COmBinatorial optimization and Related Algorithms

Graded Project   2018-2019

Variants of Vehicle Routing Problem :
Formulations, Implementations and Analysis

Consider a delivery company that is charged to carry goods on a road network from the site s (called a *source*) to the site t (called a *target*). The other sites of this network are either warehouses (depots) or junctions (crossroads). This is the context where we describe below several variations of a problem called *Vehicle Routing Problem (VRP).*

The road network can be described using a directed graph $G = (V, E)$ where the edges E are roads, while the vertices $V = D \uplus J$ are respectively the depots D, and the junctions J. Given such a graph $G = (V, E)$ and two vertices $s, t \in V$, we want to find a path from s to t in G that fulfills some optimality constraint (see below). We only want to consider *simple* (also called *elementary*) paths where each vertex appears at most once. Define the *length* of a path as its number of vertices that belong to D, i.e., that are depots.

We define below five problems (Problem 1 - Problem 5) that differ from their notion of path optimality. For each of them, your first task will be to propose Mixed Integer Linear Programming (MIP) formulation. For solving the first three ones you may make use of the techniques deployed in the 2017/18 COBRA graded project, while for solving the last two ones new ideas may be required. Furthermore, you may implement your solutions with the AMPL tool and run your codes on a set of benchmarks that will be given to you.

A brief report should be send as a pdf file named `<name1><name2>COBRAreport.pdf` to `randonov&irisa.fr` with the subject "`COBRA report`" that should contain : i) a description of the variables and the constraints you have defined for each of your MIP formulation; ii) your observations concerning your experiments for each problem and each input from the benchmark: # of variables, # of constraints, # of simplex iterations; # of branch and bound nodes that have been explored.

Also, this email should contain a separate text file containing the AMPL solutions named `<name1><name2>COBRAAMPL.txt`.

The problems you should consider are listed in the next page.

We fix a graph G and two vertices s, t on this graph.

Problem 1: Find the longest path from s to t.

Problem 2: Given a set of depots $S \subseteq D$, find a path from s to t that maximizes the number of vertices from S visited by this path[1].

Problem 3: Given an ordered set (list) of depots $L \subseteq D$, find a path from s to t that visits the vertices of L in order; the path may also visit extra vertices that do not belong to L.

Problem 4: We consider a set $\{L_1, \ldots, L_m\}$ of lists of vertices of D (only); these lists may share some elements.

The motivation for considering several lists is to represent customers where Customer i is modeled by the list L_i of depots she owns. We say that Customer i is *served* if the depots of L_i are visited in order.

The problem is to find a path from s to t that maximizes the number of served customers.

Problem 5: We consider a variant of Problem 4 where the company is now equipped with two trucks. The trucks may visit the same vertex at the same time, and each customer is served by at most one truck. We define the *workload* of a truck as the number of customers it serves.

The problem is to maximize the number of the served customers while balancing the workloads of the truck, namely by minimizing the difference between the workloads of the trucks.

[1] We say that a vertex is *visited* (or *traversed*) by a path if it belongs to it.