Control Problems for DES are Model-Checking Problems

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Plan

- Discrete Events Systems (DES)
- Control Problems for DES
- An overview of the Approach

  Control Problems are Model-Checking problems
- Control Objectives as Temporal Formulas
- Examples of Specifications
- The Synthesis Procedure Principles
- Remarks on the Results

Processes for Systems

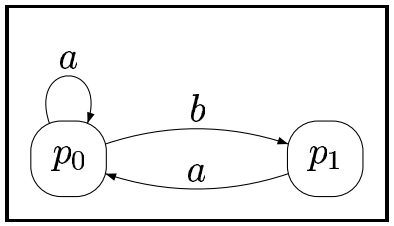
$$\mathcal{S} = \langle S, s^0, t, L \rangle \text{ on } \Gamma \subseteq AP$$

where

- $\Sigma = \{a, b, \dots\}$ events
- $AP = \{p, p', c, c', \dots\}$ (atomic) propositions
- S, $s^0 \in S$ set of states, initial state,
- $t : S \times \Sigma \to S$ transition (partial) function,
- $L : S \to 2^{\Gamma}$ labeling of states by propositions

An Example of Process

$\mathcal{S} = \langle S, s^0, t, L \rangle$ on $\{p_0, p_1\}$



$\mathcal{S}$

Synchronous Product

- $\mathcal{S}_1 = \langle S_1, s_1^0, t_1, L_1 \rangle$ on Γ_1 and $\mathcal{S}_2 = \langle S_2, s_2^0, t_2, L_2 \rangle$ on Γ_2

  (with $\Gamma_1 \cap \Gamma_2 = \emptyset$)

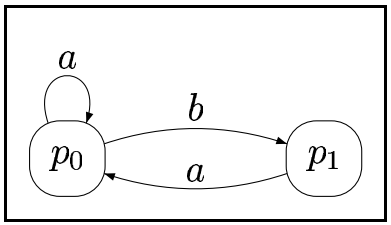
$$\mathcal{S}_1 \times \mathcal{S}_2 = \langle S_1 \times S_2, (s_1^0, s_2^0), t, L \rangle$$

$t((s_1, s_2), a) = (s_1', s_2')$ whenever

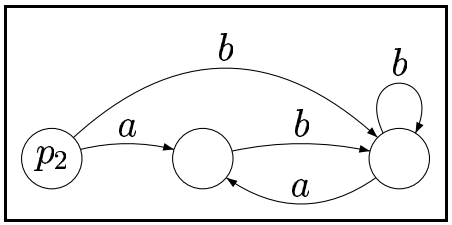
$$\begin{cases} s_1' = t_1(s_1, a), \text{ and} \\ s_2' = t_2(s_2, a) \end{cases}$$

$L(s_1, s_2) = L_1(s_1) \cup L_2(s_2)$

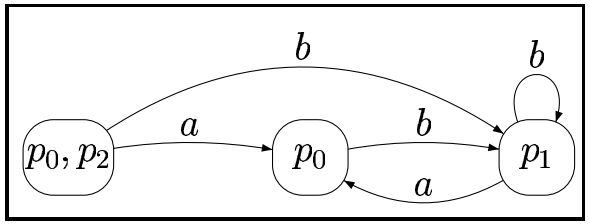
Example of Synchronous Product



$\mathcal{S}_1$



$\mathcal{S}_2$



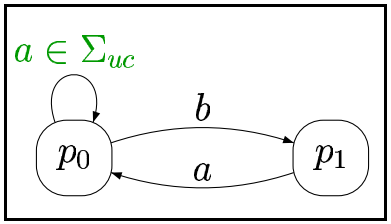
$\mathcal{S}_1 \times \mathcal{S}_2$

Control Problems

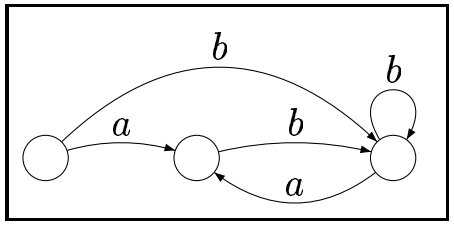
- $\Sigma = \Sigma_{uc} \uplus \Sigma_c$
- Controllers are processes on $\emptyset$
- Admissible Controllers
- $\mathcal{C}$ is an admissible controller of $\mathcal{S}$ for property P whenever

