1. Brief Self-Introduction
2. Multi-player Arenas
3. The Logic(s)
4. Strategies
5. Relativization
6. Expressing properties of strategies
Biography

- Postdoctorate Sussex University
- Lecturer U. Rennes1 and Researcher in S4* project (INRIA) (*) System Synthesis and Supervision, Scenarios)
- Marie Curie Fellowship CSL RSISE ANU (2006)
Sophie Pinchinat

Scientific coordinator of the Marie Curie Scientific Project MASLOG 021669 (FP6-2004-Mobility-6)

Advisory Board of the MCFA


Board of Directors of IFSIC (U. Rennes 1)
Research Interests

- Logics: temporal logics, second order temporal logics, mu-calculus, satisfiability, model-checking
- Automata and Games: tree automata, parity automata, games (two players, multi players)
- Synthesis: models synthesis from logical specifications
- Open and Component-based Systems
- Planning and Games
- Theory of Control: discrete-event systems, controller synthesis, architectures of control, partial observation, decentralized and modular control
- Observability Issues
- Diagnosis: diagnosis patterns, diagnosis of open systems
Current Research on Logics for Multi-player Arenas

Relevance of the topic: open systems, component-based systems, multi-agents paradigm, etc.
Current Research on Logics for Multi-player Arenas

- Interpretation of (temporal and modal) logics in multi-player arenas
- Monadic Second Order Extensions
- Commitments
- Decidability Issues for the Model-Checking Problem
e.g. Automata Constructions
- Subsumes Alternating Time Logics [Alur et al. 2002]
Multi-player Arenas

- A subclass of Kripke Structures
- A set of players $P$ who individually select in each state some subsets of successors; when all the players have chosen, a single successor is designated
- Similar to Concurrent Game Structures

Definition

$$S = \langle S, R, \bar{\Lambda}, \bar{\lambda}, P \rangle$$

- States $S$, successor binary relation $R \subseteq S \times S$, labeling of state $\lambda : \Lambda \rightarrow 2^S$
Multi-player Arenas

**Definition**

\[ S = \langle S, R, \bar{\Lambda}, \bar{\lambda}, P \rangle \]

- States \( S \), successor binary relation \( R \subseteq S \times S \), labeling of state \( \lambda : \Lambda \rightarrow 2^S \)

\[ P = \{ U, V \} \text{ where } U \]
- (resp. \( V \)) controls variable \( u \) (resp. \( v \))
Multi-player Arenas

Definition

\[ S = \langle S, R, \Lambda, \lambda, P \rangle \]

- States \( S \), successor binary relation \( R \subseteq S \times S \), labeling of state \( \lambda : \Lambda \rightarrow 2^S \)

![Diagram of states and transitions]

- \( u, (\{1, 2\}, U) \)
- \( q_u \)
- \( q_v \)
- \( q_{uv} \)
- \( (\{1\}, V) \)
- \( (\{2\}, V) \)
- \( (\{1\}, U) \)
- \( (\{2\}, U) \)
- \( (\{1\}, V) \)
- \( (\{1\}, U) \)
- \( (\{1\}, V) \)
Multi-player Arenas

Definition

\[ S = \langle S, R, \bar{\Lambda}, \bar{\lambda}, P \rangle \]

- States \( S \), successor binary relation \( R \subseteq S \times S \), labeling of state \( \lambda : \Lambda \rightarrow 2^S \)
- \( R \) has a finite maximal degree \( m \)
- Directions are elements of \( [m] \) (\( = \{1, \ldots, m\} \))
- \( \bar{\Lambda} = \Lambda \cup (\mathcal{P}([m]) \times P) \) and \( \bar{\lambda} : \bar{\Lambda} \rightarrow 2^S \)

\[ \bar{\lambda}(q_u) \text{ contains } u, \text{ but also } (\{1, 3\}, U), (\{2, 4\}, U) \]
Kripke Structures as Trees
Kripke Structures as Trees

\[ (\{1, 3\}, U), (\{2, 4\}, U), (\{1, 2\}, V), (\{3, 4\}, V) \]
Kripke Structures as Trees

(\{1, 3\}, U), (\{2, 4\}, U), (\{1, 2\}, V), (\{3, 4\}, V)

