Checking Presence Reachability Properties on Parameterized Shared-Memory Systems

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Joint work with Nathalie Bertrand, Nicolas Markey, Ocan Sankur
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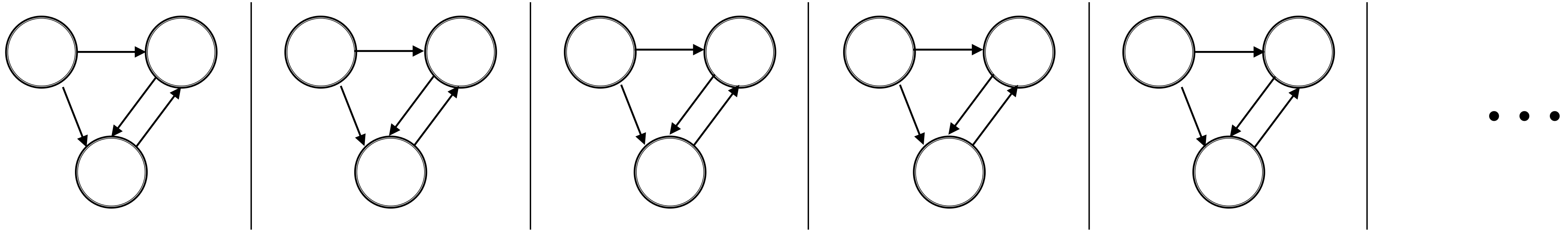
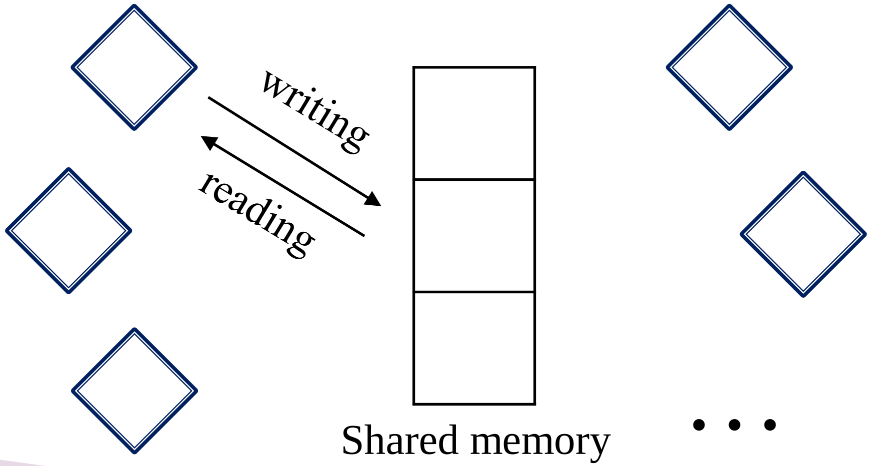
Parameterized Shared-memory Systems

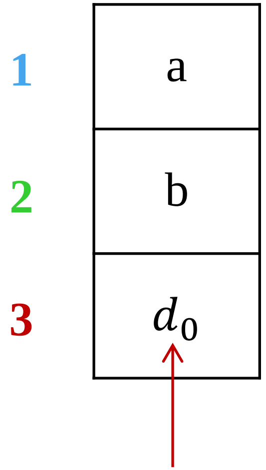


Arbitrary number of identical, anonymous agents
communicating using a shared memory

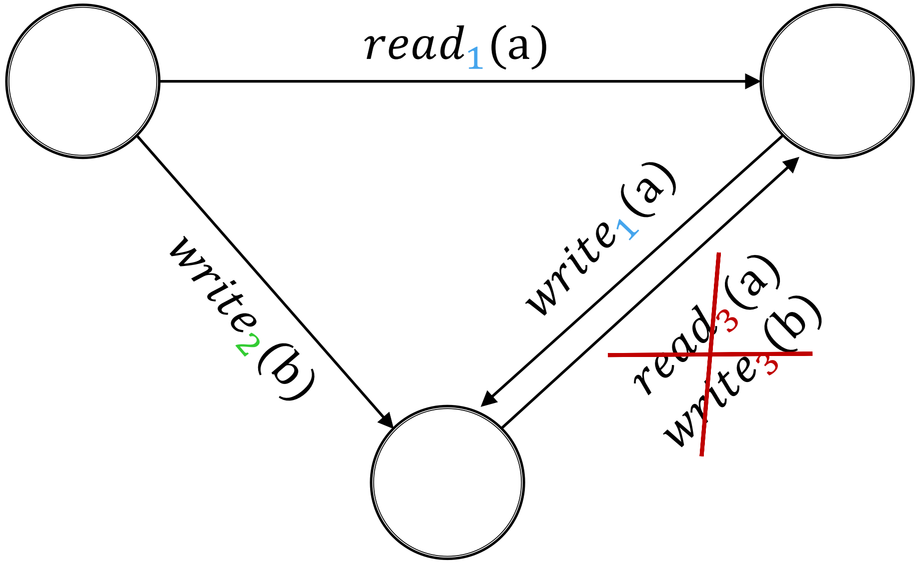


A model for shared-memory systems[1]

Each register has a value from the finite set of symbols Σ



Registers are *initialized* to value d_0



No atomic read/write combinations

1. Model inspired from: Esparza, J., Ganty, P., Majumdar, R.: *Parameterized verification of asynchronous shared-memory systems.* Journal of the ACM, 2016

Round-based algorithms

We want to model round-based distributed algorithms[2,3,4] that look like this:

Asynchronous rounds

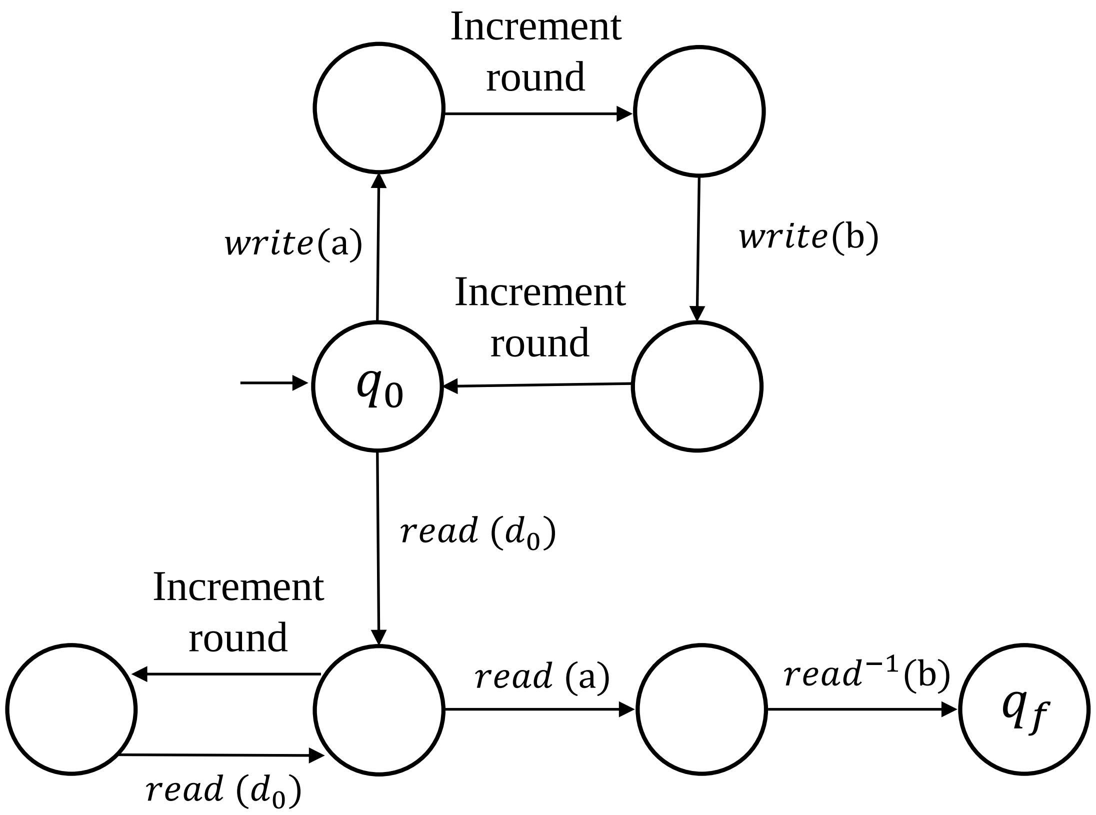
Each round has its own set of shared registers
→ unbounded memory !

read and write to registers
of nearby rounds only

```
for k = 0 to ∞ do
    read register 0 of round k ;
    if read value is a then ... else ... ;
    write to register 1 of round k ;
    ...
    read register 0 of round k − 1 ;
    ...
end
```