  $\mathcal{C}$ is admissible and $\mathcal{S} \times \mathcal{C}$ satisfies the property P

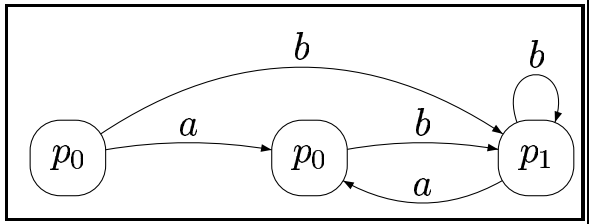
  Also $\mathcal{C}_1$ and $\mathcal{C}_2$...



$\mathcal{S}$

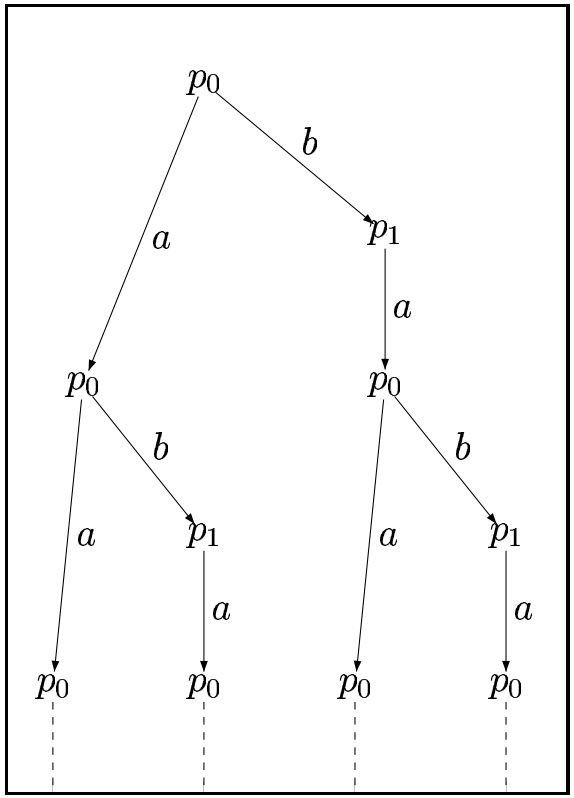


$\mathcal{C}$

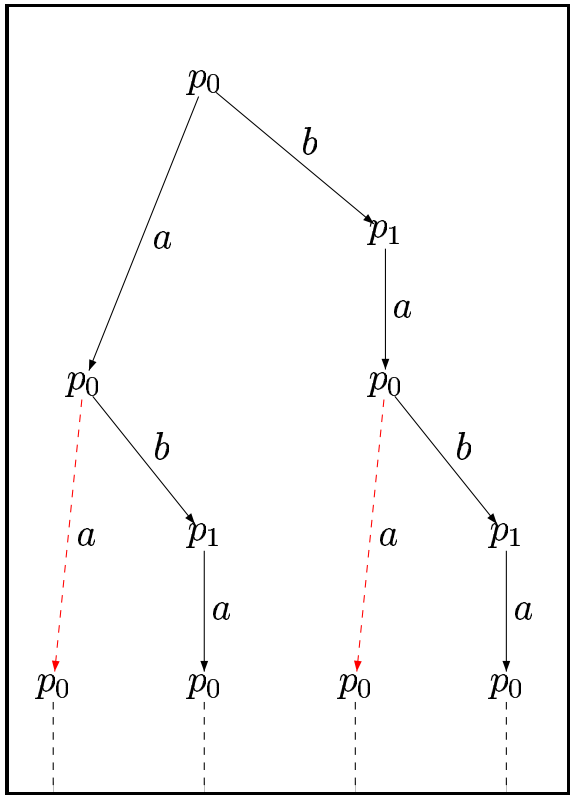


$\mathcal{S} \times \mathcal{C}$ satisfies P

Pruning

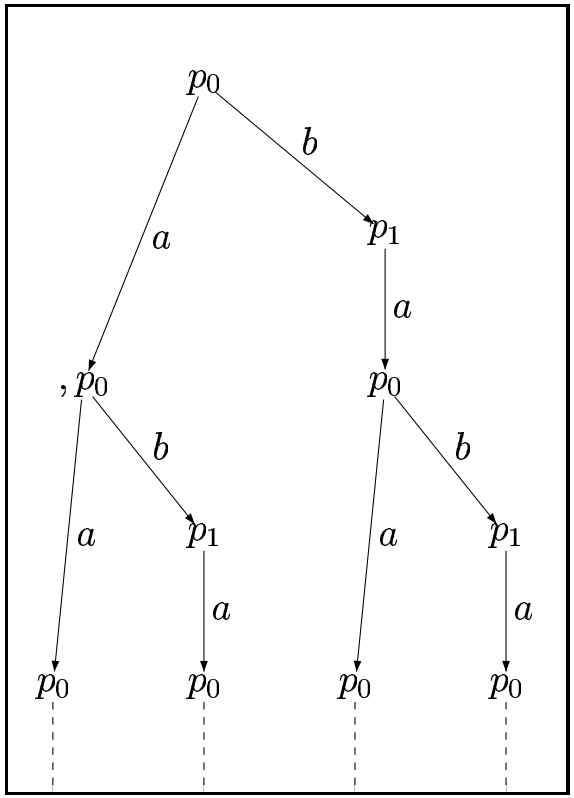


computation tree of $\mathcal{S}$

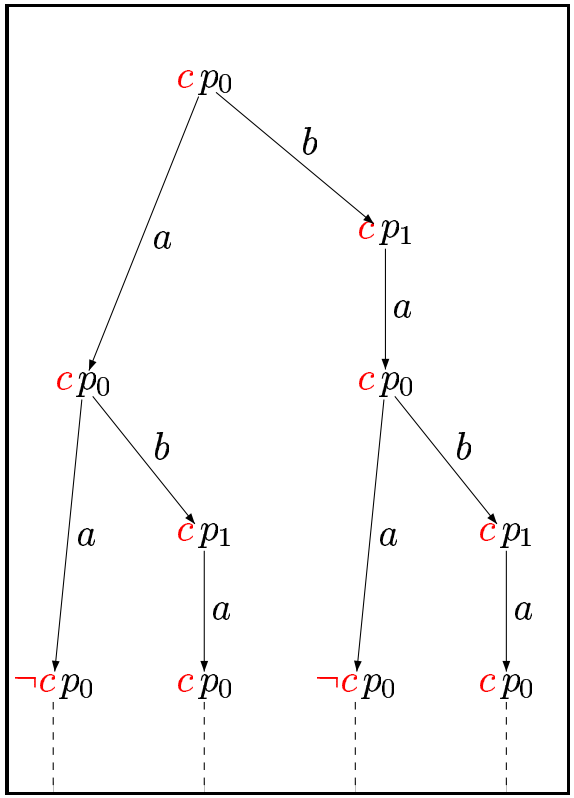


