Specifying and Synthesizing Open Systems and their Controllers

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Plan

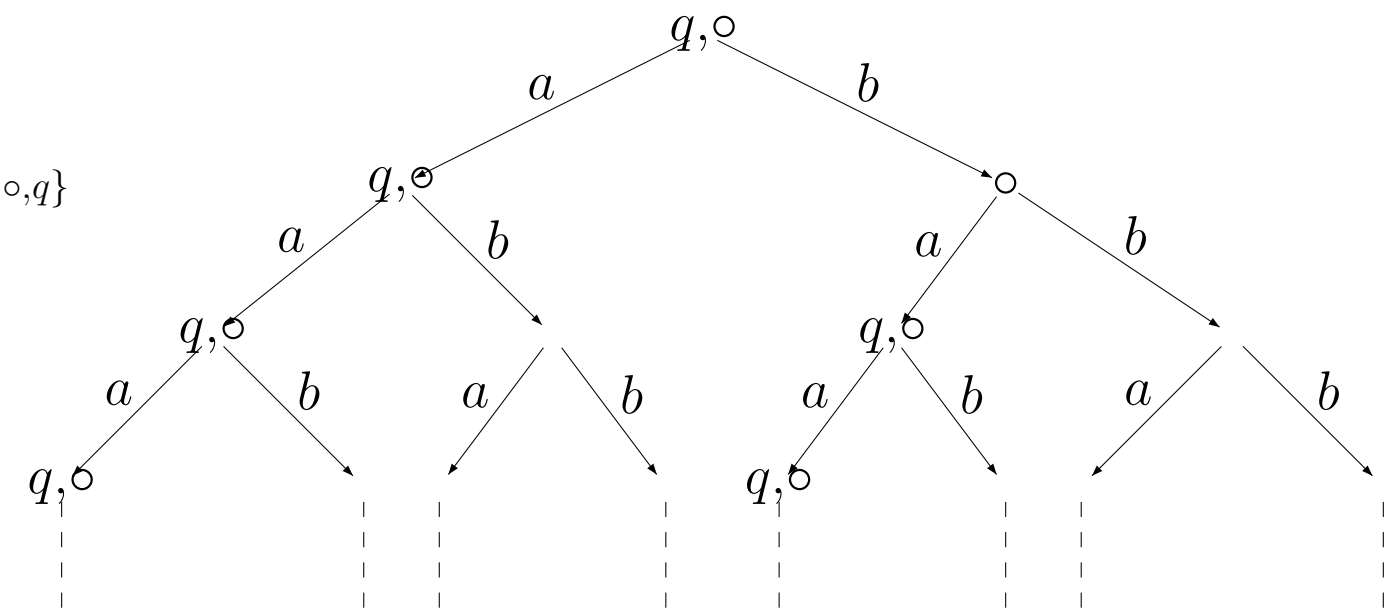
- The Framework
- Model-Checking Closed Systems
- Satisfiability
- Synthesis of Decisions for φ over Λ
- Expressing Qualitative Properties Decisions
- Expressing Constraints on Decisions
- Perspectives
- Partial Observation

The Framework

- A fixed set $\Sigma = \{a, b, c, \ldots\}$
- The full tree T
- A labeling $\lambda : \Sigma^* \longrightarrow 2^{\Lambda}$ where Λ is a set of propositions
- A decision is a labeling $p : \Sigma^* \longrightarrow 2^{p}$ where p is a proposition

