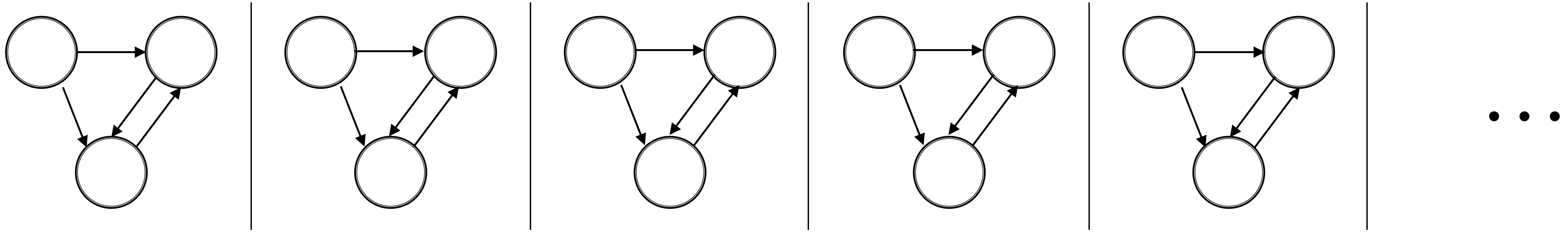
Checking Presence Reachability Properties on Parameterized Shared-Memory Systems

Nicolas Waldburger
PhD supervisors: Nathalie Bertrand, Nicolas Markey, Ocan Sankur
SynCoP23, 23/04/2023

Parameterized verification



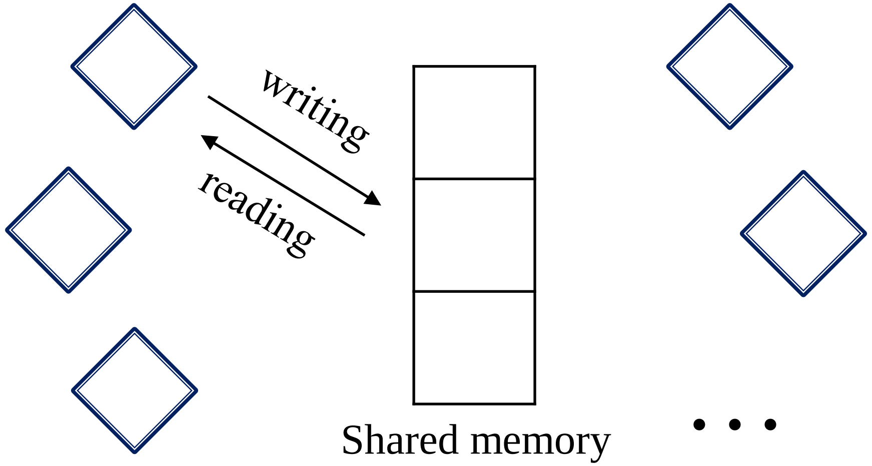
- *Arbitrary* number of processes
- Processes are *identical* agents
- No identifiers: processes are *anonymous*
- Modelled by a single, common *finite automaton*

1. Model inspired from: Esparza, J., Ganty, P., Majumdar, R.: *Parameterized verification of asynchronous shared-memory systems.* Journal of the ACM, 2016

Shared-memory systems

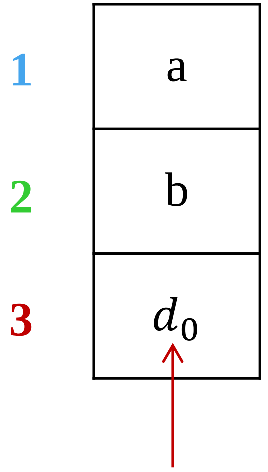
Two models in this talk:

- Simple model: shared-memory systems with finite memory
- More complex model: round-based shared-memory systems

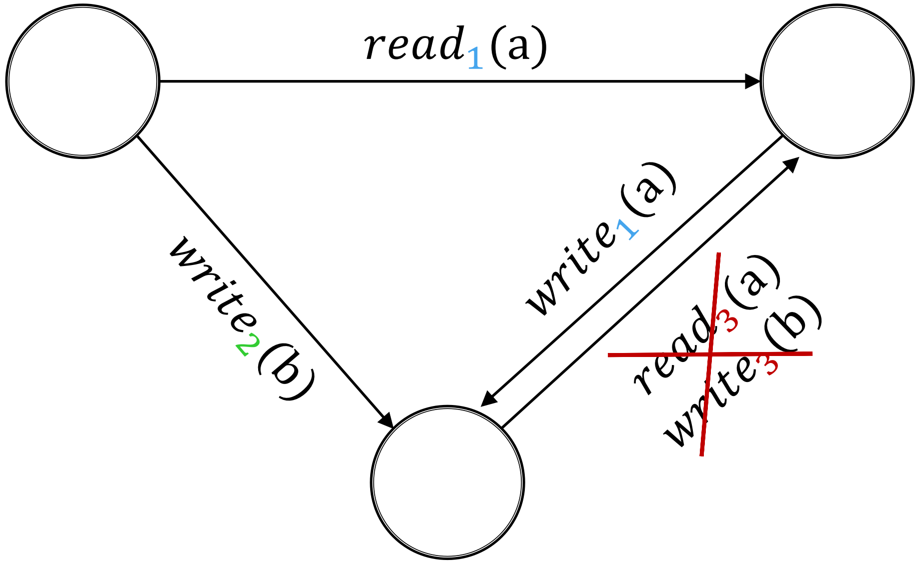


A model for shared-memory systems[1]

Finite number of shared registers, each register has a value from finite set of symbols Σ



Registers are *initialized* to value d_0

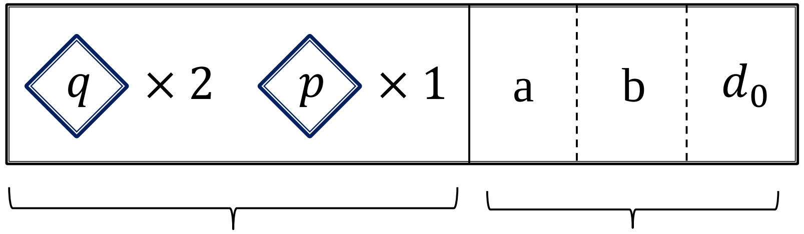


No atomic read/write combinations

1. Model inspired from: Esparza, J., Ganty, P., Majumdar, R.: *Parameterized verification of asynchronous shared-memory systems.* Journal of the ACM, 2016

Semantics

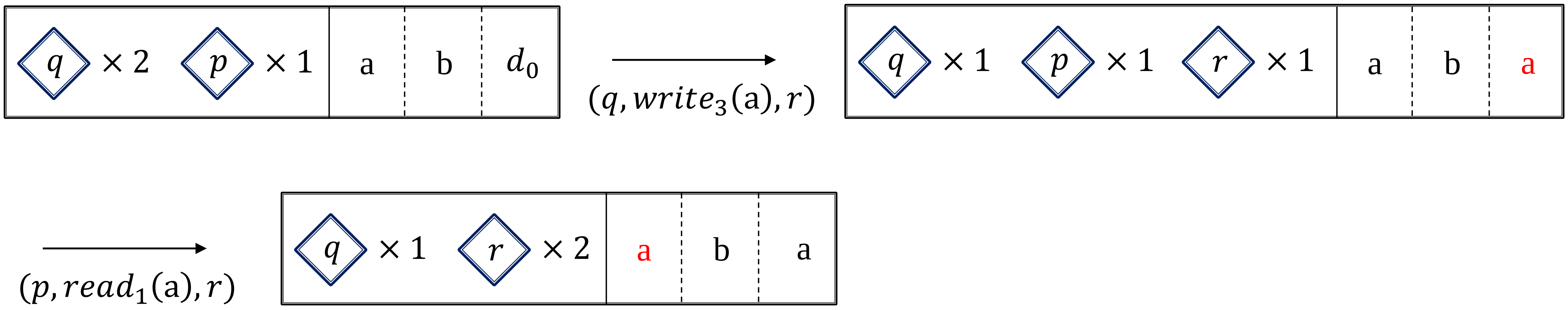
A configuration:



How many process are on each state

Content of the registers

Semantics



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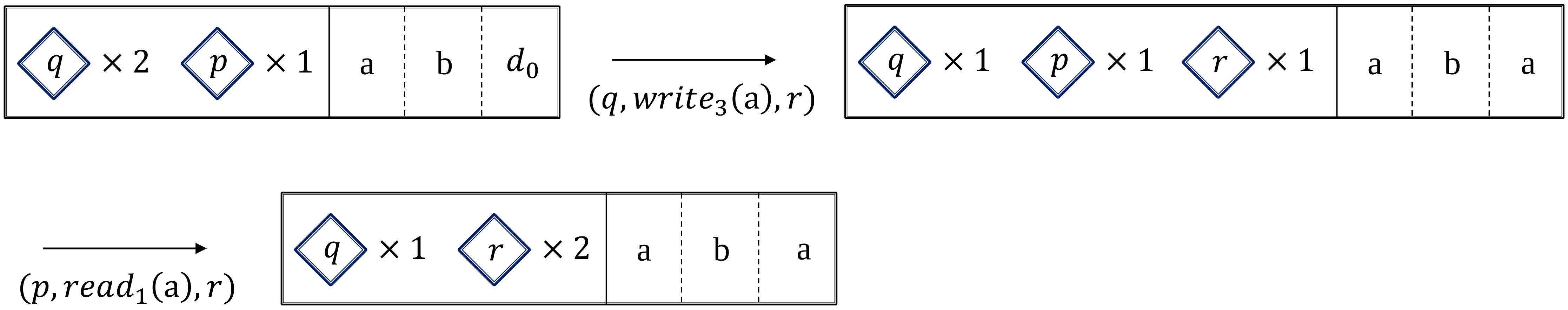
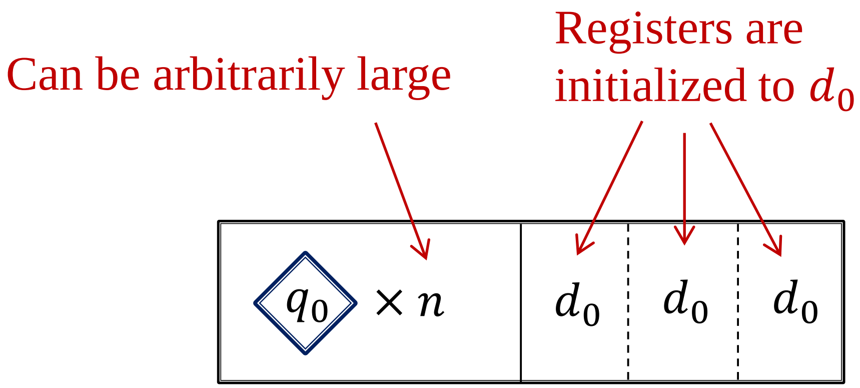
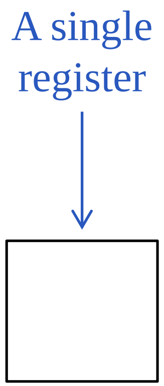


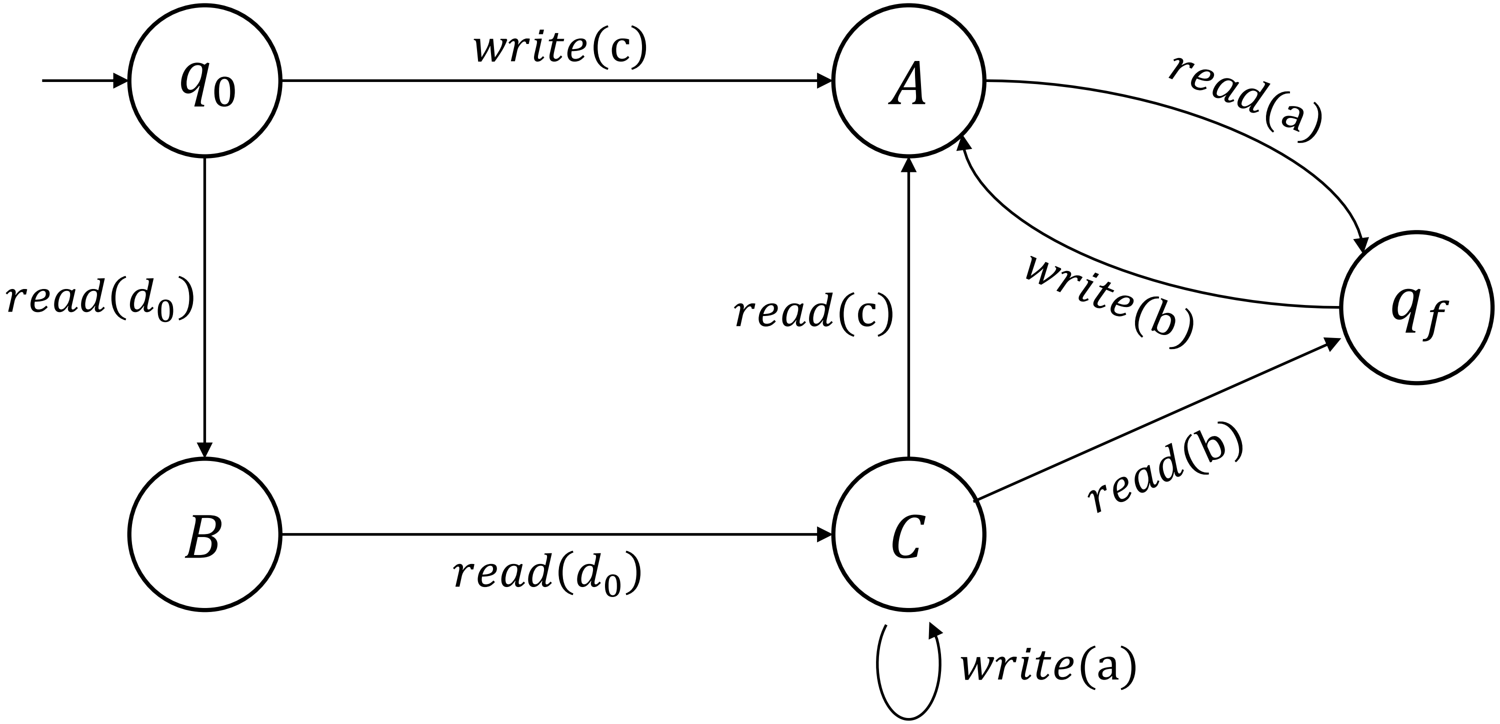
Initial configurations:



with $n \geq 0$ and q_0 the initial state

A small example





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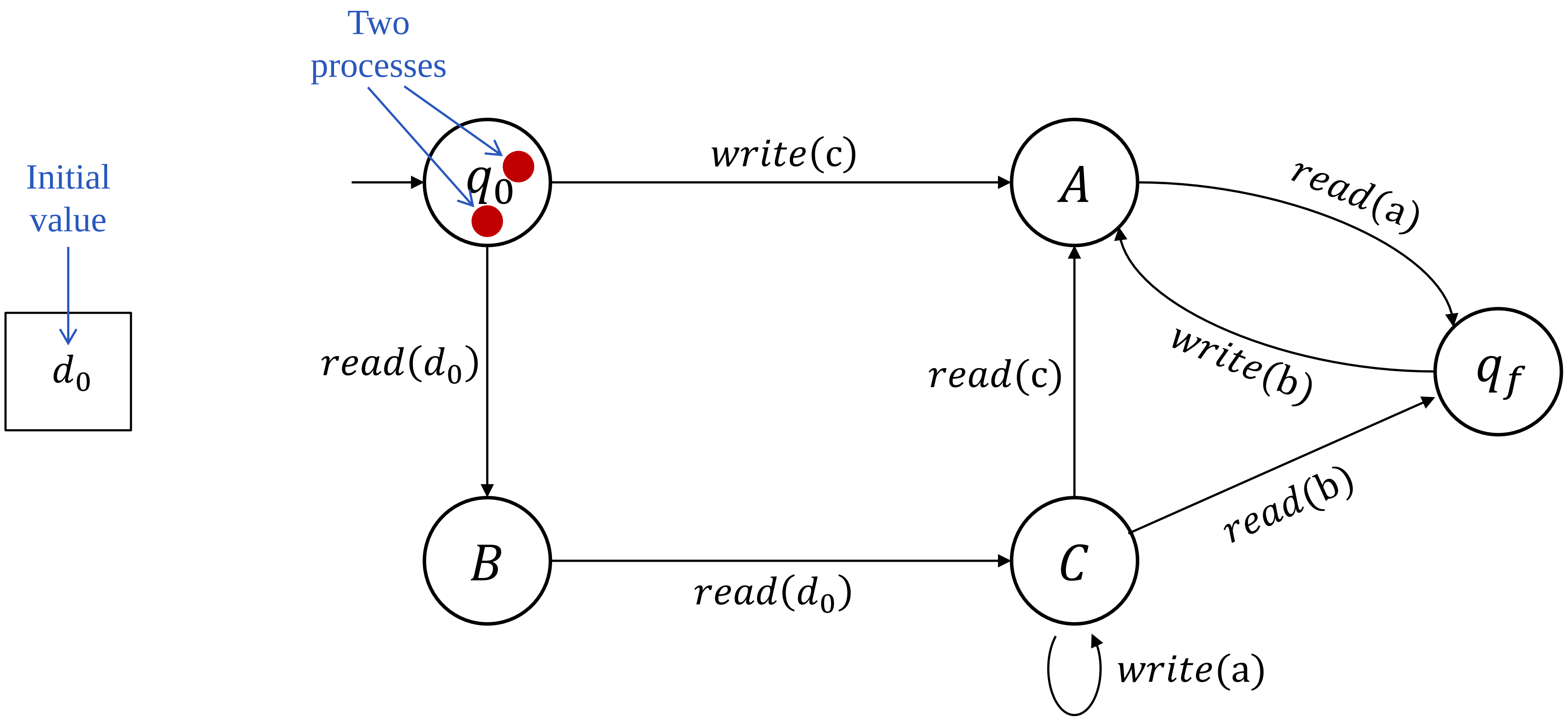

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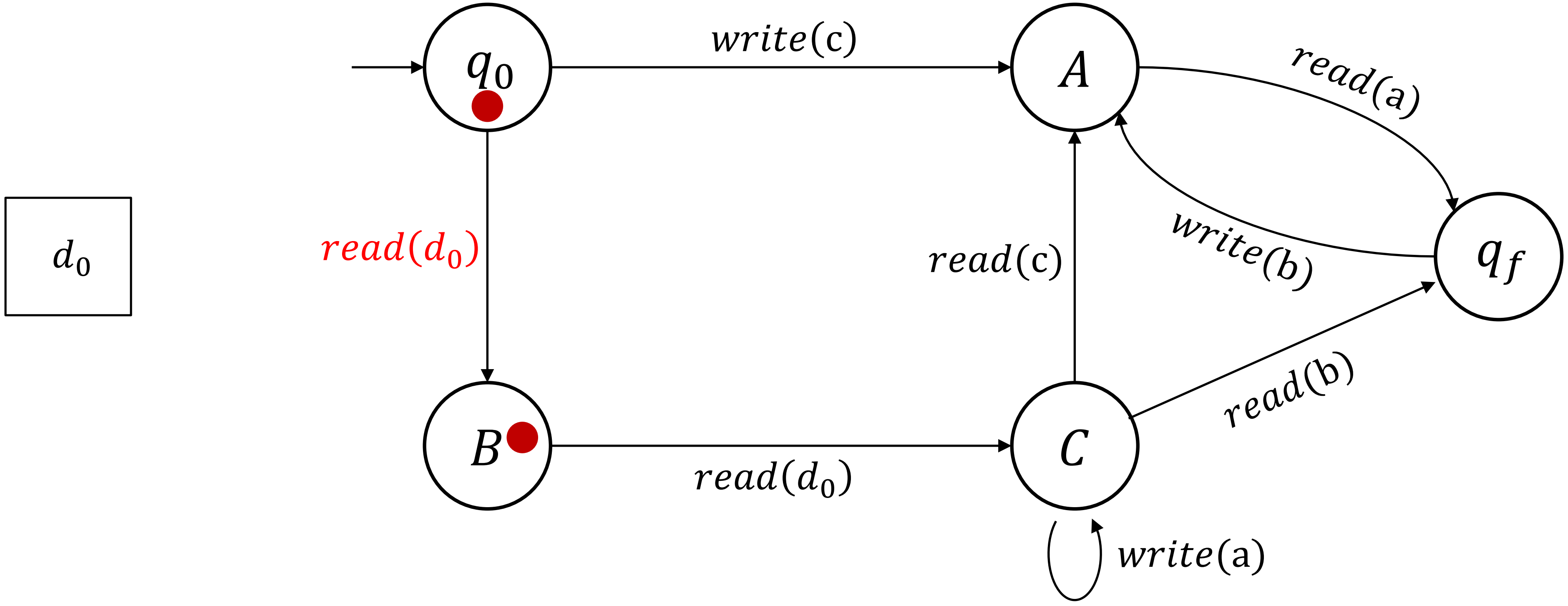

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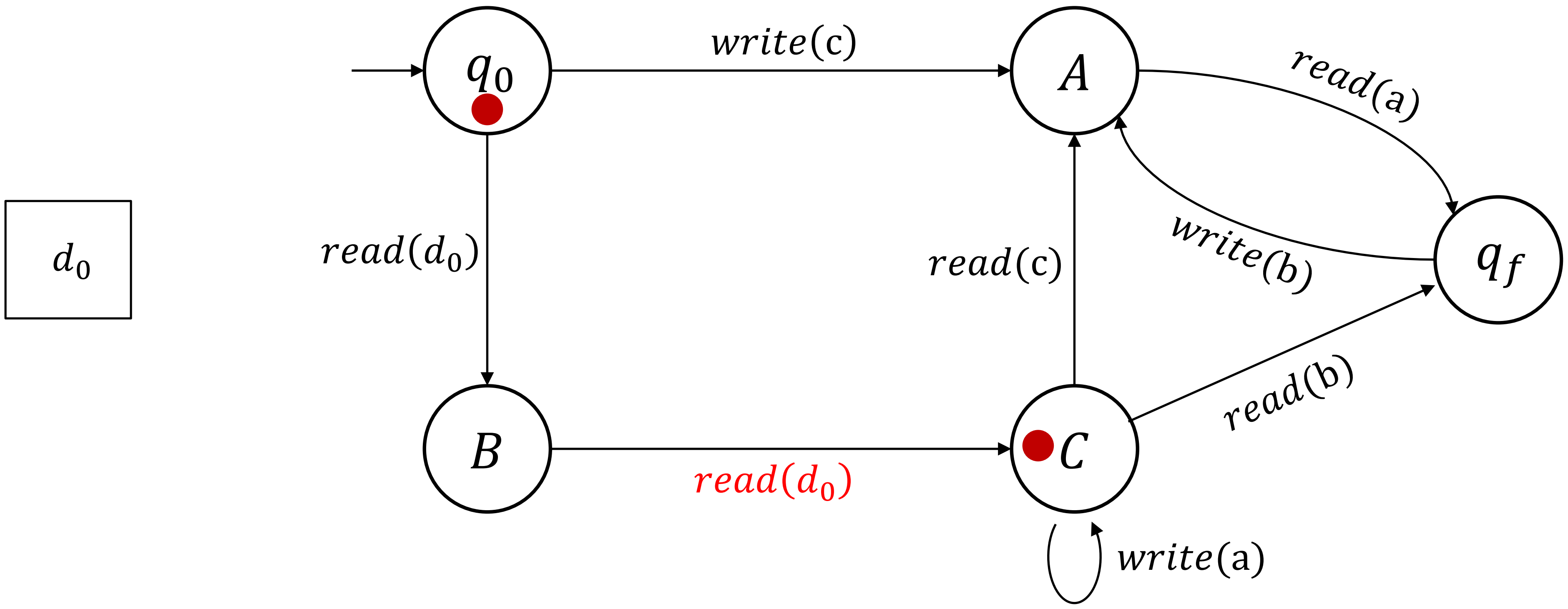
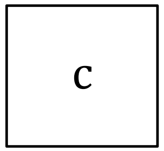
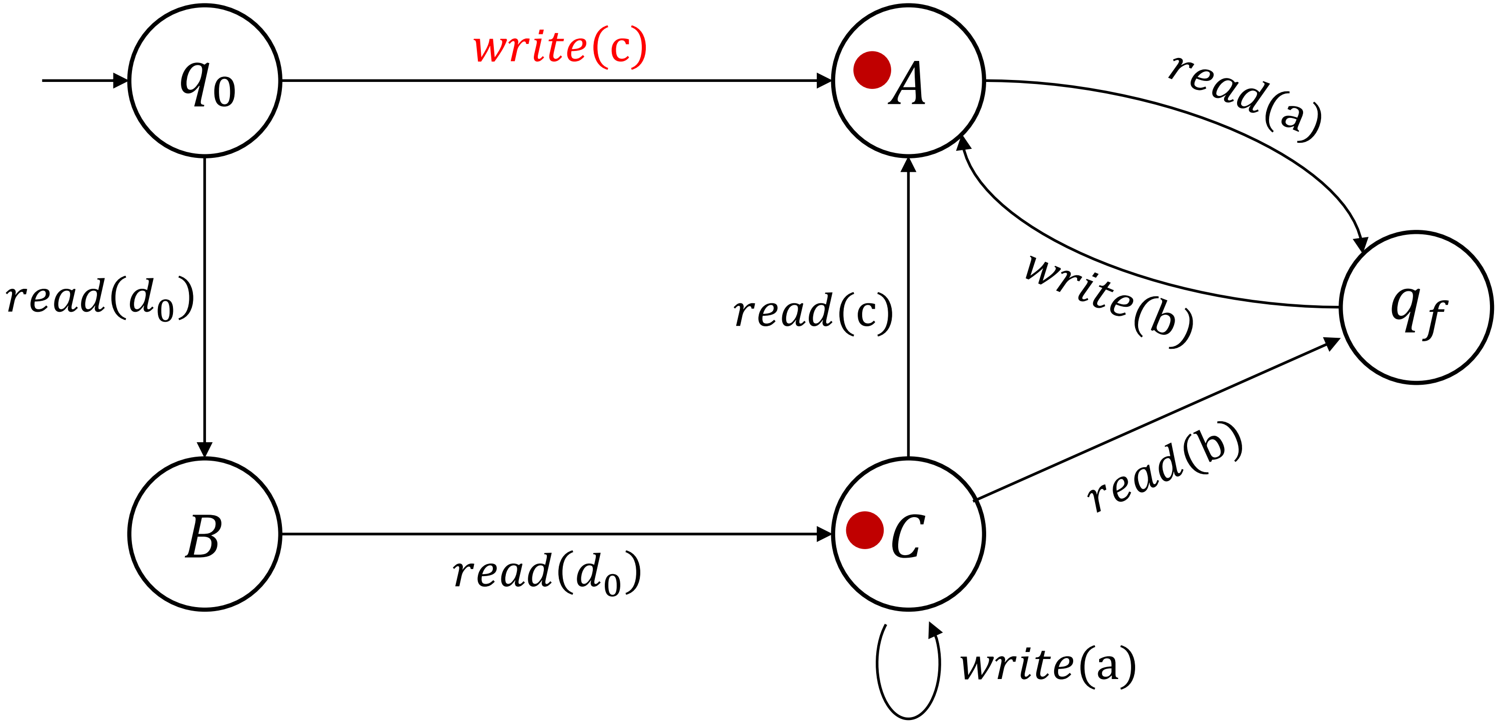