computation tree of $\mathcal{S} \times \mathcal{C}$

c-labeling of $\mathcal{S}$



computation tree of $\mathcal{S}$



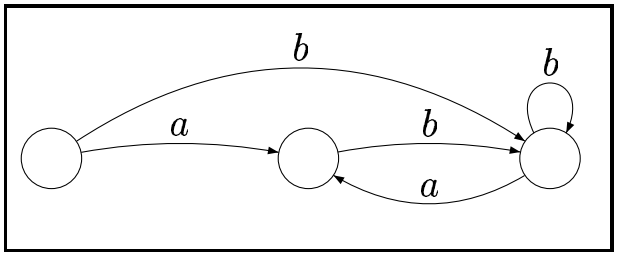
computation tree of $\mathcal{S} \times \mathcal{E}_{\mathcal{C}}$

Labeling Processes are Controllers

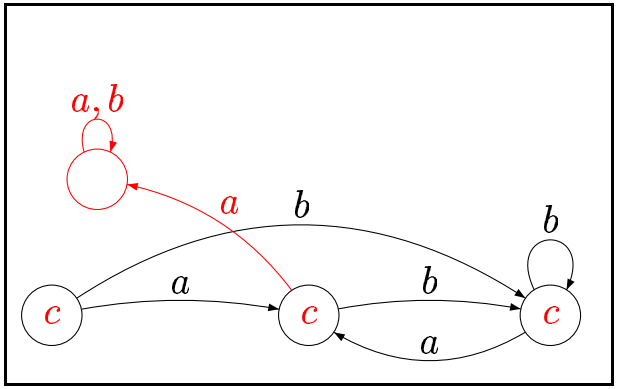
- c-labeling process

$\mathcal{E} = \langle E, e^0, t', L' \rangle$ a process on $\{c\}$

which is complete, i.e. $t'(e, a)$ si aways defined.

