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Single image super-resolution using sparse representations with structure constraints

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ABSTRACT

We propose a new structure tensor based regularization term in order to guide a single-image super-resolution problem based on sparse representation. The new formulation allows reducing the ringing artefacts which can be observed around edges reconstructed by existing methods. Our method, named SE-ASDS, achieves much better results than many state-of-the-art algorithms, showing significant improvements in terms of PSNR, SSIM and visual quality.

INTRODUCTION

The regularized structure tensor is given by

$$\mathbf{J}_r = \begin{bmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{bmatrix} = \begin{bmatrix} \left(\nabla I_x \nabla I_x^T\right) & \left(\nabla I_x \nabla I_y^T\right) \\ \left(\nabla I_x \nabla I_y^T\right) & \left(\nabla I_y \nabla I_y^T\right) \end{bmatrix} * G^{4\sigma} \quad (1)$$

where ∇I_x and ∇I_y are computed using separable Gaussian derivative kernels DoG_x^σ and DoG_y^σ on the channel I_j of the image I, G is a Gaussian kernel and $(*)$ is the convolution operation.
The eigenvalues (used to detect the edges) is given by

$$\lambda_\pm = \frac{g_{11} + g_{22} \pm \sqrt{(g_{11} - g_{22})^2 + 4g_{12}^2}}{2} \quad (2)$$

and the eigenvectors (used to find the stream line) is given by

$$\theta_\pm = \begin{bmatrix} 2g_{12} \\ g_{22} - g_{11} \pm \sqrt{(g_{11} - g_{22})^2 + 4g_{12}^2} \end{bmatrix} \quad (3)$$

In order to detect salient edges, we use Equation (4), where $p = (x, y)$ represent the pixel coordinates.

$$S(p) = \frac{\lambda_+(p)}{\max_{p \in I} \lambda_+(p)} \quad (4)$$

EDGENESS TERM

We propose the regularization term presented in Equation (5)

$$E_{Edg}(I_h) = \phi(p) \frac{\beta_2}{2} \left(I_h(p) - \tilde{I}_h^{edg}(p) \right)^2 \quad (5)$$

where β_2 is a regularization parameter and $\phi(p)$ (given by Equation (6)) allows to apply this constraint only on salient edges (on the stream line).

$$\phi(p) = \begin{cases} 1, & S(p) > \nu \\ 0, & otherwise \end{cases} \quad (6)$$

The stream line is presented and explained in Figure 1.

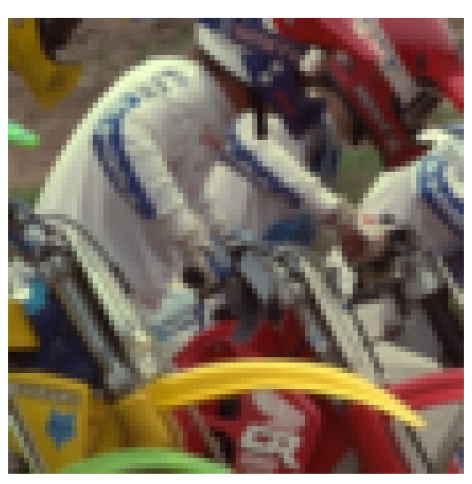

FIGURE 1: The yellow box corresponds to the current pixel p_{sl}^0. The stream line is given in blue; The energy term E_{Edg} forces the value of the current pixel to be as close as possible to pixel values having lowest saliency (i.e., meaning that pixel belongs to flat area).

In Equation (5), $\tilde{I}_h^{edg}$ is the linear combination of pixel values of the stream line in the direction $\pm\theta_+$, defined by

$$\tilde{I}_h^{edg} = \sum_{i=-sl}^{sl} \alpha_i I^h(p_{sl}^i) \quad (7)$$

where p_{sl}^i are the pixels values located on the stream line defined by direction $\pm\theta_+$ in p_{sl}^0. The weights α_i are computed as

$$\alpha_i = \xi_i \exp\left(- \frac{\left[I_h(p_{sl}^i) - I_h(p_{sl}^0) \right]^2}{h} \right) \quad (8)$$

where $\left[I_h(p_{sl}^i) - I_h(p_{sl}^0) \right]^2$ is squared difference between the pixel $I_h(p_{sl}^i)$ belonging to the stream line and the central pixel p_{sl}^0; and h is a decay factor. In this work, h is adaptively computed as the instantaneous power $h = \|.\|_2$ for each stream line. Weights α_i are positive and normalized such that $\sum_{i=-sl}^{sl} \alpha_i = 1$.
The weights $\{\xi_i\}$ are binary and computed using the following equation:

$$\xi(p_{sl}^i) = \begin{cases} 1, & S(p_{sl}^i) \le S(p_{sl}^0) \\ 0, & otherwise \end{cases} \quad (9)$$

The main idea is to sharpen salient edges by forcing the current pixel value to be as close as possible to values of pixels belonging to less salient edges.