Example

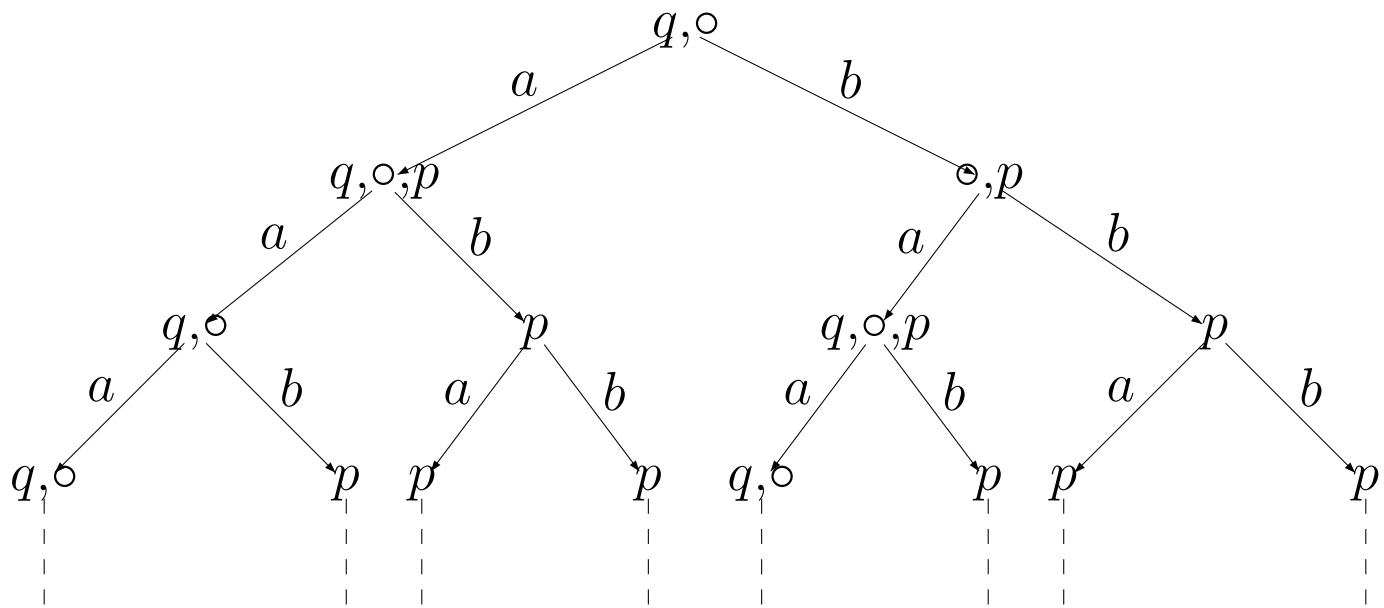
$\lambda : \{a, b\}^* \longrightarrow 2^{\{\circ, q\}}$



Use labels $\circ$ and q

Example

$p : \Sigma^* \longrightarrow 2^p$



Example

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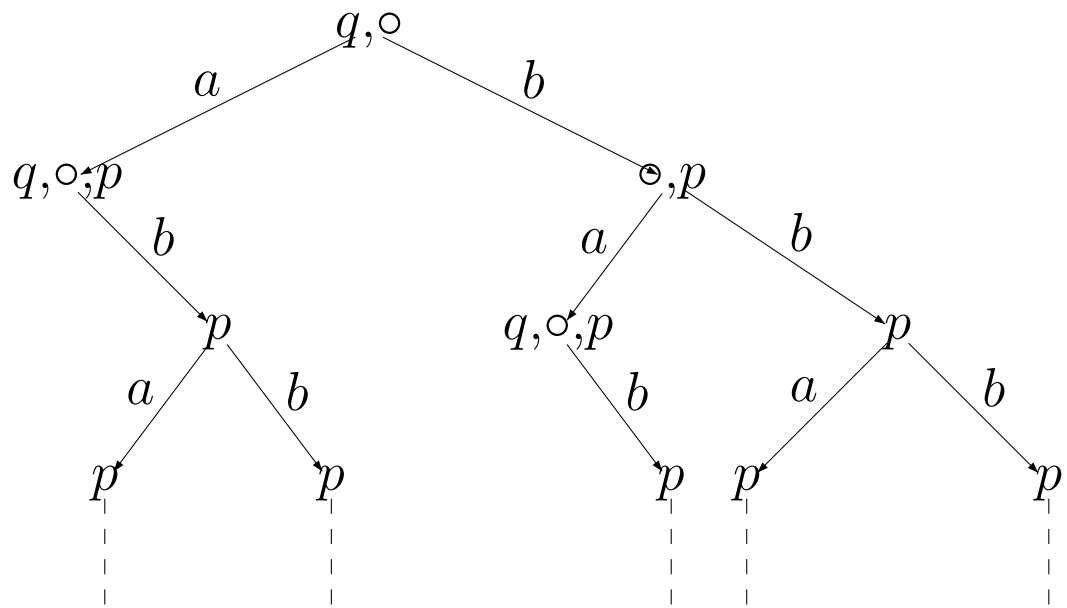