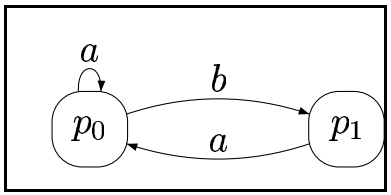


$\mathcal{C}$

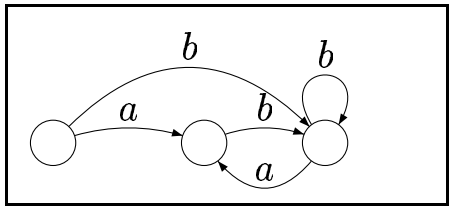


$\mathcal{E}_\mathcal{C}$

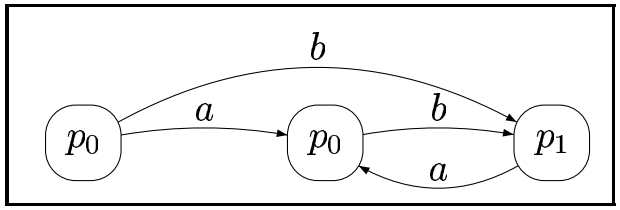
Labeling Processes and Controllers



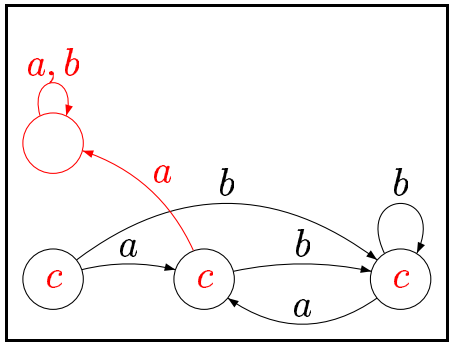
$\mathcal{S}$



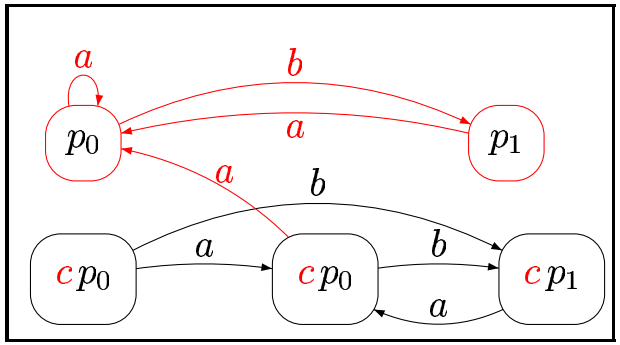
$\mathcal{C}$



$\mathcal{S} \times \mathcal{C}$



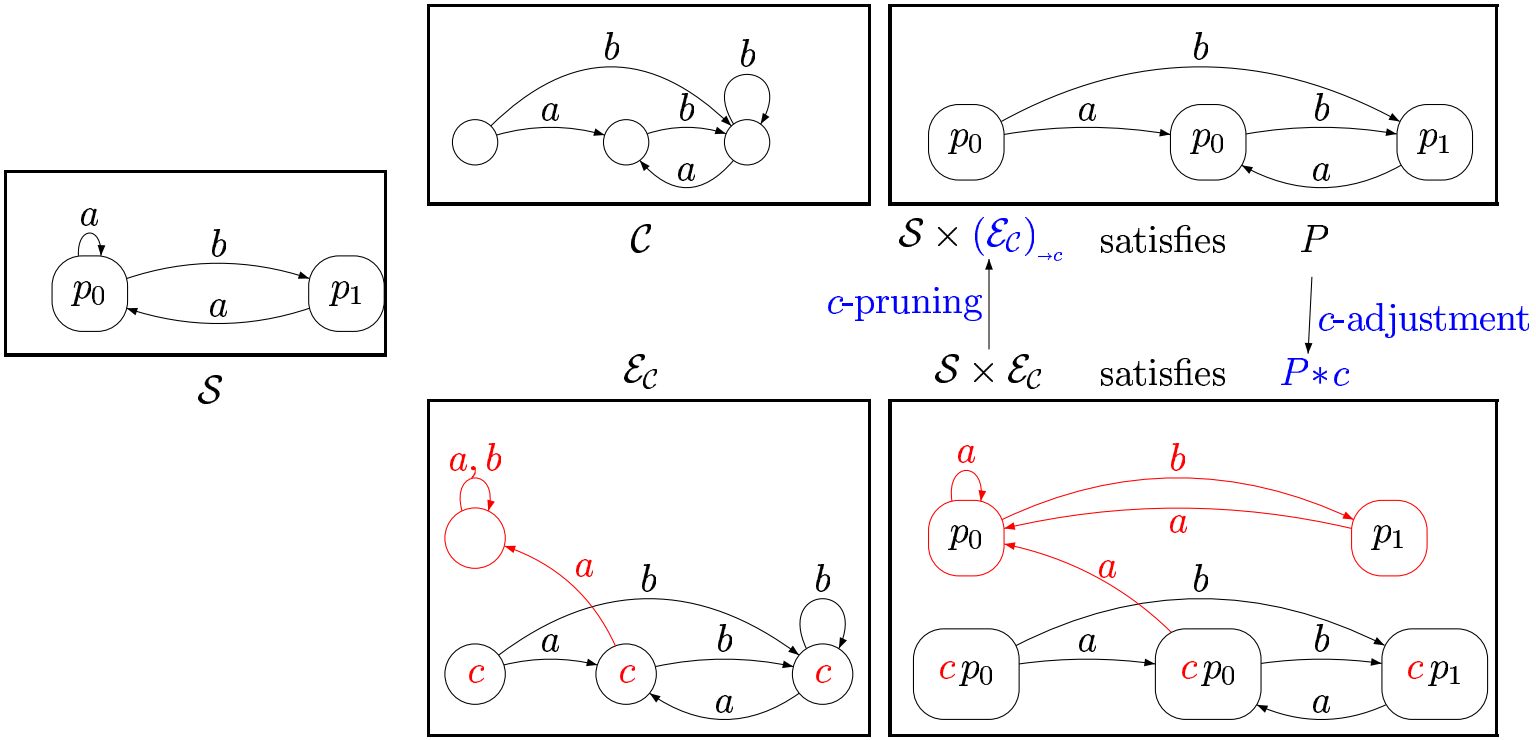
$\mathcal{E}_{\mathcal{C}}$



$\mathcal{S} \times \mathcal{E}_{\mathcal{C}}$

c-Adjustment and c-Pruning

$\mathcal{S} \times \mathcal{E}$ satisfies $P * c$ iff $\mathcal{S} \times \mathcal{E}_{\to c}$ satisfies P



The Property P

- Mu-calculus L_μ

  [Kozen 1983], [Arnold & Niwinski 2001]

- Fix-point operators to build your own modalities

  L_μ subsumes CTL, LTL, CTL*,...

  Safety, Liveness, Fairness, ...

- But you can use your favorite logic HML, CTL ....

The Mu-Calculus L_μ

- Syntax of L_μ $\quad \top \mid p \mid \neg\beta \mid \beta \vee \beta' \mid <a>\beta \mid X \mid \mu X.\beta(X)$

  where $p \in AP$, $a \in \Sigma$, conditions on $\beta(X)$, ...

- Semantics of L_μ $\quad \mathcal{S}, s \models \beta$ for "state s of $\mathcal{S}$ satisfies β"

- Semantics $\quad \mathcal{S}, s \models \top$ always

  $\mathcal{S}, s \models p$ iff s has label p

  $\mathcal{S}, s \models <a>\beta$ iff there exists $s' \in S$, $\begin{cases} t(s,a) = s' \\ \mathcal{S}, s' \models \beta \end{cases}$

  $\mathcal{S}, s \models \mu X.\beta(X)$ is technical ....