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Logics for Multi-player Arenas
Temporal Logics with Decision Modalities

- Branching time propositional mu-calculus [Kozen83]

\[ g \mid \top \mid \neg \beta \mid \beta_1 \lor \beta_2 \mid \text{EX} \beta \mid Z \mid \mu Z. \beta(Z) \]
Temporal Logics with Decision Modalities

- Branching time propositional mu-calculus [Kozen83]
  \[ g \mid T \mid \neg \beta \mid \beta_1 \lor \beta_2 \mid \text{EX} \beta \mid Z \mid \mu Z. \beta(Z) \]

- easier with CTL
  \[ g \mid T \mid \neg \beta \mid \beta_1 \lor \beta_2 \mid \text{EX} \beta \mid \text{E} \beta_1 \text{U} \beta_2 \mid \ldots \]
Temporal Logics with Decision Modalities

- Branching time propositional mu-calculus [Kozen83]
  \[
g | \top | \neg \beta | \beta_1 \lor \beta_2 | \mathbf{E} \mathbf{X} \beta | Z | \mu Z.\beta(Z)
  \]

- easier with \textit{CTL}
  \[
g | \top | \neg \beta | \beta_1 \lor \beta_2 | \mathbf{E} \mathbf{X} \beta | \mathbf{E} \beta_1 \mathbf{U} \beta_2 | \ldots
  \]

- Add \textit{decision modalities}
Temporal Logics with Decision Modalities

- Branching time propositional mu-calculus [Kozen83]
  \[ g \mid T \mid \neg \beta \mid \beta_1 \lor \beta_2 \mid EX \beta \mid Z \mid \mu Z. \beta(Z) \]

- easier with $CTL$
  \[ g \mid T \mid \neg \beta \mid \beta_1 \lor \beta_2 \mid EX \beta \mid E \beta_1 U \beta_2 \mid \ldots \]

- Add decision modalities

**Definition**

A decision modality is written $(p)f$

- where $f$ is a proposition, and $p \in Players$

- “All propositions in $f$ matches a decision of $p$”
Temporal Logics with Decision Modalities

- $g \models T \models (p) \models \neg \beta \models \beta_1 \lor \beta_2 \models EX \beta \models \ldots$

\[
[[g]]_S = \{s \in S | s \in \lambda(g)\} \quad [[\top]]_S = S
[[\neg \beta]]_S = S \setminus [[\beta]]_S \quad [[\beta_1 \lor \beta_2]]_S = [[\beta_1]]_S \cup [[\beta_2]]_S
[[EX \beta]]_S = \{s \in S | \exists s' \in sR \cap [[\beta]]_S\}
Temporal Logics with Decision Modalities

- **$g \mid \top \mid (p)f \mid \neg \beta \mid \beta_1 \lor \beta_2 \mid \mathbf{E} \mathbf{X} \beta \mid \ldots$**

  $[[g]]_S = \{s \in S \mid s \in \lambda(g)\}$
  $[[\top]]_S = S$
  $[[\neg \beta]]_S = S \setminus [[\beta]]_S$
  $[[\beta_1 \lor \beta_2]]_S = [[\beta_1]]_S \cup [[\beta_2]]_S$
  $[[\mathbf{E} \mathbf{X} \beta]]_S = \{s \in S \mid \exists s' \in sR \cap [[\beta]]_S\}$

**Definition**

A *decision modality* is written $(p)f$ where $f$ is a proposition, and $p \in \text{Players}$.

- $[[ (p)f ]]_S = \{s \in S \mid sR \cap \lambda(f) \in \text{dec}(s, p)\}$
Temporal Logics with Decision Modalities

- $g \mid \top \mid (p)f \mid \neg \beta \mid \beta_1 \lor \beta_2 \mid \text{EX} \beta \mid \ldots$

\[
[[g]]_S = \{s \in S \mid s \in \lambda(g)\} \quad [[\top]]_S = S
\]
\[
[[\neg \beta]]_S = S \setminus [[\beta]]_S \quad [[\beta_1 \lor \beta_2]]_S = [[\beta_1]]_S \cup [[\beta_2]]_S
\]
\[
[[\text{EX} \beta]]_S = \{s \in S \mid \exists s' \in sR \cap [[\beta]]_S\}
\]
\[
[[ (p)f ]]_S = \{s \in S \mid sR \cap \lambda(f) \in \text{dec}(s, p)\}
\]