(T, λ) IN p

The Logic

- Mu-calculus $\varphi \rightsquigarrow \varphi \,\text{IN}\, p$, modalities refer to trajectories "inside p"

$$
\begin{array}{ll}
<a>\varphi \,\text{IN}\, p \;=\; <a>(p \wedge \varphi \,\text{IN}\, p) & q \,\text{IN}\, p \;=\; q \\
(\neg\varphi) \,\text{IN}\, p \;=\; \neg(\varphi \,\text{IN}\, p) & (\varphi \wedge \varphi') \,\text{IN}\, p \;=\; \varphi \,\text{IN}\, p \wedge \varphi' \,\text{IN}\, p \\
(\mu Y.\varphi) \,\text{IN}\, p \;=\; \mu Y.(\varphi \,\text{IN}\, p) & Y \,\text{IN}\, p \;=\; Y
\end{array}
$$

- Theorem $(T, \lambda) \,\text{IN}\, p \models \varphi$ iff $(T, \lambda) \models \varphi \,\text{IN}\, p$

- Let $\mathcal{S}$ be represented by $(T, \lambda) \,\text{IN}\, \circ$ with $\Lambda \supseteq \{\circ\}$

$\mathcal{S} \models \varphi$ if and only if $(T, \lambda) \,\text{IN}\, \circ \models \varphi$ if and only if $(T, \lambda) \models \varphi \,\text{IN}\, \circ$

Model-Checking Closed Systems

Given $\lambda : \Sigma^* \longrightarrow \Lambda$ and φ over Λ

$$(T, \lambda) \models \varphi?$$

1) Provided (T, λ) finitely represented (λ is regular)
2) Consider the alternating (parity) tree automaton $\mathcal{A}_\varphi$
3) Find a (memoryless) winning strategy in $\mathcal{G}((T, \lambda), \mathcal{A}_\varphi)$

Satisfiability of φ over Λ

Does $(T, \lambda) \models \varphi$ for some $\lambda : \Sigma^* \to \Lambda$?

Find a memoryless w.s. in $\mathcal{G}(\mathcal{A}_\varphi)$

$(T, \emptyset) \models \exists \Lambda . \varphi$?

1) Compute $\mathcal{B}$ a ndta equivalent to $\mathcal{A}_\varphi$ Simulation Theorem

2) Get $\mathcal{B}'$ by projecting $\mathcal{B}$ in order to forget Λ

3) Find a memoryless w.s. in $\mathcal{G}((T, \emptyset), \mathcal{B}')$ delivering the propositions in Λ

Complexity: EXPTIME in φ and PTIME in λ

Synthesizing a Decision for φ over Λ

Does $(T, \lambda) \,\text{IN}\, p \models \varphi$ for some decision p ?

Does $(T, \lambda) \models \varphi \,\text{IN}\, p$ for some decision p ?

$(T, \lambda) \models \exists p.\varphi \,\text{IN}\, p$?

1) Compute $\mathcal{B}'$ as $\mathcal{A}_{\varphi \,\text{IN}\, p} \downarrow p$ Simulation Theorem

2) Find a memoryless w.s. in $\mathcal{G}((T, \lambda), \mathcal{B}')$ delivering the values of p

Complexity: EXPTIME in φ and PTIME in λ

Expressing Qualitative Properties of the Decision p

$p \sqsubseteq p'$ $\begin{cases} \text{means labeled by } p \text{ implies labeled by } p' \\ \text{is } L_\mu \text{ definable} \end{cases}$

Maximal permissiveness $(T, \lambda) \models \exists p.\ \varphi \text{ IN } p \ \wedge\ \forall p'.\ [\, p \sqsubset p' \Rightarrow \neg \varphi \text{ IN } p' \,]$

Minimal permissiveness

Uniqueness

Expressing Constraints on the Decision

Taking into account e.g. inputs, environment's move, other players', decision of other agents... Given by some "states", some propositions or some events...