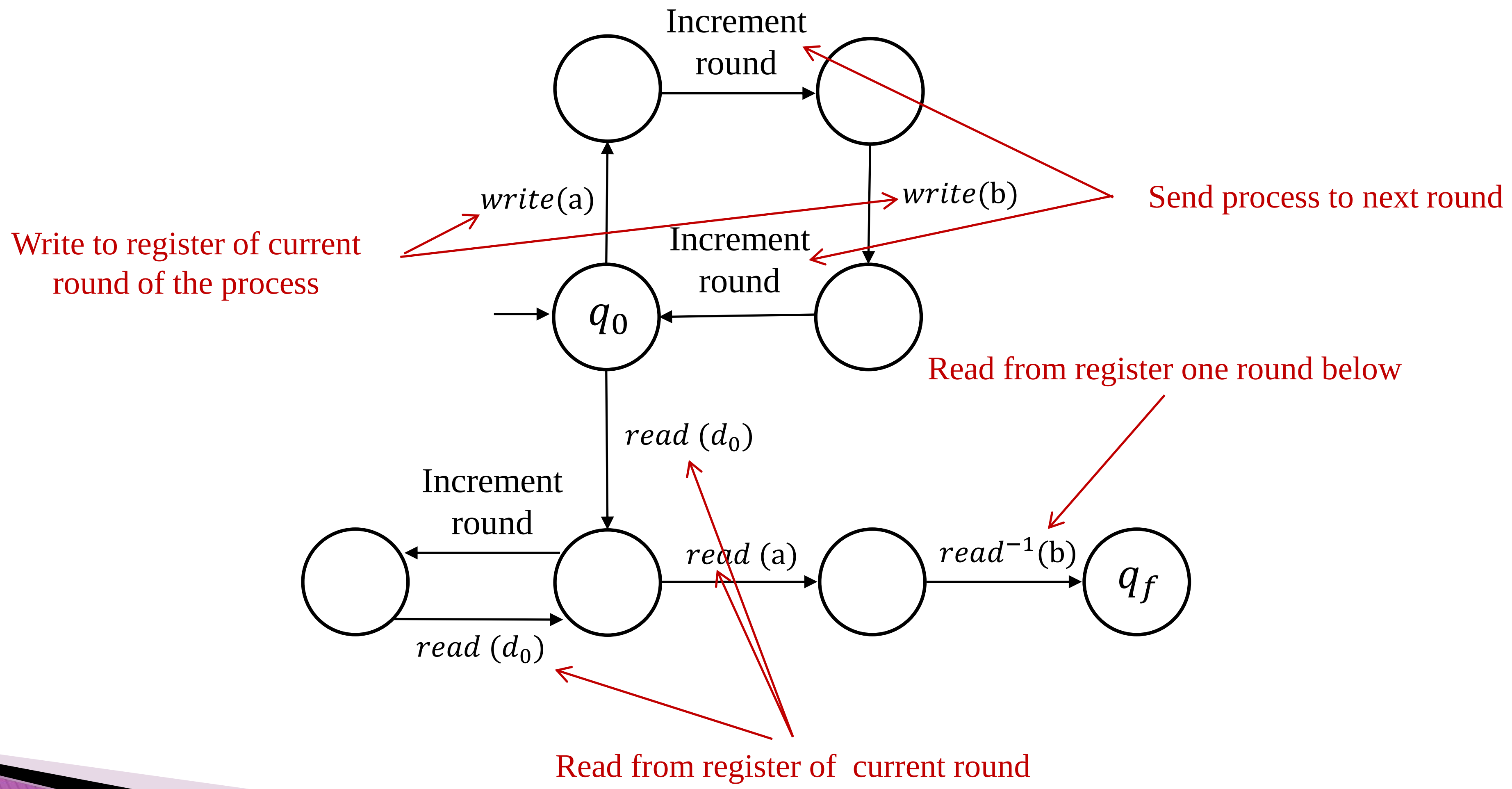
2. Aspnes, J.: *Fast deterministic consensus in a noisy environment.* Journal of Algorithms, 2002
3. Guerraoui, R., Ruppert, E.: *Anonymous and fault-tolerant shared-memory computing.* Distrib. Comput., 2007
4. Raynal, M., Stainer, J.: *A Simple Asynchronous Shared Memory Consensus Algorithm Based on Omega and Closing Sets. CISIS, 2012*

An example



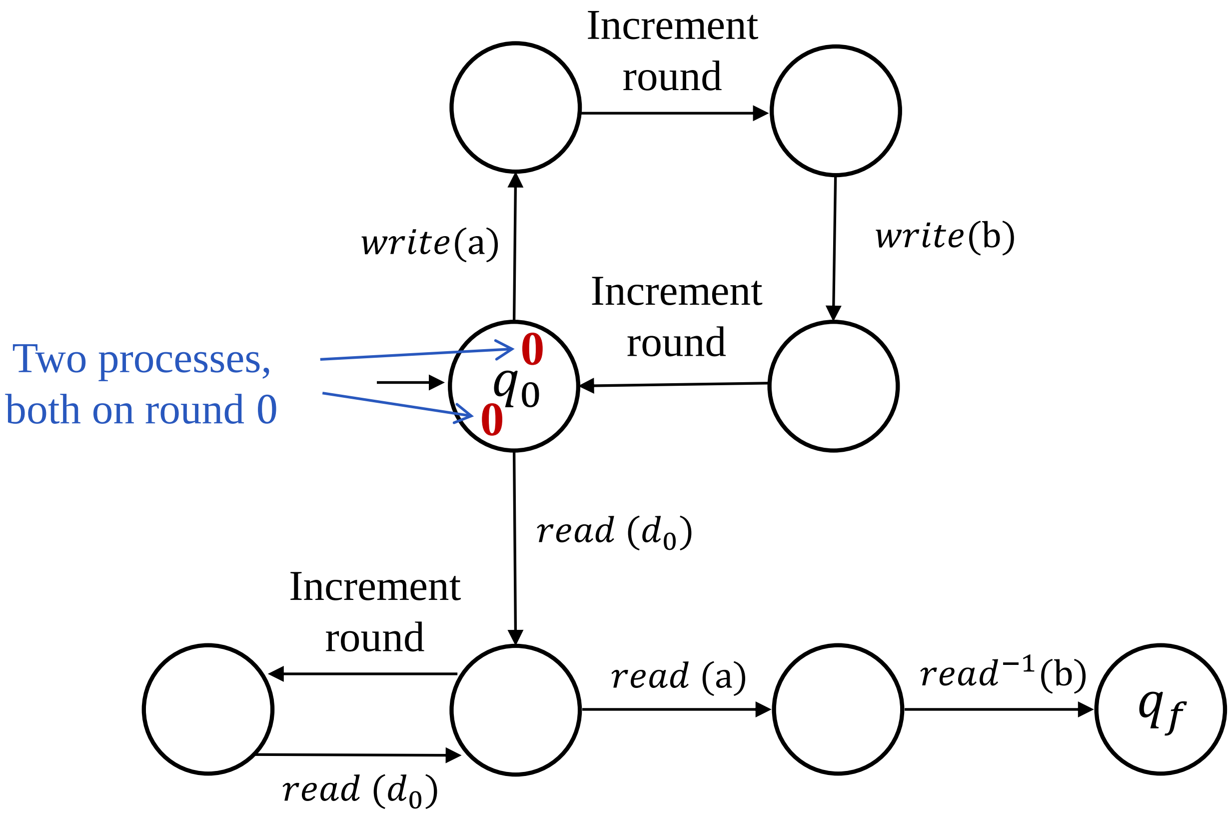
Increment round
$write$(a)
$write$(b)
Increment round
q_0
$read\ (d_0)$
Increment round
$read$ (a)
$read^{-1}$(b)
q_f
$read\ (d_0)$

An example

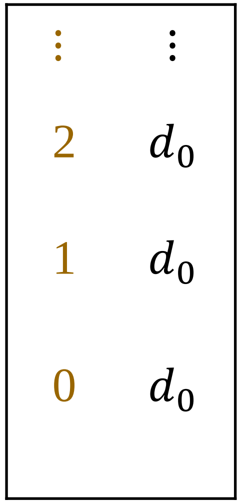


Increment
round
write(a)
write(b)
Send process to next round
Write to register of current
round of the process
Increment
round
q_0
Read from register one round below
$read\ (d_0)$
Increment
round
$read$ (a)
$read^{-1}$(b)
q_f
$read\ (d_0)$
Read from register of current round