ALGORITHM

The pseudo-code of the proposed algorithm for computing the Edgeness Term is

Algorithm 1: Implementation of the I_h^{edg} for SE-ASDS

input : HR image I_h^0; iterations N; constant ζ.
output: sharper HR image I_h^{edg}.
for $i \leftarrow 0$ **to** N **do**
  Compute the structure tensor $\mathbf{J}$;
  Compute the regularized structure tensor $\mathbf{J}_r$;
  Compute eigenvectors and eigenvalues;
  Compute the energy term $E = \frac{\partial E_{Edg}(I_h)}{\partial I_h}$;
  foreach *pixel p of the HR picture* **do**
    $I_h^{i+1}(p) = I_h^i(p) - \zeta E(p)$

MINIMIZATION

Using Iterative Shrinkage Thresholding (IST), the proposed method minimizes the cost function $E(I_h) = E(I_h|I_l) + E_{Edg}(I_h) + E_{NL}(I_h) + E_\alpha(I_h)$, where the term $E_{Edg}(I_h)$ denotes the new regularization term: Edgeness Term.
The starting point of this iterative scheme is given by a first HR guess noted I_h^0 (using bicubic interpolation):

$$I_h^{t+1} = I_h^t - \frac{\partial E(I_h^t)}{\partial I_h} \quad (10)$$

where

$$\begin{aligned} \frac{\partial E(I_h^t)}{\partial I_h} \approx \; & \beta_1 \left((I_h * G) \downarrow - I_l \right) \uparrow * G \\ & + \phi(p) \frac{\beta_2}{2} \left(I_h - I_h^{edg} \right) \quad \text{"equal to } E_{Edg}(I_h)\text{"} \\ & + \beta_3 \left(I_h - I_h^{NL} \right) \\ & + \beta_4 |\alpha|_1 \end{aligned} \quad (11)$$

and α is a sparse representation of I_h on a sub-dictionary Φ_k.
Note that Equation (11) is an approximation since the derivative of the new Edgeness Term $\frac{\partial E_{Edg}(I_h)}{\partial I_h}$ is not rigorously equal to $\beta_2 \phi(p) \left(I_h - I_h^{edg} \right)$. In order to derive the second term (in red) of Equation (11), we made the assumption that $I_h(p_{sl}^i) \approx I_h(p_{sl}^0)$ in Equation (8). To the best of our knowledge, this strategy is reasonable and locally valid when we choose a short length stream line as in performed experiments.