- Control of Closed systems $\Sigma_u \subseteq \Sigma$ uncontrollable events

$(T, \lambda) \models \exists p.\ AG(\bigwedge_{u \in \Sigma_u} <u>p) \ \wedge\ \varphi \,\mathrm{IN}\, p$

- Module Checking/MC Open Systems $\{S, E\} \subseteq \Lambda$ exclusive

$(T, \lambda) \models \forall e.\ AG(S \ \Rightarrow\ \bigwedge_{a \in \Sigma}[a]e) \ \wedge\ \varphi \,\mathrm{IN}\, e$

- Control of Open Systems/Computing Strategies

$$(T, \lambda) \models \exists p.\ AG(E \ \Rightarrow\ \bigwedge_{a \in \Sigma}[a]p) \\ \wedge \forall e. AG(S \ \Rightarrow\ \bigwedge_{a \in \Sigma}[a]e) \wedge \varphi \,\mathrm{IN}\, (p \wedge e)$$

Complexity: 2EXPTIME in φ and PTIME in λ

Maximal Permissiveness 3EXPTIME

General Comments

- Model-Checking
  $Q_1\Lambda_1.Q_2\Lambda_2.\ldots Q_n\Lambda_n.\varphi$ is $|\lambda|^{(n-1)EXP(|\varphi|)} \times EXP(|\varphi|)$
- Satisfiability $\begin{cases} \exists\Lambda_1.Q_2\Lambda_2.\ldots Q_n\Lambda_n.\varphi \text{ is } nEXP(|\varphi|) \\ \forall\Lambda_1.Q_2\Lambda_2.\ldots Q_n\Lambda_n.\varphi \text{ is } (n+1)EXP(|\varphi|) \end{cases}$
- Succinctness w.r.t. the mu-calculus

Perspectives

- Game Structures with labelings. Take $A = \{1, 2\} \subseteq Agt = \{1, 2, 3\}$

  $out(F_A, q)$ is given by 3 propositions p_1 (choice of gent 1), p_2, and p_3

  $\langle\langle A, a\rangle\rangle \bigcirc \varphi$ translates $\exists p_1 \exists p_2 \forall p_3.\ (<a>\varphi)\ \text{IN}\ (p_1 \wedge p_2 \wedge p_3)$

  Complexity Issue (size a models)

- Components Given classes $\gamma_1, \gamma_2, \gamma_3$ for components

  $\exists p_1 \in \gamma_1. \forall p_2 \in \gamma_2. \forall p_3 \in \gamma_3.\ \varphi\ \text{IN}\ (p_1 \wedge p_2 \wedge p_3)$

- Fusioning Decisions

  $\exists p_1 \in \gamma_1. \exists p_2 \in \gamma_2. \forall p_3 \in \gamma_3.\ \varphi\ \text{IN}\ \alpha(p_1, p_2, p_3).$

  e.g. $\alpha(p_1, p_2, p_3) = p_1 \vee p_2 \vee p_3$

  e.g. $\alpha(p_1, p_2, p_3) = p_1 \wedge <a>p_2 \wedge p_3$

  Expressiveness?

Partial Observation

- Given $I \subset \Sigma$ of internal moves

  Define $\sim \subseteq \Sigma^*$ for the congruence generated by $\epsilon \sim i$ for all $i \in I$

- $(T, \lambda) \models \exists^{\sim} p.\varphi$ means $\begin{cases} \text{there exists } p \text{ s.t. } (T, \lambda \cup p) \models \varphi, \text{ and} \\ \text{moreover } p(w) = p(w') \text{ whenever } w \sim w' \end{cases}$

  p is a uniform strategy

- The MC of $Q_1^{\sim_1} p_1 . Q_2^{\sim_2} p_2 \cdots Q_m^{\sim_m} p_m . \varphi$ is decidable whenever $\sim_m \subseteq \cdots \subseteq \sim_2 \subseteq \sim_1$

  [Pinchinat&Riedweg05]

- Corollary (with $\sim_1 = \sim_2$) Maximal Permissiveness is decidable

- Corollary Open Systems under Partial Observation

More Regarding Partial Observation

- Interplay between labelings over the Full Trees $\Sigma' \cup \mathcal{I} \subseteq \Sigma$

  [Pinchinat&Riedweg05] Weak Synchronous Product

  [AVW03] [Briand06] Propositions $\circlearrowleft^i$ and Loop Automata

- Observational equivalences between labelings (τ-bisimulations)

  Non-deterministic Models with deterministic decisions [Pinchinat&Raclet05]

  Undistinguihable moves as in [Briand06]

- Games to decide Observability Properties

  ⋆ Exhibit $w \sim w'$ but $w \in Acc$ and $w' \notin Acc$

  ($w \sim_1 w_1$ and $w \sim_2 w_2$ but $w \in Acc$ and $w_1, w_2 \notin Acc$)

  ⋆ Extend/Restrict a property to make it observable

  ⋆ Extend the perception to observe certain properties