An example

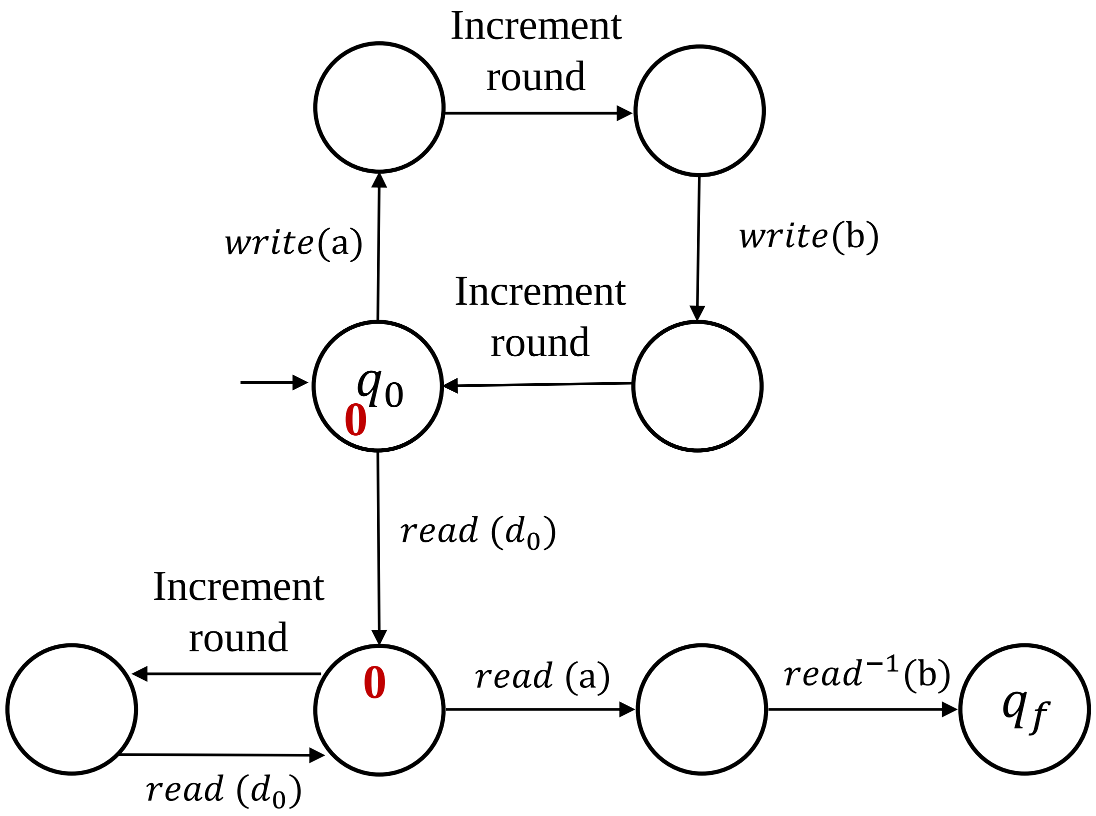


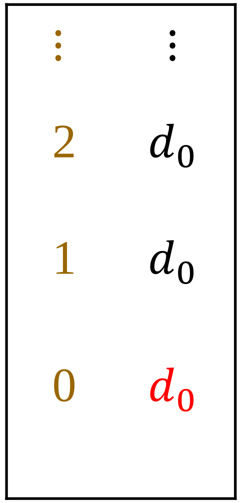
Increment
round
write(a)
write(b)
Increment
round
Two processes,
both on round 0
0
0
q_0
read (d_0)
Increment
round
read (a)
$read^{-1}$(b)
q_f
read (d_0)



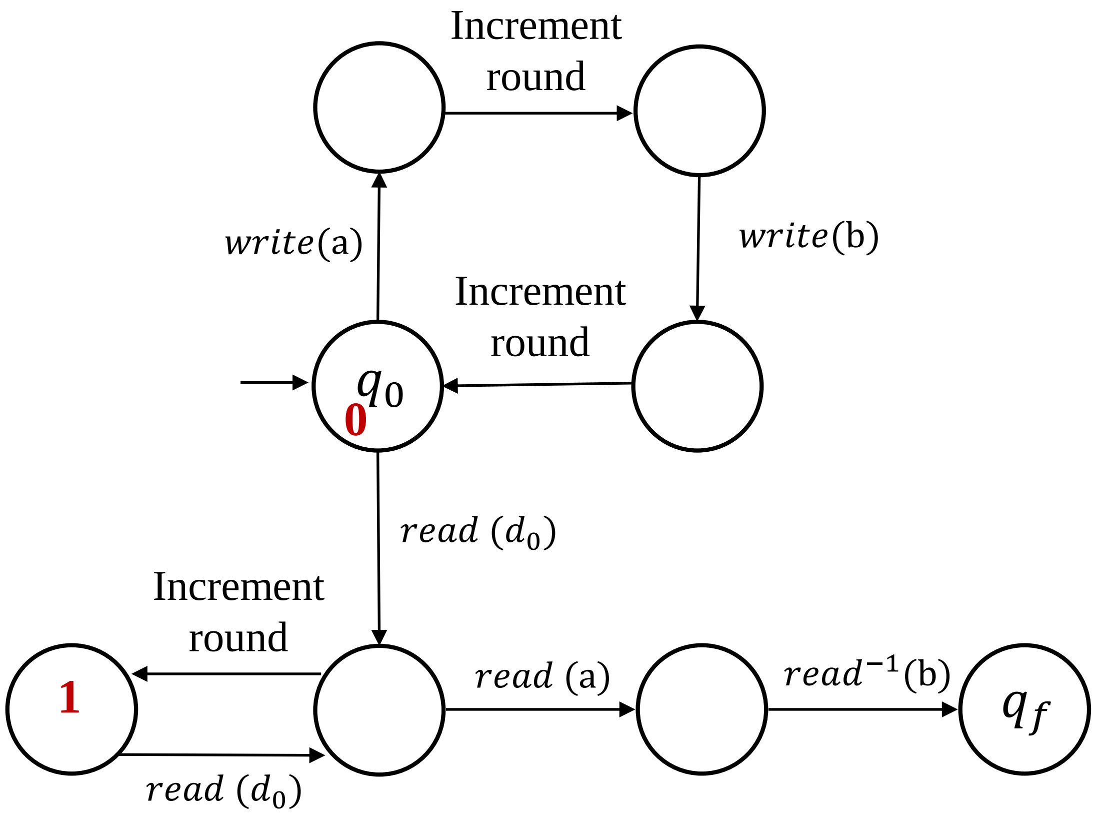
⋮ ⋮
2 d_0
1 d_0
0 d_0

An example

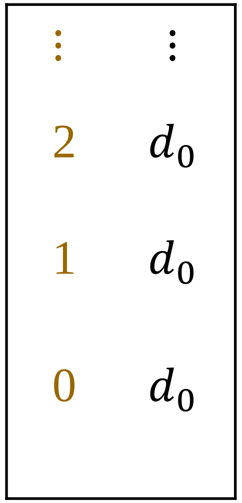

Increment
round
write(a)
write(b)
Increment
round
q_0
0
read (d_0)
Increment
round
0
read (a)
$read^{-1}$(b)
q_f
read (d_0)


⋮ ⋮
2 d_0
1 d_0
0 d_0

An example

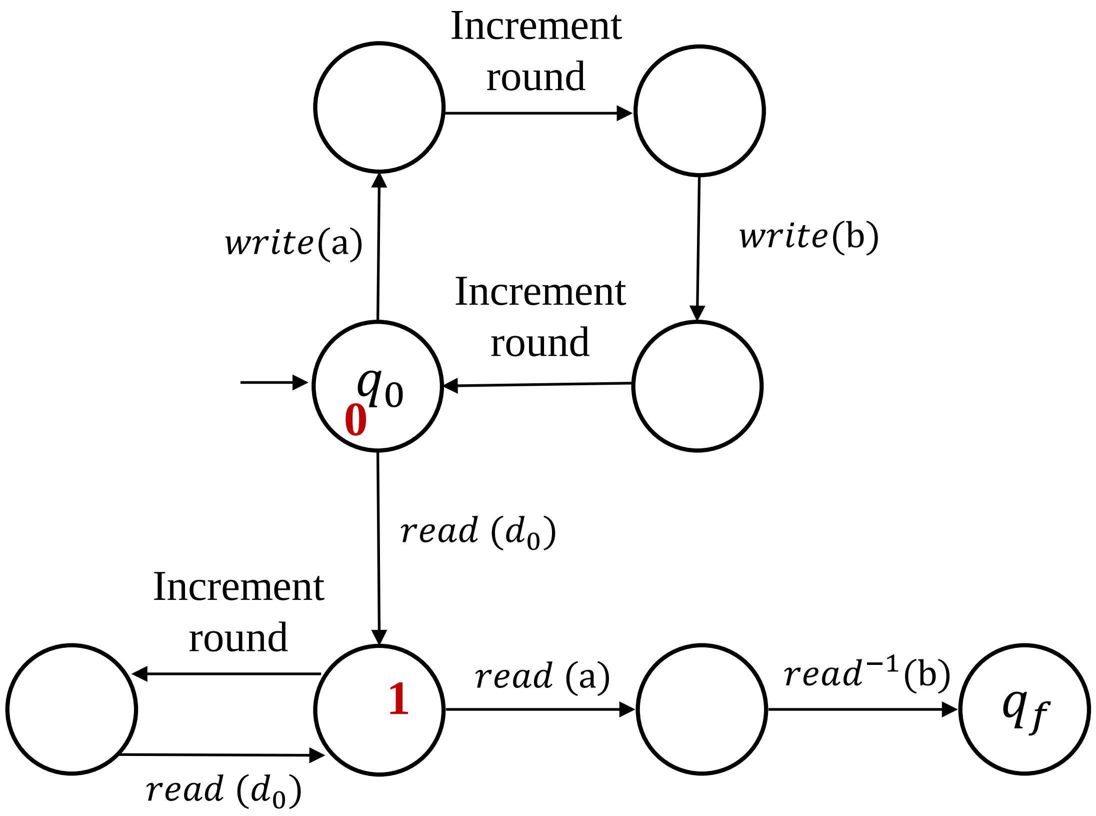


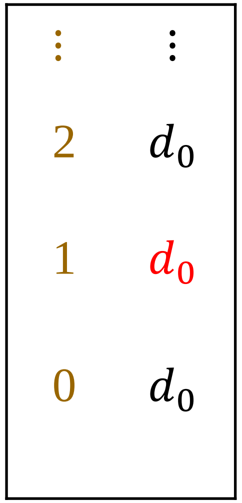
Increment round
write(a)
write(b)
Increment round
q_0
0
read (d_0)
Increment round
1
read (a)
$read^{-1}$(b)
q_f
read (d_0)