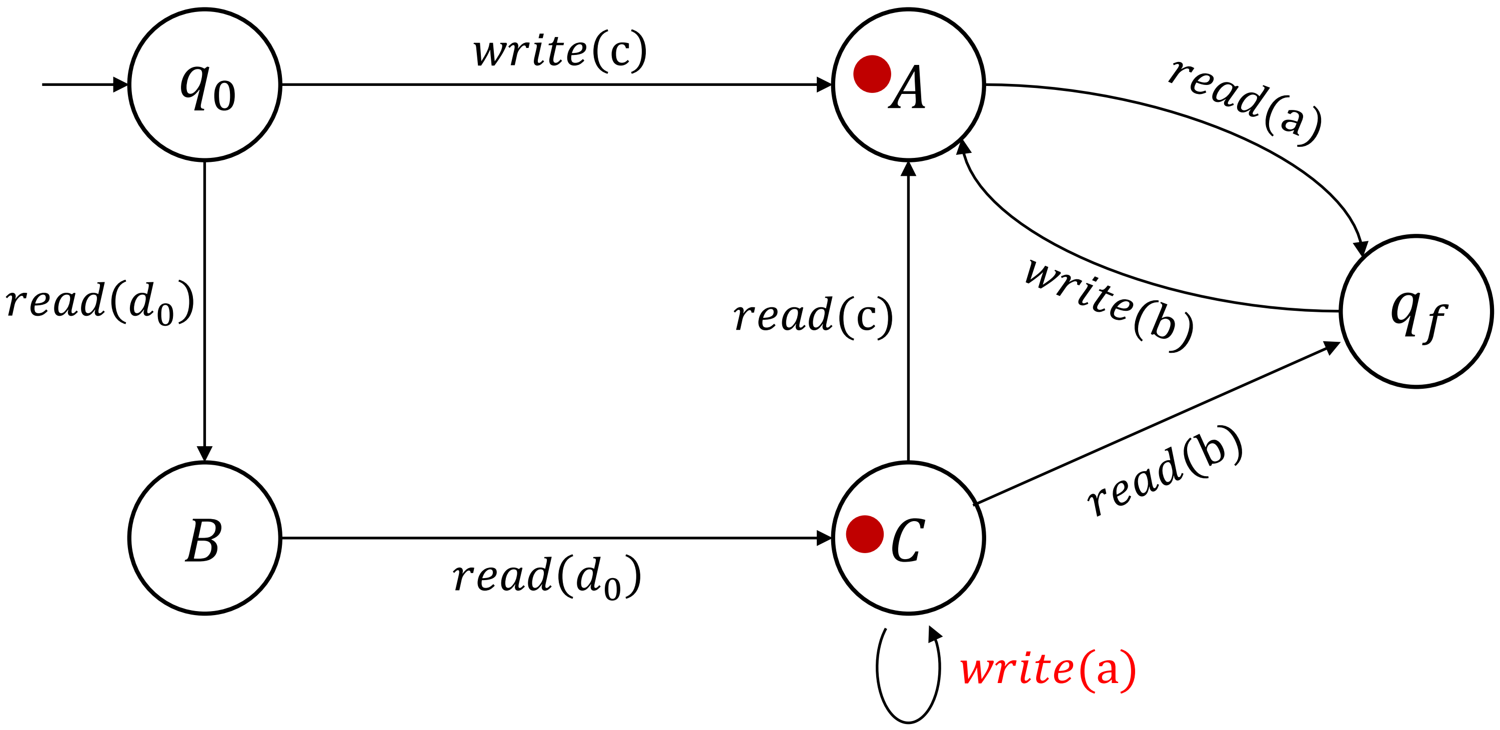
A small example



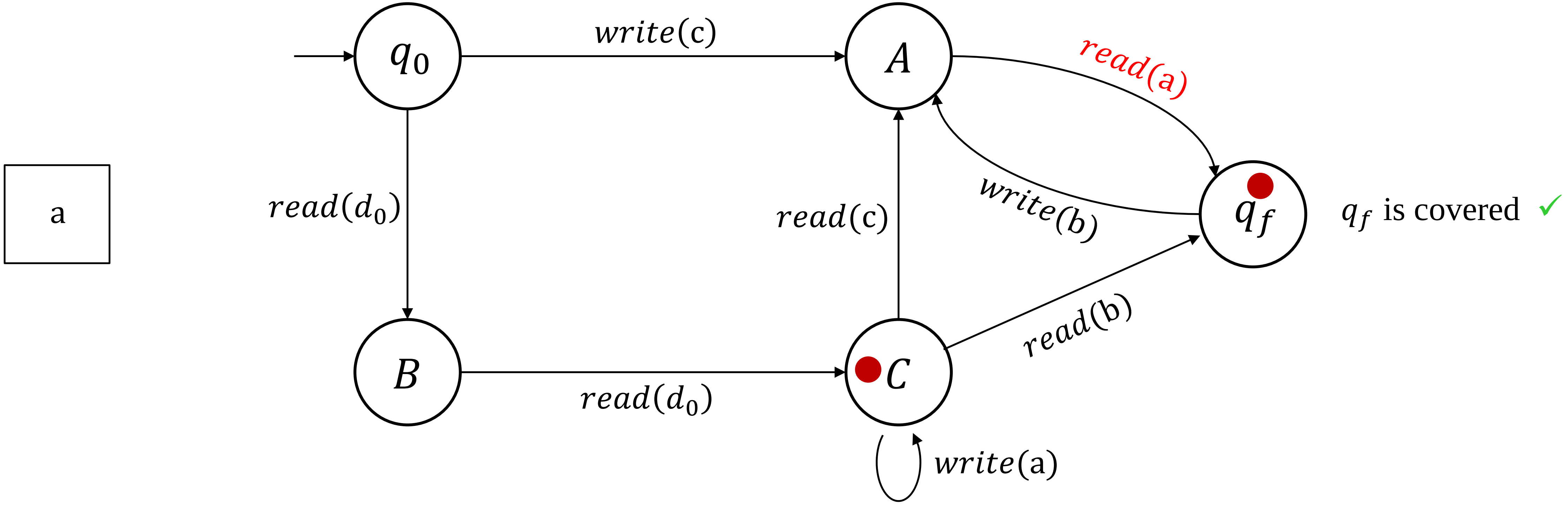


A small example

a



A small example



Reachability problems

An initial configuration

A least one process on q_f: "error state"

COVER: $\exists n,\ \exists \gamma_0,\ \exists \rho:\ \gamma_0 \rightarrow^* \gamma\ ,\ \ \gamma(q_f) > 0$?

Parameterized: arbitrarily many processes

Execution

Reachability problems

COVER: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \to^* \gamma\ ,\ \ \gamma(q_f) > 0$?

TARGET: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \to^* \gamma\ ,\ \ \forall q \neq q_f,\ \gamma(q) = 0$?

All processes
"synchronize" on q_f

Reachability problems

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TARGET: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \to^* \gamma\ ,\ \ \forall q \neq q_f,\ \gamma(q) = 0$?

PRP[2]: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \to^* \gamma\ ,\ \ \gamma \vDash \phi$?

with $\phi \in \mathcal{B}(\{\#q = 0, \#q > 0\}, \{\mathbf{reg}_i = d, \mathbf{reg}_i \neq d\})$

↑ Presence Reachability Problem

$\#q$ = number of processes on q

2. Inspired from CRP in: Delzanno, G., Sangnier, A., Traverso, R., Zavattaro, G.: *On the Complexity of Parameterized Reachability in Reconfigurable Broadcast Networks*, Tech. Rep., 2012

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↑

Presence Reachability Problem

with $\phi \in \mathcal{B}(\{\#q = 0, \#q > 0\}, \{\mathbf{reg}_i = d, \mathbf{reg}_i \neq d\})$

Examples: $\phi =$ ``$\#q_f > 0$'' (COVER),

$\phi =$ ``$\bigwedge_{q \neq q_f} \#q = 0$'' (TARGET)

$\phi =$ ``$(\#q_1 > 0)\ \vee\ ([\#q_2 = 0]\ \wedge\ [\mathbf{reg}_1 = d_0])$''

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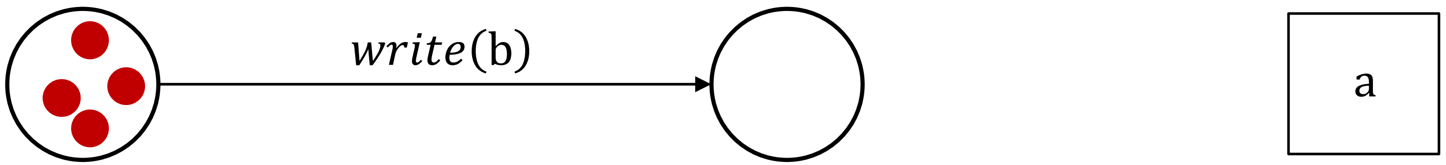
Monotonicity

A process may "copy" the behavior of another process on the same state.



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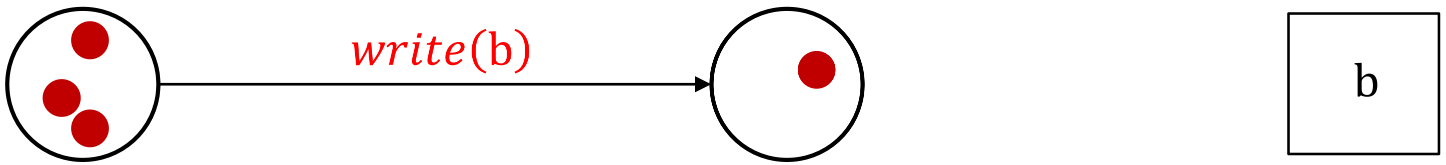

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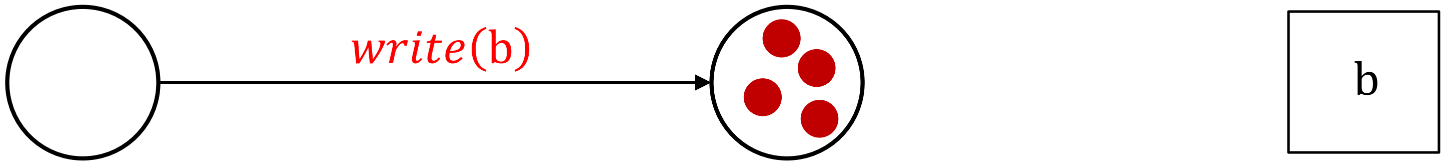

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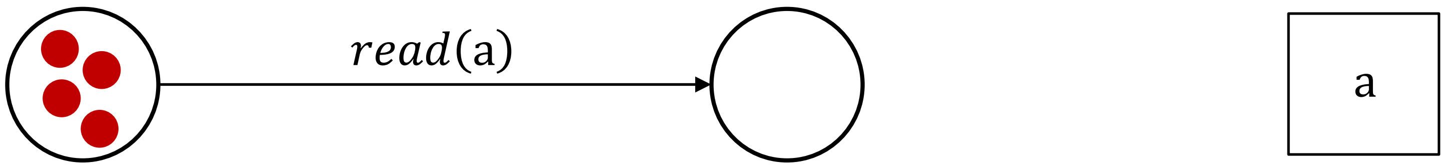

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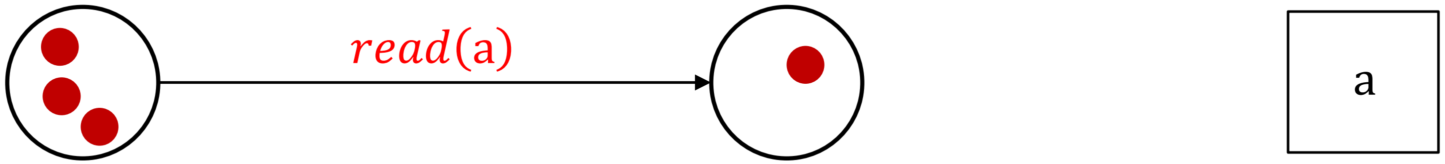

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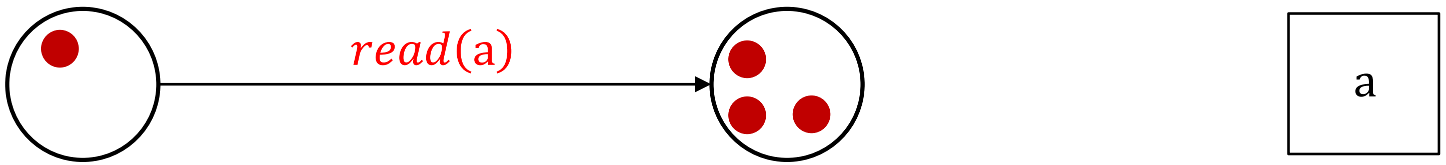

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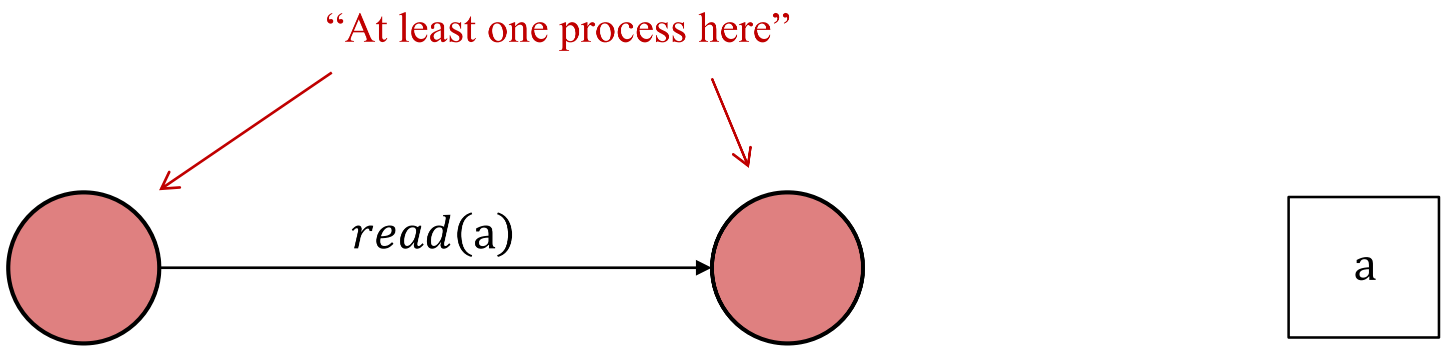
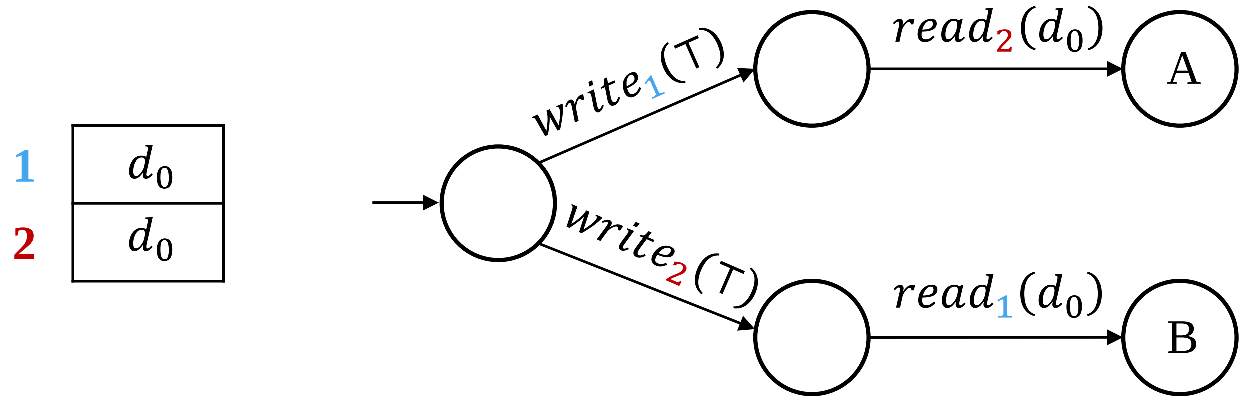


Abstraction: remember whether there is at least one process on a given state.

Sound and *Complete* for PRP because of monotonicity property

NP-completeness of COVER

COVER: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \rightarrow^* \gamma\ ,\ \ \gamma(q_f) > 0$?



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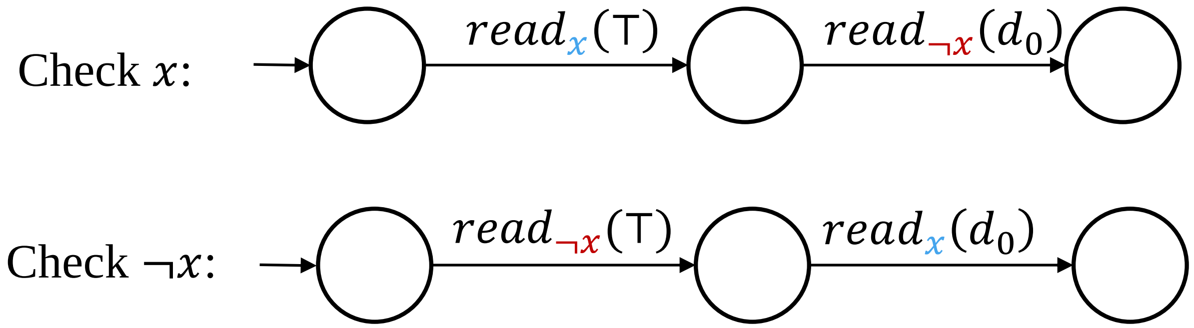
Reduction from 3-SAT:

x	d_0
$\neg x$	d_0



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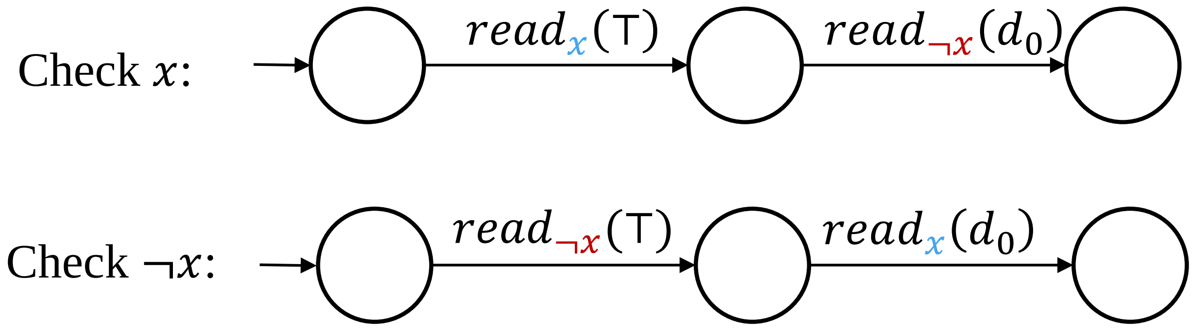


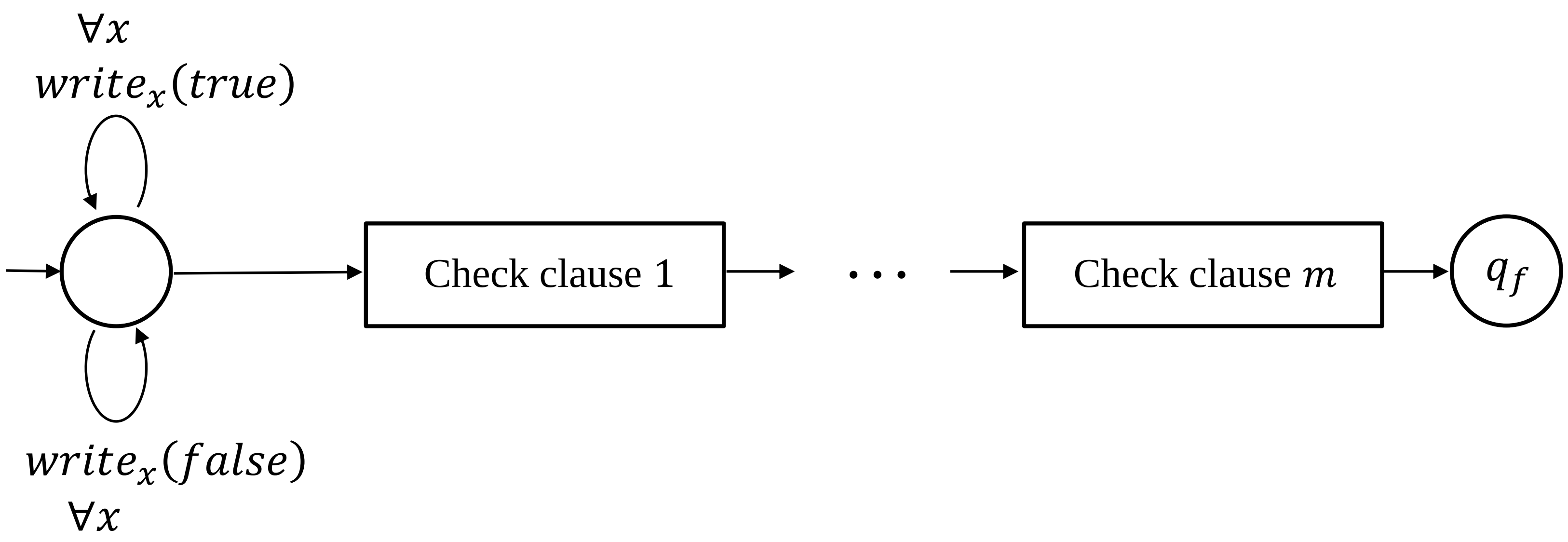
Directly relies on initialization of registers!

COVER drops down to PTIME when the registers are not initialized (applying a simple saturation technique).

TARGET when registers are not initialized

TARGET: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \rightarrow^* \gamma\ ,\ \ \forall q \neq q_f,\ \gamma(q) = 0$?

TARGET is still NP-complete when registers are not initialized. Reduction from 3-SAT:



TARGET with a single register

TARGET: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \rightarrow^* \gamma\ ,\ \ \forall q \neq q_f,\ \gamma(q) = 0$?

TARGET is PTIME when only one register.
One can reduce the problem to the case when the register is not initialized.
Algorithm inspired from broadcast protocols[4].

4. Fournier, P.: *Parameterized verification of networks of many identical processes*. PhD thesis, 2015

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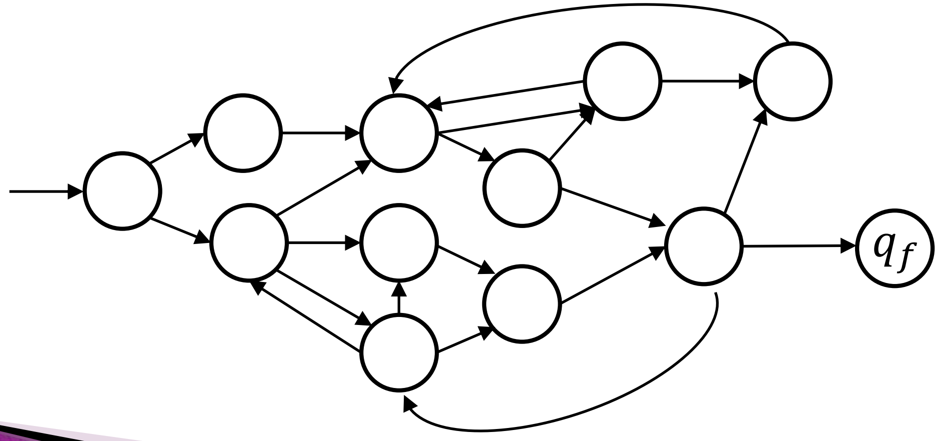
Compute *coverable states* (the state can be covered from initial configurations)
and *backwards coverable states* (q_f may be reached from some configuration containing the state).



4. Fournier, P.:
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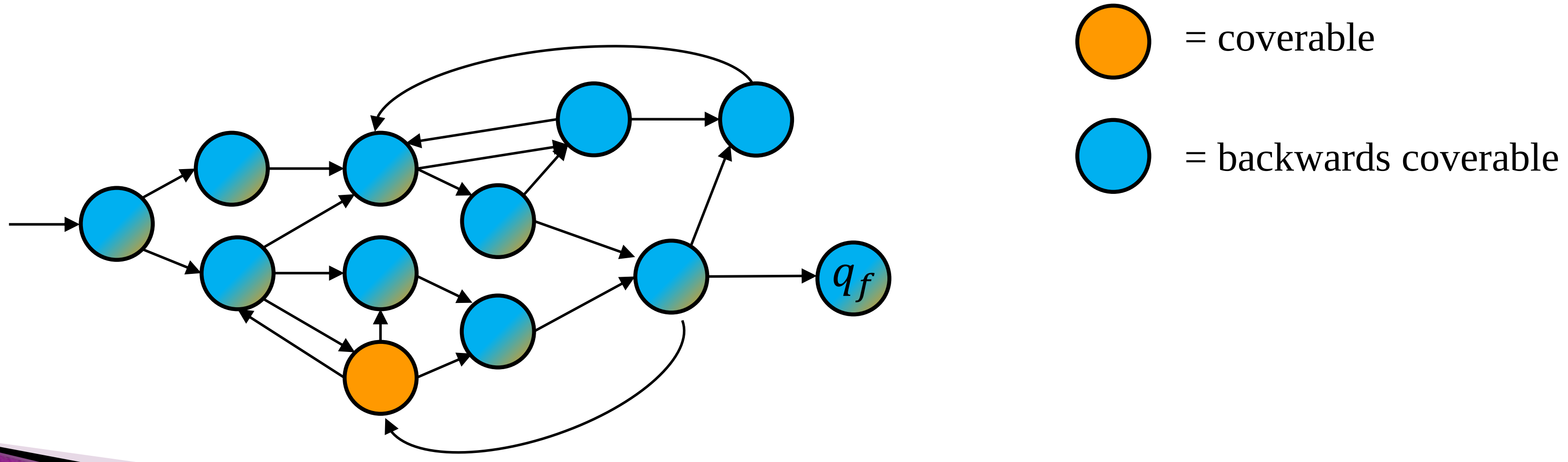

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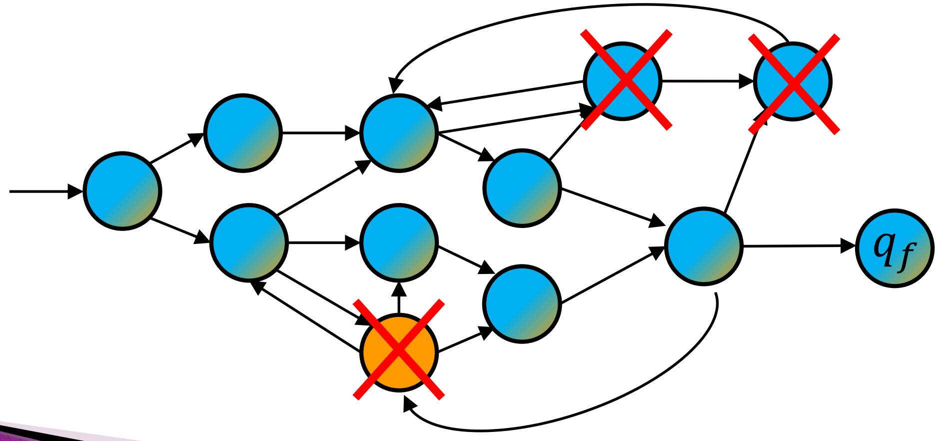
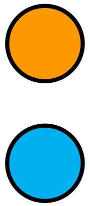
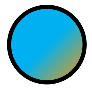




Iteratively remove all states that are not 

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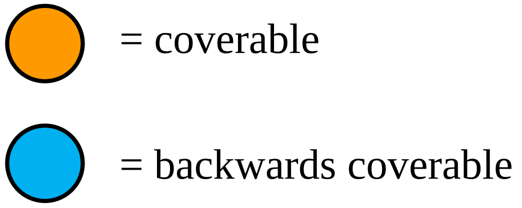
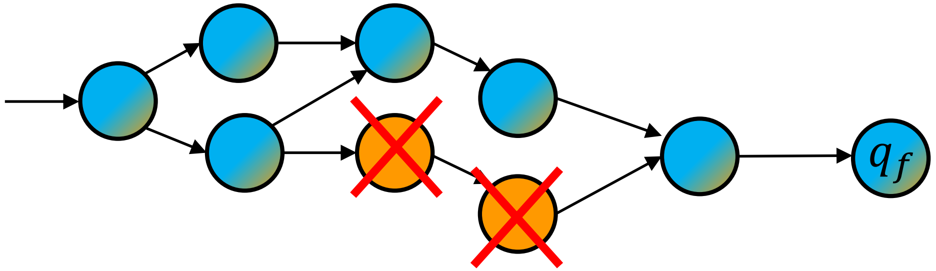
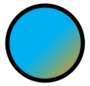




Iteratively remove all states that are not 

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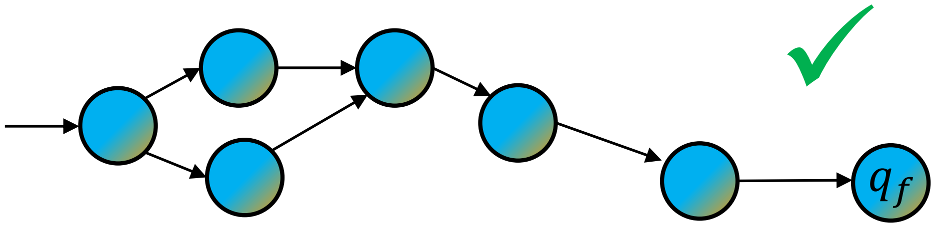
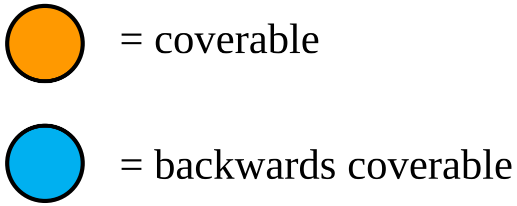
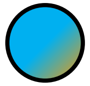




Iteratively remove all states that are not 

4. Fournier, P.: *Parameterized verification of networks of many identical processes*. PhD thesis, 2015

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For simplicity: the register is not initialized. Algorithm inspired from broadcast protocols[4].