EXPERIMENTAL RESULTS

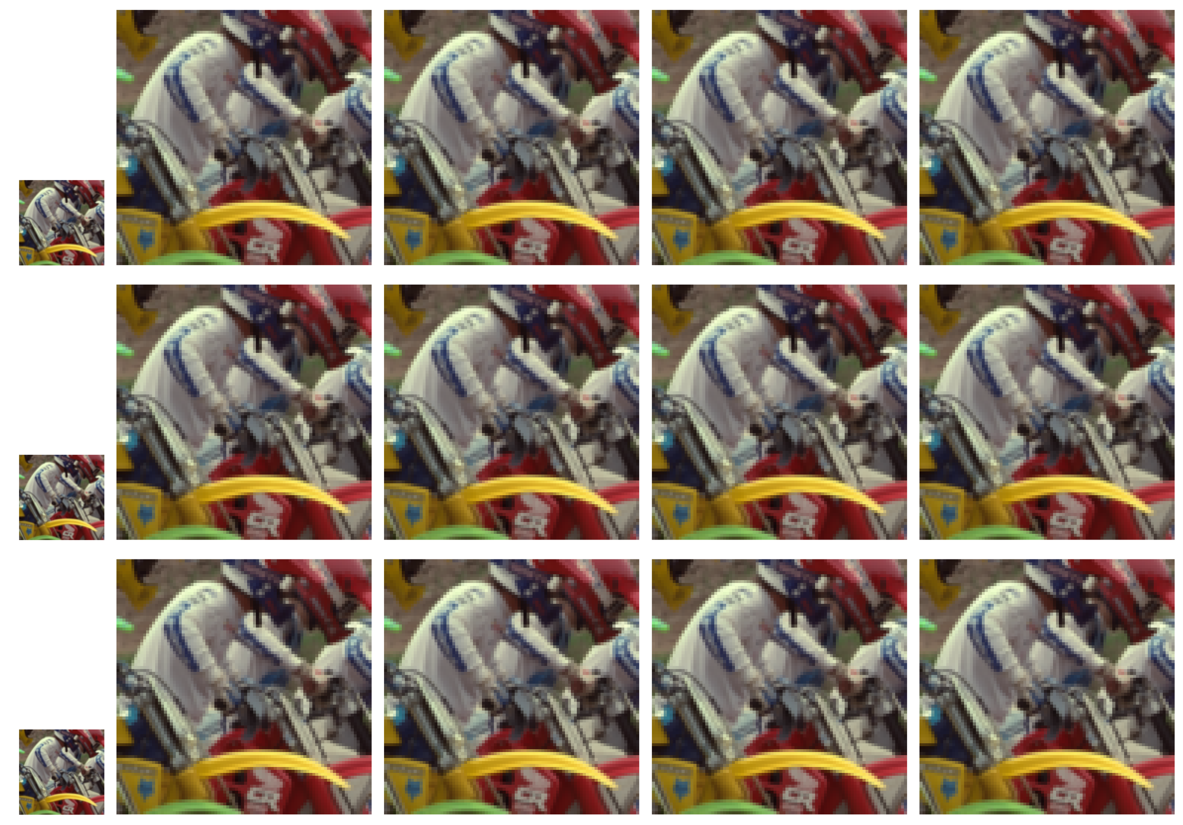

FIGURE 2: 1st column: LR image ($\div 3$); 2nd: NN; 3rd: Dong et al.'s ASDS results; 4th: Our results; 5th: comparison between 3rd and 4th columns on patches: edges of 4th column are more contrasted than 3rd.

Images	[1]	[2]	[3]	[4]	[5]	SE-ASDS
Girl	32.93	31.94	32.51	31.21	33.54	**33.56**
	0.8102	0.7704	0.7912	0.7878	0.8242	**0.8252**
Parrot	28.78	27.71	27.98	27.59	30.00	**30.29**
	0.8845	0.8682	0.8665	0.8856	0.9093	**0.9136**
Butterfly	25.16	25.19	23.73	26.60	27.34	**28.48**
	0.8336	0.8623	0.7942	0.9036	0.9047	**0.9236**
Leaves	24.59	24.34	24.35	24.58	26.80	**27.69**
	0.8310	0.8372	0.8170	0.8878	0.9058	**0.9261**
Parthenon	26.32	25.87	24.08	25.89	26.83	**27.05**
	0.7135	0.6791	0.6305	0.7163	0.7349	**0.7446**
Flower	28.16	27.50	27.76	27.38	29.19	**29.29**
	0.8120	0.7800	0.7929	0.8111	0.8480	**0.8511**
Hat	29.92	29.68	29.65	29.19	30.93	**31.53**
	0.8438	0.8389	0.8362	0.8569	0.8707	**0.8805**
Raccoon	28.80	27.96	28.49	27.53	29.24	**29.27**
	0.7549	0.6904	0.7273	0.7076	0.7677	**0.7686**
Bike	23.48	23.31	23.20	23.61	24.62	**24.97**
	0.7438	0.7219	0.7188	0.7567	0.7962	**0.8098**
Plants	31.87	31.45	31.48	31.28	33.47	**34.17**
	0.8792	0.8617	0.8698	0.8784	0.9095	**0.9163**
Average	28.03	27.49	27.69	27.49	29.19	**29.63**
	0.8115	0.7910	0.7954	0.8190	0.8471	**0.8559**

Table 1: The PSNR (dB) and SSIM results (luminance components) of super-resolved HR images.

CONCLUSION

The proposed SE-ASDS approach gives better results than Daubechies [1], Yang et al. [3], Dai et al. [2], Marquina and Osher [4] and Dong et al. [5] in terms of PNSR, SSIM and visual quality for all benchmark images. In our experiments, SE-ASDS is faster and gives in average $0.44\ dB$ improvement compared to Dong et al.'s method in terms of PSNR.

References

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