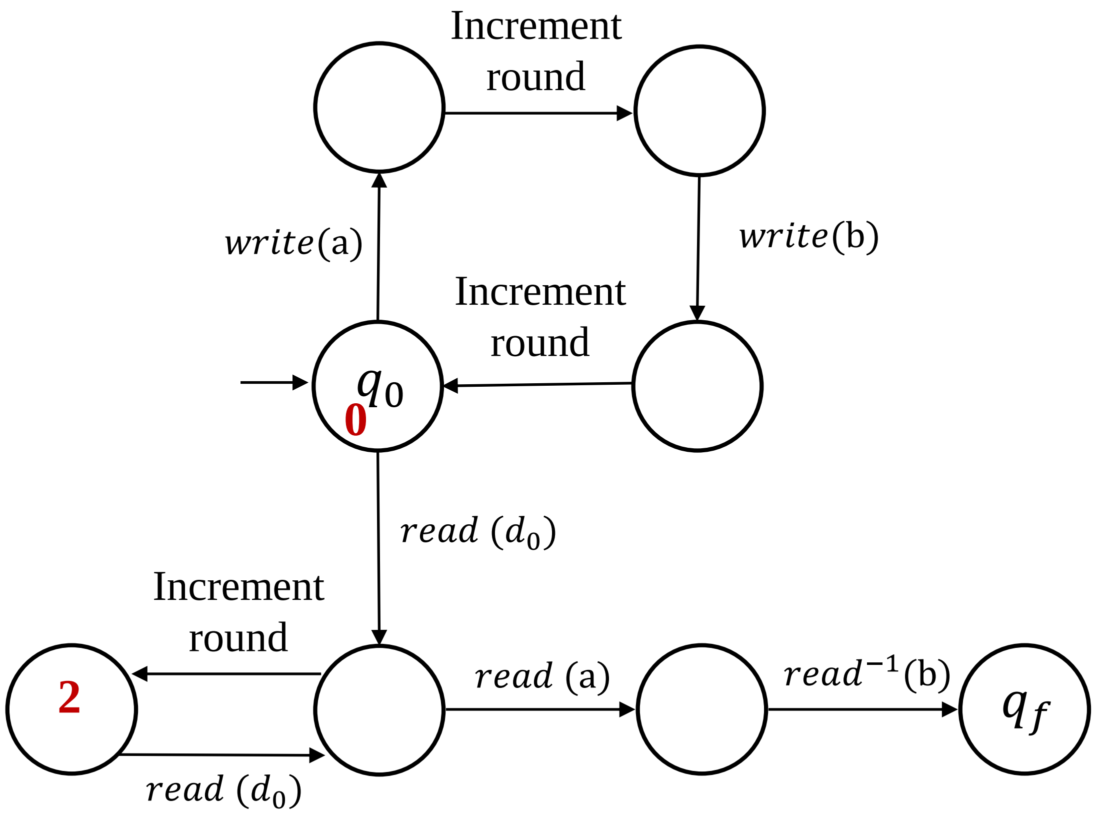
⋮ ⋮
2 d_0
1 d_0
0 d_0

An example


Increment round
write(a)
write(b)
Increment round
q_0
0
read (d_0)
Increment round
1
read (a)
$read^{-1}$(b)
q_f
read (d_0)

