A Topological Perspective on Diagnosis

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WODES, Gothenburg, Sweden, May 28-30, 2008
The Framework

The Problem

The Contribution

A Zoom In: The Saturation Problem

Last Slide
Introduction

• We consider a set of infinite sequences (of events).

• A property is given by a set of $\omega$-sequences.

• A particular infinite sequence is partially observed.

What can we infer from this incomplete information about the sequence regarding the property?

Applications: Diagnosis and Control of DES, Game Theory
The Framework

Mathematical Framework

• $\Sigma \ni a, b, l, \ldots$ an alphabet

\[ \Sigma^\omega \ni w, w', \ldots \] \(\omega\)-Sequences

\[ \mathcal{P}(\Sigma^\omega) \supseteq L, S, \ldots \] Properties (\(\omega\)-languages)

• $\Delta$ an observation alphabet

\[ \Delta^\omega \ni \pi, \ldots \] \(\omega\)-Observations

• A morphism $m : \Sigma \to \Delta^*$ (like $P : \Sigma \to \Sigma_o \subseteq \Sigma$)

• The Observational Equivalence $\approx \subseteq \Sigma^\omega \times \Sigma^\omega$

  "having the same $m$-image"
The Problem

Given \( L \subseteq \Sigma^\omega \) and an incremental infinite chain of observations

\[
\tau_1 < \tau_2 < \ldots < \lim_{i \to \infty} (\tau_i) = \pi
\]

when and what can we infer regarding membership in \( L \)?

That is: if an \( \omega \)-sequence \( w \) is consistent with the \( \omega \)-observation \( \pi \), when and what can we infer about \( w \in L \)?
Abstraction: $[w]_\approx = \{ w' \in \Sigma^\omega | w' \approx w \}$

- The $\approx$-saturation of $L$
  
  \[(L)_\approx := \bigcup_{w \in L} [w]_\approx\]

- $(L)_\approx$ and $m(L)$ are the same objects: $(L)_\approx = m^{-1}(m(L))$
In the paper

1. As if $L$ is not $\approx$-saturated, we cannot tell much, deciding $L = (L)_{\approx}$ is crucial

   The Saturation Problem

   - The results are given for an arbitrary $\omega$-regular language $L$ and an arbitrary $\sim \in Rat(\sum^\omega \times \sum^\omega)$
   - Lower bound complexity of the problem, reached when $\sim$ is $\approx$
   - An optimal decision procedure
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2. $w \in L$ cannot be inferred in a finite amount of time in general (eg, infinitely often $a$), unless $L = B\Sigma^\omega$ where $B \subseteq \Sigma^*$
   $\Leftrightarrow$ $L$ is an open set in the Cantor Topology
   The Openness Problem
   - Prediagnosability [Jiang-Kumar04]
   - Decision procedure for Openness [Landweber69,Alpern-Schneider87]
     Lower bound is unknown for non-deterministic Buchi
Next in the paper

- Merging Points 1 and 2 by an adequate topology

  Open sets are \((B) \simeq \Sigma^\omega\) (with \(B \subseteq \Sigma^*\))
  
  \[=\text{Open sets in the space } \Delta^\omega\]

- We distinguish a subspace \(S\) of \(\Sigma^\omega\)
  
  - \(\simeq\)-saturation in \(S\): \(L \cap S = (L) \simeq \cap S\)
  
  - Openness in \(S\): \(L \cap S = B \Sigma^\omega \cap S\) (with \(B \subseteq \Sigma^*\))
  
  - Decision Problems (\(S\) is a close set)

- Relation with classic Diagnosis
  
  [Sampath-Sengupta-Lafortune-Sinaamohideen-Teneketzis96]:
  a comparison with the twin-plant algorithm

  [Jiang-Huang-Chandra-Kumar04]

  see also [Jeron-Marchand-P-Cordier06]
The (≈-)Saturation Problem

Given

1. An $\omega$-regular language $L$
   (given by a non-deterministic Buchi automaton $A$ over $\Sigma$)
2. A relation $\approx \subseteq \Sigma^\omega \times \Sigma^\omega$
   (given by $m: \Sigma \rightarrow \Delta^*$ alphabetic, ie $|m(a)| \leq 1$, $\forall a \in \Sigma$)

Decide $L = (L)\approx$?

- Lower bound: if NO OBSERVATION then $(L)\approx$ equals $\Sigma^\omega$
  The Universality Problem is PSPACE-complete

- Upper bound: there exists a PSPACE algorithm, as
  - $\approx \in \text{Rat}(\Sigma^\omega \times \Sigma^\omega)$, and
  - $L = (L)\approx$ is equivalent $\approx \cap (L \times L^c) = \emptyset$
\[ \approx \in \text{Rat}(\Sigma^\omega \times \Sigma^\omega) \]

\( P : \{a, b, l\} \rightarrow \{a, b\}, \) let \( w = alblaa \) and \( w' = llaballa \)

Read input on \( \begin{cases} 
\text{Tape 1} & \text{if state is grey}, \\
\text{Tape 2} & \text{otherwise.} 
\end{cases} \)

The 2-automaton has \(|\Sigma_o| + 4\) states
\[ \approx \cap (L \times L^c) \in \text{Rat}(\Sigma^\omega \times \Sigma^\omega) \]

Decide emptiness by projecting on each component
A PSPACE algorithm (case $\approx$)

- Construct a non-deterministic Buchi $B$ which accepts $L^c$
  
  It has $O(2^{|A| \log |A|})$ states, eg [klarlund91]

- The 2-automaton $\Theta'$ is like $\Theta$ but componentwise constrained by $A$ and $B$
  
  It has $O((|\Sigma_0| + 4) \cdot |A| \cdot 2^{|A| \log |A|})$ states encoded in space $O(\log(|\Sigma_0| + 4)) + \log |A| + |A| \log |A|)$

- The algorithm guesses an accepting run of $\Theta'$ a sequence of states $r_0 r_1 \ldots r_i \ldots r_n$ with $r_0$ initial, $r_i$ accepting, and $r_i = r_n$.

+ NPSPACE = PSPACE [Savitch70]
The Algorithm

1. Let $r$ be the initial state of $\Theta'$
2. Choose a state $r'$
3. If $r'$ is a successor of $r$, let $r = r'$
   else halt (without accepting)
4. If $r$ is accepting, goto 5 or 2, else goto 2
5. Let $r_A = r$ // guess it is $r_i$
6. Choose a state $r'$
7. If $r'$ is a successor of $r$, let $r = r'$
   else halt (without accepting)
8. If $r = r_A$, accept, else goto 6
Concluding Remarks

- An adequate topology on $\omega$-languages to handle partial observation, with two concepts
  
  Openness and Saturation

- Application to diagnosis of DES with non-deterministic $\omega$-regular supervision patterns

- A machinery to decide Saturation. Extension to $k + 1$-automata for decentralized control under partial observation.