Compute *coverable states* (the state can be covered from initial configurations)
and *backwards coverable states* (q_f may be reached from some configuration containing the state).

The algorithm is generalizable to PRP when the formula is in Disjunctive Normal Form (DNF).

DNF-PRP: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \to^* \gamma\ ,\ \ \gamma \vDash \phi,$

ϕ in DNF: $\phi = \bigvee_i \left(t_{i,1} \wedge t_{i,2} \wedge \cdots \wedge t_{i,m_i}\right),$

$t_{i,j} \in \{\#q = 0, \#q > 0\} \cup \{\mathbf{reg}_i = d, \mathbf{reg}_i \neq d\}$

4. Fournier, P.:
Parameterized verification of networks of many identical processes. PhD thesis, 2015

Summary of complexity results[5]

	COVER	TARGET	DNF-PRP	PRP
General case	NP-complete	NP-complete	NP-complete	NP-complete
Not initialized	PTIME-complete	NP-complete	NP-complete	NP-complete
One register	PTIME-complete	PTIME-complete	PTIME-complete	NP-complete

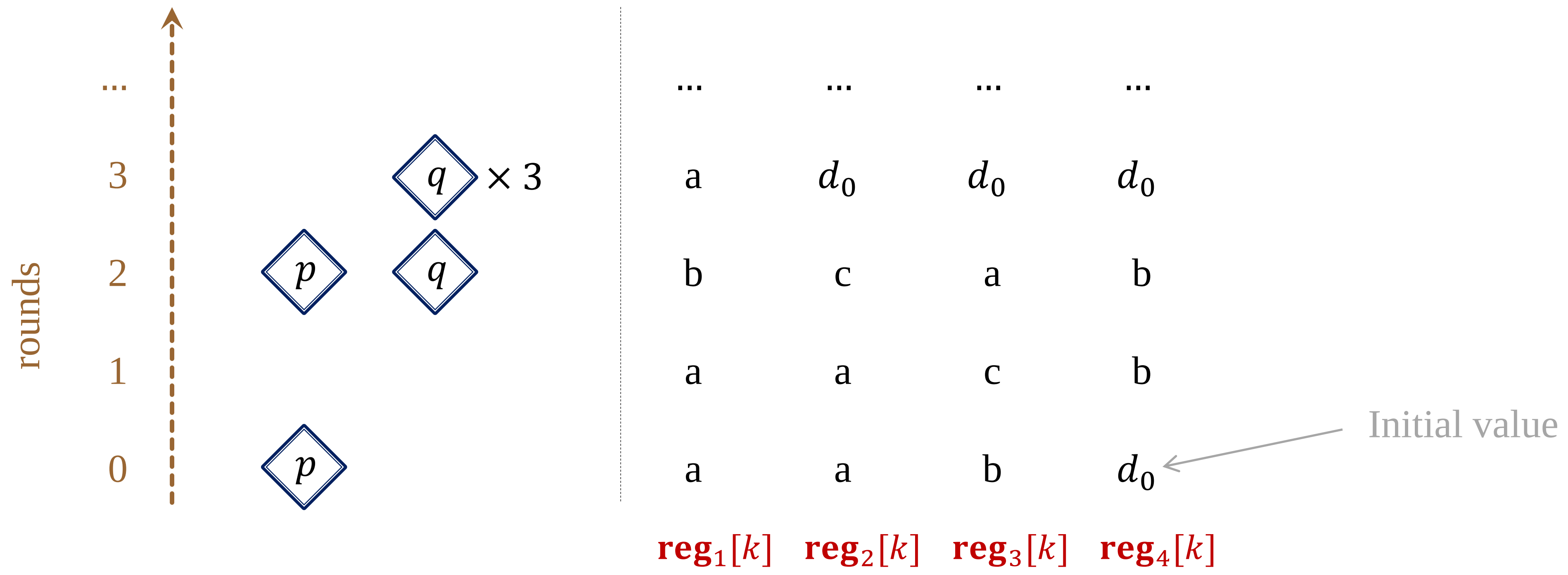
5. W: *Checking Presence Reachability Properties on Parameterized Shared-Memory Systems*, submitted

Round-based shared-memory systems

Round-based shared-memory systems

Model inspired by round-based algorithms from the literature[678].

Process progress in asynchronous rounds, each round having its own finite set of registers.



6. Aspnes, J.: *Fast deterministic consensus in a noisy environment.* Journal of Algorithms, 2002

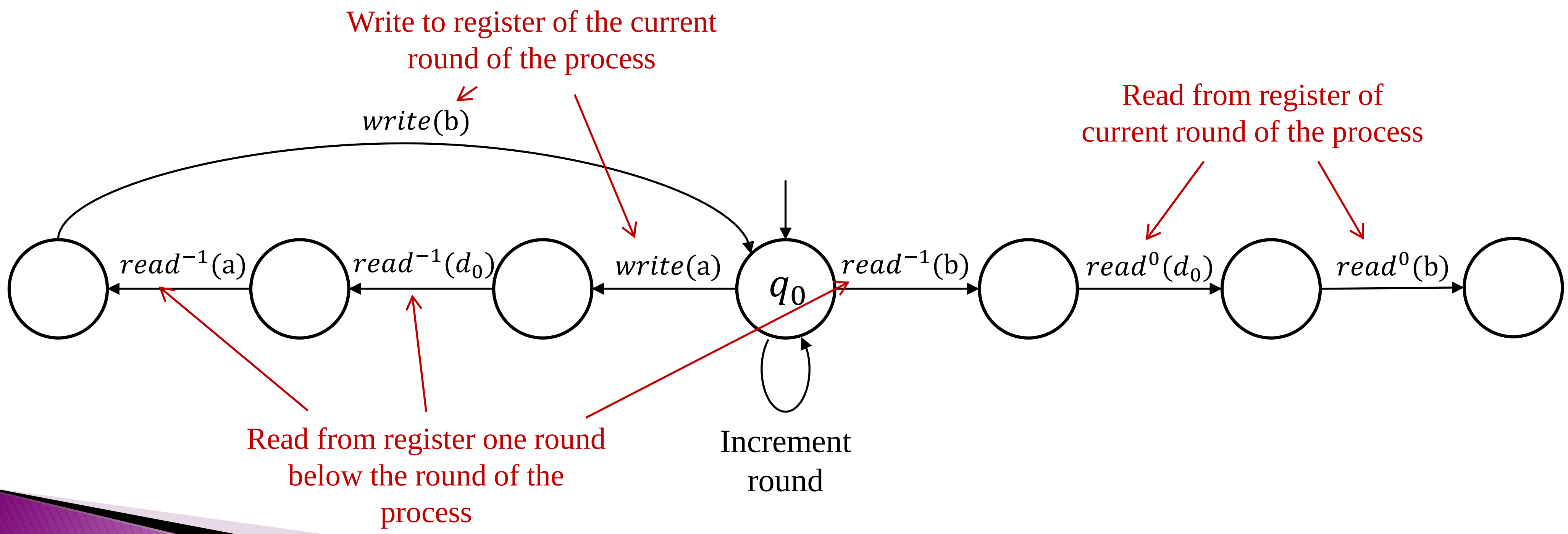
7. Guerraoui, R., Ruppert, E.: *Anonymous and fault-tolerant shared-memory computing.* Distrib. Comput., 2007

8. Raynal, M., Stainer, J.: *A Simple Asynchronous Shared Memory Consensus Algorithm Based on Omega and Closing Sets. CISIS, 2012*

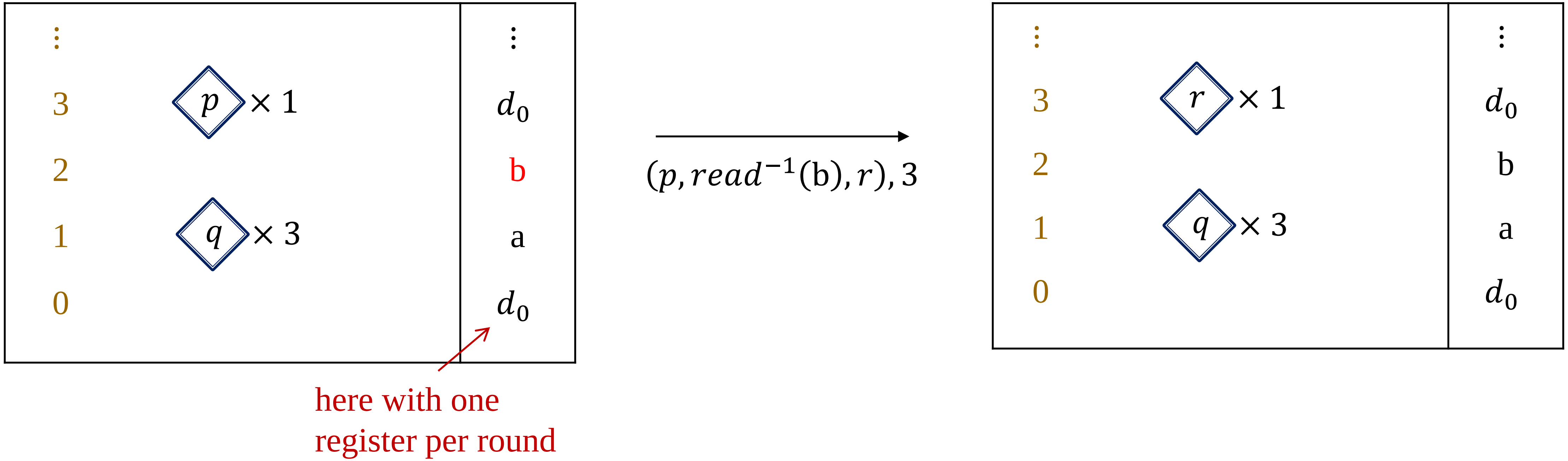
The round-based model

- Read transitions now mention from which round they are reading, relatively to the current round of the process
- A new type of transitions: *round increments*, which send the process to the next round

Example with one register per round:

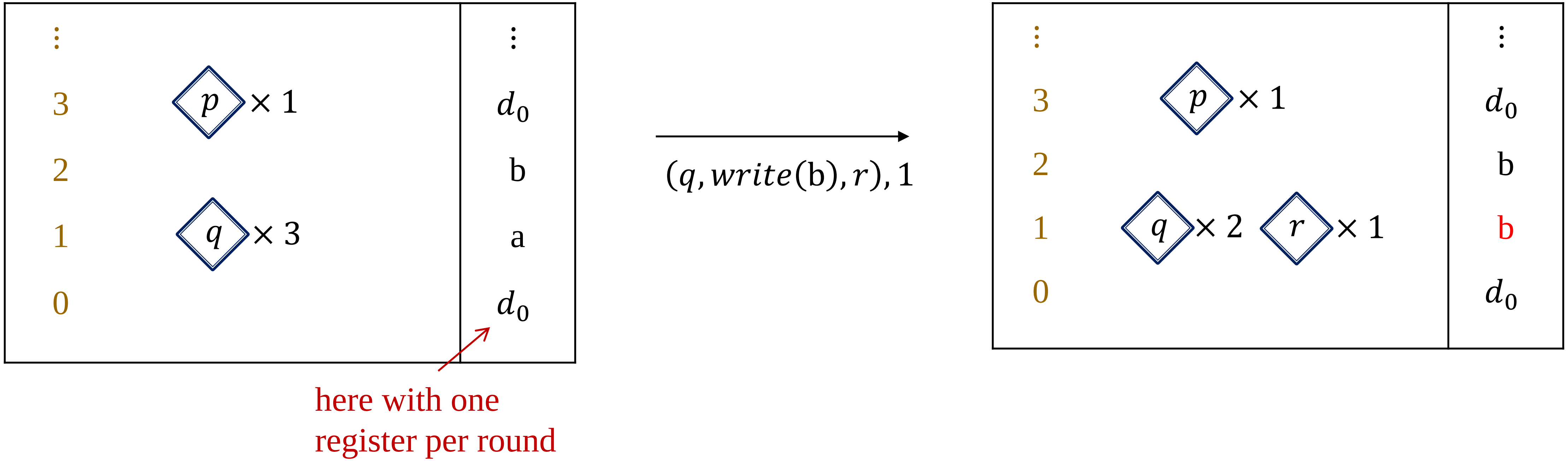


Semantics

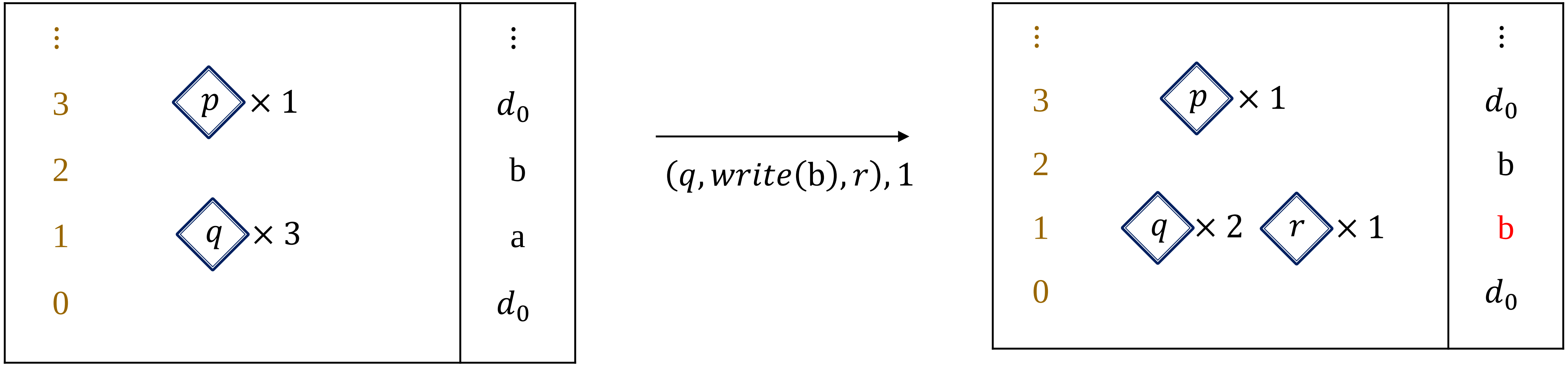


⋮
3
p × 1
d_0
2
b
1
q × 3
a
0
d_0
here with one
register per round
$(p, read^{-1}(\mathrm{b}), r), 3$
⋮
3
r × 1
d_0
2
b
1
q × 3
a
0
d_0

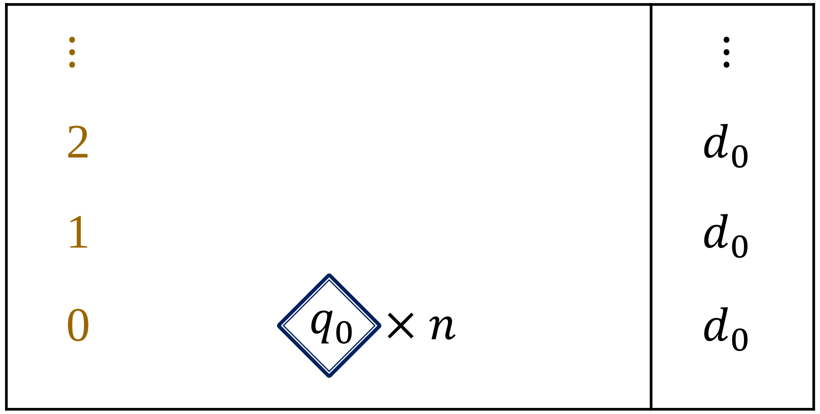
Semantics


⋮
3
2
1
0
p × 1
q × 3
⋮
d_0
b
a
d_0
here with one
register per round
$(q, write(\mathrm{b}), r), 1$
⋮
3
2
1
0
p × 1
q × 2
r × 1
⋮
d_0
b
b
d_0

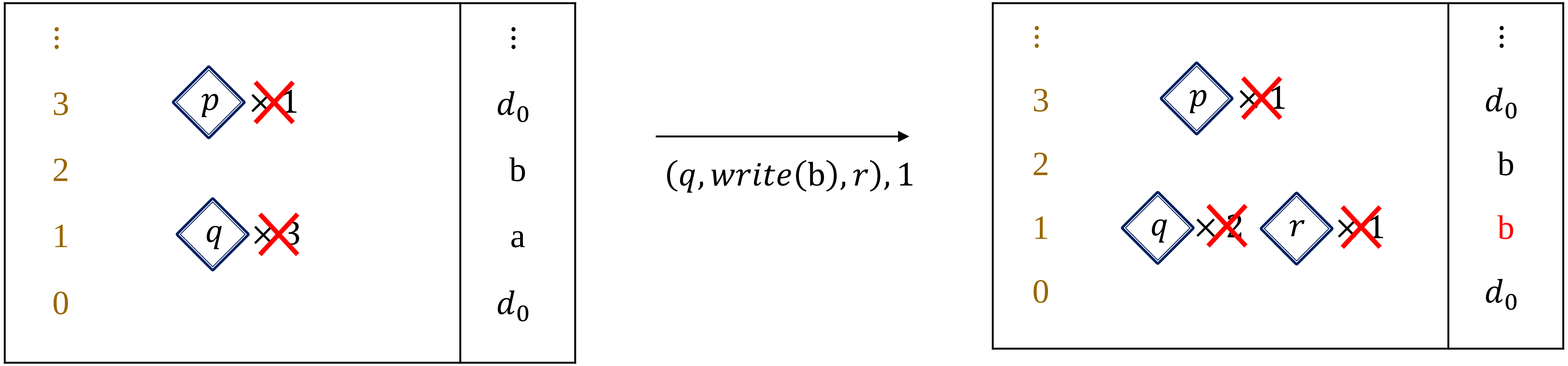
Semantics



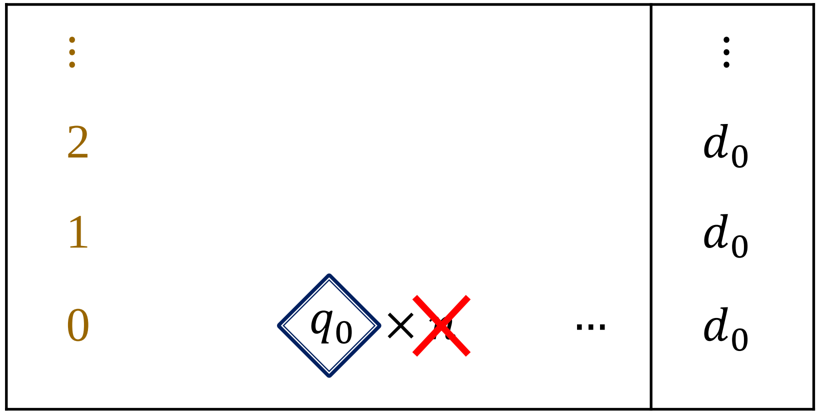
Initial configurations:



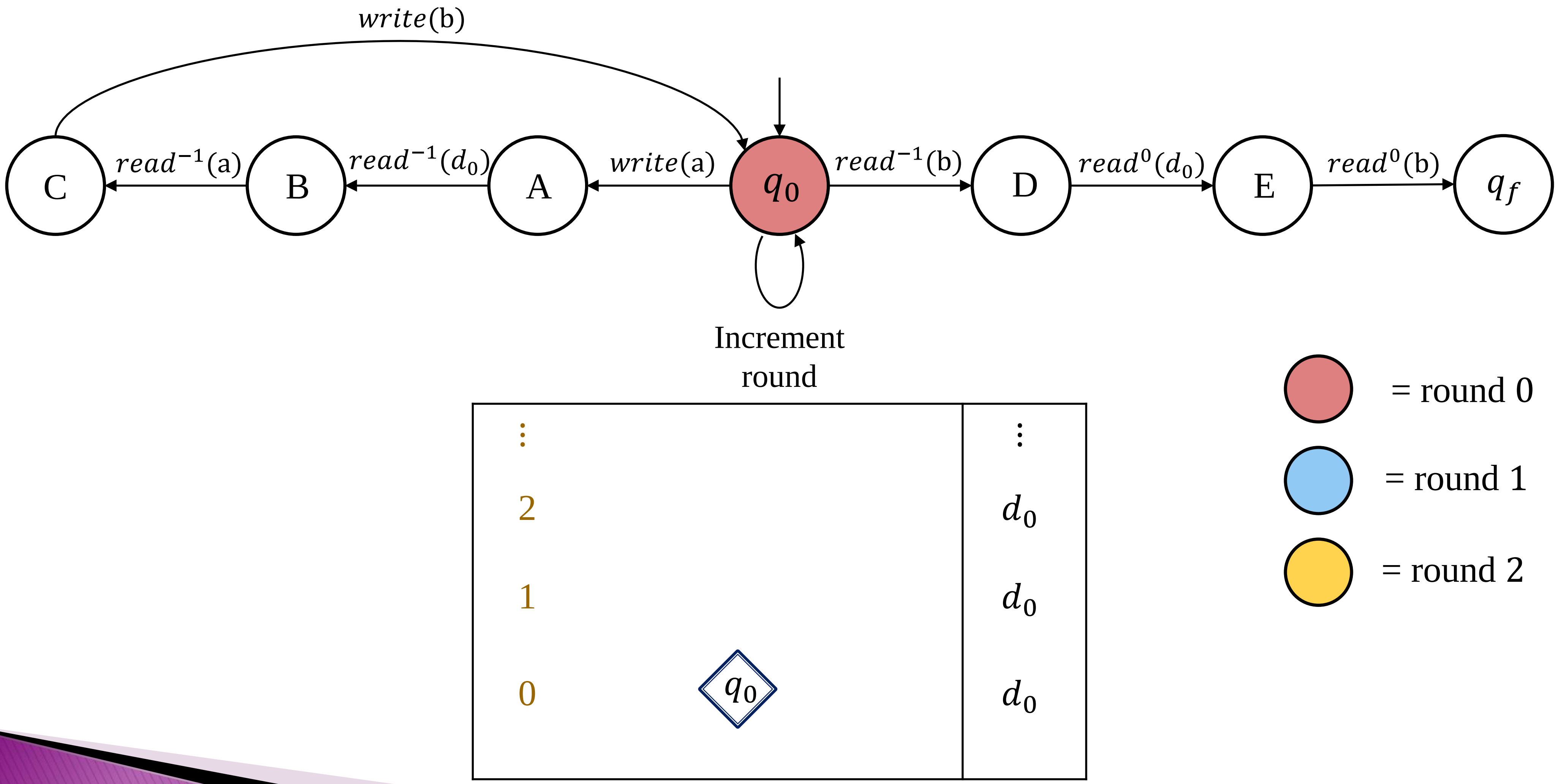
Abstraction



Initial configurations:



An example of round-based register protocol



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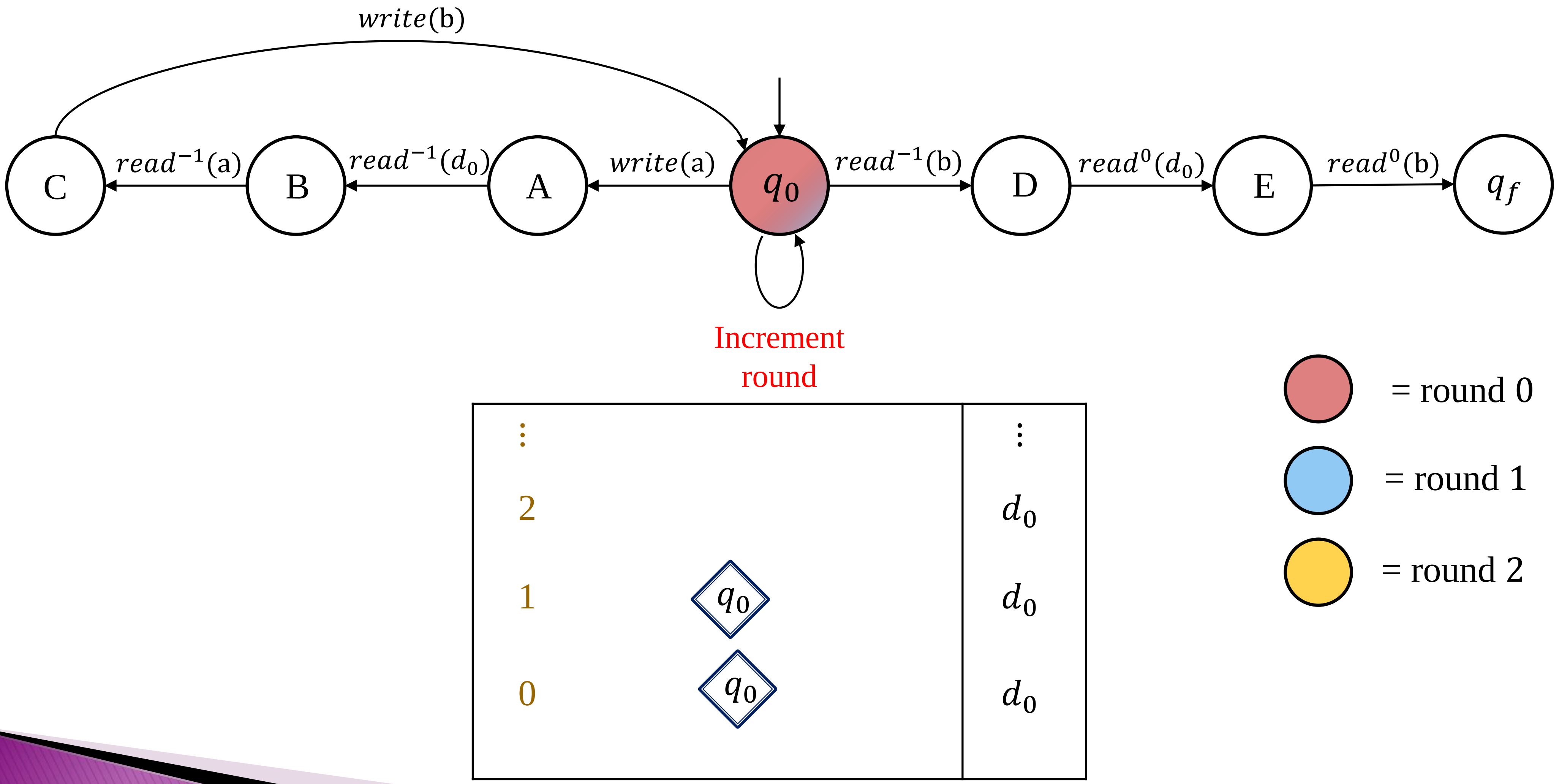