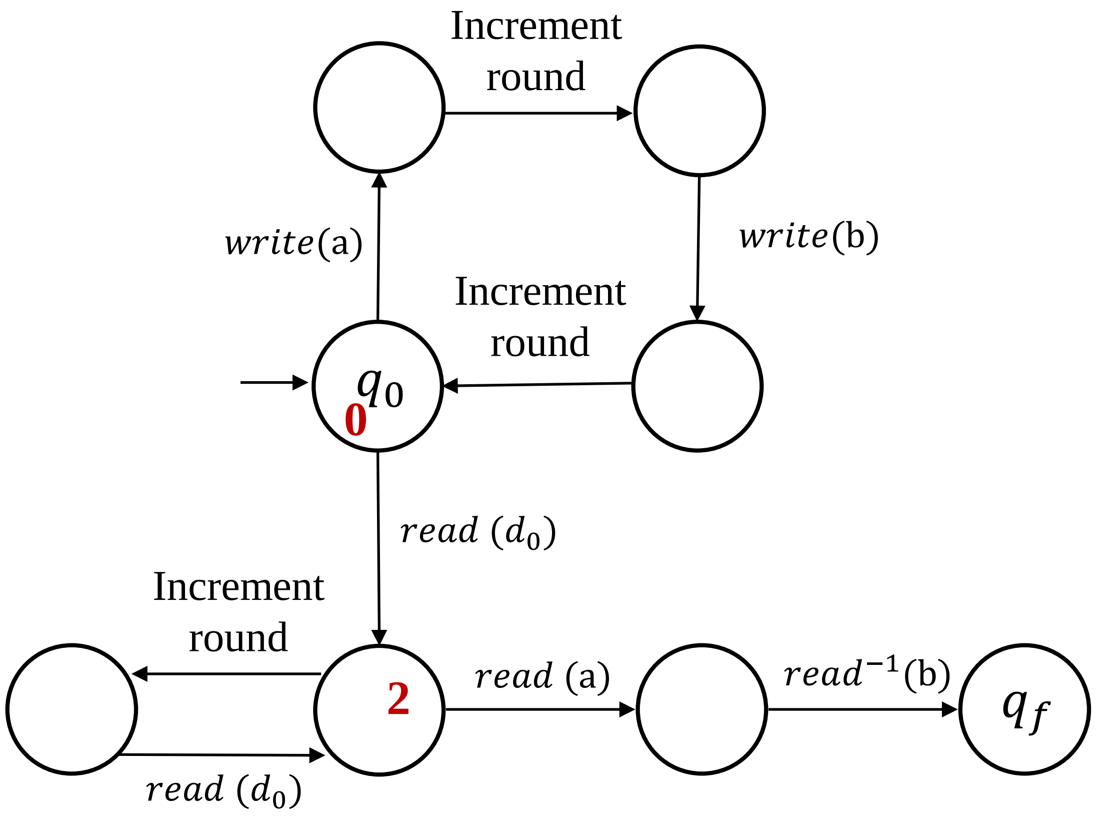

⋮ ⋮
2 d_0
1 d_0
0 d_0

An example



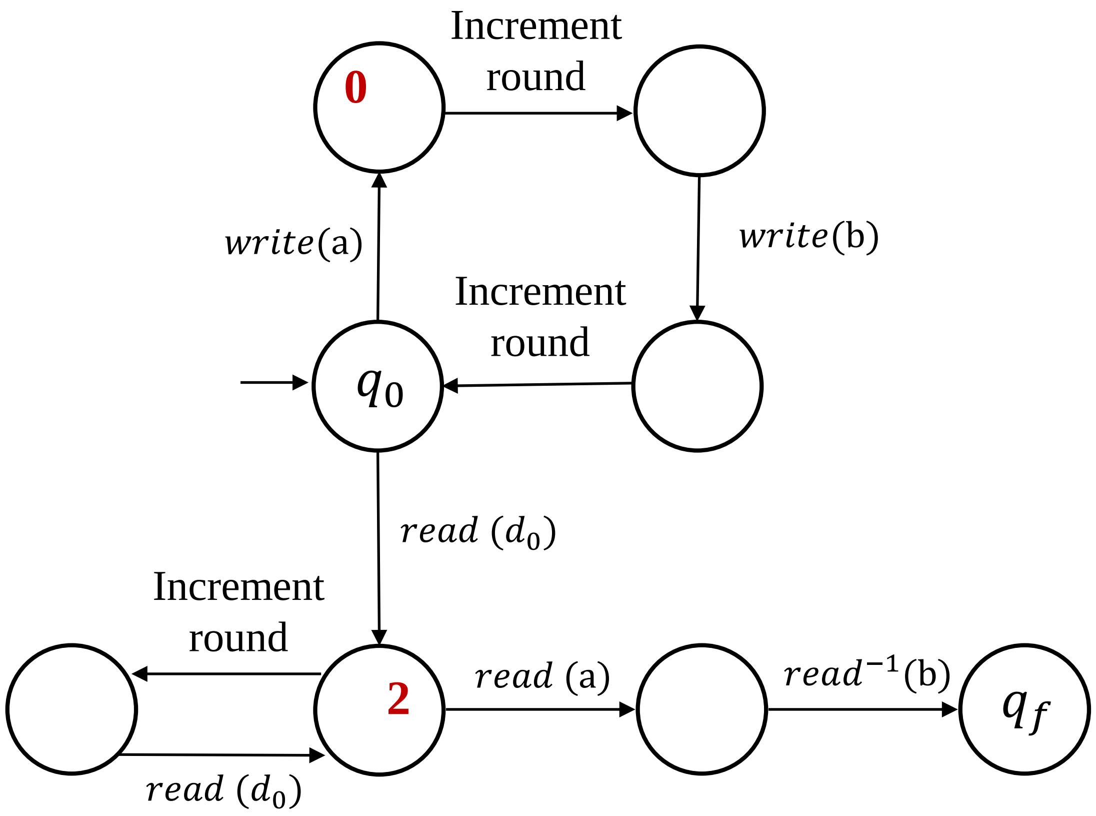
⋮	⋮
2	d_0
1	d_0
0	d_0

An example



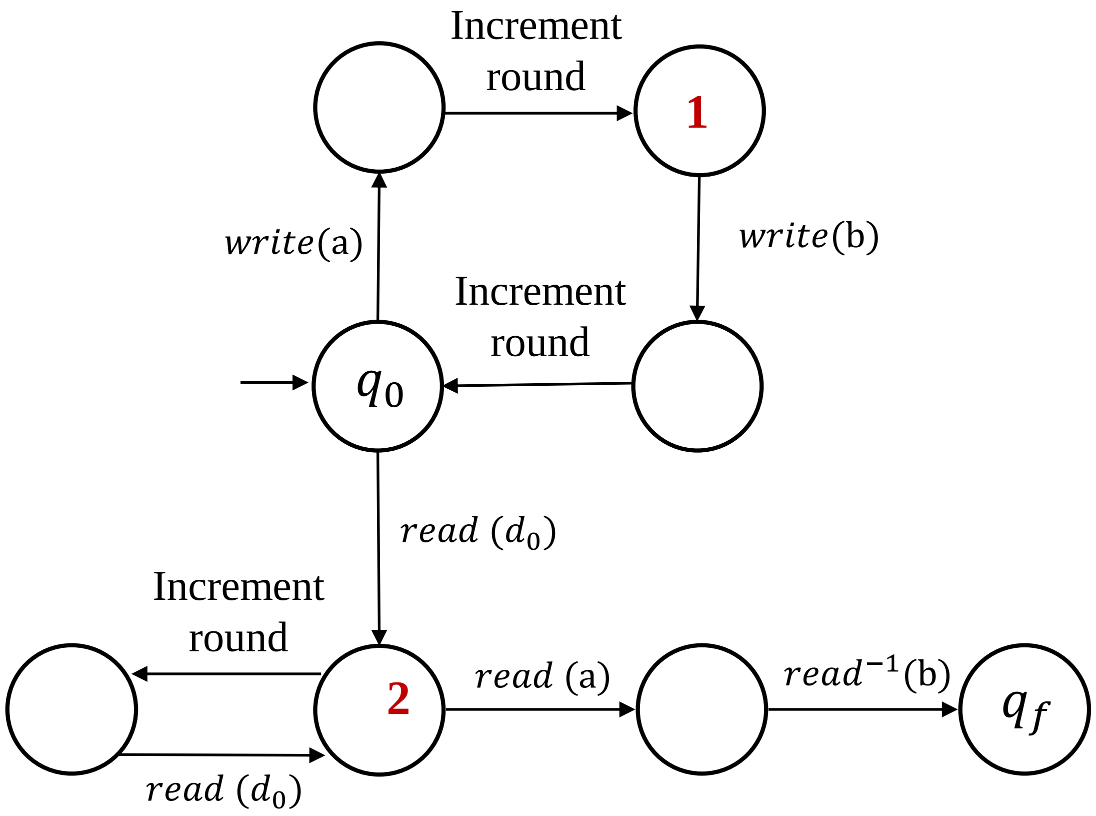
⋮	⋮
2	d_0
1	d_0
0	d_0

An example

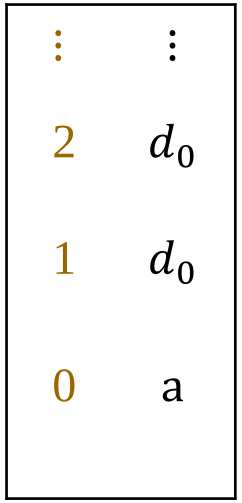


⋮	⋮
2	d_0
1	d_0
0	a

An example

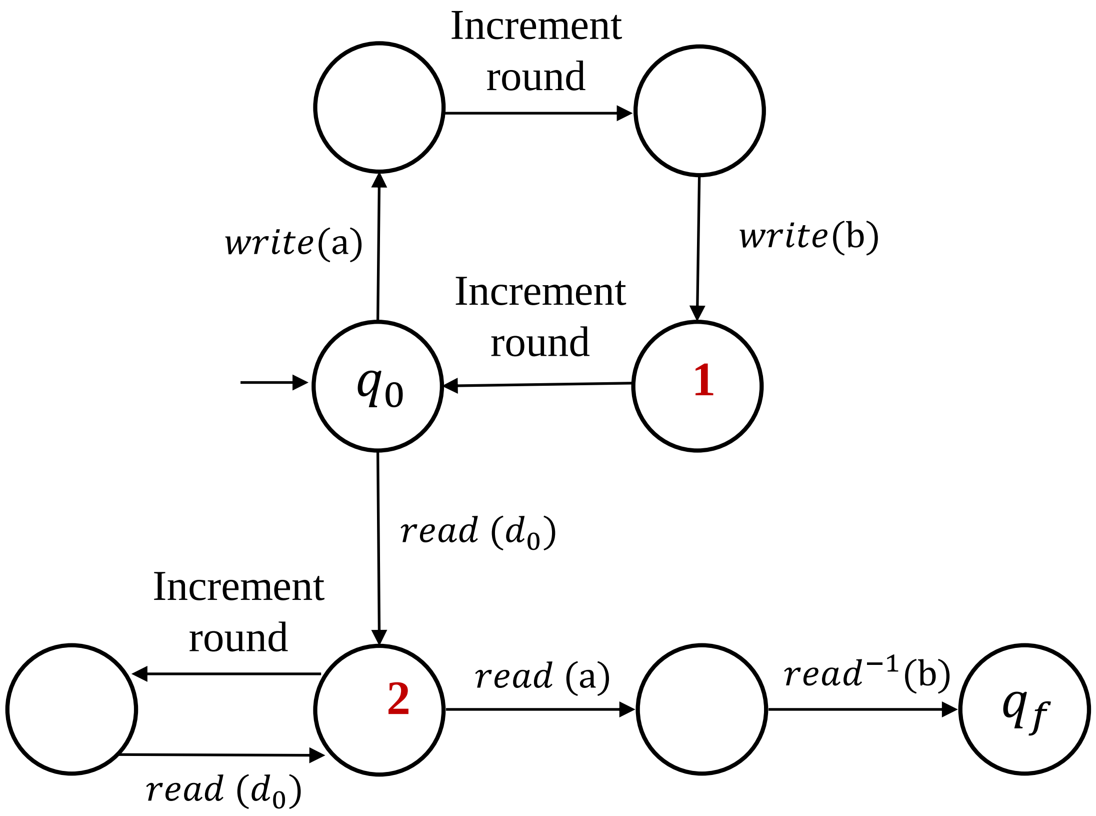


Increment round
1
write(b)
write(a)
Increment round
q_0
read (d_0)
Increment round
2
read (a)
$read^{-1}$(b)
q_f
read (d_0)



