Connected Component Trees for Multivariate Image Processing and Applications in Astronomy

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Abstract

In this paper, we investigate the possibilities offered by the extension of the connected component trees (cc-trees) to multivariate images. We propose a general framework for image processing using the cc-tree based on the lattice theory and we discuss the possible applications depending on the properties of the underlying ordered set. This theoretical reflexion is illustrated by two applications in multispectral astronomical imaging: source separation and object detection.

1 Introduction

The cc-tree has become a popular model for the analysis of gray-level images. It provides a hierarchical representation of images with various applications: connected filters [10], segmentation [5], object detection and characterization [7], source separation [3]. Moreover, algorithms to efficiently compute the cc-tree for integer images [8, 10] or floating point images [2] are available. Nevertheless, all these approaches were limited to gray-level images while multivariate images are becoming more and more popular. Recently, the authors of [6] have begun to investigate about the extensions of the cc-trees for color images but limited to connected filters. Compared to this work, we aim at studying the properties of the cc-trees for more general applications involving multivariate images.

In this paper, the definition of the cc-tree compatible with multivariate images and based on the lattice theory is given in Sec. 2. The use of this definition is then discussed and its possible applications depending on the properties of the underlying ordering relation are examined. This theoretical work is illustrated with two applications in multispectral astronomical imaging (Sec. 3): source separation and object detection.

2 Connected Component Tree

We now recall some preliminary definitions and give a rigorous definition of the cc-tree. Then, the general framework for multivariate image processing using the cc-tree is discussed.

Let $\mathcal{L}$ be a join bounded semi-lattice with the order relation $\leq$, the supremum operator $\bigvee$ and its least and greatest elements $\perp$ and $\top$. Let $I : \mathcal{D} \mapsto \mathcal{L}$ be an image from a discrete domain $\mathcal{D}$ (generally $\mathcal{D} = \mathbb{Z}^2$) to $\mathcal{L}$. In the sequel, we assume that $\mathcal{L} = \overline{\mathbb{R}}^n$ with $n \geq 1$ and $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$.

We also assume that the support $\mathrm{s}(I) = \{x \in \mathcal{D} \,|\, I(x) \neq \perp\}$ of the image I is finite. We define the thresholding $\overline{I}^t \subseteq \mathcal{D}$ of the image I at level $t \in \mathcal{L}$ by $\overline{I}^t = \{x \in \mathcal{D} \,|\, t \leq I(x)\}$.

Moreover, $\mathcal{D}$ is equipped with a connexion $\mathcal{C}$ [11] (for example the 4- or 8-neighborhood if $\mathcal{D} = \mathbb{Z}^2$). Let $E \subseteq \mathcal{D}$, we denote by $\mathrm{CC}(E) \subseteq \mathcal{P}(E)$ the set of connected components of E with respect to $\mathcal{C}$, with $\mathcal{P}(E)$ the set of all subsets of E. Let $G = \bigcup_{t \in \mathcal{L}} \mathrm{CC}\left(\overline{I}^t\right) \subseteq \mathcal{P}(\mathcal{D})$ be the set of all connected components of the thresholdings of I at all possible levels. Then, we consider the Hasse diagram (G, V) of the set inclusion partial order relation $\subseteq$. The set of edges V is defined by $V = \left\{(x, y) \in G^2 \,|\, y \subset x, \forall z, (y \subseteq z \subset x) \Rightarrow (z = y)\right\}$. As $\mathrm{s}(I)$ is a finite set, the set G is also finite ensur-