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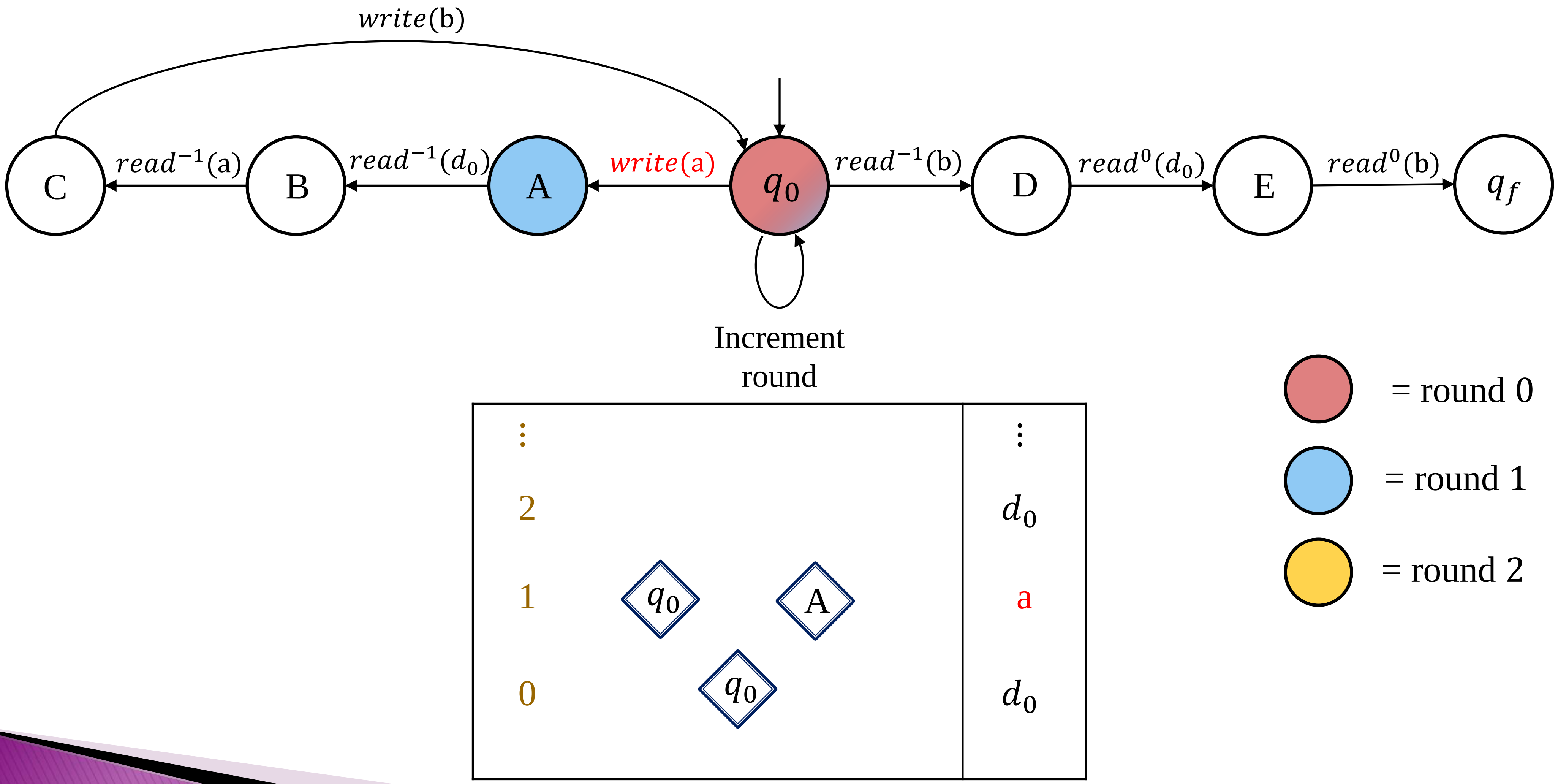

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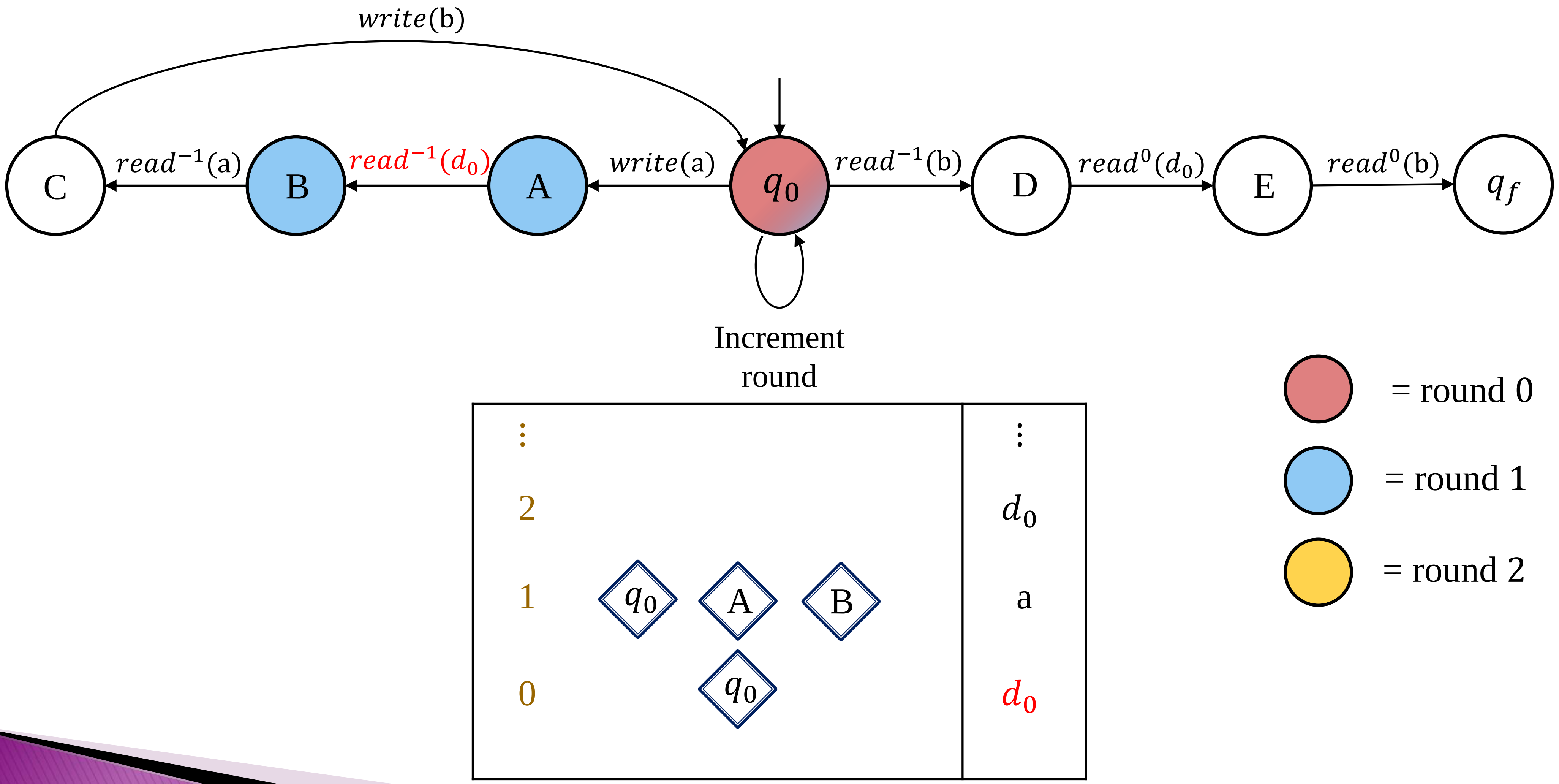

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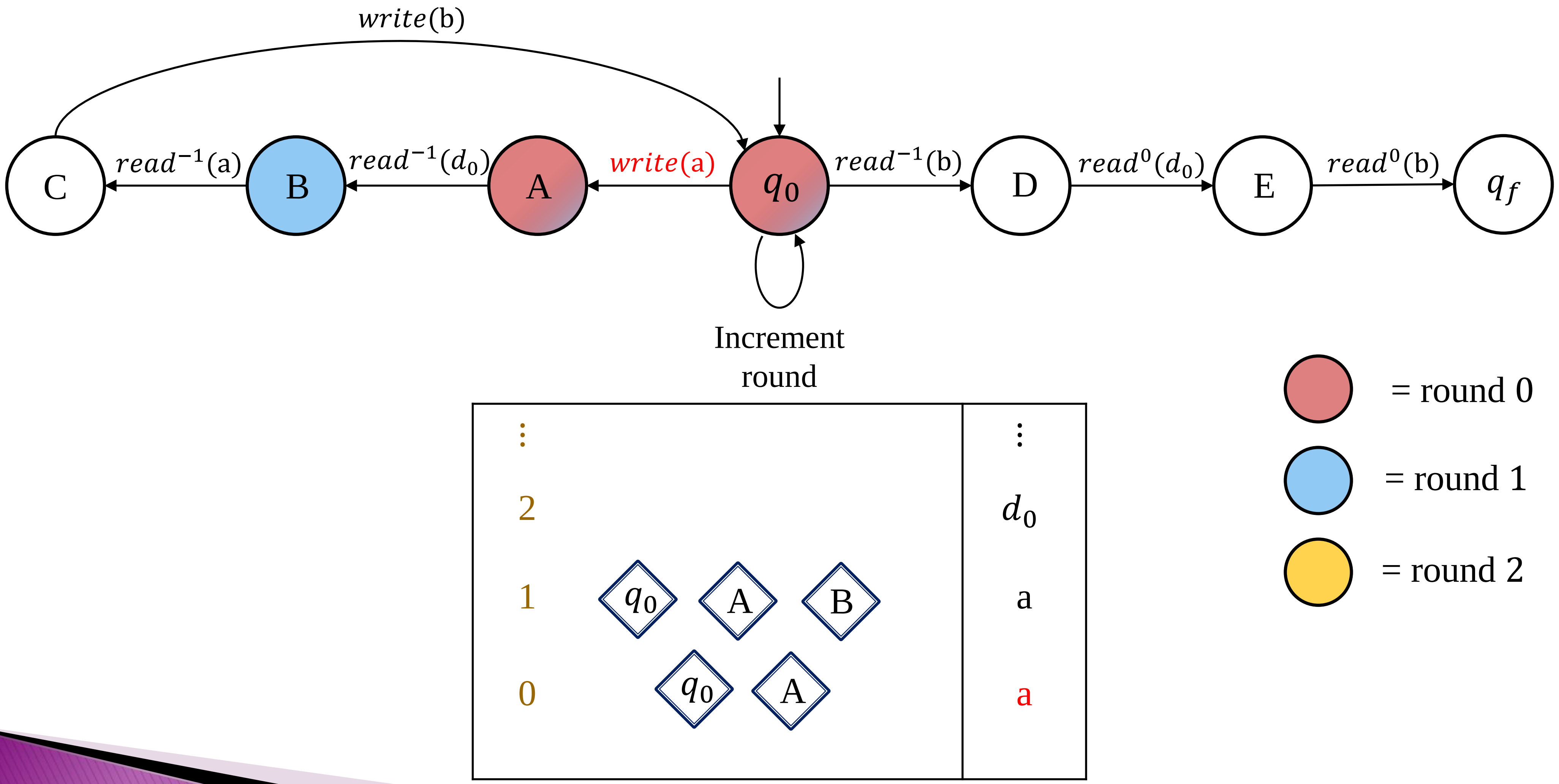

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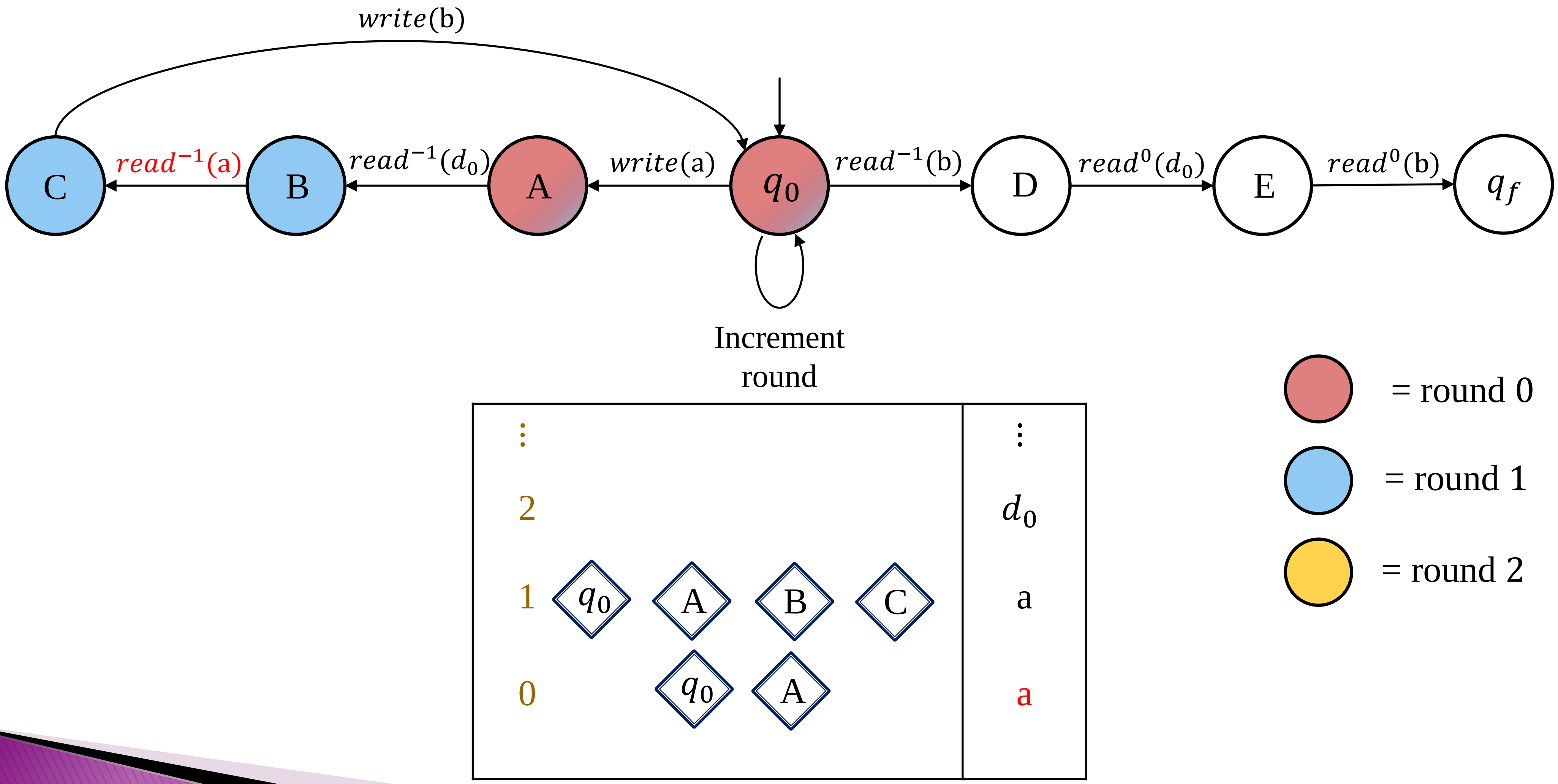

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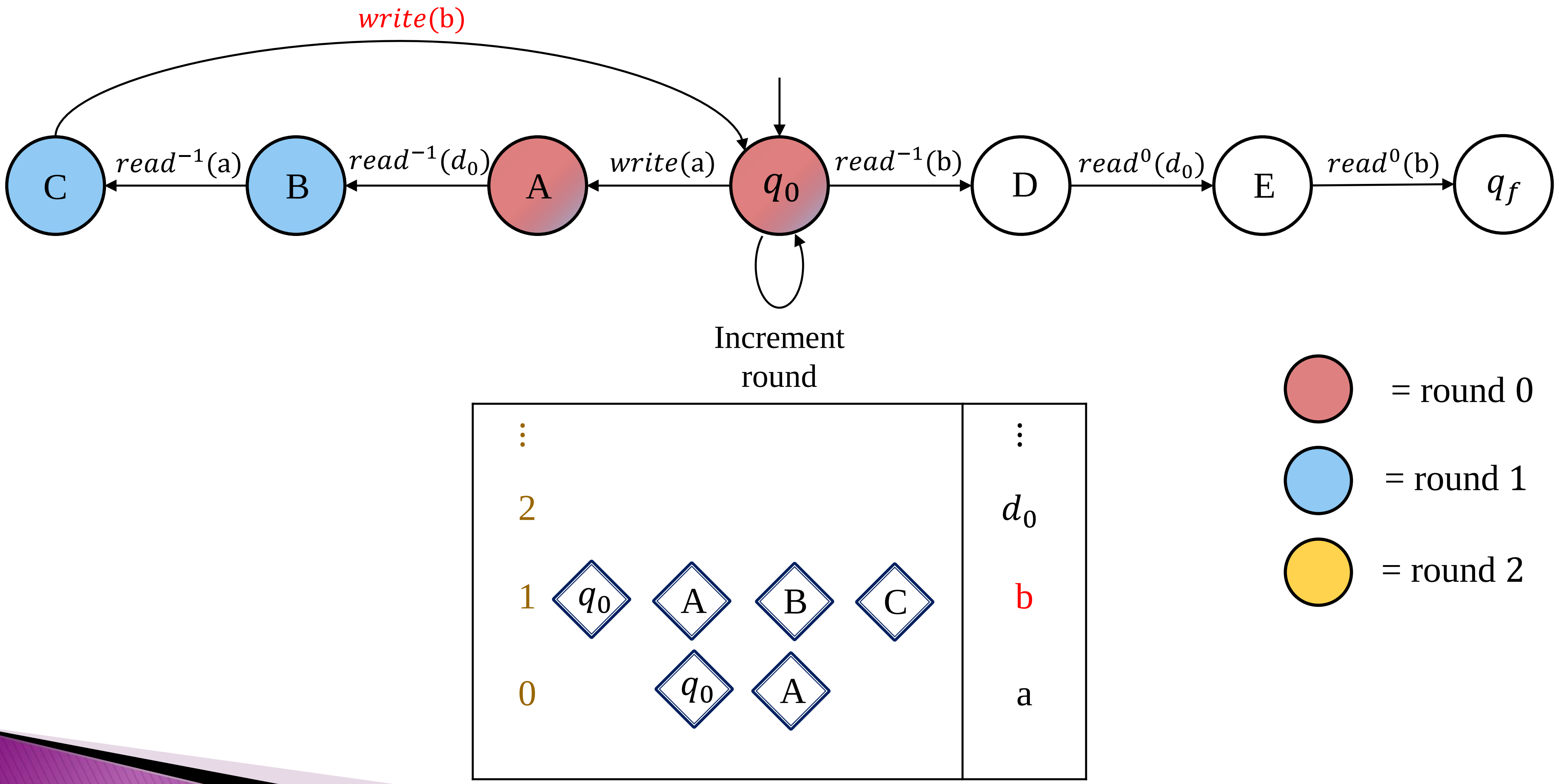

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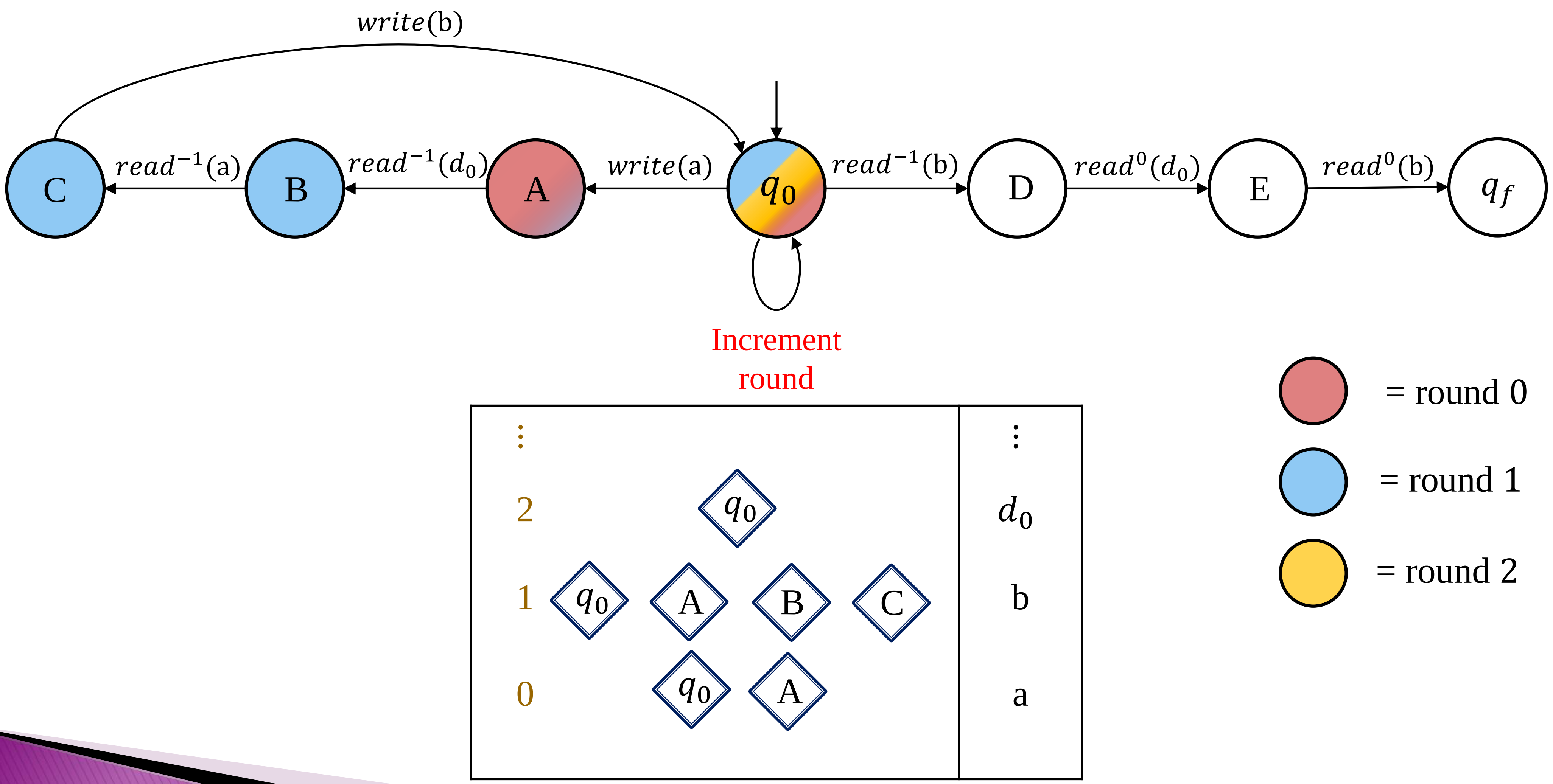

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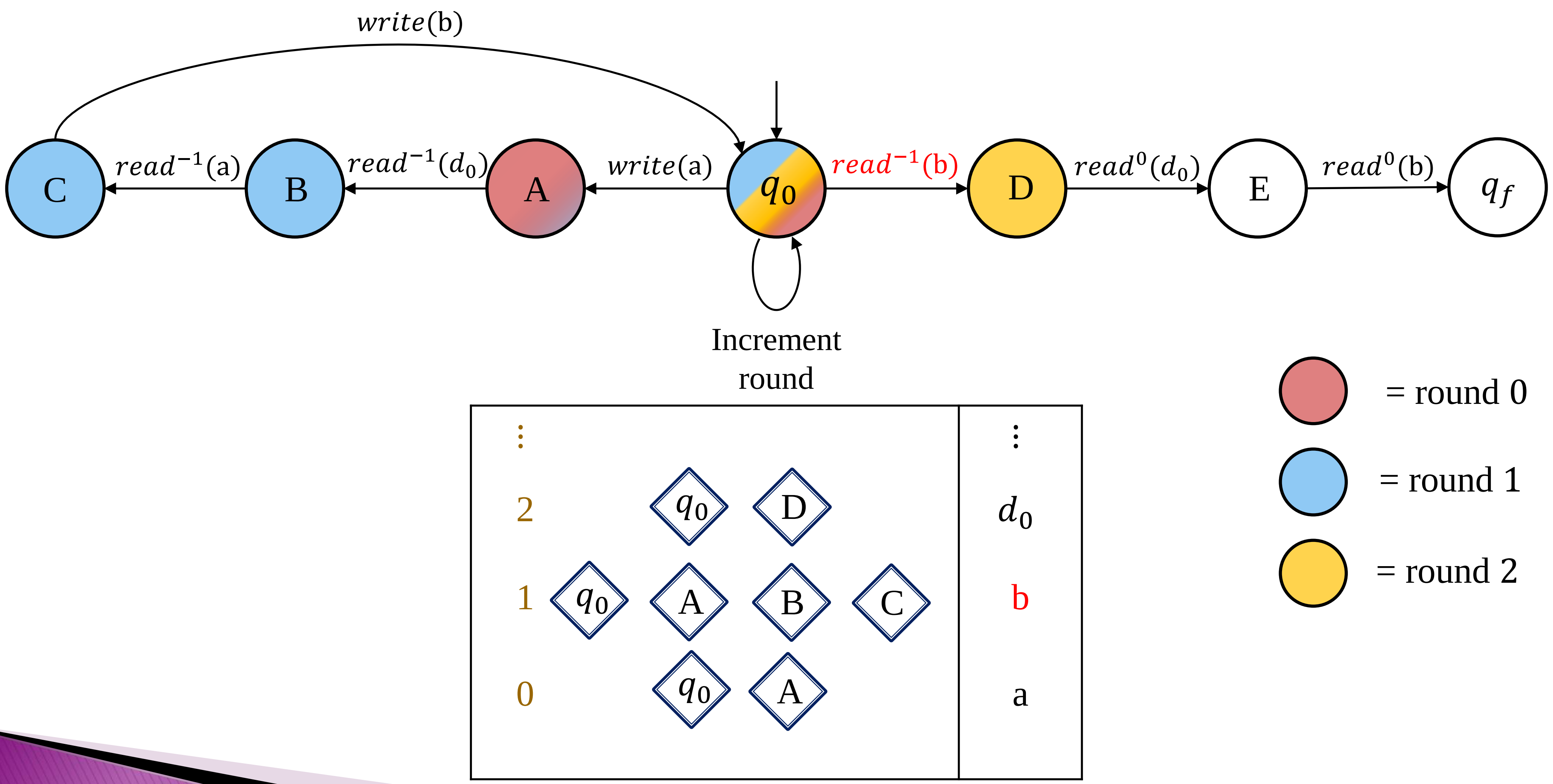