$[a]\alpha \stackrel{\text{def}}{=} \neg <a>\neg\alpha$ means "if any a-successeurs then it satisfies α"

$\mu X.\ <a>X \vee m$ means "some a^* trace reaches a marked state"

$\mathbf{AG}\,(\alpha) \stackrel{\text{def}}{=} \neg\mu X.\neg([\,]\neg X \wedge \beta)$ means "property α is invariant"

The Quantified Mu-Calculus $\mathrm{Q}L_\mu$

- Syntaxe of $\mathrm{Q}L_\mu$ $\quad \exists c.\alpha \mid \neg\alpha \mid \alpha \vee \alpha' \mid \beta$

  where $c \in AP$ and $\beta \in L_\mu$

- $\mathcal{S}, s \models \exists c.\alpha$ iff there exists $\mathcal{E} = \langle E, \varepsilon^0, t', L' \rangle$ on $\{c\}$ s.t.

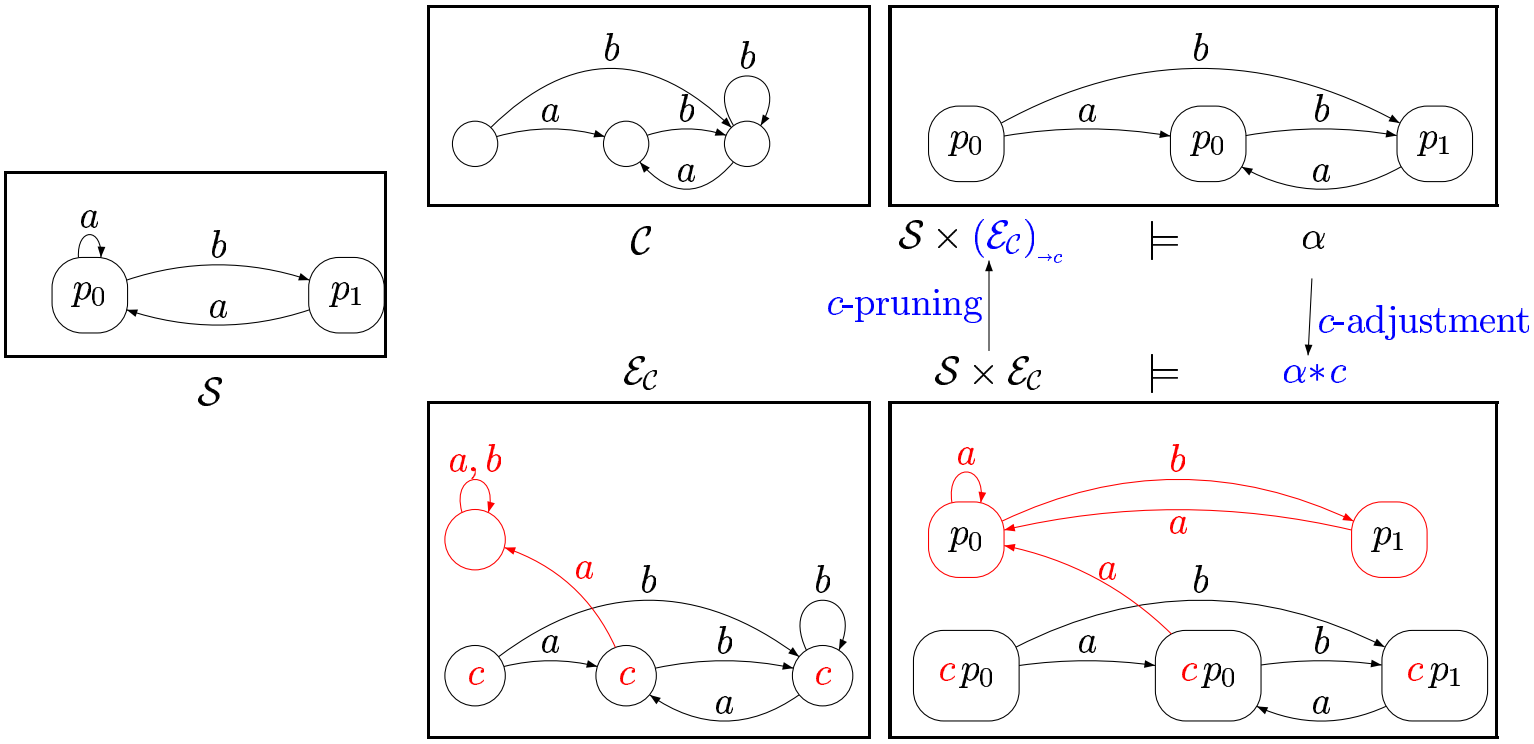
$$\mathcal{S} \times \mathcal{E}, (s, \varepsilon^0) \models \alpha$$