“The proposition in $f$ matches a decision of $p$”
Temporal Logics with Decision Modalities

- $g \mid \top \mid (p)f \mid \neg \beta \mid \beta_1 \lor \beta_2 \mid EX \beta \mid \ldots$

$[[g]]_S = \{s \in S \mid s \in \lambda(g)\}$
$[[\top]]_S = S$
$[[\neg \beta]]_S = S \setminus [[\beta]]_S$
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$[[EX \beta]]_S = \{s \in S \mid \exists s' \in sR \cap [[\beta]]_S\}$

- $[[ (p)f ]]_S = \{s \in S \mid sR \cap \lambda(f) \in dec(s, p)\}$

$[[ (p)\Gamma ]]_S = \{s \in S \mid sR \cap \bigcap_{f \in \Gamma} \lambda(f) \in dec(s, p)\}$
Temporal Logics with Decision Modalities

- $g \models \top \models (p) \models \neg \beta \models \beta_1 \lor \beta_2 \models \text{EX} \beta \models \ldots$

\[
[[g]]_S = \{s \in S \mid s \in \lambda(g)\} \\
[[\top]]_S = S \\
[[\neg \beta]]_S = S \setminus [[\beta]]_S \\
[[\beta_1 \lor \beta_2]]_S = [[\beta_1]]_S \cup [[\beta_2]]_S \\
[[\text{EX} \beta]]_S = \{s \in S \mid \exists s' \in sR \cap [[\beta]]_S\} \\
[[ (p)f ]]_S = \{s \in S \mid sR \cap \lambda(f) \in \text{dec}(s, p)\}
\]

- Write $S, s \models \beta$ for $s \in [[\beta]]_S$
Brief Self-Introduction
Multi-player Arenas

The Logic(s)
Strategies
Relativization
Expressing properties of strategies

Example of Decision Modalities
Example of Decision Modalities

\[
(\{1, 3\}, U), (\{2, 4\}, U), (\{1, 2\}, V), (\{3, 4\}, V)
\]
Example of Decision Modalities
Example of Decision Modalities

$(V)f_1 \land \neg(U)f_1$
Example of Decision Modalities

$$(V)f_1 \text{ and } \neg(U)f_1$$

$$\neg(V)f_2 \text{ and } \neg(U)f_2$$
Example of Decision Modalities

\[(V)f_1 \text{ and } \neg(U)f_1\]
\[\neg(V)f_2 \text{ and } \neg(U)f_2\]
\[\neg(V)f_3 \text{ and } \neg(U)f_3\]
Labelings and Strategies

Definition

A \textit{f-labeling} of $S$ from $s_0$ is a (complete) Kripke Structure $\mathcal{E}$ with the only proposition $f$.

To label $S$, take $S \times \mathcal{E}$ which starts from $(s_0, r)$. 

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Labelings and Strategies

Consider a labeling of $S$

![Diagram of a labeled graph with nodes and edges labeled with $q$, $q_u$, $q_v$, $q_{uv}$, and transitions labeled with $f_V$]
Labelings and Strategies

\[ S \]

\[ q \]

\[ u \]

\[ q_u \]

\[ q_v \]

\[ q_{uv} \]

\[ u, v \]

\[ \mathcal{E} \]

\[ r \]

\[ 1, 2 \]

\[ 3, 4 \]

\[ f_V \]
Definition

A \( f \)-labeling \( \mathcal{E} \) of \( S \) from \( s_0 \) is a **p-strategy** whenever

\[
S \times \mathcal{E}, (s_0, r) \models \text{AG} (p)f
\]
Given a $p$-strategy, say using proposition $f_p$, cut everything which leaves $f_p$. Obtain $\text{OUT}(f_p, S, s_0)$.
Summary

1. The tree of $S$ from $s_0$ can be decorated with a proposition $f$, via a $f$-labeling, $E$.

2. We can express that the decoration by $f$ respects (everywhere) a possible choice of some player $p$, with $\text{AG}[(p)f]$.