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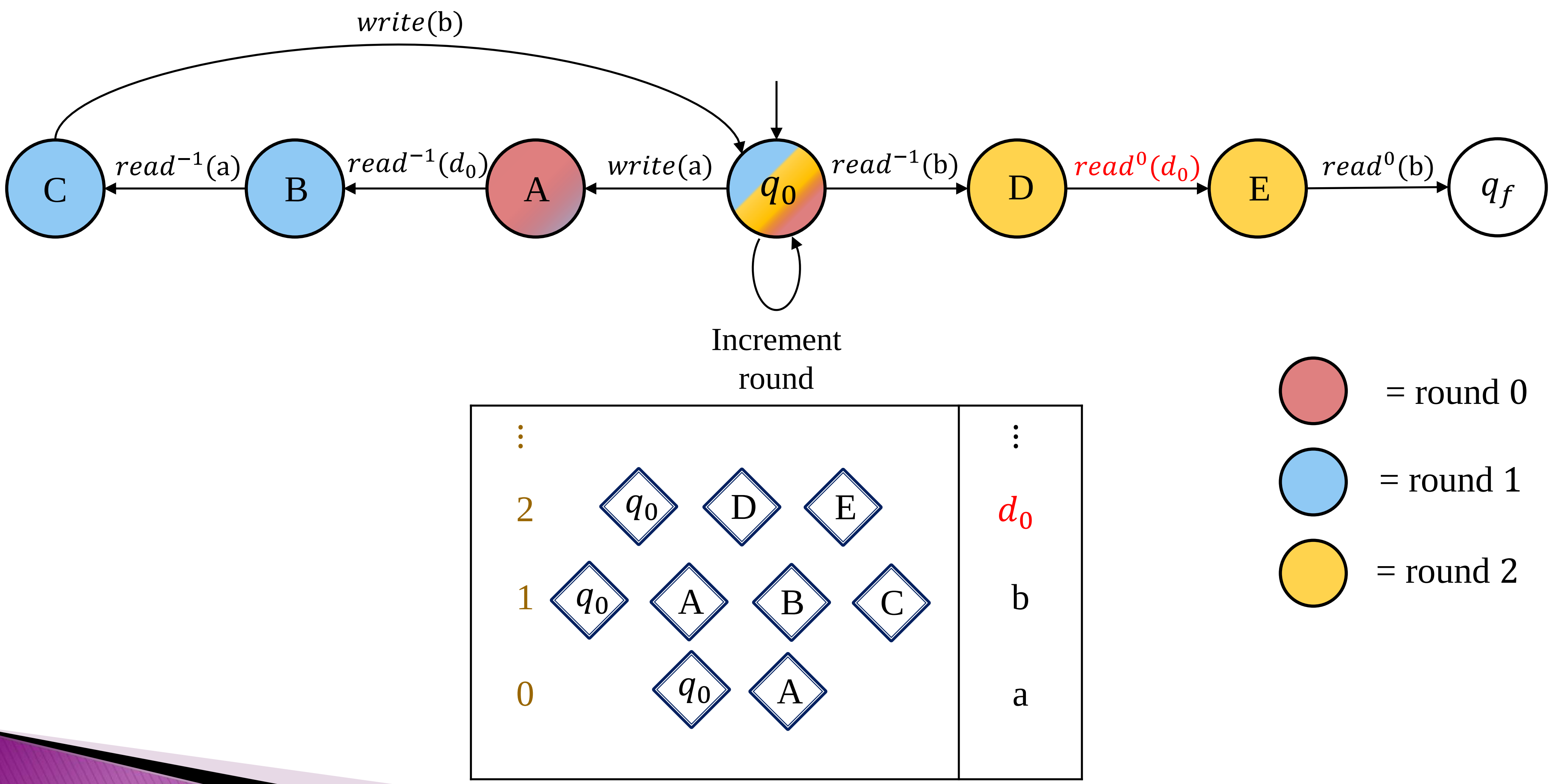

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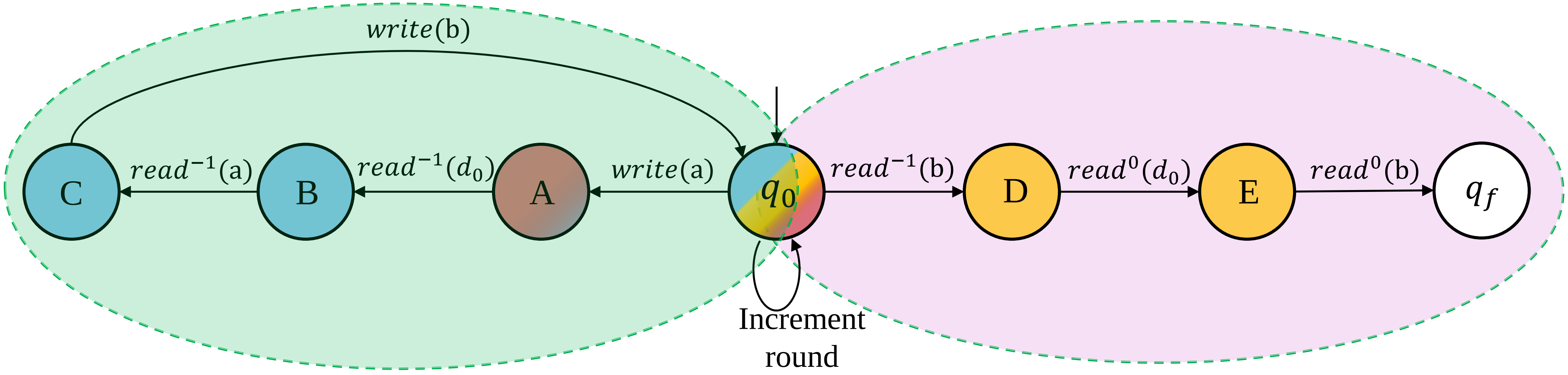

To write b to **reg**$[k]$, one must write to **reg**$[k]$ while **reg**$[k-1]$ still has value d_0

To cover q_f at round k, one must have written b to **reg**$[k-1]$ while **reg**$[k]$ still has value d_0

q_f cannot be covered !

Reachability problems in round-based setting

Round-based COVER: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \rightarrow^* \gamma\ ,\ \exists k\ \ \gamma(q_f, k) > 0$?

↑

There exists a round k such that some process is at round k and on state q_f

Reachability problems in round-based setting

Round-based COVER: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \rightarrow^* \gamma\ ,\ \exists k\ \ \gamma(q_f, k) > 0$?

Round-based TARGET: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \rightarrow^* \gamma\ ,\ \ \forall k, \forall q \neq q_f,\ \gamma(q, k) = 0$?

↑

Every process is on state q_f regardless of its round

Reachability problems in round-based setting

Round-based COVER: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \to^* \gamma\ ,\ \exists k\ \ \gamma(q_f, k) > 0$?

Round-based TARGET: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \to^* \gamma\ ,\ \ \forall k, \forall q \neq q_f,\ \gamma(q, k) = 0$?

Round-based PRP: $\exists n,\ \exists \gamma_0,\ \exists \rho\colon \gamma_0 \to^* \gamma\ ,\ \ \gamma \vDash \psi$?

with ψ a first-order formula on rounds with no nested quantifiers

Examples: $\psi =$ ``$\exists k\ (\#(q_1, k+1) > 0 \land \mathbf{reg}_i[k] = d)\ \lor\ \forall k\ \#(q_0, k) = 0''$

At some round, there is a process on state q_1 while register i of previous round has value d

no process is on q_0

Complexity results

Theorem[9]*:* Round-based COVER is PSPACE-hard.

9. Bertrand, N., Markey, N., Sankur, O., Waldburger, N.: *Parameterized safety verification of round-based shared-memory systems.* ICALP, 2022

Complexity results

Theorem[9]*:* Round-based COVER is PSPACE-hard.

Theorem[9][10]*:* Round-based PRP is PSPACE-complete.

9. Bertrand, N., Markey, N., Sankur, O., Waldburger, N.:
Parameterized safety verification of round-based shared-memory systems. ICALP, 2022
10. W: *Checking Presence Reachability Properties on Parameterized Shared-Memory Systems*, submitted

Complexity results

Theorem[9]*:* Round-based COVER is PSPACE-hard.

Theorem[9,10]*:* Round-based PRP is PSPACE-complete.

Challenge: the number of rounds relevant at the same time may need to be exponential.

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A non-deterministic polynomial-space algorithm

Witness execution: $\sigma_0 \xrightarrow{\theta_0} \sigma_1 \xrightarrow{\theta_1} \sigma_2 \xrightarrow{\theta_2} \sigma_3 \xrightarrow{\theta_3} \sigma_4 \xrightarrow{\theta_4} \sigma_5 \xrightarrow{\theta_5} \sigma_6 \xrightarrow{\theta_6} \sigma_7 \vDash \psi$

A non-deterministic polynomial-space algorithm

Witness execution: $\sigma_0 \xrightarrow{\theta_0} \sigma_1 \xrightarrow{\theta_1} \sigma_2 \xrightarrow{\theta_2} \sigma_3 \xrightarrow{\theta_3} \sigma_4 \xrightarrow{\theta_4} \sigma_5 \xrightarrow{\theta_5} \sigma_6 \xrightarrow{\theta_6} \sigma_7 \models \psi$

$$\Downarrow$$

Actions:	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
Rounds:	1	4	3	2	0	1	4

A non-deterministic polynomial-space algorithm

Witness execution: $\sigma_0 \xrightarrow{\theta_0} \sigma_1 \xrightarrow{\theta_1} \sigma_2 \xrightarrow{\theta_2} \sigma_3 \xrightarrow{\theta_3} \sigma_4 \xrightarrow{\theta_4} \sigma_5 \xrightarrow{\theta_5} \sigma_6 \xrightarrow{\theta_6} \sigma_7 \models \psi$

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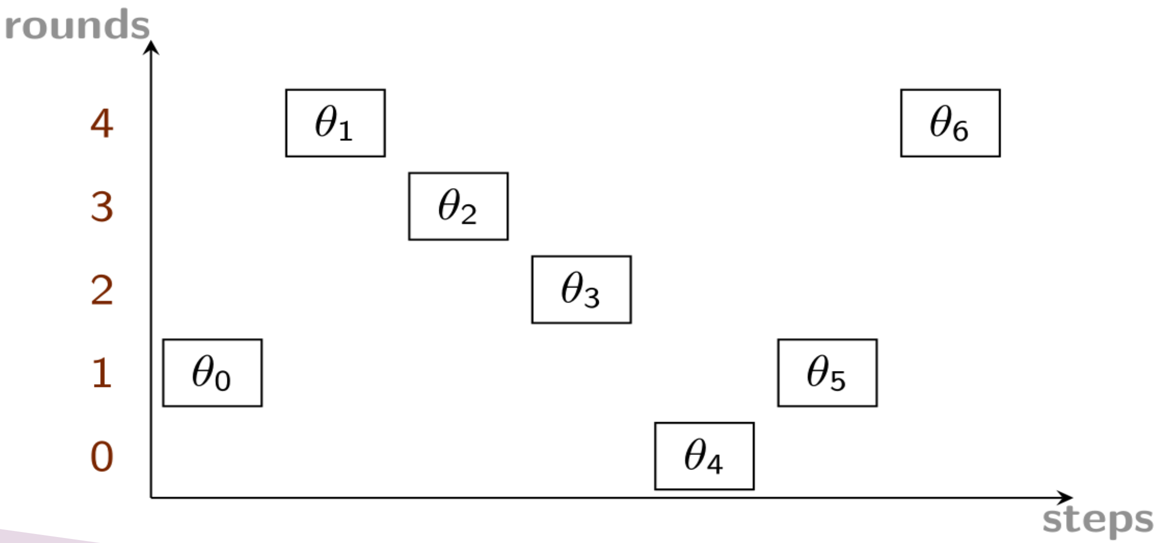
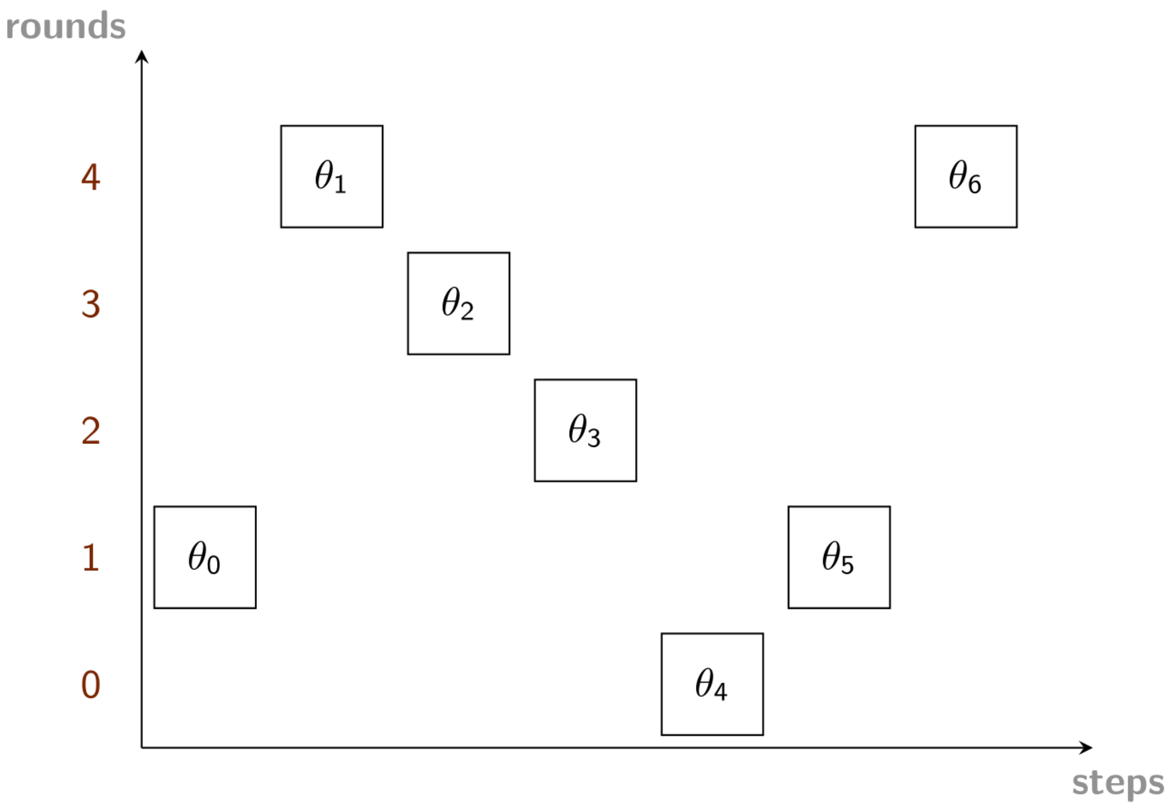


A non-deterministic polynomial-space algorithm



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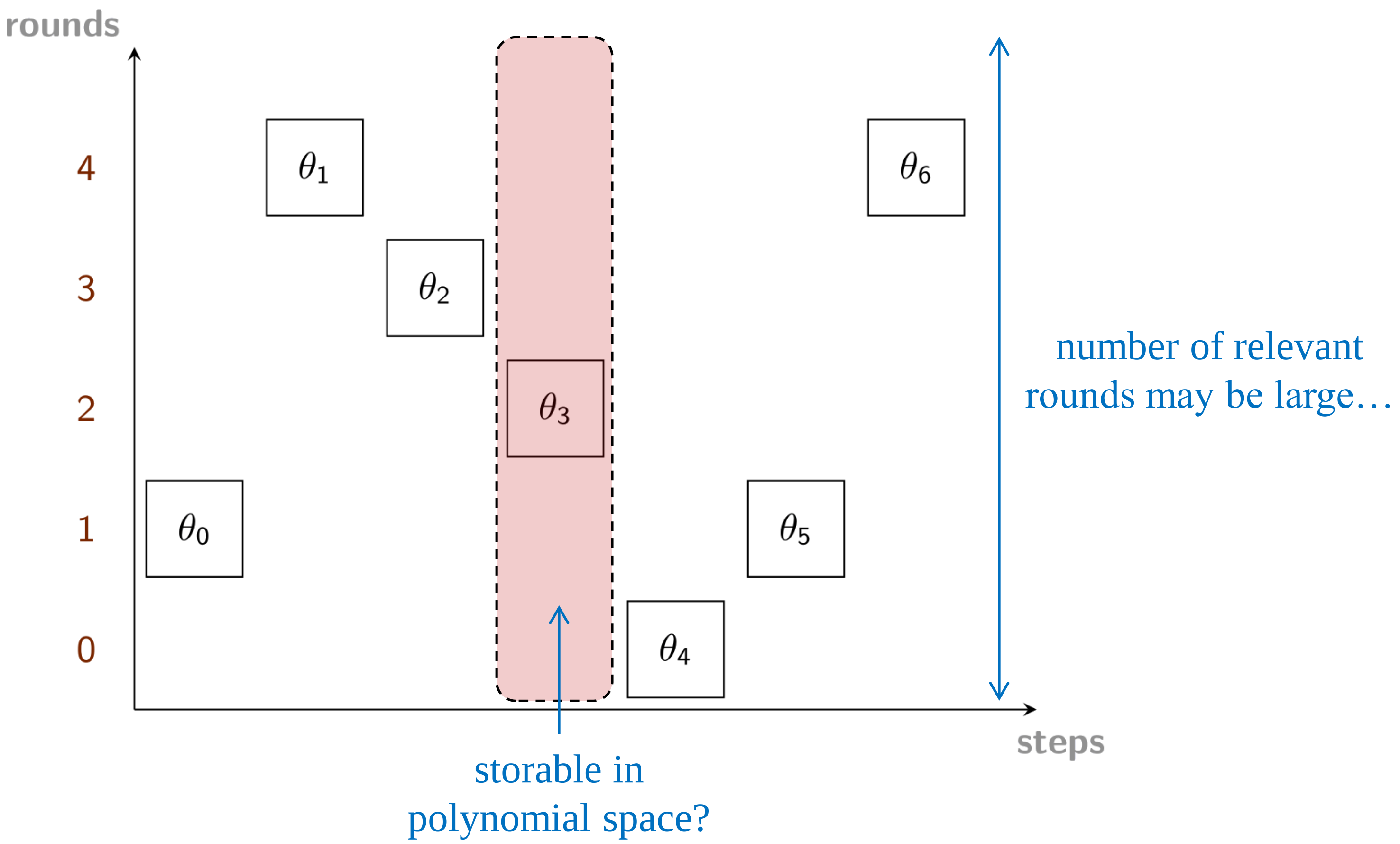
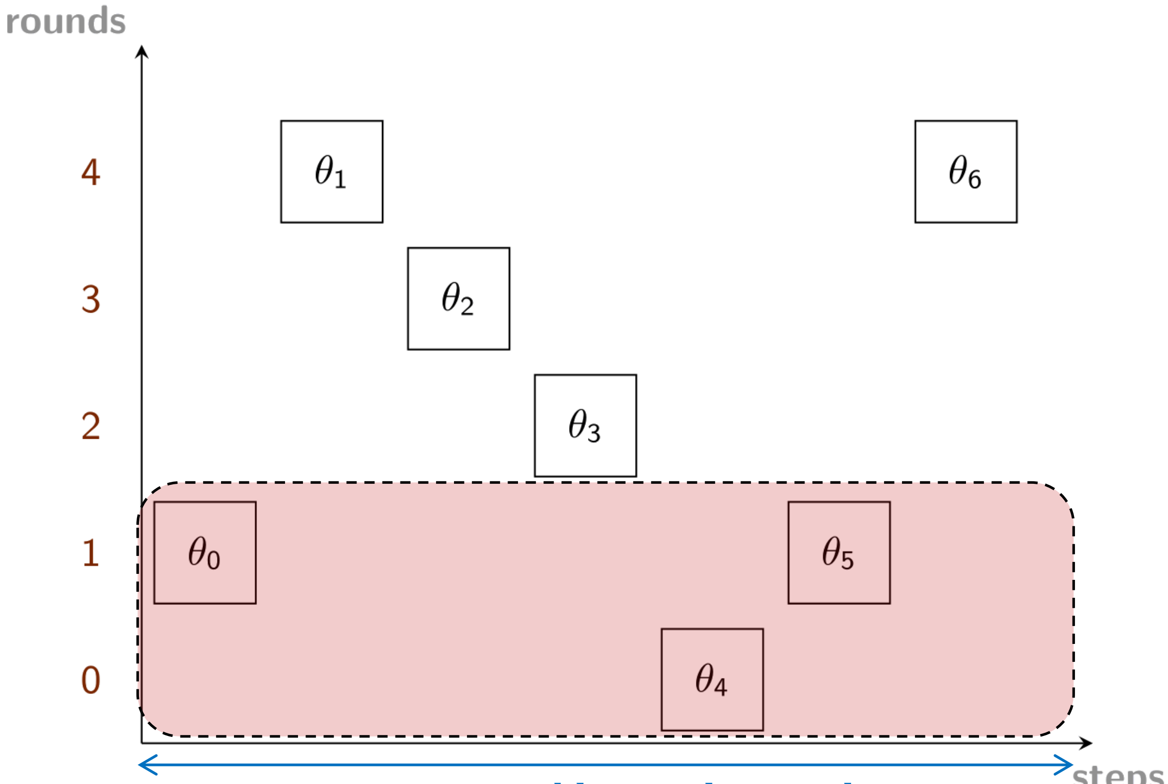


A non-deterministic polynomial-space algorithm

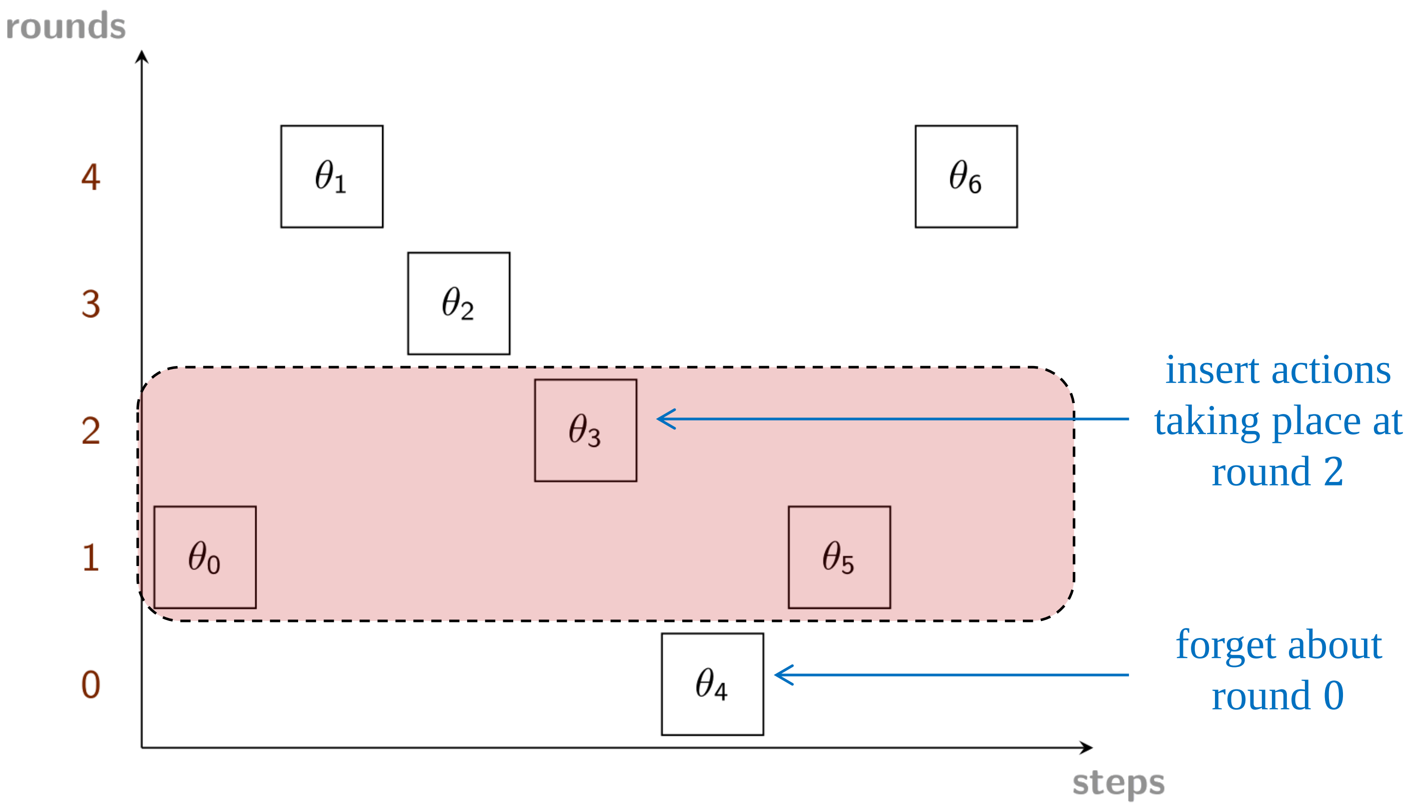


storable in polynomial space using abstract representation

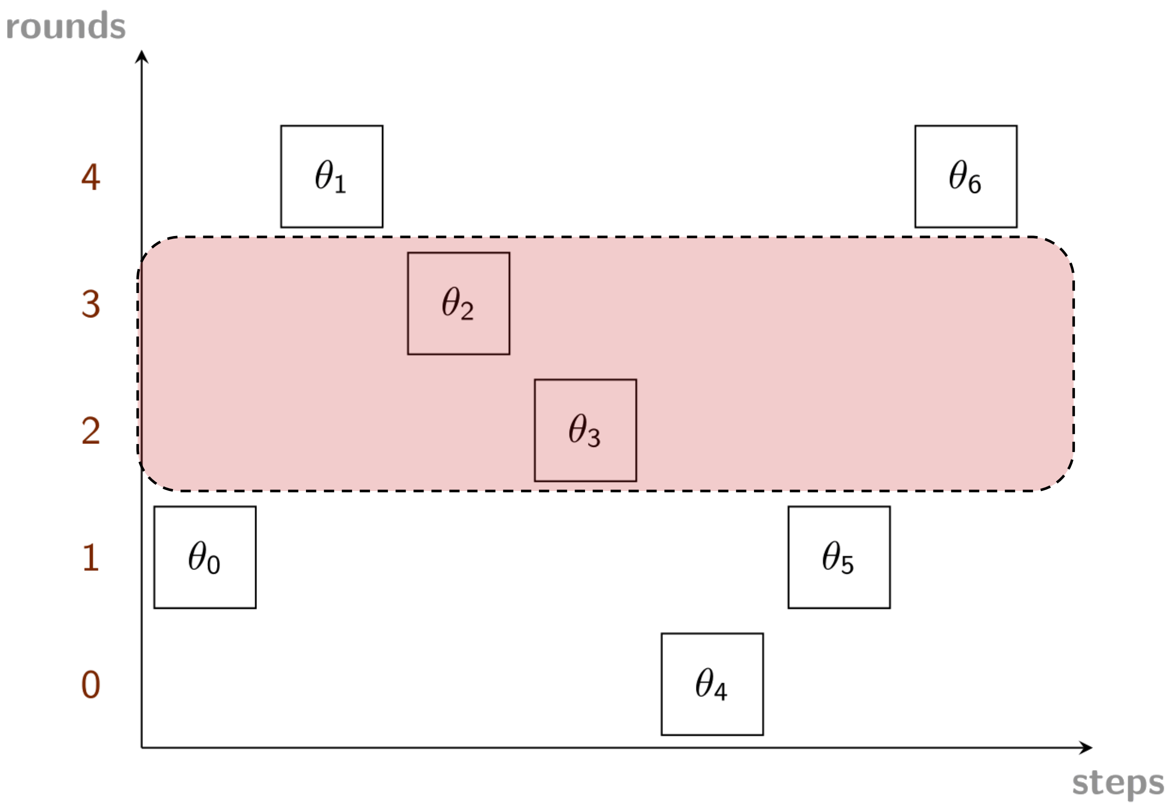
sliding window on $v+1$ rounds where v is the highest i such that some $read^{-i}(x)$ appears in the protocol

v is assumed to be given in unary (here $v = 1$)

A non-deterministic polynomial-space algorithm

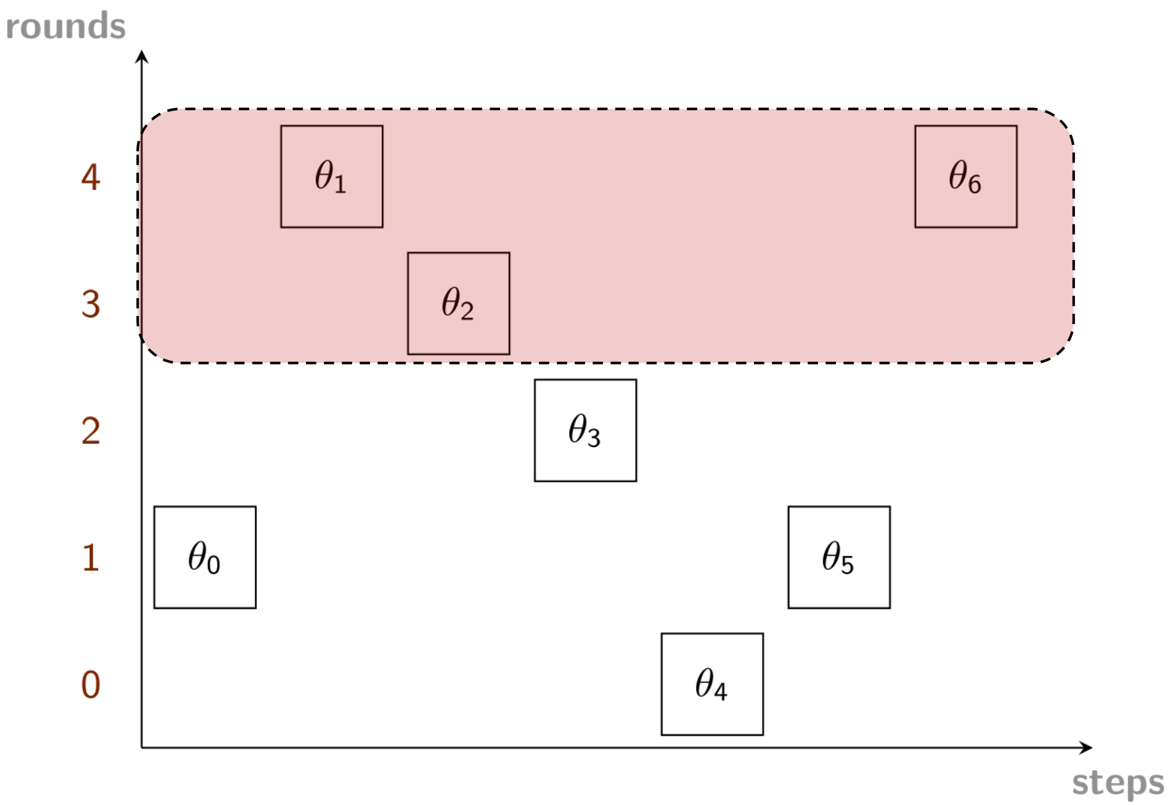


A non-deterministic polynomial-space algorithm



A non-deterministic polynomial-space algorithm

As the execution is guessed, we progressively guess why the configuration reached will satisfy ψ.



