hp-bisimilar if exists relation $R$ between cube paths in $X$ and cube paths in $Y$ s.t. for all $(\rho, \sigma) \in R$:
  - for all $\rho \rightsquigarrow \rho'$, there is $\sigma \rightsquigarrow \sigma'$ with $(\rho', \sigma') \in R$,
  - for all $\sigma \rightsquigarrow \sigma'$, there is $\rho \rightsquigarrow \rho'$ with $(\rho', \sigma') \in R$,
  - for all $\rho \sim \rho'$, there is $\sigma \sim \sigma'$ with $(\rho', \sigma') \in R$,
  - for all $\sigma \sim \sigma'$, there is $\rho \sim \rho'$ with $(\rho', \sigma') \in R$,

- Theorem: HDA are hp-bisimilar iff they are om-bisimilar.

- Proof via unfoldings of HDA into higher-dimensional trees (universal covering)
Next Steps

- Coalgebraic characterization?
- Relation to Staton-Winskel’s (LICS 2010) unfolding of HDA into presheaves over symmetric event structures?
- Hereditary hp-bisimilarity?