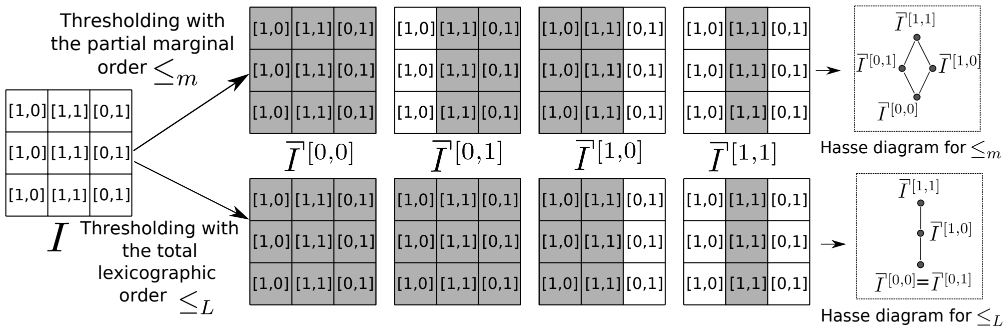

Figure 1. This example shows the Hasse diagrams of the thresholds with the marginal partial order $\leq_m$ and a lexicographic total order $\leq_L$. From left to right we have: the bi-valued image, its thresholding (gray) at levels $[0,0]$, $[0,1]$, $[1,0]$ and $[1,1]$, and the resulting Hasse diagrams for the two orders.

ing that the graph is correctly defined. Moreover, (G,V) is valued by a function $l : G \mapsto \mathcal{P}\left(\overline{\mathbb{R}}^n\right)$, $\forall g \in G,\ l(g) = \max\left\{t \in \overline{\mathbb{R}}^n \mid g \subseteq \mathrm{CC}\left(\overline{I}^t\right)\right\}$. For a node g, we call $l(g)$ the *levels* of g (i.e., the maximum threshold values so that g is included in the thresholded image).

In the general case, $\leq$ is a partial order and the Hasse diagram of G contains cycles [9]. Fig. 1 (top line) shows an example using the marginal partial order: $v, v' \in \overline{\mathbb{R}}^n,\ v \leq_m v' \Leftrightarrow v_i \leq v'_i,\ i = 0, \ldots, n$. In this case, the different thresholdings partially overlap, leading to a cycle in the Hasse diagram.

On the contrary, a total order or pre-order (i.e., without anti-symmetry) on $\mathcal{L}$ ensures that the Hasse diagram of G is an acyclic graph. Thus, in the sequel, we consider only total (pre-)order, and with no loose of generality, we assume that $\mathrm{CC}(\mathcal{D}) = \{\mathcal{D}\}$ so (G,V) can be considered as a tree rooted by its greatest element $\overline{I}^{\perp} = \mathcal{D}$ (Fig. 1 bottom line).

We now present, in a multivariate context, how the tree can be filtered to extract relevant components and finally, how to reconstruct an image from the filtered tree.

2.1 Filtering and Reconstruction

The nodes of the tree are equipped with any quantitative or qualitative information called *attributes* that may help to achieve the given purpose (e.g., Sec. 3). Then, let P be a predicate on G based on these attributes. We define a pruning function F_P which removes all nodes which do not fulfill P and provides a new tree $F_P(G,V) = (G',V')$.

Usually the predicate is based on a threshold function and removes all nodes having an attribute lower than a given value. This approach, based on a local decision criterion, naturally leads to idempotent operators, but more complex predicates can also rely on global decisions [10]. In this case, we can always derive the idempotent filter: F_P^∞ which is the filter F_P recursively applied until convergence (ensured by the finite size of G).

Finally, the processing can be completed (if necessary) by a reconstruction of the tree which is an operator from the tree space back to the image space defined by:

$$\forall x \in \mathcal{D},\ R_{(G',V')}(x) = \bigvee_{g \in G'} f(g,x) \qquad (1)$$

where $f : G \times \mathcal{D} \mapsto \overline{\mathbb{R}}^{n'}$ can be any value computed from g and x. The overall framework is summarized in Fig. 2.

2.2 Influence of the Order Type

Given the previous definition of the cc-tree (filtering and reconstruction), we now explore the differences implied by the choice of a total order or a total pre-order to build the tree.