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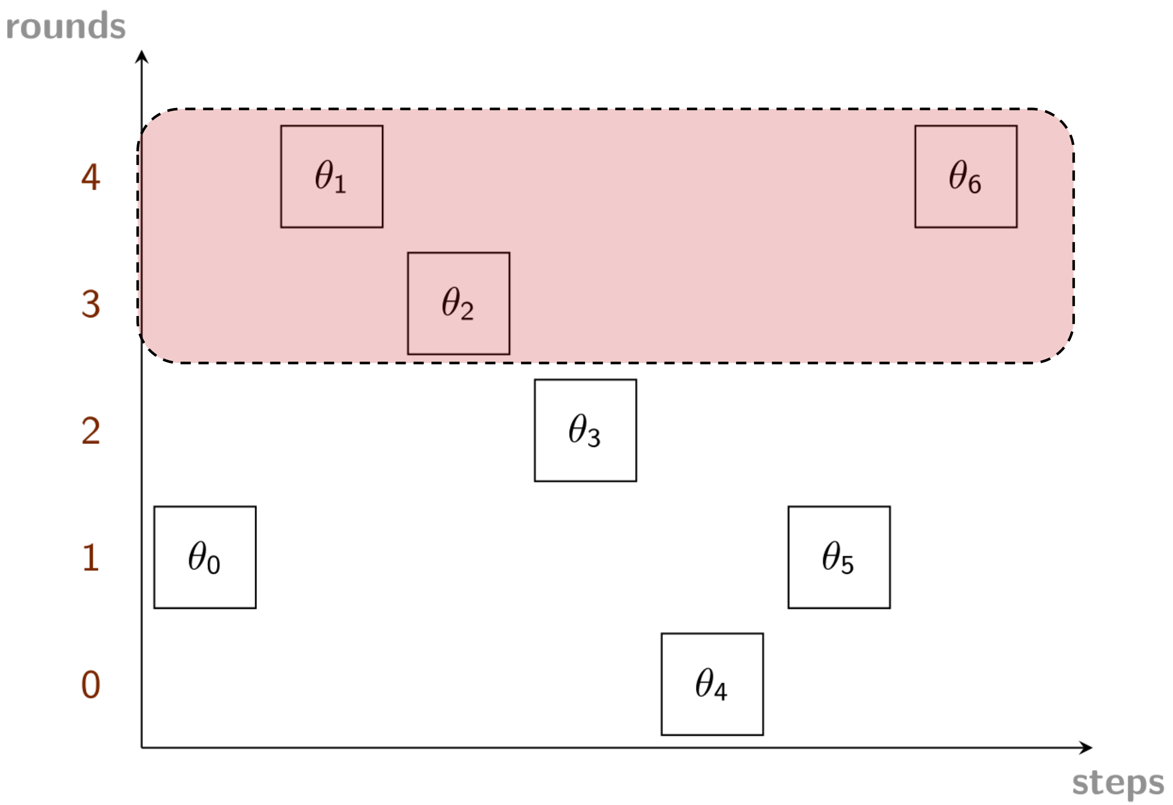


From this algorithm, we obtain exponential upper bounds on the number of processes and rounds needed.

Conclusion

Summary

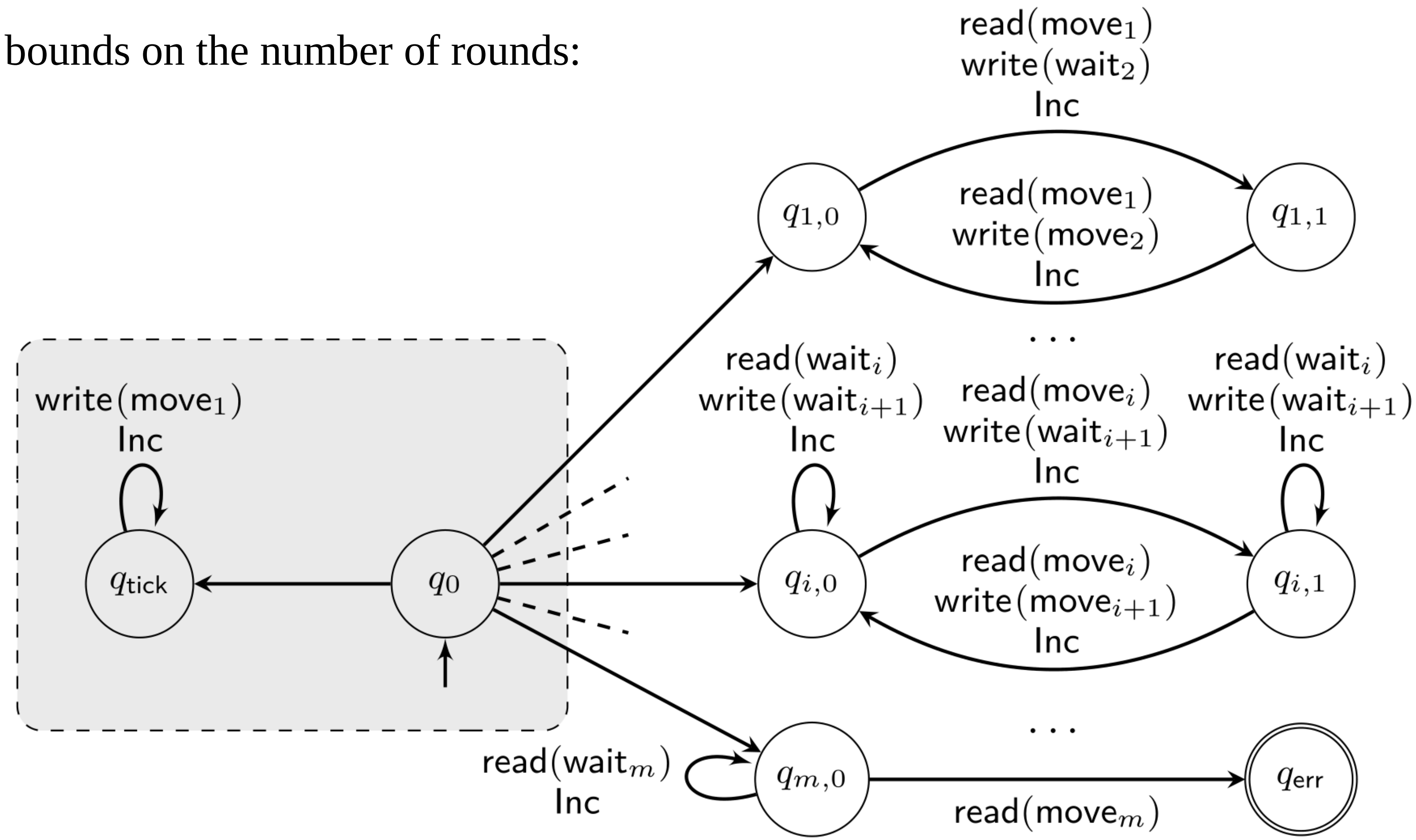
- Two models in this talk: *roundless* register protocols and *round-based* register protocols.
- Properties studied are *reachability properties* which do not “count” processes. Two classical such problems are COVER and TARGET; PRP is a general class which encompasses these two problems.
- In the first model, despite its simplicity, PRP is NP-complete, but some restrictions make it PTIME.
- In the second model, PRP is PSPACE-complete, and similar restrictions do not decreases the complexity.

Future work

- Introducting stochastic schedulers and study almost-sure reachability (work in progress, some weird behaviors occur that make it very different from the roundless case)
- Weak memory

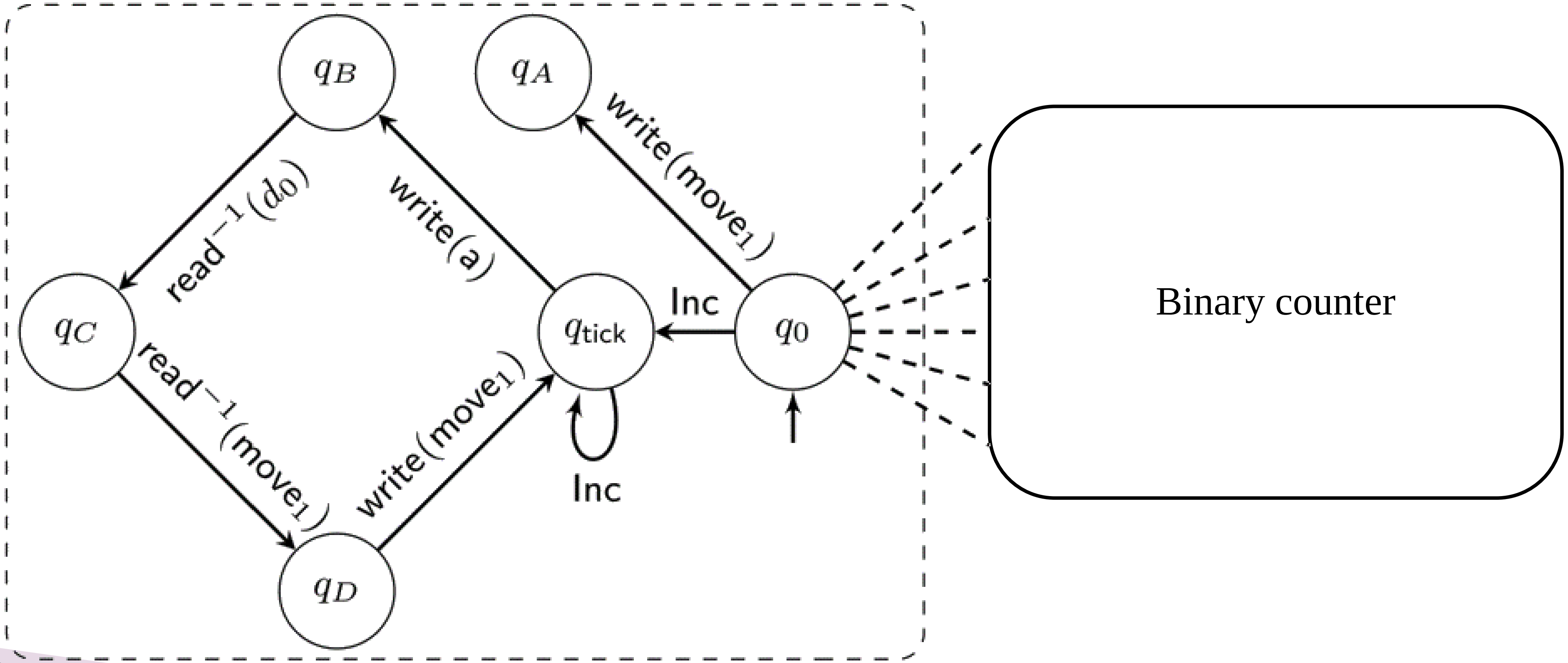
A challenge: exponential lower bounds

Exponential lower bounds on the number of rounds:



A challenge: exponential lower bounds

Exponential lower bounds on the number *active* rounds:



A motivating example

Binary consensus problem:
Make all processes agree on a common value, each process starting an initial preference p.
Validity: If a process decided value p, some process started with value p
Agreement: Two processes that decide decide of the same value
Termination: All processes eventually decide of a value

Aspnes' consensus algorithm[3]:

```
int k := 0, bool p ∈ {0,1}, (rg_b[r])_{b∈{0,1}, r∈ℕ} all initialized to no;
while true do
    read from rg_0[k] and rg_1[k] ;
    if rg_0[k] = yes and rg_1[k] = no then p := 0;
    else if rg_0[k] = no and rg_1[k] = yes then p := 1;
    write yes to rg_p[k] ;
    if k > 0 then
        read from rg_{1-p}[k-1] ;
        if rg_{1-p}[k-1] = no then return p;
    k := k+1;
```

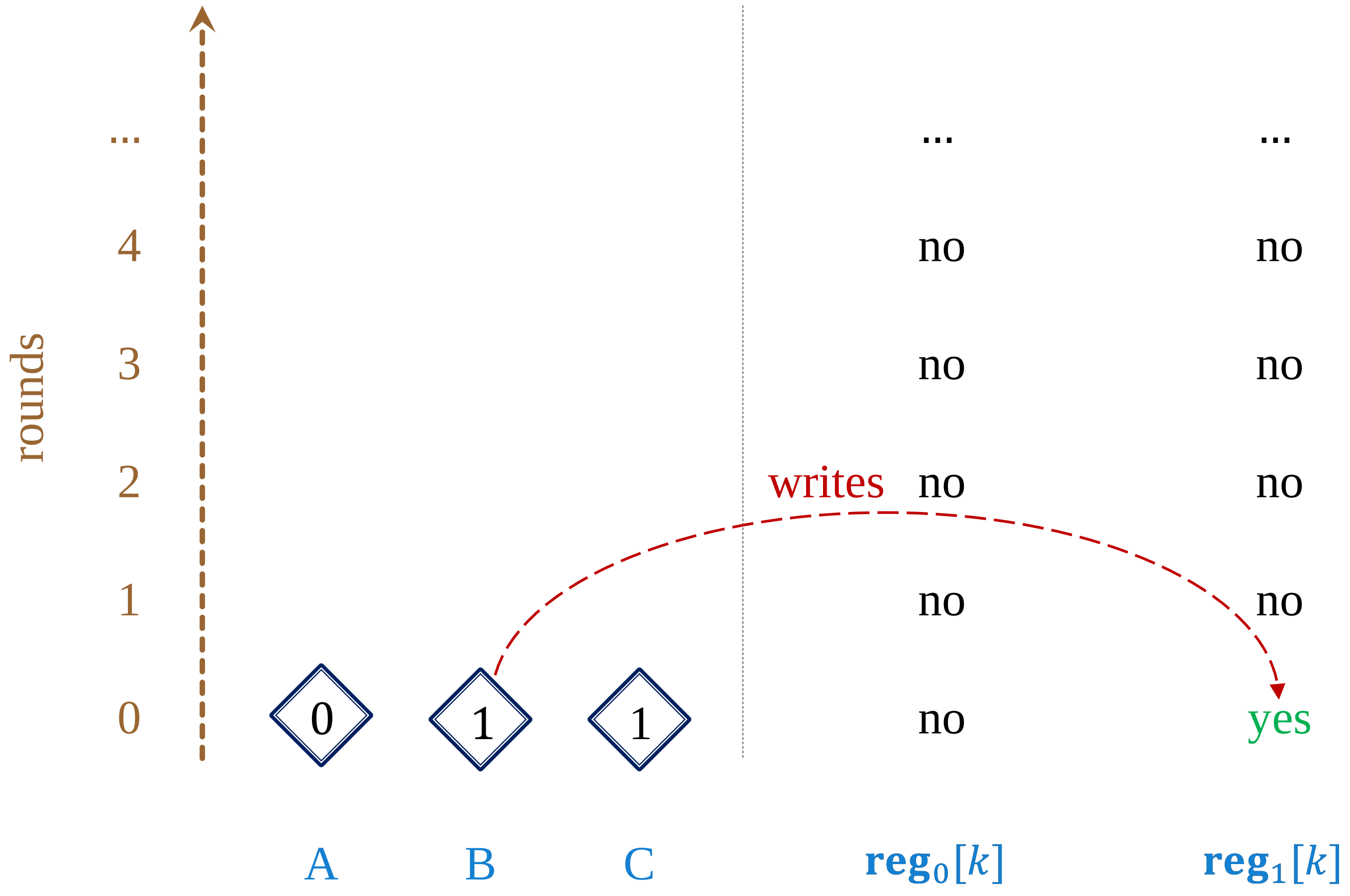
read from registers of rounds k and $k-1$

write to registers of round k

Example of execution of the algorithm

rounds	A	B	C	$\mathbf{reg}_0[k]$	$\mathbf{reg}_1[k]$
…				…	…
4				no	no
3				no	no
2				no	no
1				no	no
0	0	1	1	no	no

Example of execution of the algorithm



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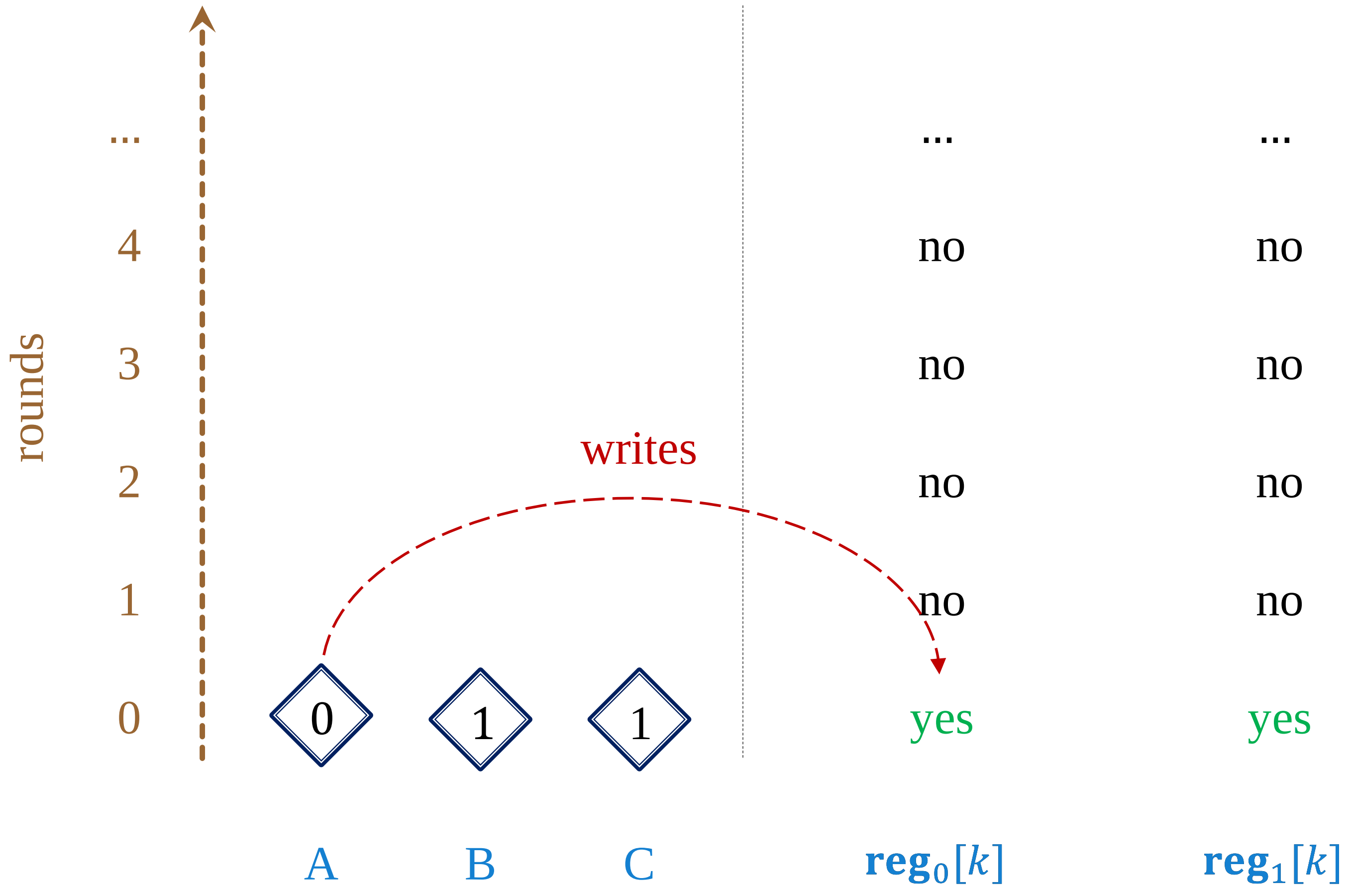

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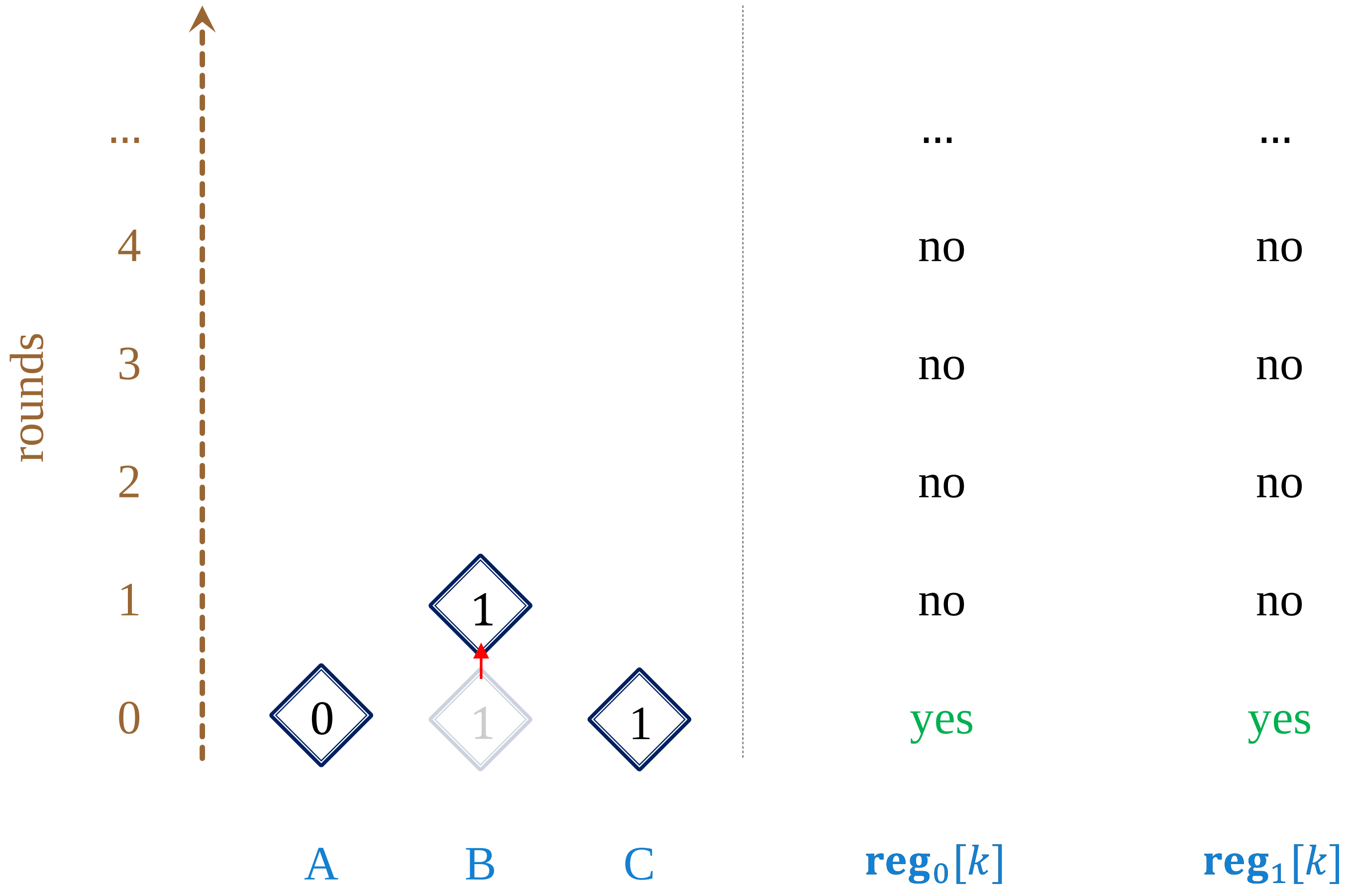

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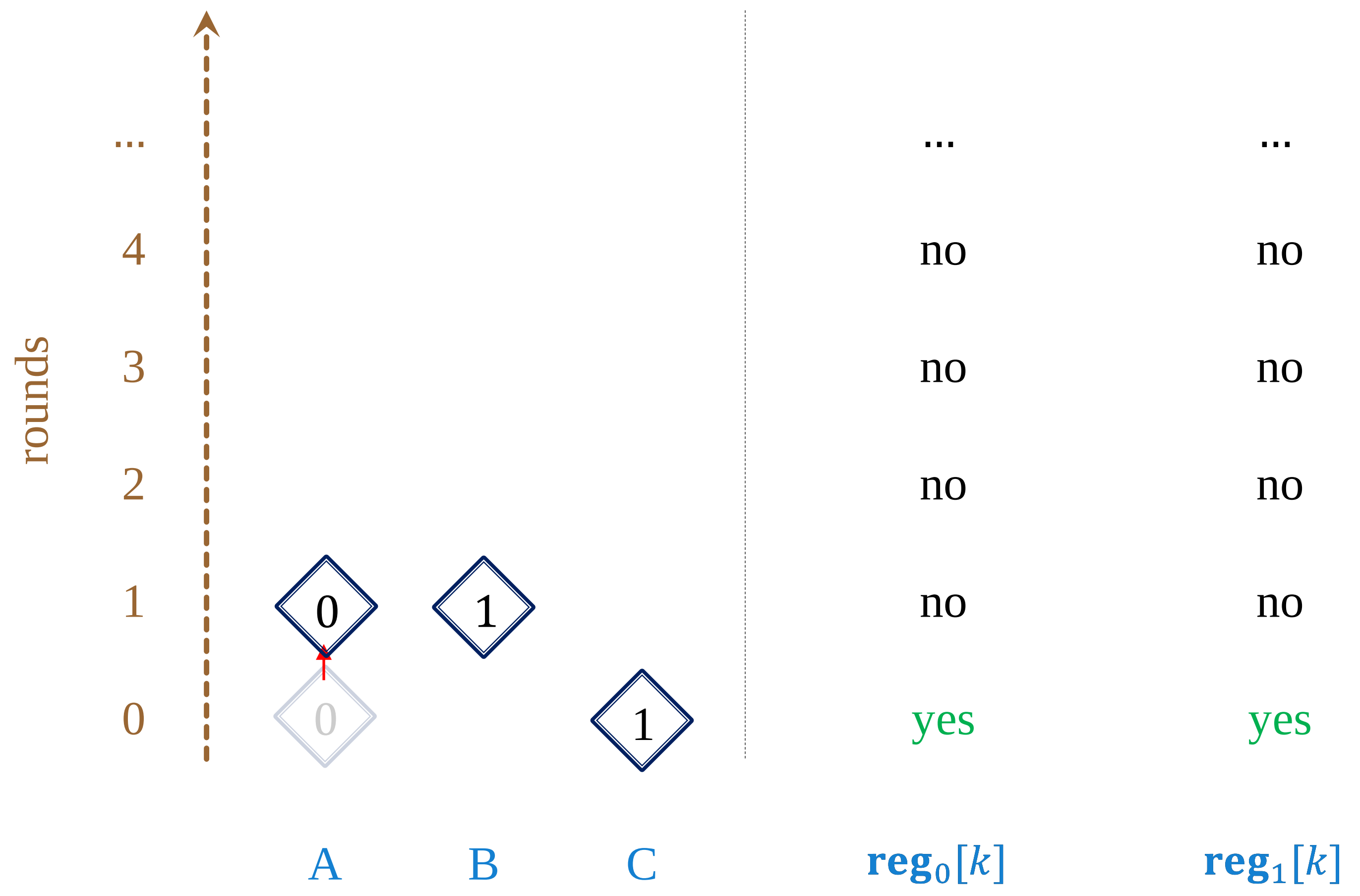