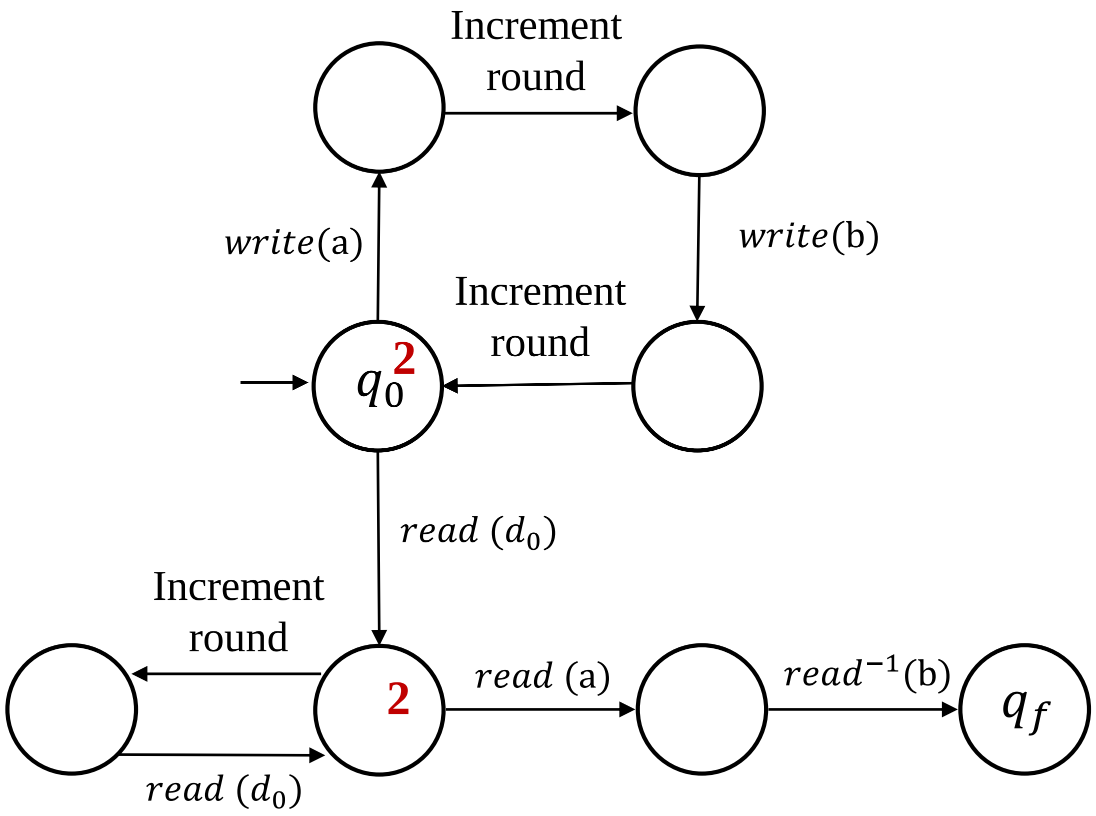
⋮ ⋮
2 d_0
1 d_0
0 a

An example



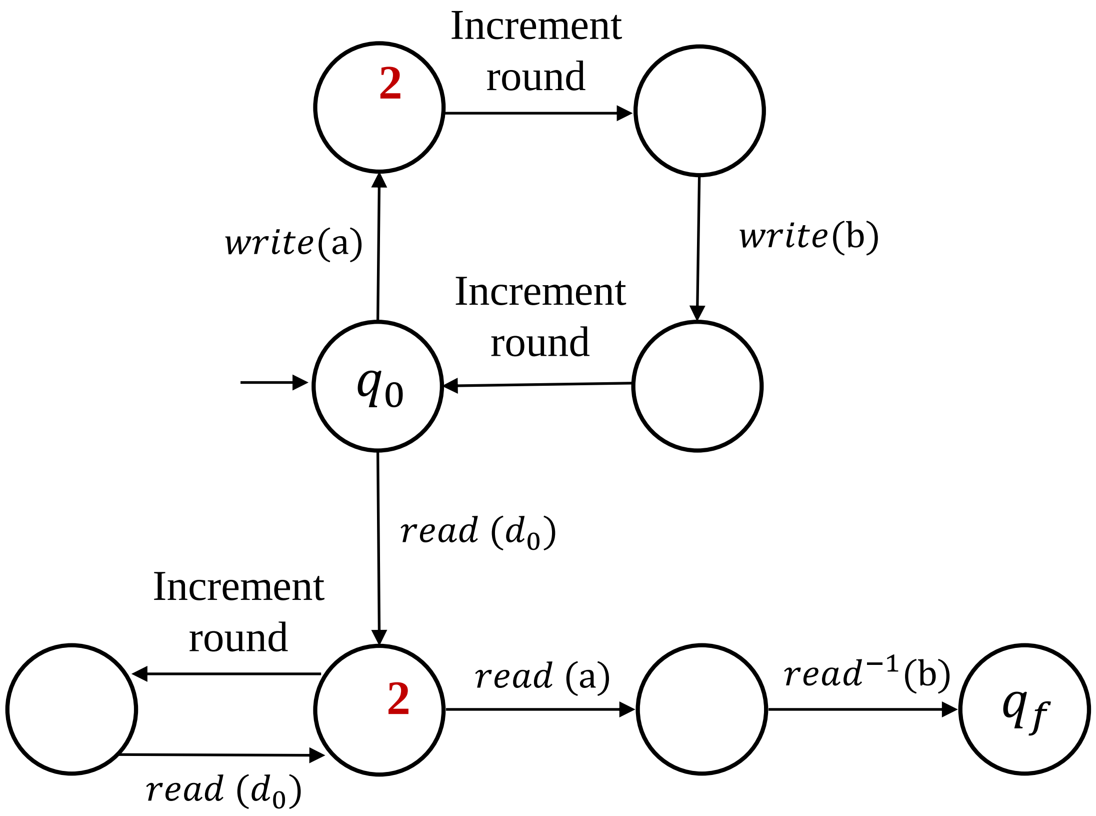
⋮	⋮
2	d_0
1	b
0	a

An example



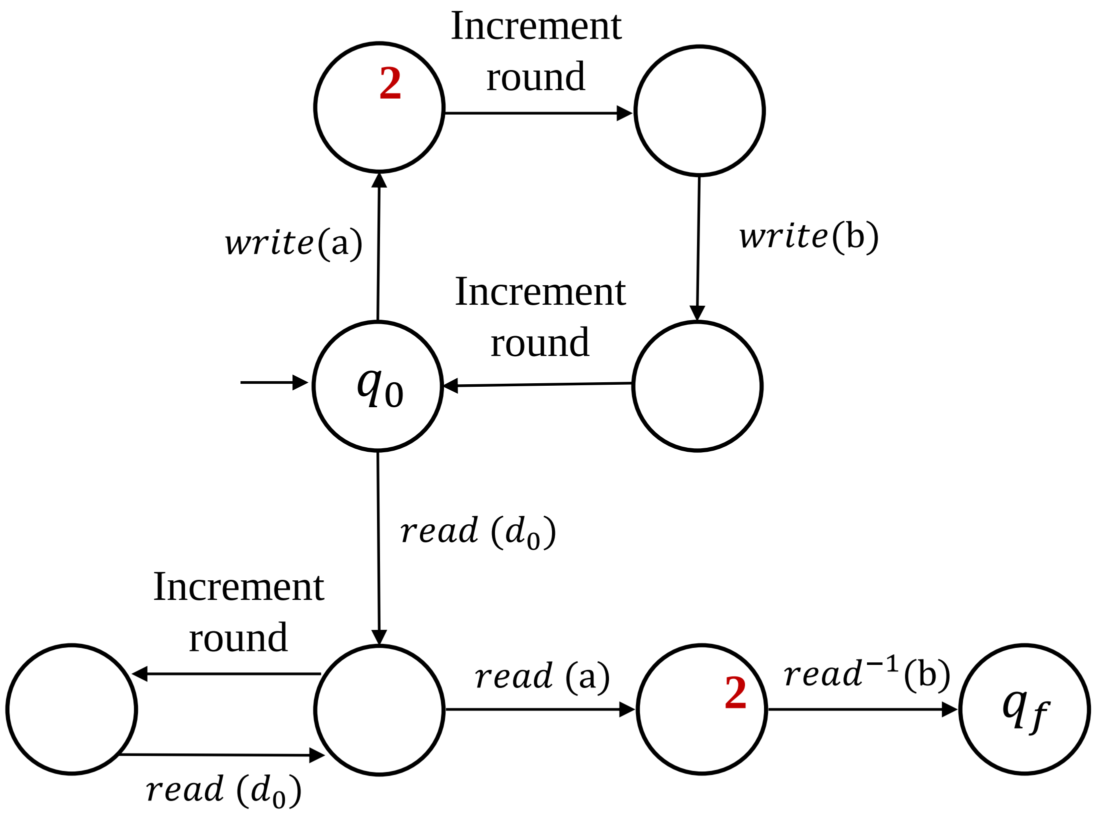
⋮	⋮
2	d_0
1	b
0	a

An example



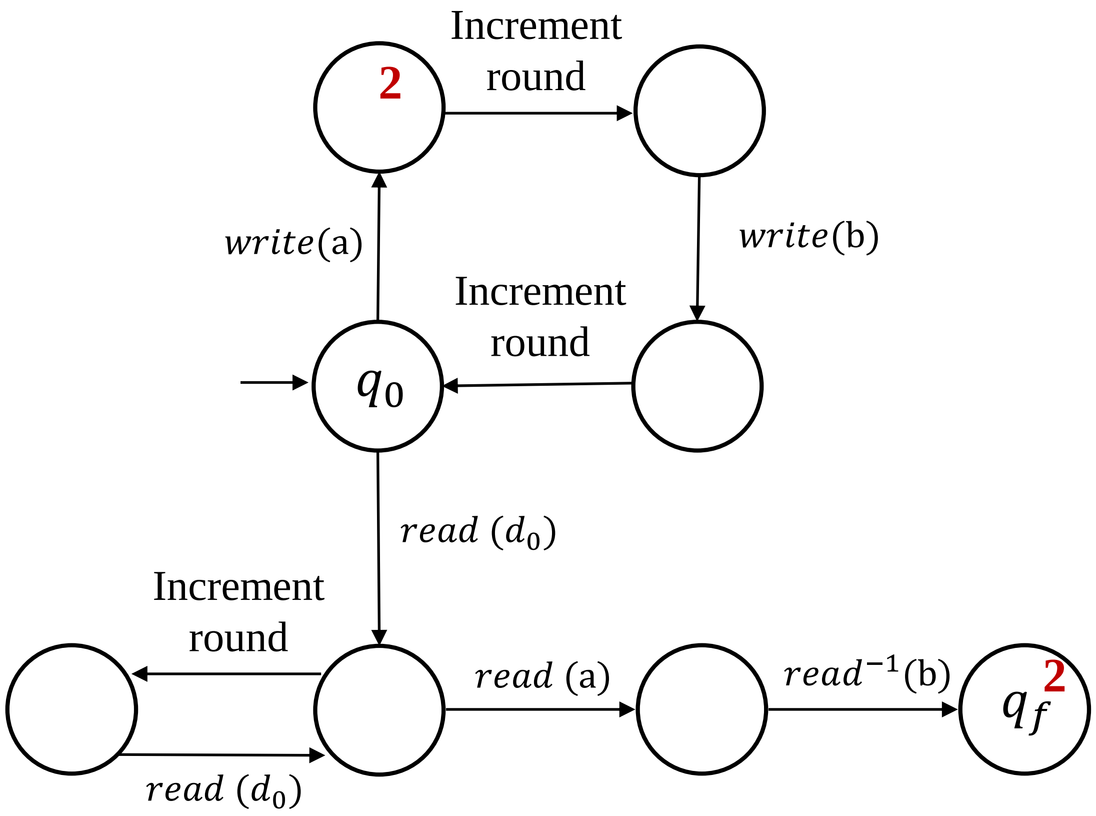
⋮	⋮
2	a
1	b
0	a

An example



⋮	⋮
2	a
1	b
0	a

An example



⋮	⋮
2	a
1	b
0	a

Reachability problems in round-based setting

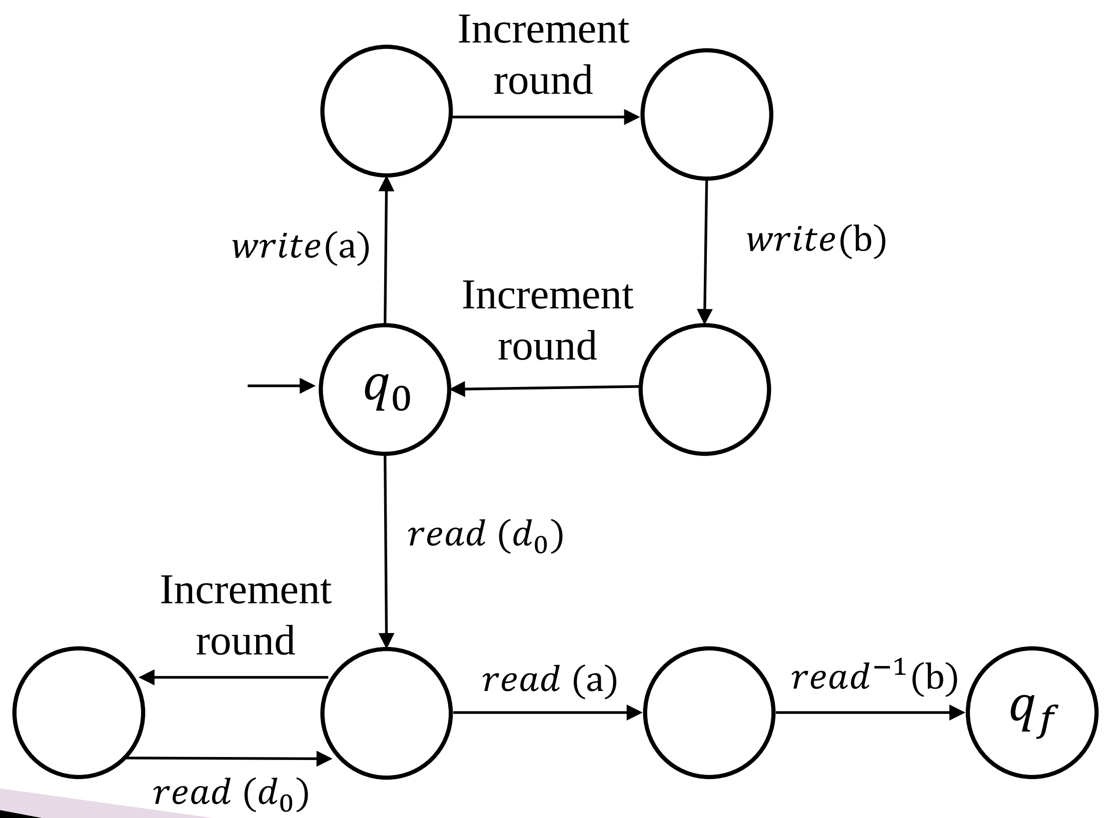