In the case of a total order, the valuation function l returns a singleton for each node and thus, it can be used directly in the reconstruction step to design connected filters. In fact, this case is nearly equivalent to a gray level image, nevertheless, the multivariate information may still be very useful to define relevant attributes.

On the other hand, if the order is a total pre-order, then the valuation function may return a set

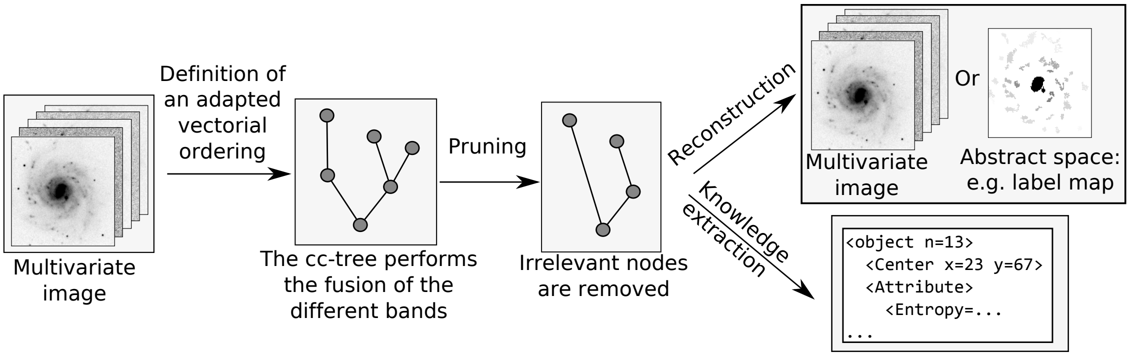

Figure 2. General framework using cc-trees

of levels $\{t_i\}$ such that $\forall i, j$, $i \neq j$ then $t_i \leq t_j$ and $t_j \leq t_i$ but $t_i \neq t_j$. This is an issue to design connected filters as the valuation function cannot be used as it in the reconstruction process. In [6], the authors proposed to compute a new value from the set $\{t_i\}$, for example the mean or the median. But the mean may introduce new values, which can be considered as a drawback while the median requires the use of another total order.

Nevertheless, if the reconstruction step is not needed or if it is not based on the pixel values of the original image, and if the attributes do not require that each node has a unique level, a pre-order does not lead to any difficulty. For example in detection or segmentation tasks, we might be only interested by the boundaries of detected objects.

3 Applications

We propose two applications in 5-band astronomical images to demonstrate the capabilities of cc-trees in multivariate frameworks. The bands come from ultra-violet to near infrared and were extracted from the Sloan Digitalized Sky Survey. Each band has a different signal to noise ratio and a different seeing (size of the point spread function).

We define a vectorial order based on the combination of several techniques [1]: a reduced order defined as a normalized truncated energy function, a quantified and normalized order and finally, a lexicographic order. Then, let $v, v' \in \overline{\mathbb{R}}^n$, we define the pre-order $\leq_{Ap}$ by:

$$v \leq_{Ap} v' \Leftrightarrow \left[\lfloor \mathrm{E}_n(v) \rfloor, \left\lfloor \frac{v_1}{k\sigma_1} \right\rfloor, \ldots, \left\lfloor \frac{v_n}{k\sigma_n} \right\rfloor\right] \leq_L \left[\lfloor \mathrm{E}_n(v) \rfloor, \left\lfloor \frac{v'_1}{k\sigma_1} \right\rfloor, \ldots, \left\lfloor \frac{v'_n}{k\sigma_n} \right\rfloor\right] \quad (2)$$

where $\leq_L$ is the lexicographic order, $\sigma_1, \ldots, \sigma_n$ are the standard deviation of the noise in each band, k is a confidence factor ($k = 3$), $\lfloor \cdot \rfloor$ is the floor function. Moreover, the bands are sorted by seeing size (best seeing at first). $\mathrm{E}_n(v)$ is the normalized energy defined as: $\mathrm{E}_n(v) = \left\| \frac{v_1}{k\sigma_1}, \ldots, \frac{v_n}{k\sigma_1} \right\|$.