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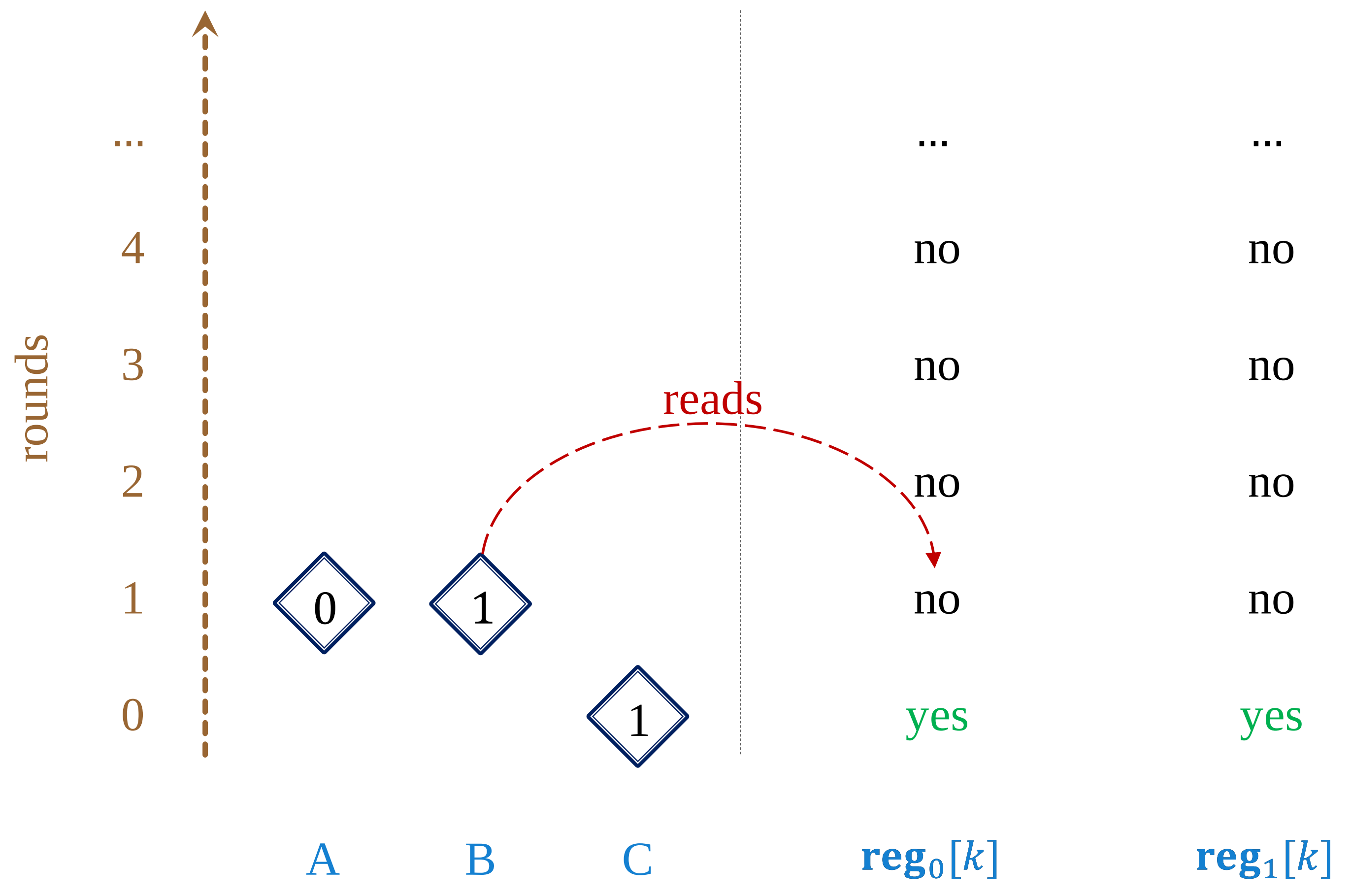

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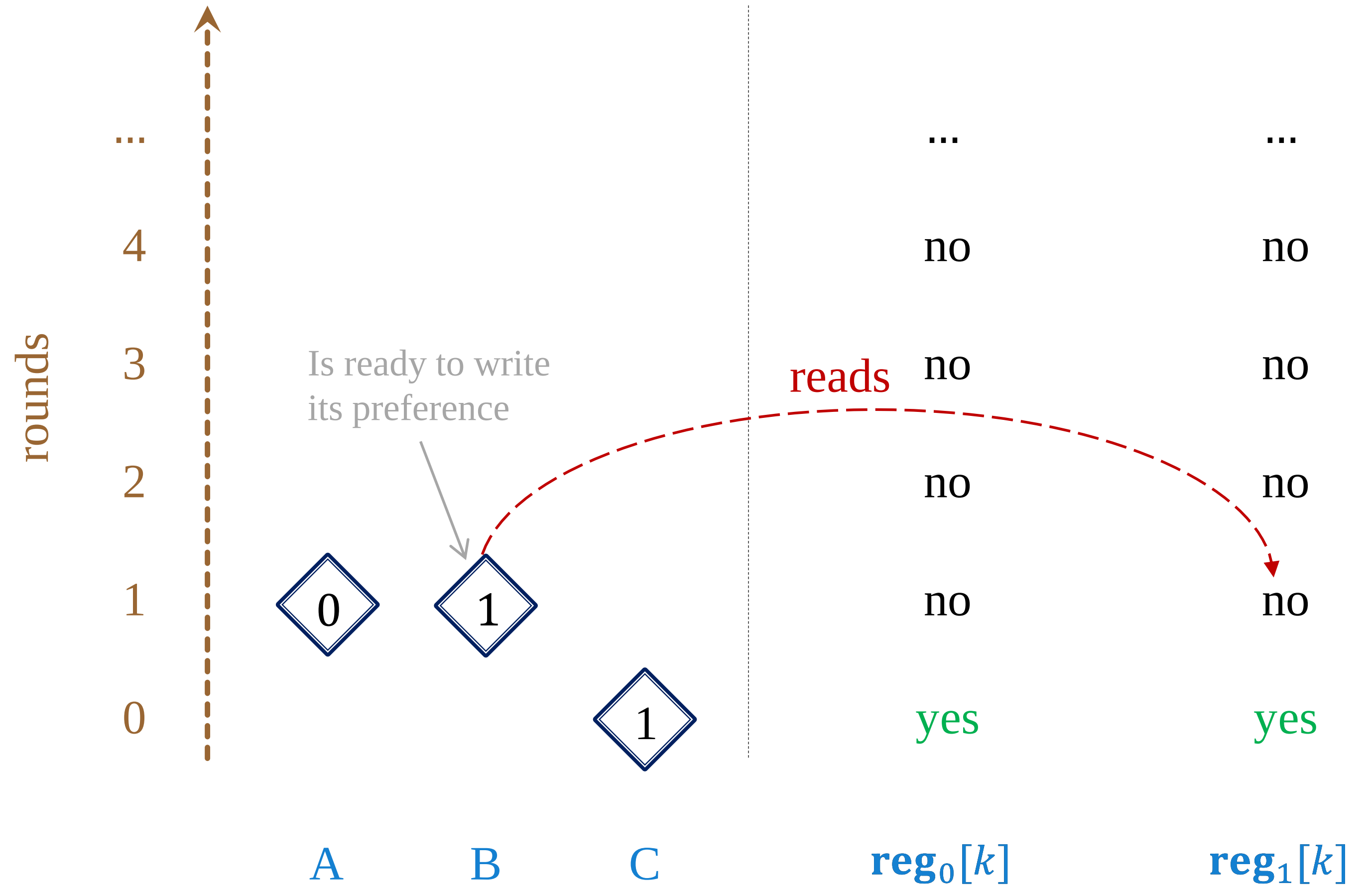

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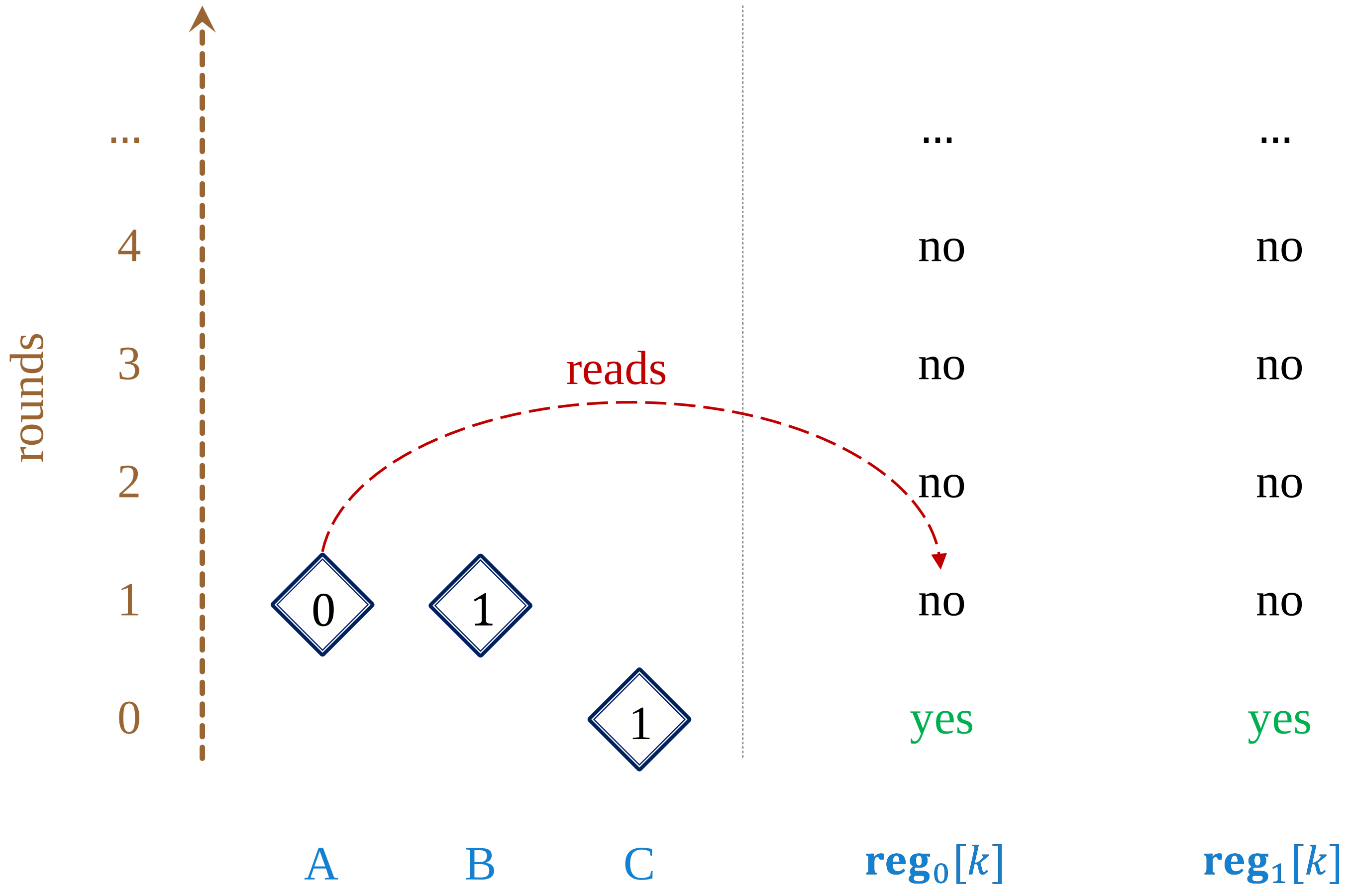

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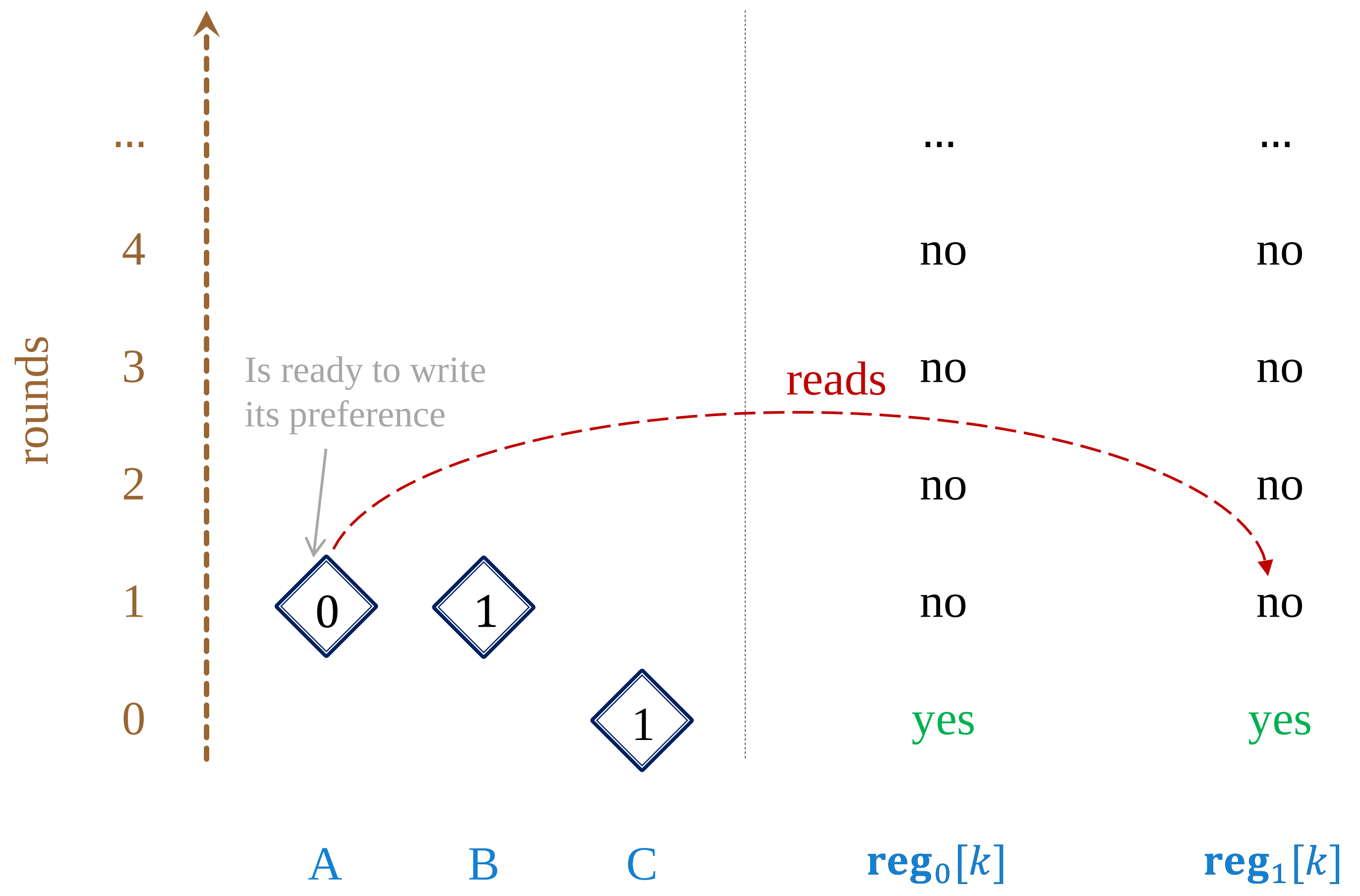

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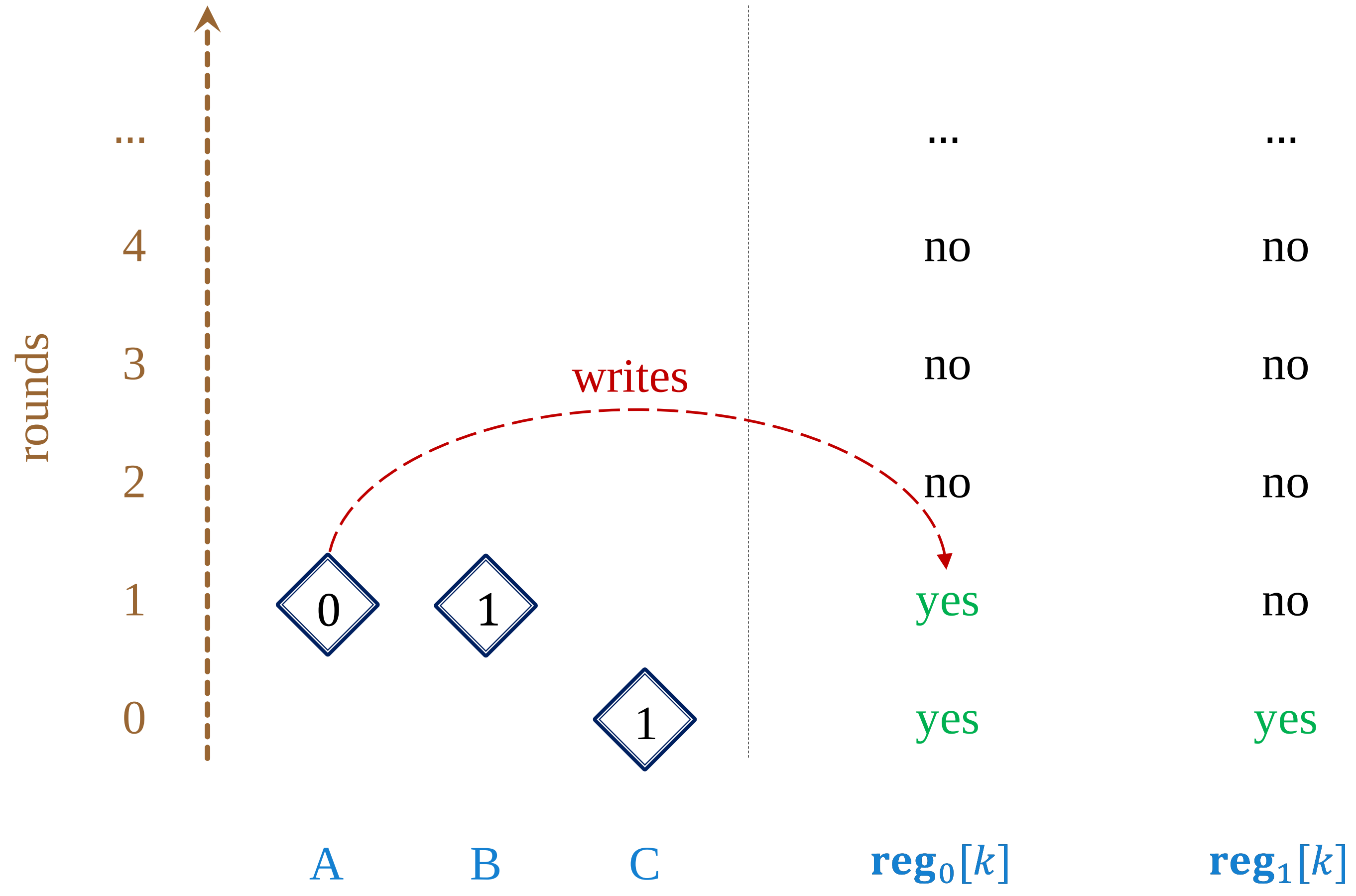

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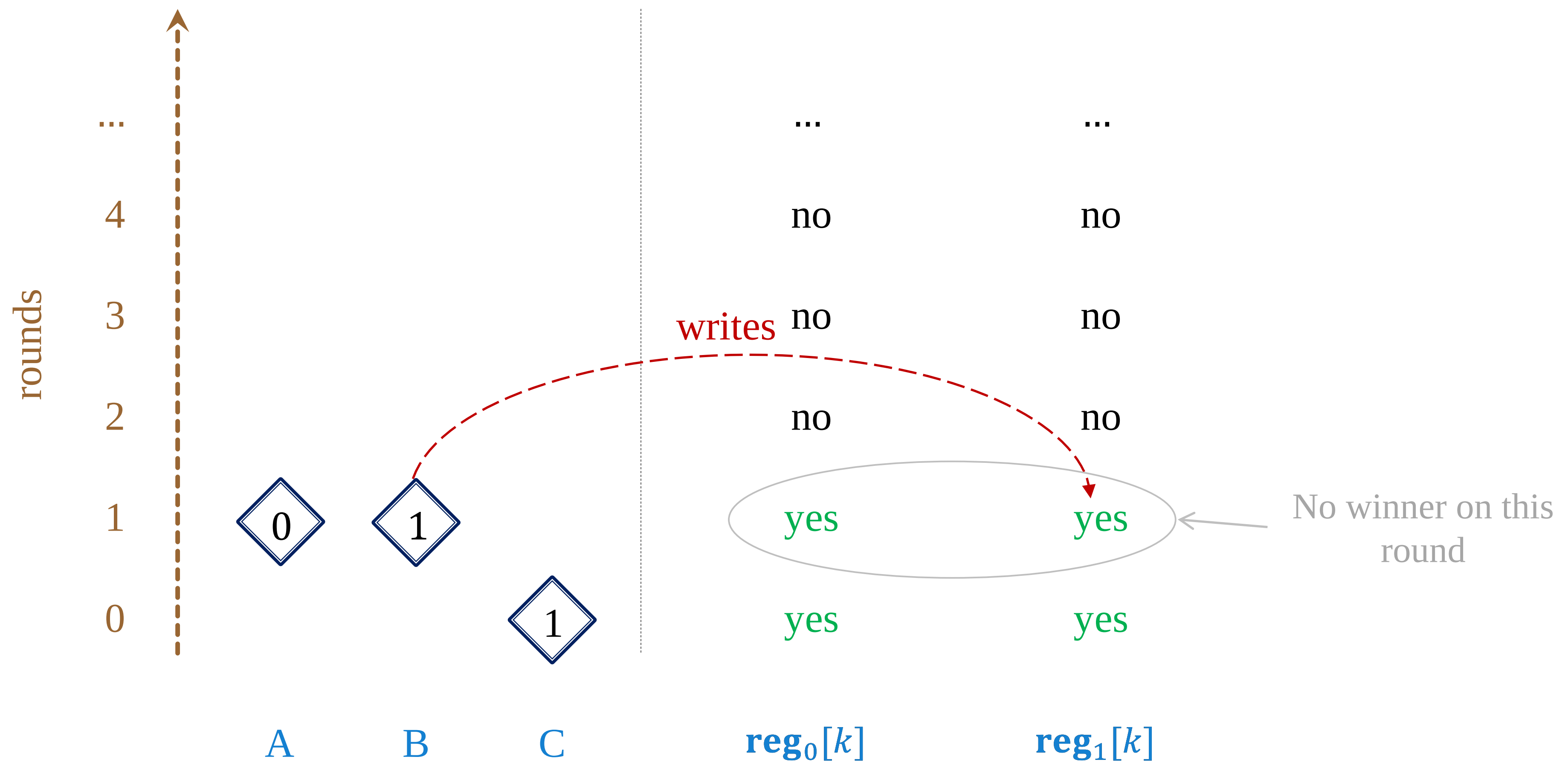

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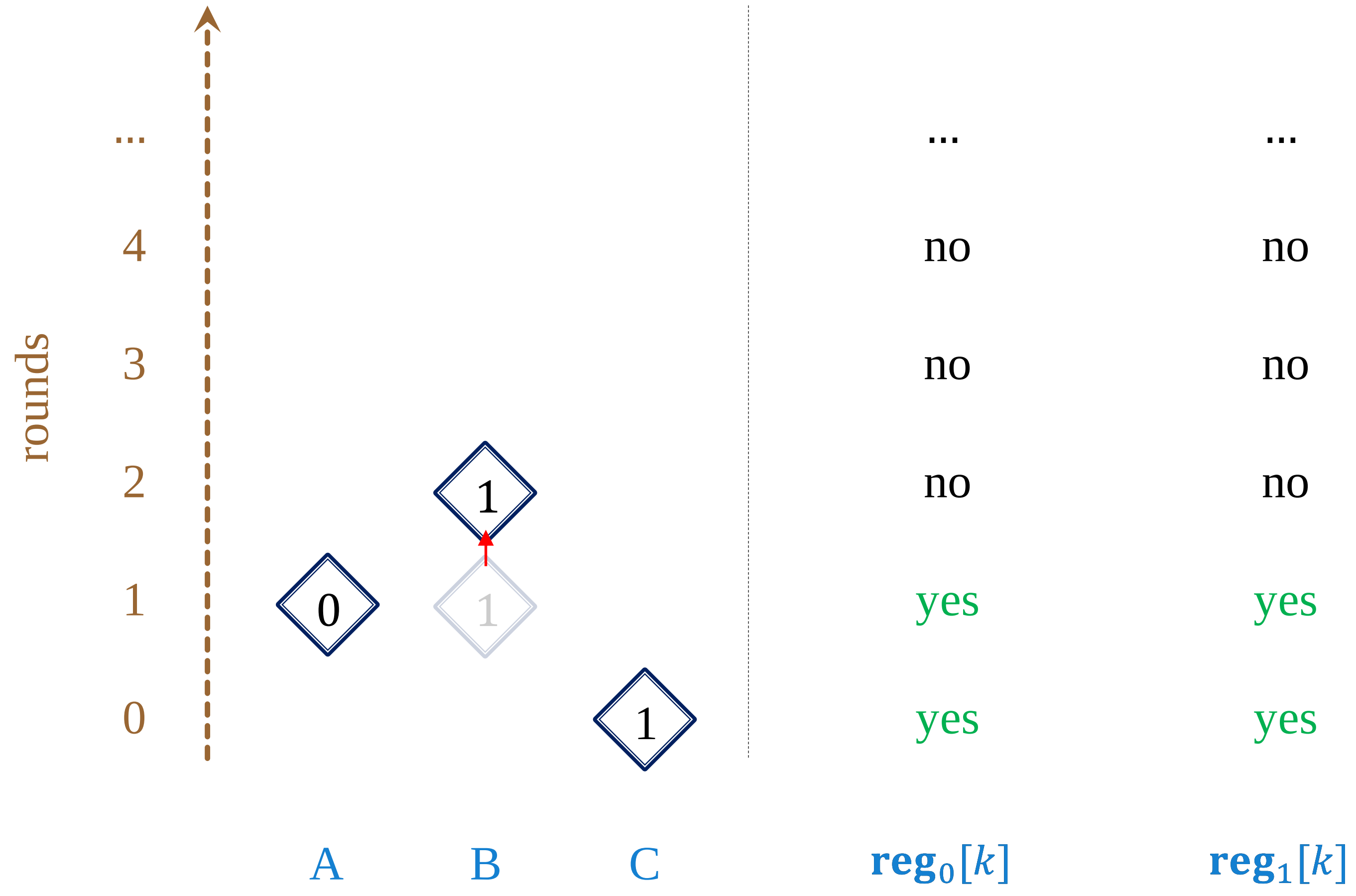