3. We can prune $(S, s_0)$ according to $f$ to get $\text{OUT}(f, S, s_0)$ the outcome of this policy of the player $p$

   + By considering several propositions $f_{p_1}, f_{p_2}, \ldots, f_{p_k}$, we can handle coalitions.
Given \((S, s_0)\) and a \(p\)-strategy \(\mathcal{E}\) (using proposition \(f\)), we have

**Theorem**

*For any formula \(\beta\),*

\[
\text{OUT}(f, S, s_0), s_0 \models \beta \iff S, s_0 \models (\beta \ast f)
\]

where

**Definition**

\((\beta \ast f)\), the \(f\)-relativization of \(\beta\), is defined by

\[
(EX \beta \ast f) = EX [f \land (\beta \ast f)]
\]
Back to the logic - Relativization

Given \((S, s_0)\) and a \(p\)-strategy \(E\) (using proposition \(f\)), we have

**Theorem**

*For any formula \(\beta\),*

\[
\text{OUT}(f, S, s_0), s_0 \models \beta \text{ iff } S, s_0 \models (\beta \ast f)
\]

where

**Definition**

\((\beta \ast f)\), the \(f\)-relativization of \(\beta\), is defined by

\[
(\text{EX} \beta \ast f) = \text{EX} [f \land (\beta \ast f)] \\
(g \ast f) = g \\
(\beta_1 \lor \beta_2 \ast f) = (\beta_1 \ast f) \lor (\beta_2 \ast f) \\
((p)f' \ast f) = (p)(f \land f')
\]

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Logics for Multi-player Arenas
Back to the logic - Relativization

Given \((S, s_0)\) and a \(p\)-strategy \(E\) (using proposition \(f\)), we have

**Theorem**

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**Definition**

\((\beta \ast f)\), the \(f\)-relativization of \(\beta\), is defined by

\[
(\text{EX} \ \beta \ast f) = \text{EX} [f \land (\beta \ast f)]
\]

\[
(E \ \beta_1 \ \text{U} \ \beta_2 \ast f) = E [f \land (\beta_1 \ast f)] \ \text{U} \ [f \land (\beta_2 \ast f)]
\]
Expressing properties of strategies

\[ S, s_0 \models \textbf{AG}[(p)f] \land (\beta \ast f) \]

means

“player \( p \) has a strategy to guarantee \( \beta \)”

- Generalizes to a set of players
- Existence of strategies is captured by the monadic second order extension of the logic
Monadic second order extension of the logic

- $\mathcal{S}, s \models \exists f. \alpha(f)$ means there exists a $f$-labeling $\mathcal{E}$ s.t.

  $$\mathcal{S} \times \mathcal{E}, (s_0, r) \models \alpha(f)$$
Monadic second order extension of the logic

- $S, s \models \exists f. \alpha(f)$ means there exists a $f$-labeling $E$ s.t.
  $$S \times E, (s_0, r) \models \alpha(f)$$

- Automata constructions (based on the projection)
Monadic second order extension of the logic

- \( S, s \models \exists f. \alpha(f) \) means there exists a \( f \)-labeling \( \mathcal{E} \) s.t.
  \[
  S \times \mathcal{E}, (s_0, r) \models \alpha(f)
  \]

- Automata constructions (based on the projection)
  \( \Rightarrow \) Decidability
Monadic second order extension of the logic

- \( S, s \models \exists f. \alpha(f) \) means there exists a \( f \)-labeling \( \mathcal{E} \) s.t.
  \[
  S \times \mathcal{E}, (s_0, r) \models \alpha(f)
  \]

- Automata constructions (based on the projection)
  \[\Rightarrow\] Decidability
  \[\Rightarrow\] Polynomial in the size of \( S \)
Monadic second order extension of the logic

- \( S, s \models \exists f. \alpha(f) \) means there exists a \( f \)-labeling \( E \) s.t.

  \[
  S \times E, (s_0, r) \models \alpha(f)
  \]

- Automata constructions (based on the projection)
  - \( \Rightarrow \) Decidability
  - \( \Rightarrow \) Polynomial in the size of \( S \)
  - \( \Rightarrow \) Can compute \( E \), hence strategies
Monadic second order extension of the logic