COVER: $\exists n, \exists \rho\colon \gamma_0 \rightarrow^* \gamma\,,\ \exists k\ \ \gamma(q_f, k) > 0$?

$\uparrow$

Parameterized by n: arbitrarily large number of agents

Reachability problems in round-based setting

COVER: $\exists n, \exists \rho\colon \gamma_0 \rightarrow^* \gamma\,,\ \exists k\ \gamma(q_f, k) > 0$?



✓

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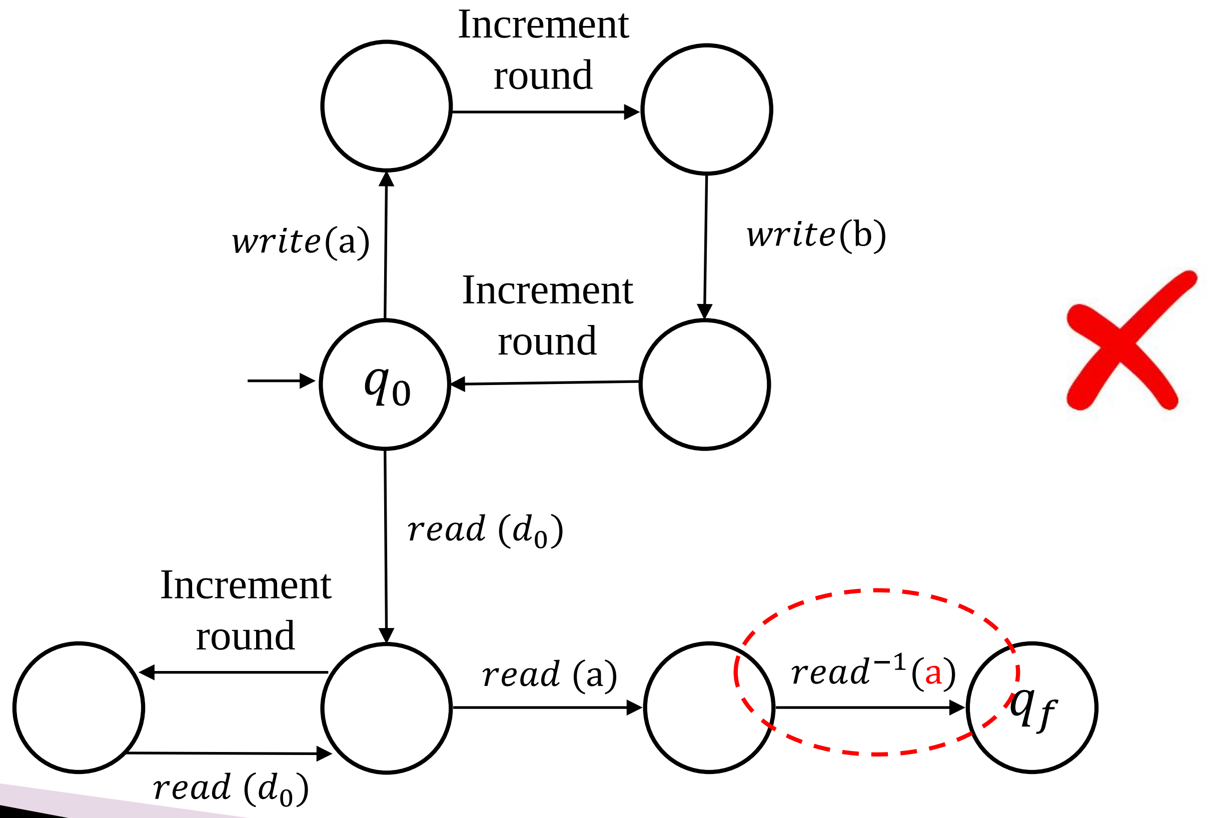