Remember

- $\mathcal{S} \times \mathcal{E}$ is a c-labeling of $\mathcal{S}$

  that is (an unfolding of) $\mathcal{S}$ with the new label c put somehow.
- This labeling denotes a control policy

Remember the Pruning and the Adjustment



c-Adjustment of Mu-calculus formulas, but actually of $\mathrm{Q}L_\mu$-formulas

- $\mathcal{E}_{\to c}$ is c-pruning the of $\mathcal{E}$ keeps states "inside" c and forget c
- the c-adjustment of α adjusts the formula to talk only about trajectories "inside" c

  $\alpha * c$ essentially relies on $(<a>\ldots) * c \stackrel{\text{def}}{=} <a>(c \wedge \ldots)$

  Theorem (c-Pruning and c-Adjustment)

  $$\mathcal{S} \times \mathcal{E}_{\to c} \models \alpha \text{ iff } \mathcal{S} \times \mathcal{E} \models \alpha * c$$

- RMK: CTL logic $(\mathbf{EX}\ldots) * c \stackrel{\text{def}}{=} \mathbf{EX}(c \wedge \ldots)$, $(\mathbf{AX}\ldots) * c \stackrel{\text{def}}{=} \mathbf{AX}(c \Rightarrow \ldots)$, $(\mathbf{E}\ldots\mathbf{U}\ldots) * c \stackrel{\text{def}}{=} \mathbf{E}(c \wedge \ldots\mathbf{U}\, c \wedge \ldots)$, $(\mathbf{A}\ldots\mathbf{U}\ldots) * c \stackrel{\text{def}}{=} \mathbf{A}(c \Rightarrow \ldots\mathbf{U}\, c \Rightarrow \ldots)$

The Theorem for Basic Controllers Specification

Write $\texttt{Admissible}(c) \in L_\mu$ for $\mathbf{AG}\,(\bigwedge_{u \in \Sigma_{uc}} [u]c)$

For any $\alpha \in \mathrm{Q}L_\mu$,

there exists an admissible controller $\mathcal{C}$ of $\mathcal{S}$ for α

if and only if

$$\mathcal{S} \models \exists c.\ \texttt{Admissible}(c) \wedge \alpha * c$$

Proof sketch for $\Leftarrow$)

There exists an admissible controller $\mathcal{C}$ of $\mathcal{S}$ for α

if and only if

$$\mathcal{S} \models \exists c.\ \texttt{Admissible}(c) \wedge \alpha * c$$

1. $\mathcal{S} \times \mathcal{E} \models \texttt{Admissible}(c) \ \Leftrightarrow\ \mathcal{E}_{\to c}$ is admissible.
2. $\mathcal{S} \times \mathcal{E} \models \alpha * c \ \Leftrightarrow\ \mathcal{S} \times \mathcal{E}_{\to c} \models \alpha$

Other Examples of Specifications

Use the pattern

$$\exists c.\ \boxed{\cdots} * c \ \wedge \boxed{\cdots}$$

- Admissible we've seen $\texttt{Admissible}(c)$

  $\mathbf{AG}\,(\bigwedge_{a\in\Sigma} UnCont(a) \ \Rightarrow\ [a]c)$

- Non-Blocking

  $\mathbf{AG}\,(\mathbf{EF}\,(m) * c)$ where $\mathbf{EF}\,m$ means "$Reachable(m)$",