- \( S, s \models \exists f. \alpha(f) \) means there exists a \( f \)-labeling \( \mathcal{E} \) s.t.
  \[
  S \times \mathcal{E}, (s_0, r) \models \alpha(f)
  \]

- Automata constructions (based on the projection)
  - Decidability
  - Polynomial in the size of \( S \)
  - Can compute \( \mathcal{E} \), hence strategies
  - Always regular solutions!
Specializing the logic

Define $\exists (f \text{ of } p).\alpha \overset{\text{def}}{=} \exists f.[AG[(p)f] \land \alpha]$
Define $\exists (f \text{ of } p).\alpha \overset{\text{def}}{=} \exists f. [\mathbf{AG} [(p)f] \land \alpha]$

$\exists (f \text{ of } p). (\alpha \ast f)$ expresses a commitment of $p$: 

Specializing the logic
Specializing the logic

- Define $\exists f (\text{of } p). \alpha \overset{\text{def}}{=} \exists f. [AG [(p)f] \land \alpha]$
- $\exists f (\text{of } p). (\alpha \ast f)$ expresses a commitment of $p$:
  - $\exists f (\text{of } p). (EF [\exists (f' \text{ of } p') (\alpha \ast f')] \ast f)$
  Inside the “$f$-subtree” there is a position from which a $f'$-subtree exists where $\alpha$ holds.
Specializing the logic

- Define \( \exists f \text{ of } p. \alpha \overset{\text{def}}{=} \exists f. [AG[(p)f] \land \alpha] \)
- \( \exists f \text{ of } p. (\alpha * f) \) expresses a commitment of \( p \):
  - \( \exists f \text{ of } p. (EF[\exists f' \text{ of } p'(\alpha * f')]*f) \)
    Inside the “\( f \)-subtree” there is a position from which a \( f' \)-subtree exists where \( \alpha \) holds.
  - However, we can express alternating time logics properties

\[
\langle \langle U \rangle \rangle G[\neg u \land \langle \langle U, V \rangle \rangle F(u \land v)]
\]

by using the \textit{bounded relativization}
Conclusion

- Multi-player arenas as Kripke structures
- Temporal logics with decision modalities
Conclusion

- Multi-player arenas as Kripke structures
- Temporal logics with decision modalities
- Second order extension to specify strategies
Conclusion

- Multi-player arenas as Kripke structures
- Temporal logics with decision modalities
- Second order extension to specify strategies
  - Address $S, s_0 \models \exists f. (\alpha * f)$
Multi-player arenas as Kripke structures

Temporal logics with decision modalities

Second order extension to specify strategies

Address $S, s_0 \models \exists \exists \exists (f \text{ of } p). (\alpha \ast f)$

(Parity Tree) Automata constructions
Conclusion

- Multi-player arenas as Kripke structures
- Temporal logics with decision modalities
- Second order extension to specify strategies
  - Address $S, s_0 \models \exists (f \text{ of } p). (\alpha \ast f)$
  - (Parity Tree) Automata constructions
  - Needs the Simulation Theorem of [Muller & Schupp 95]
Conclusion

- Multi-player arenas as Kripke structures
- Temporal logics with decision modalities
- Second order extension to specify strategies
  - Address $\mathcal{S}, s_0 \models \exists (\exists (f \text{ of } p).(\alpha \ast f))$
  - (Parity Tree) Automata constructions
    - Needs the Simulation Theorem of [Muller & Schupp 95]
  - Associated parity games to model-check
Conclusion

- Multi-player arenas as Kripke structures
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  - Associated parity games to model-check
    - A winning strategy delivers a labeling, hence a $p$-strategy
Conclusion

- Multi-player arenas as Kripke structures
- Temporal logics with decision modalities
- Second order extension to specify strategies
  - Address $S, s_0 \models \exists (f \text{ of } p). (\alpha \ast f)$
  - (Parity Tree) Automata constructions
    - Needs the Simulation Theorem of [Muller & Schupp 95]
    - Associated parity games to model-check
      - A winning strategy delivers a labeling, hence a $p$-strategy
- Commitments easily specified
- Commitment not considered in Alternating Time Logics