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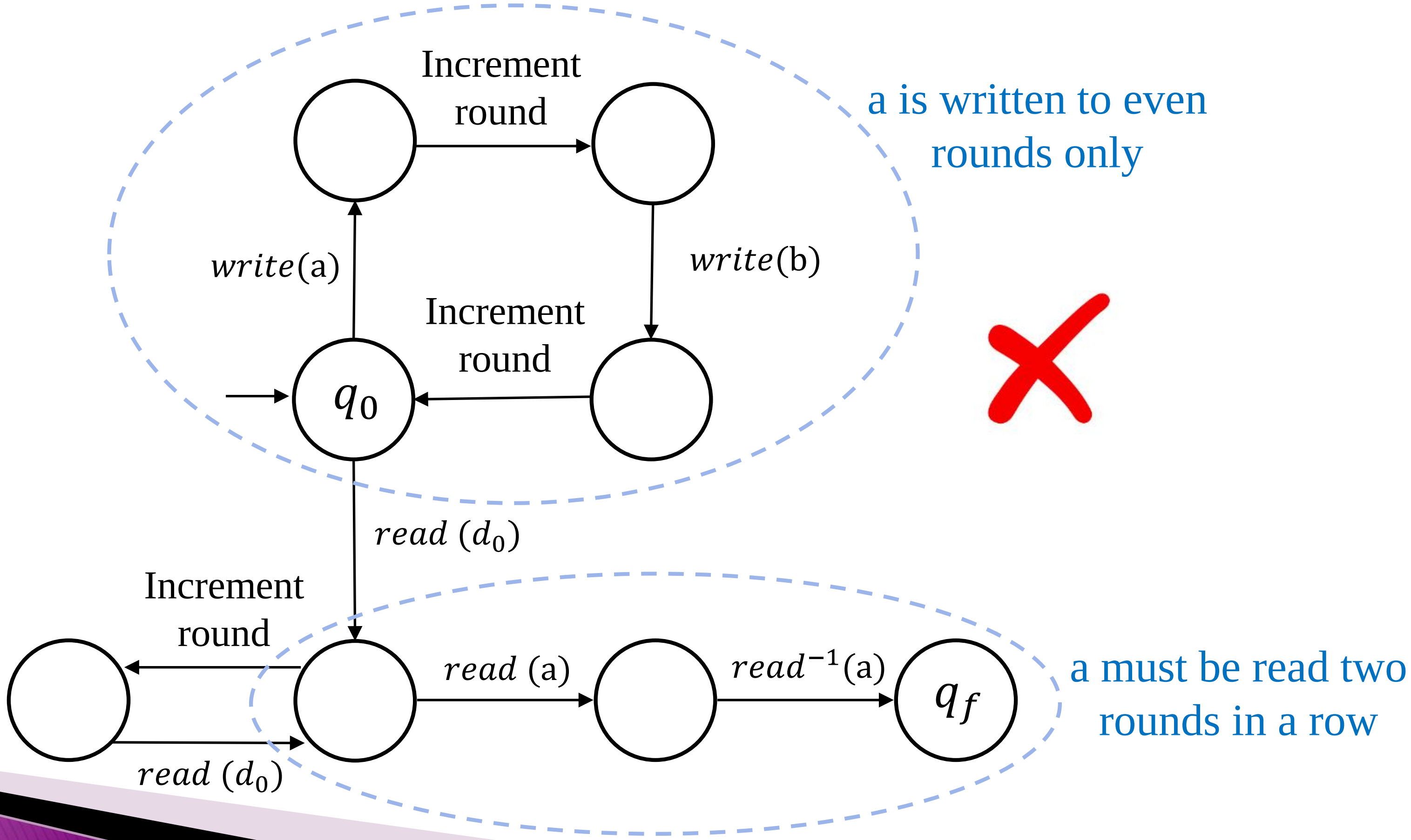


Reachability problems in round-based setting

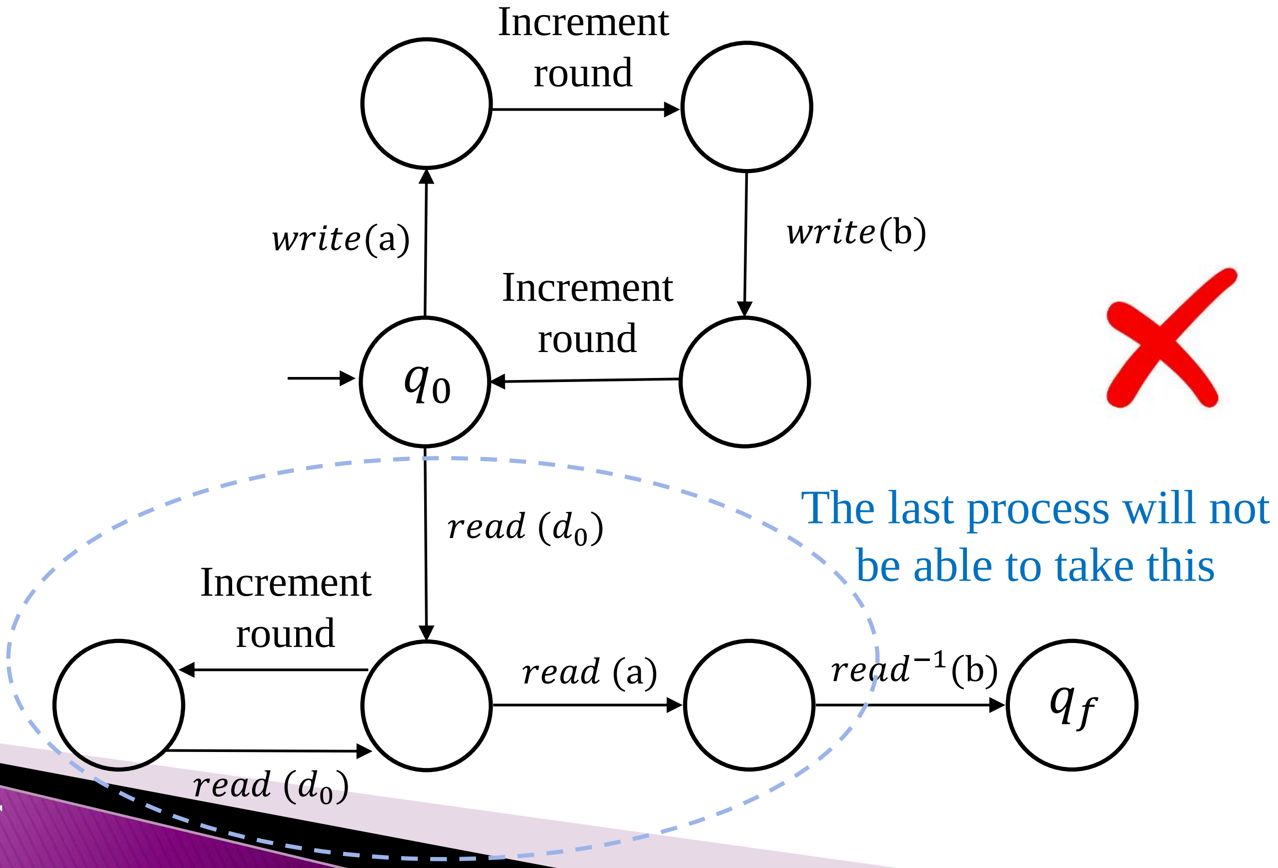
COVER: $\exists n, \exists \rho \colon \gamma_0 \rightarrow^* \gamma\,,\ \exists k\ \ \gamma(q_f, k) > 0$?

TARGET: $\exists n, \exists \rho \colon \gamma_0 \rightarrow^* \gamma\,,\ \forall k, \forall q \neq q_f,\ \gamma(q, k) = 0$?

Reachability problems in round-based setting

COVER: $\exists n, \exists \rho \colon \gamma_0 \rightarrow^* \gamma \,,\ \exists k \ \ \gamma\left(q_f, k\right) > 0$?

TARGET: $\exists n, \exists \rho \colon \gamma_0 \rightarrow^* \gamma \,,\ \forall k, \forall q \neq q_f,\ \gamma(q, k) = 0$?



Reachability problems in round-based setting

COVER: $\exists n, \exists \rho\colon \gamma_0 \rightarrow^* \gamma\,,\ \exists k\ \ \gamma\big(q_f, k\big) > 0$?

TARGET: $\exists n, \exists \rho\colon \gamma_0 \rightarrow^* \gamma\,,\ \ \forall k, \forall q \neq q_f,\ \gamma(q, k) = 0$?

Presence Reachability Problem: $\exists n, \exists \rho\colon \gamma_0 \rightarrow^* \gamma\,,\ \ \ \gamma \vDash \psi$?

with ψ a first-order formula on rounds with no nested quantifiers

Example: $\psi =$ ``$\exists k\ (\#(q_1, k+1) > 0 \wedge \#(q_1, k) = 0)\ \ \vee\ \ \forall k\ \#(q_0, k) = 0''$

For some k, (q_1, k) is not populated but $(q_1, k+1)$ is

no process is on q_0

Complexity results

Theorem[5]*:* COVER is PSPACE-hard.

Theorem[6]*:* The Presence Reachability Problem is PSPACE-complete.

5. Bertrand, N., Markey, N., Sankur, O., W.: *Parameterized safety verification of round-based shared-memory systems.* ICALP, 2022
6. W: *Checking Presence Reachability Properties on Parameterized Shared-Memory Systems*, MFCS'22

Thanks for your attention!
Any questions?

Round-based shared-memory systems