  $\mathbf{AG}\,(\bigvee_{a\in\Sigma} <a>\top) * c$ for liveness

- Open Systems $\boxed{\cdots}$ is $\texttt{Admissible}(c)$

  $\boxed{\cdots}$ is $\forall e.\mathbf{AG}\,(\bigwedge_{\bar{u}\in\Sigma_c}[\bar{u}]e) \ \Rightarrow\ (\alpha * e) * c$

  where $\forall e$ is $\neg\exists e\neg$

My Favorite : the Theorem for Maximally Permissive Controllers

For any $\alpha \in \mathrm{Q}L_\mu$,

there exists a controller of $\mathcal{S}$ for α which is maximally permissive

iff

$$\mathcal{S} \models \exists c.\ \underbrace{\mathtt{Admissible}(c) \wedge \alpha * c}_{\mathtt{Solution}(c,\alpha)}\ \wedge\ \forall c'.\ [\, c \sqsubset c' \Rightarrow \neg \mathtt{Solution}(c', \alpha)\,]$$

- $c \sqsubseteq c'$ means "if a state is labeled by c, it is also labeled by c'" ;
  it is L_μ definable

Proof sketch

- $\mathcal{C}$ is more permissive than $\mathcal{C}'$ whenever $\mathcal{S} \times \mathcal{C}' \preceq \mathcal{S} \times \mathcal{C}$

  ($\preceq$ means there exists a simulation)

- $c \sqsubseteq c' \stackrel{\text{def}}{=} ([\,]\mathbf{AG}\,(c'))*c$ and $c \sqsubset c' \stackrel{\text{def}}{=} (c \sqsubseteq c') \wedge \neg(c' \sqsubseteq c)$

- In fact,

$$\mathcal{S} \times \mathcal{E} \times \mathcal{E}' \models c \sqsubseteq c' \;\Leftrightarrow\; \mathcal{S} \times \mathcal{E}_{\to c} \preceq \mathcal{S} \times \mathcal{E}'_{\to c'}$$

Model-Checking and Synthesis

We want to model-check $\boxed{\mathcal{S} \models \exists c.\beta}$ for $\beta \in L_\mu$

- Mu-calculus formulas are equivalent to (parity) tree automata

  [Emerson & Jutla 1991] $\quad \beta \rightsquigarrow \mathcal{A}_\beta$

  $\mathcal{S} \models \beta$ iff $\mathcal{S} \in \mathcal{L}(\mathcal{A}_\beta)$

  iff $\exists$ a winning strategy in the two-players game $\mathcal{G}(\mathcal{S}, \mathcal{A}_\beta)$

- [RP03] $\mathrm{Q}L_\mu$ formulae also have their automata semantics.

  $\mathcal{A}_{\exists c.\alpha}$ is the c-projection of $\mathcal{A}_\alpha$ [Rabin69]

Model-Checking and Synthesis for a formula $\exists c.\beta$

- 1. build the parity tree automaton $\mathcal{A}_{\exists c.\beta}$;
- 2. compute the parity game $G(\mathcal{A}, \mathcal{S})$;
- 3. find a winning memoryless strategy (if any) (determined by [EJ91])
- 4. it tells how to c-label the comp. tree of $\mathcal{S}$ in a regular manner
- 5. take the finite state corresponsding $\mathcal{E}$ to compute $\mathcal{C} = \mathcal{E}_{\rightarrow c}$

Complexity $O(|\mathcal{S}|^{O(|\beta|)}2^{O(|\beta|)})$

Polynomial in the size of $\mathcal{S}$

A few Remarks

- Max Perm Controllers difficult problem in the full temporal logic setting [Thistle94]

  Remember $\exists c.\texttt{Solution}(c,\beta) \ \wedge \forall c'.[\, c \sqsubset c' \Rightarrow \neg \texttt{Solution}(c',\beta)\,]$

  Independent of the plant

  Complexity of MPC $O(|\mathcal{S}|^{2^{|\beta|}} 2^{2^{|\beta|}})$

  Still Polynomial in the size of $\mathcal{S}$

- Decentralized Controllers

  $\exists c_1.\exists c_2.\beta * (c_1 \wedge c_2) \wedge \beta'(c_1,c_2)$

  Other Achitecture ? $(c_1 \vee c_2)$, $Bool(c_1,\ldots,c_n)$

- We know how to deal with partial observation