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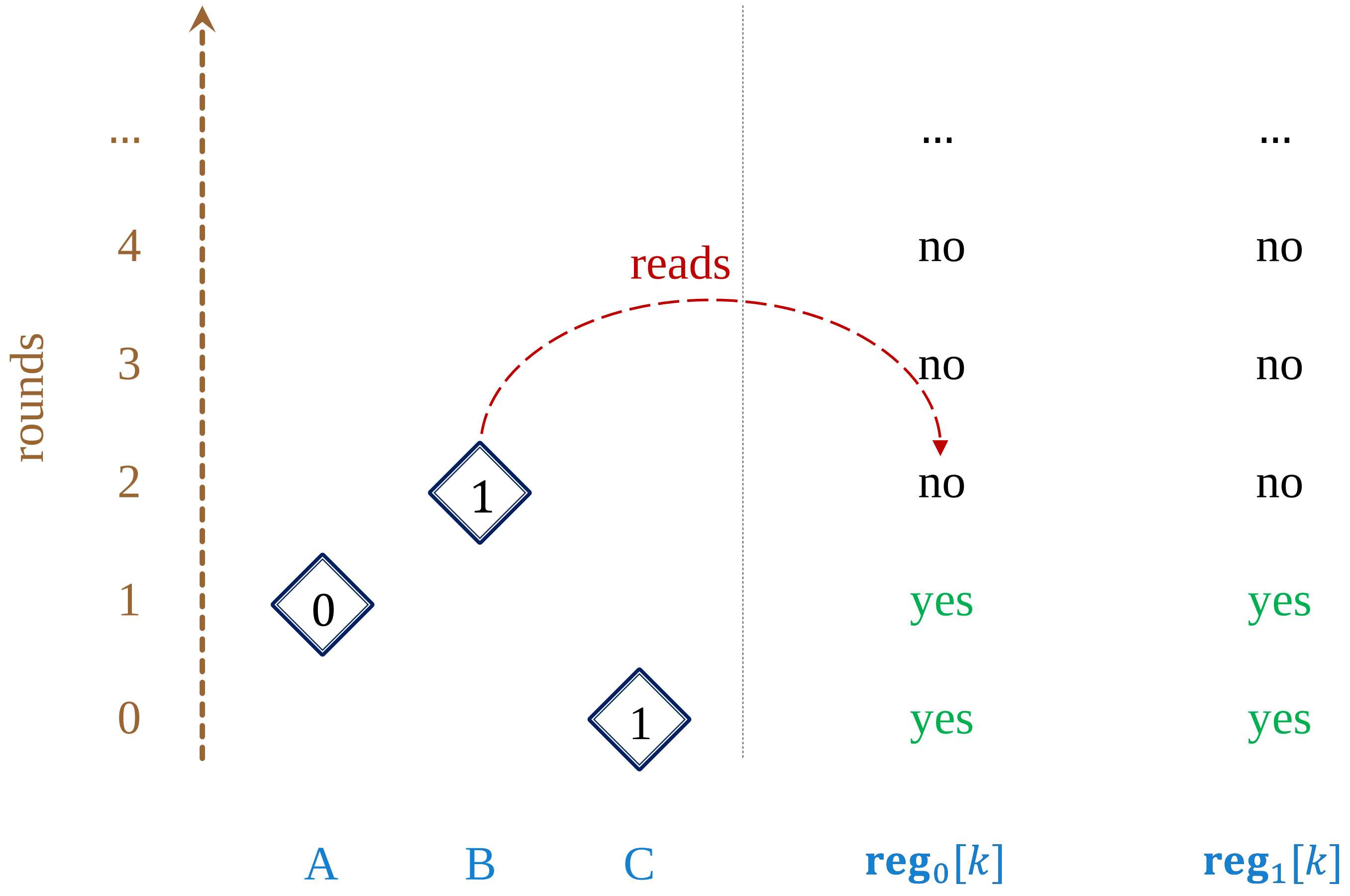

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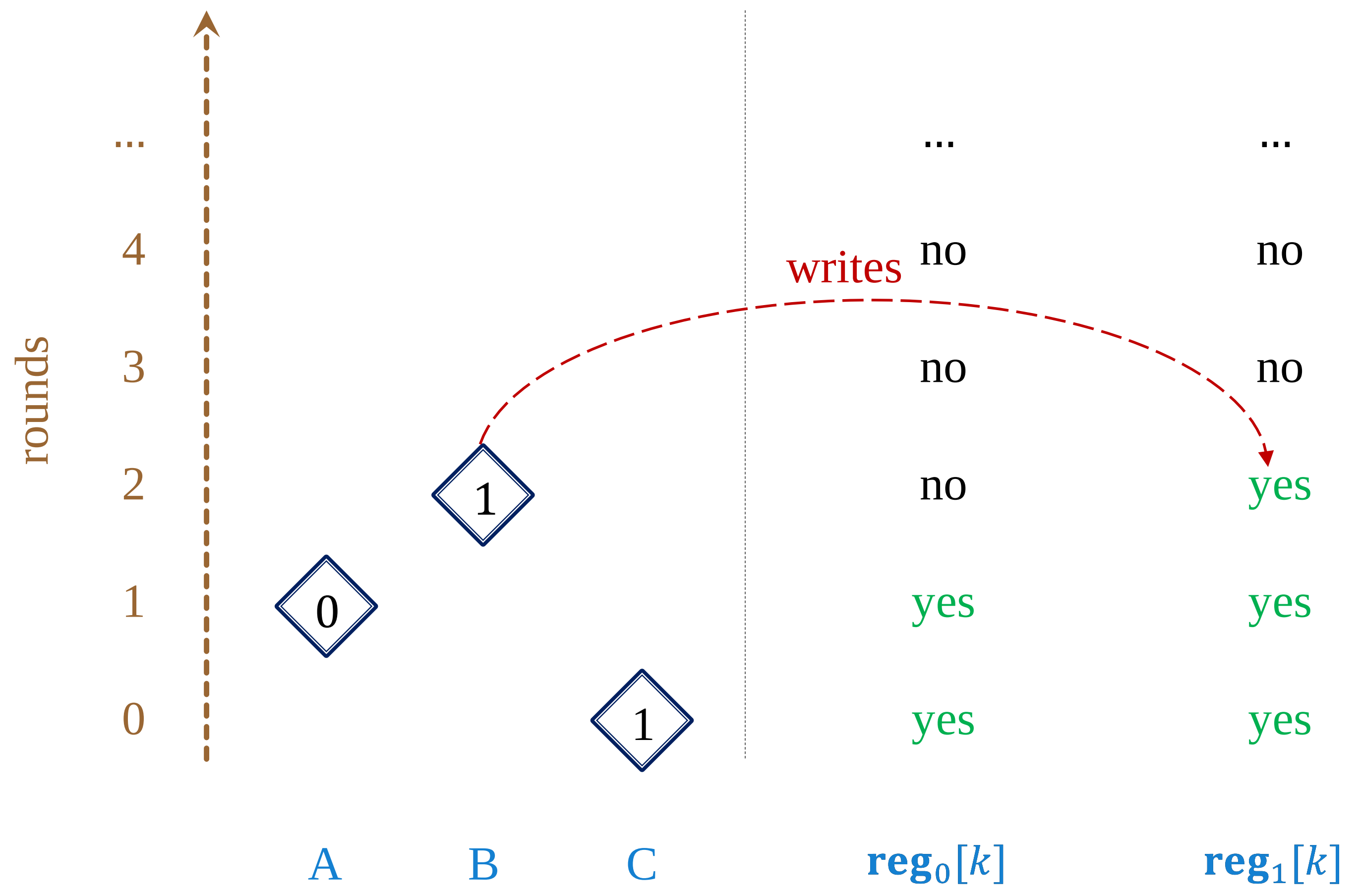

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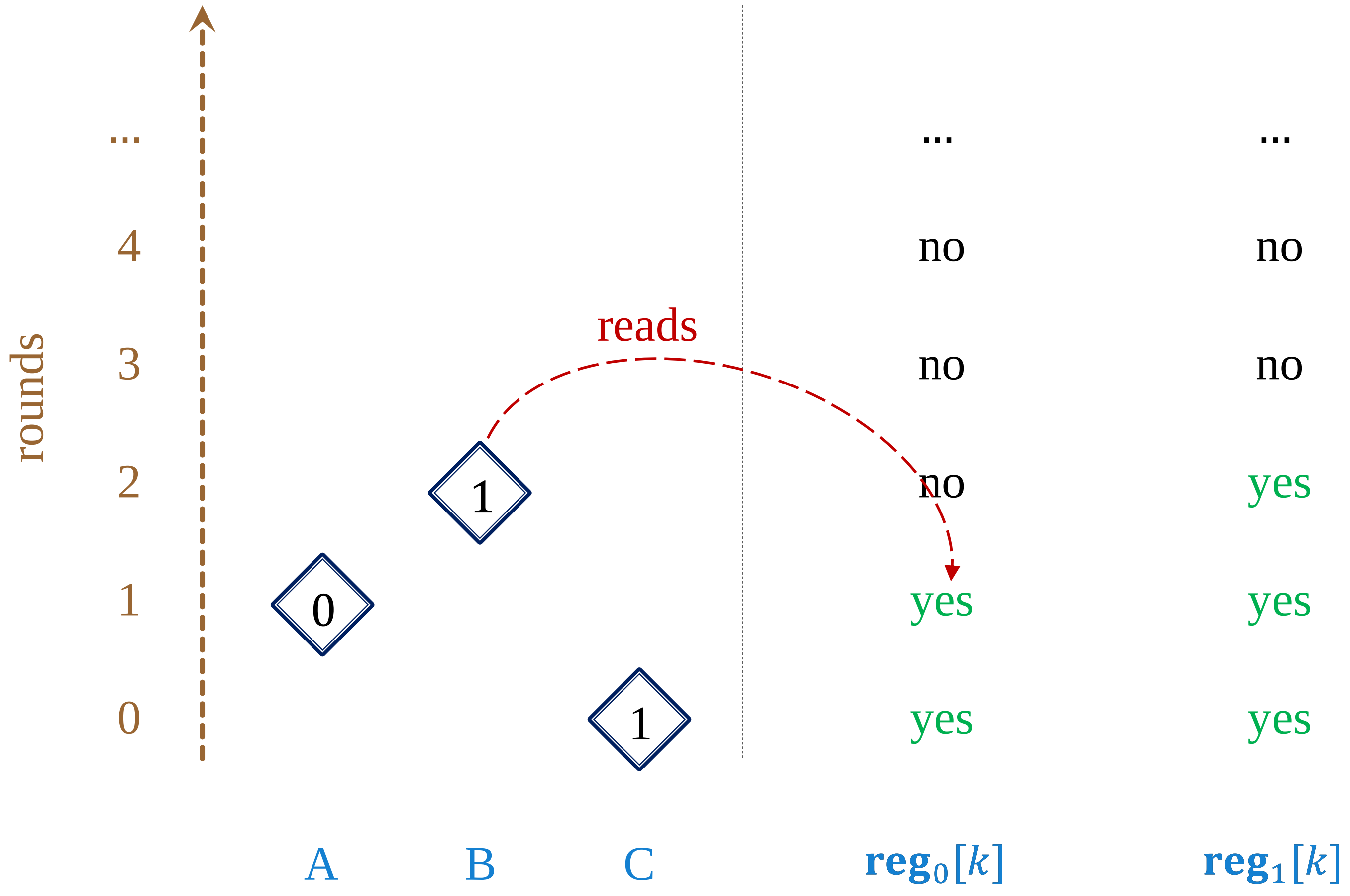

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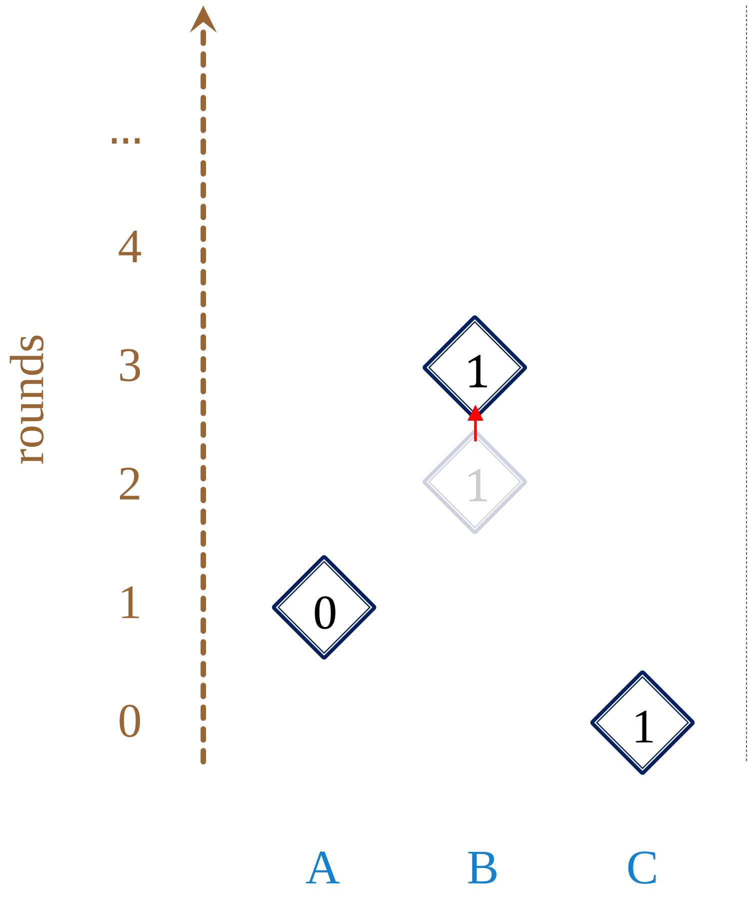

$\mathbf{reg}_0[k]$	$\mathbf{reg}_1[k]$
...	...
no	no
no	no
no	yes
yes	yes
yes	yes

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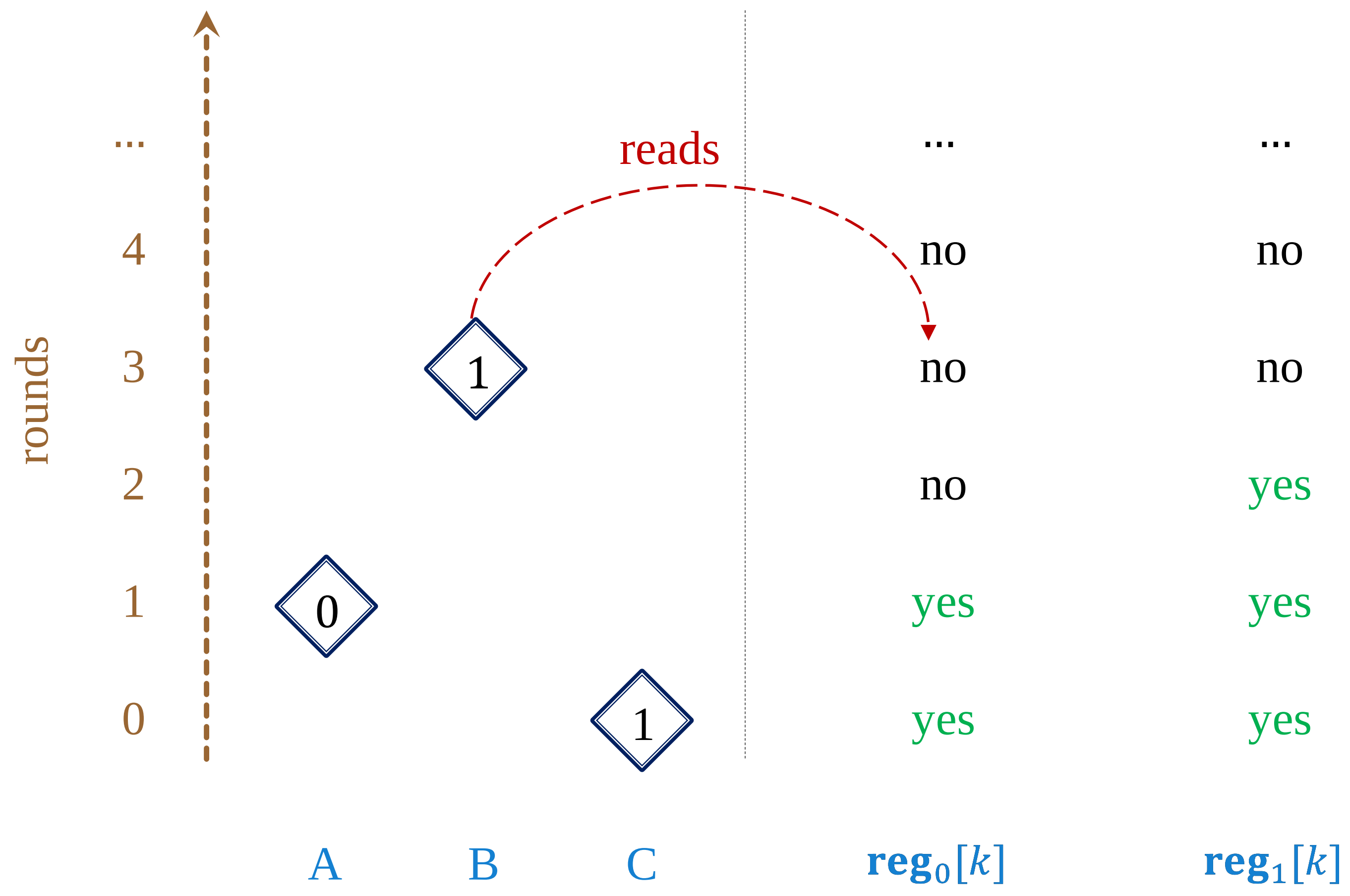

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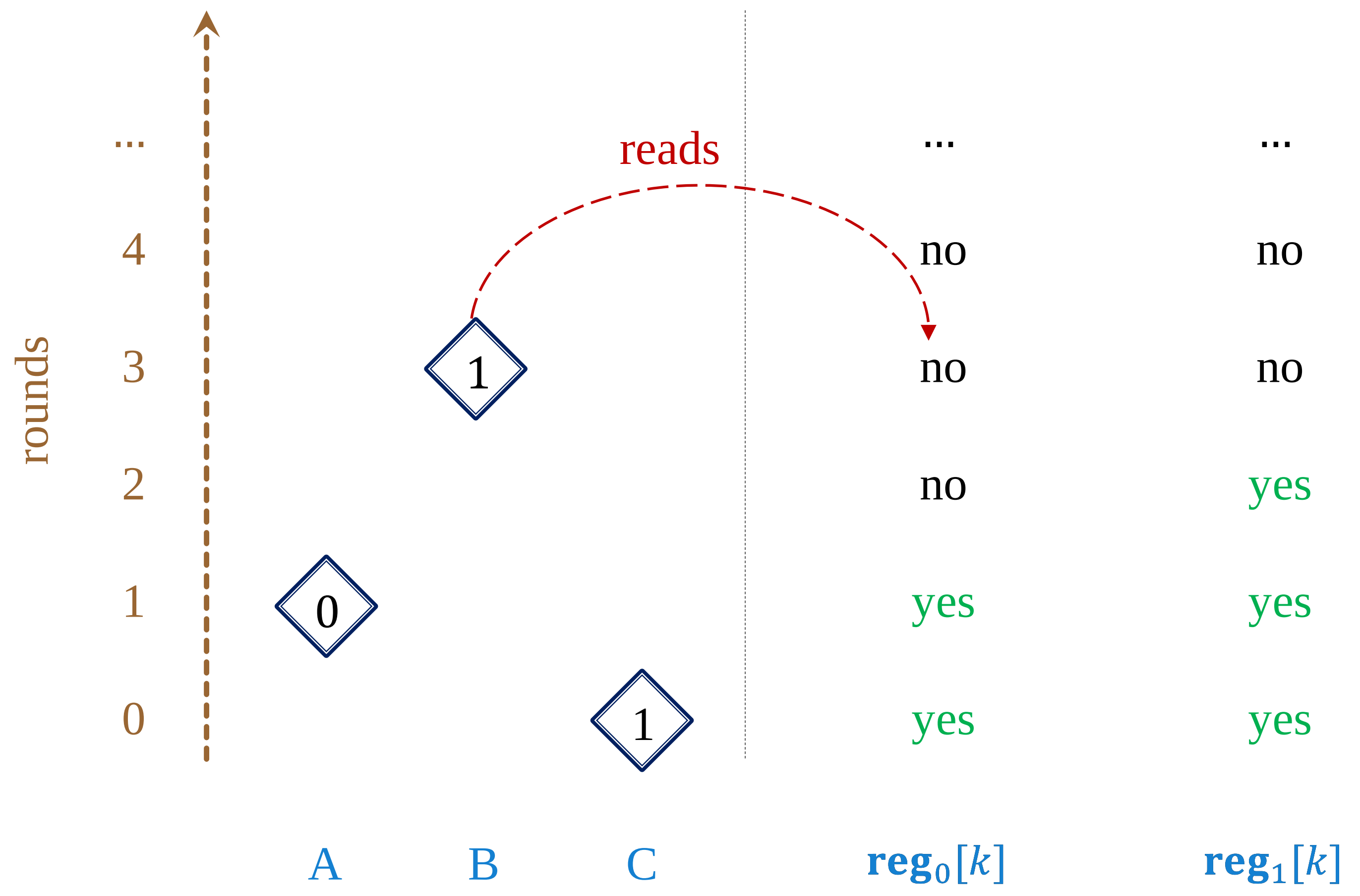

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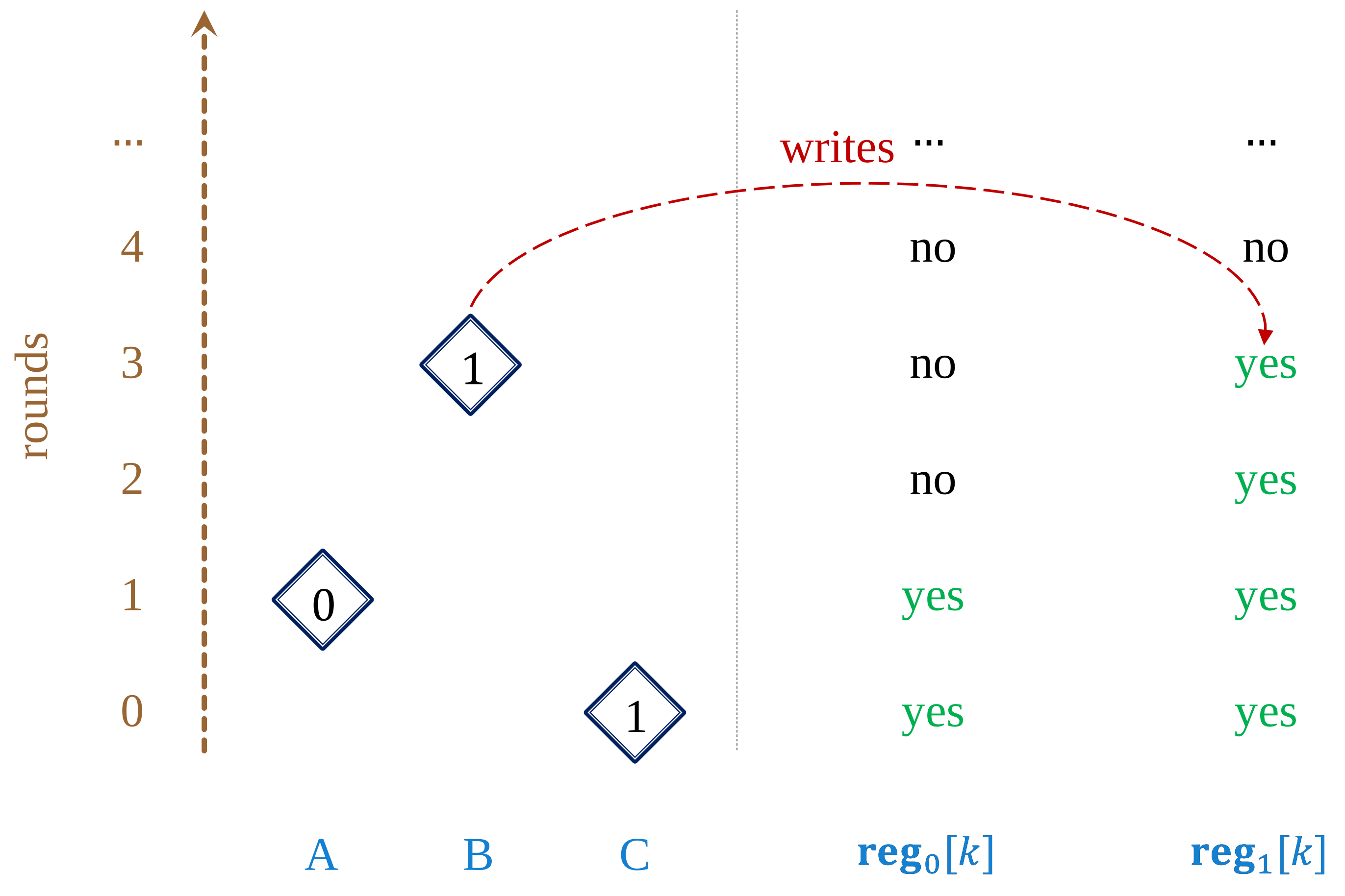

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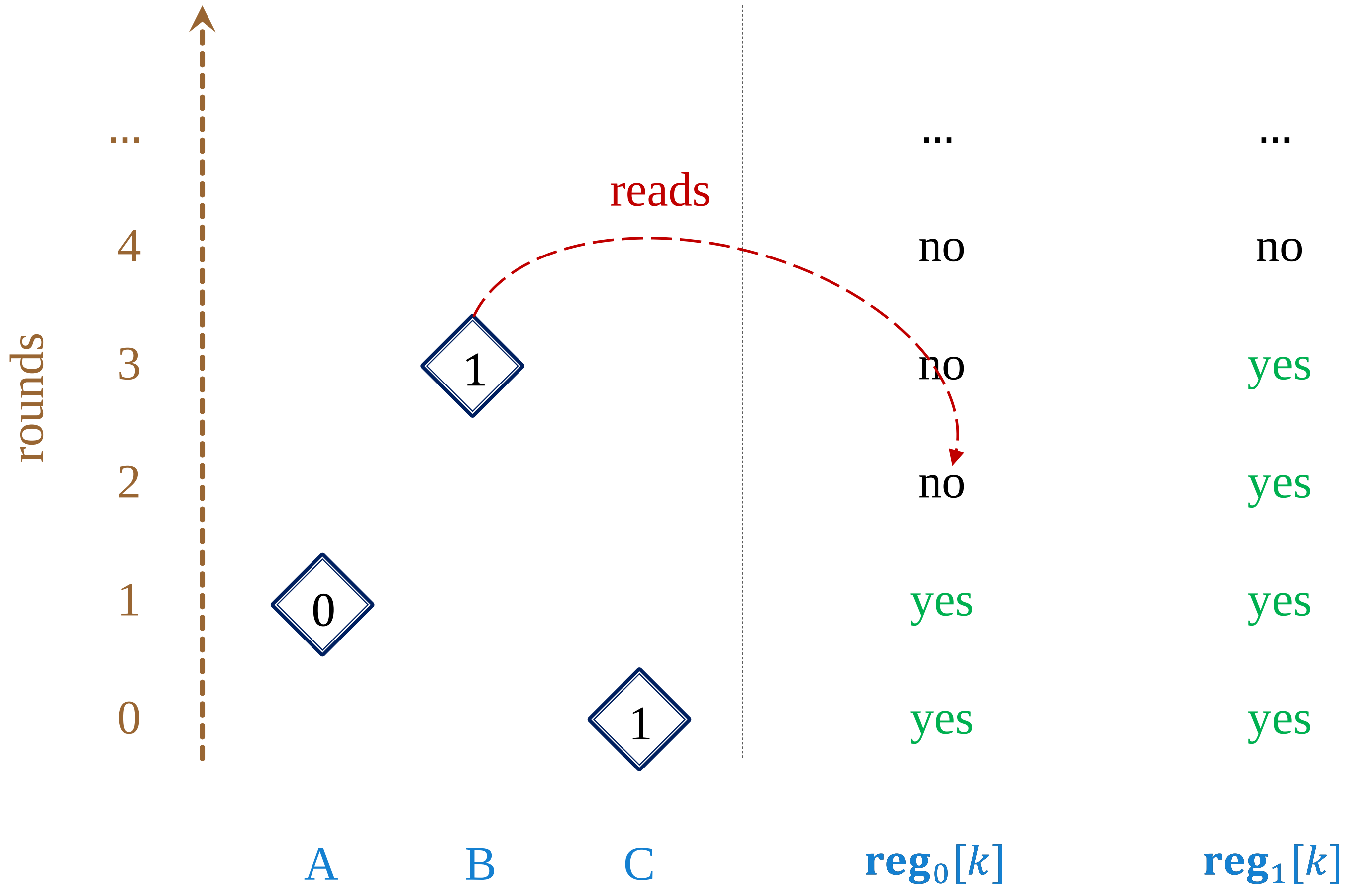

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