Homework TVA - due November 21, 2016[1]

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Exercice 1 Fragments of CTL

Consider the fragment ECTL of CTL which consists of formulas built according to the following grammar:

$$\Phi ::= a \mid \neg a \mid \Phi \wedge \Phi \mid \exists \varphi$$

$$\varphi ::= \bigcirc \Phi \mid \Box \Phi \mid \Phi \mathtt{U} \Phi$$

Therefore, ECTL-formulas are built by atomic propositions, negated atomic propositions, the Boolean connective $\wedge$ and the path quantifier $\exists$ together with the modalities $\bigcirc$, $\Box$ and $\mathtt{U}$.

For two transition systems $TS_1 = (S_1, Act, \rightarrow_1, I_1, AP, L_1)$ and $TS_2 = (S_2, Act, \rightarrow_2, I_2, AP, L_2)$, we write $TS_1 \subseteq TS_2$ whenever $S_1 \subseteq S_2$, $\rightarrow_1 \subseteq \rightarrow_2$, $I_1 = I_2$ and $L_1(s) = L_2(s)$ for all $s \in S_1$.

1. Prove that, for all ECTL-formulas Φ and all transition systems TS_1, TS_2 with $TS_1 \subseteq TS_2$, its holds:

$$TS_1 \models \Phi \text{ implies } TS_2 \models \Phi$$

2. Give a CTL-formula which is not equivalent to any ECTL-formula. Justify your answer!

Exercice 2 CTL with fairness assumption

Let $fair = \Box\Diamond\forall \bigcirc (a \wedge \neg b) \rightarrow \Box\Diamond\forall \bigcirc (b \wedge \neg a) \wedge \Diamond\Box\exists\Diamond b \rightarrow \Box\Diamond b$. Check that the transition system TS depicted in Figure 1 verifies $TS \models_{fair} \forall\Box(a \rightarrow \forall\Diamond(b \wedge \neg a))$.

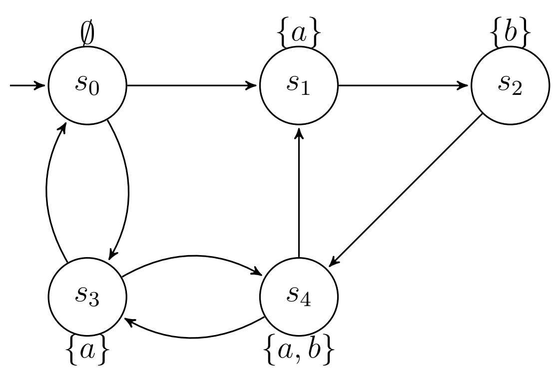

Figure 1: TS

[1]It is recommended to work in pairs

Exercice 3 Timed Automata Modeling

You have to model a smart light switch with a timed automaton, where the description of the switch is as follows.

The ligth can be in three modes: off, dim and bright. The switch sense whether a user is touching it or not. There are two ways to cycle through the light modes: each new short tap makes it go from off to dim, from dim to bright and from bright to off. A long touch follows the cycle backwards.

Exercice 4 Markov decision processes

Let M be an MDP. Let $Goal$ be a subset of S. Let s be a state. We want to find an algorithm to determine the probability (=weight of the set of infinite paths) P_s(Always some day $Goal$) of the set of infinite paths that see infinitely often any state of $Goal$ from s.

1. Show that the set "Always some day Goal" of infinite paths is mesurable in the algebra of cylinders.

2. Consider that M is a Markov Chain (MDP with only one action, hence no choice). Consider the strongly connected components of S.

2.1. What are the states of S with P_s(Always some day $Goal$) $= 0$?

2.2. What are the states of S with P_s(Always some day $Goal$) $= 1$?

2.3. How to compute P_s(Always some day $Goal$) for the other states?

3. Consider the general case for M a MDP.

3.4. Give an algorithm to compute the states of S with P_s(Always some day $Goal$) $= 0$.

3.5. How to compute P_s(Always some day $Goal$) $= 0$ for the other states?

4. Compute P_s(Always some day $\{2, 5\}$) for all states of the MDP M depicted Figure 2.

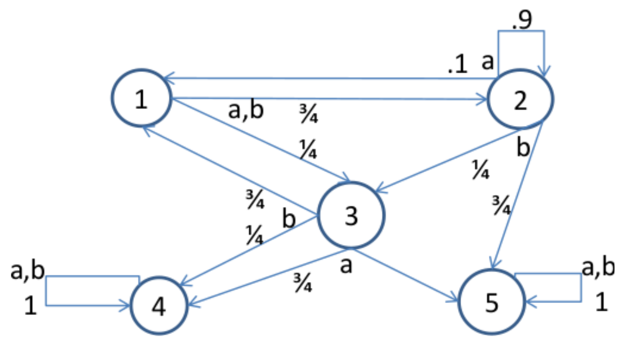

Figure 2: MDP M