To obtain a total order $\leq_{Ao}$, we extend $\leq_{Ap}$ with a lexicographic order applied to the initial spectral bands.

3.1 Source Separation

A typical problem in astronomy is to identify the different sources, this task is made difficult by the projection effects and the intrinsic difficulty to estimate the distance of the sources.

The authors of [3] describe a method based on the cc-tree in strongly quantified gray-level images. The algorithm identifies the branches of the tree having a sufficient volume relatively to their father.

We have extended this method to floating multispectral images using the order $\leq_{Ao}$ and a spatial 4-neighborhood. Nodes are equipped with a single attribute: the multispectral volume defined as $V(g) = \sqrt{\sum_{x \in g} (I(x) - l(g))^T (I(x) - l(g))}$. Then, a node of the tree is kept if its volume is greater than a fraction ρ of its father volume and if this condition also holds for at least one of its brothers. Finally, the different sources are simply identified by the leaves of the tree. This approach is both scale and flux invariant so that ρ is a very robust threshold.

Fig. 3 shows some results. Images are color compositions with enhanced contrast, the different sources are located by the red squares; we can observe that the objects superimposed on the galaxies were correctly identified as separate objects, moreover the multispectral processing allows to naturally take account of structures shining at different wavelengths.

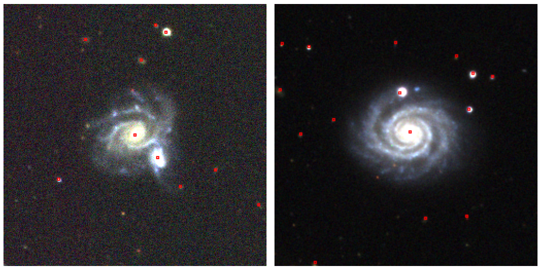

Figure 3. Source separation in two 5-band astronomical images.

3.2 H II Regions Detection

In galaxies, H II regions are made of very bright young and massive stars plunged into ionized hydrogen clouds and are an important marker of the galaxy activity. H II regions are characterized by their color, their relatively small size and their brightness. These features suggest 3 attributes to separate these regions from other bright objects: the area of the component, its color (difference between 2 bands in the astronomical context) and its energy per pixel. The pruning criteria is then simply defined as a threshold on these 3 attributes. This application uses a 4-neighborhood and the pre-order $\leq_{Ap}$ (since neither the reconstruction nor the attributes require a unique component level).

Fig. 4 shows two examples, the images are color compositions with enhanced contrast and the different H II regions are located in red (boundaries). The H II regions are correctly identified among other bright sources thanks to the multispectral processing which brings the discriminant color information.

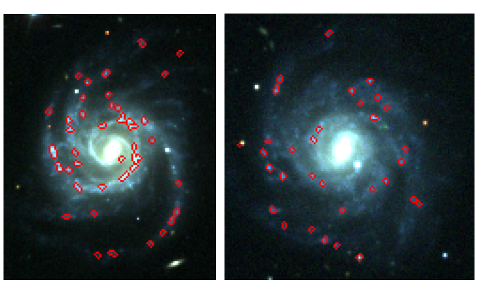

Figure 4. H II regions detection in two 5-bands astronomical images.

4 Conclusion

In this paper we have discussed the possible applications of the extension of cc-trees to multivariate images within a general framework valid for various image processing applications. We have investigated the possibilities and the difficulties inherent to the choice of a total order or pre-order related to the pixel values. Two applications in multispectral astronomical images are developed with the definition of relevant astronomical vectorial total order and pre-order.

In future works, we plan to explore the definition of the cc-trees with more general definitions of the connectivity based on non flat connected components [4].

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