Parameterized safety verification of round-based shared-memory systems

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Round-based shared-memory algorithms

The distributed systems considered

- **Parallel**, **identical** processes communicating via **shared memory**

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Consensus with shared memory is difficult: there is no wait-free consensus protocol with shared memory and two processes.

A motivating example: Aspnes' consensus algorithm

int $k := 0$, bool $p \in \{0, 1\}$, $(\mathrm{rg}_b[r])_{b\in\{0,1\}, r\in\mathbb{N}}$ all initialized to no;
while true **do**
  read from $\mathrm{rg}_0[k]$ and $\mathrm{rg}_1[k]$;
  if $\mathrm{rg}_0[k] =$ yes and $\mathrm{rg}_1[k] =$ no **then** $p := 0$;
  else if $\mathrm{rg}_0[k] =$ no and $\mathrm{rg}_1[k] =$ yes **then** $p := 1$;
  write yes to $\mathrm{rg}_p[k]$;
  if $k > 0$ **then**
    read from $\mathrm{rg}_{1-p}[k-1]$;
    if $\mathrm{rg}_{1-p}[k-1] =$ no **then** return p;
  $k := k+1$;

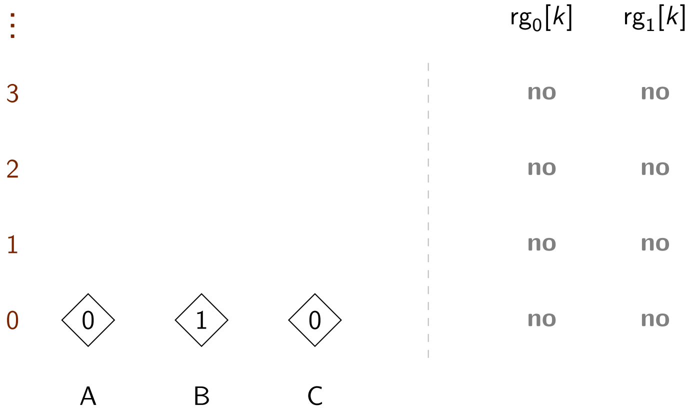
read from registers of rounds k and $k-1$

write to registers of round k

Algorithm 1: Aspnes' consensus algorithm[1].

[1] James Aspnes, Fast deterministic consensus in a noisy environment, *Journal of Algorithms*, 2002.

An example of execution of Aspnes' consensus algorithm



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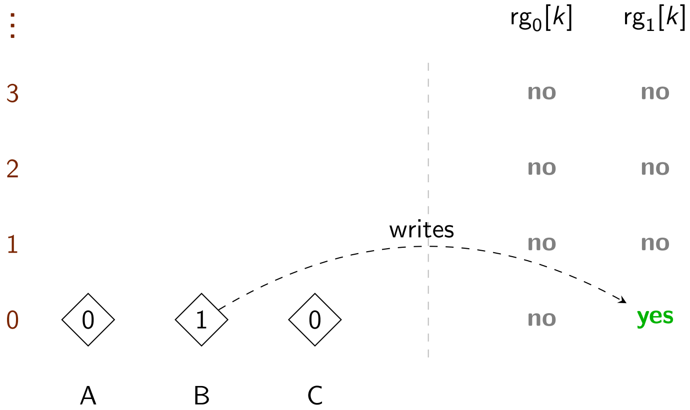

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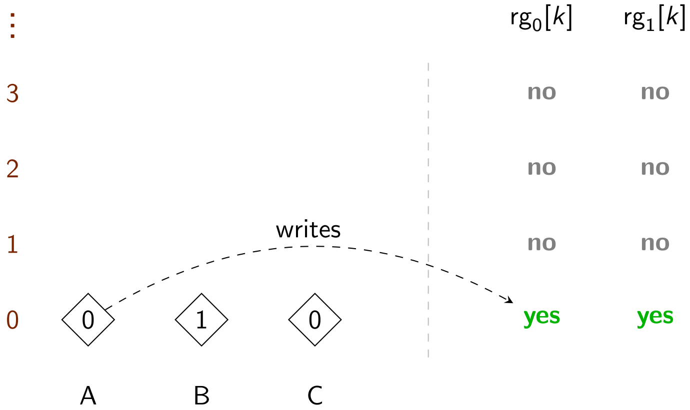

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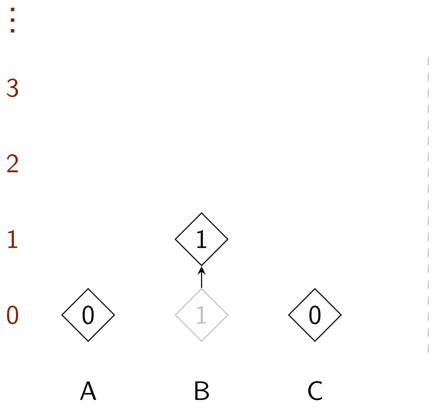

$rg_0[k]$	$rg_1[k]$
no	**no**
no	**no**
no	**no**
yes	**yes**

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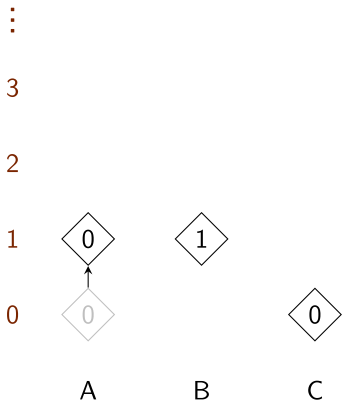

k	$rg_0[k]$	$rg_1[k]$
3	no	no
2	no	no
1	no	no
0	yes	yes

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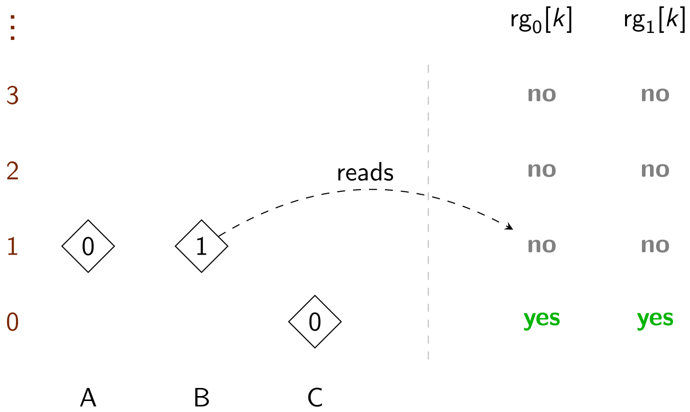

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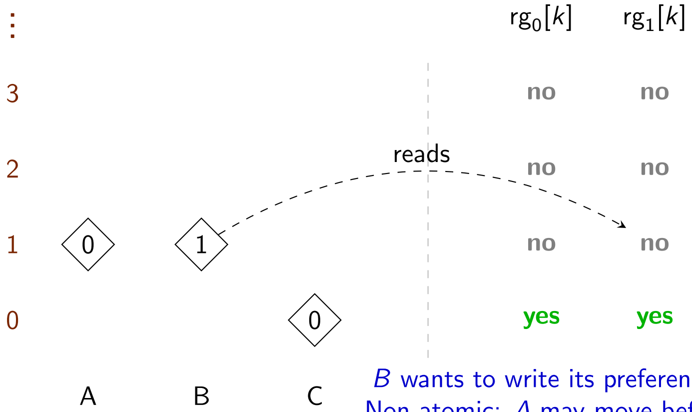

B wants to write its preference on $rg_1[k]$

Non-atomic: A may move before B writes

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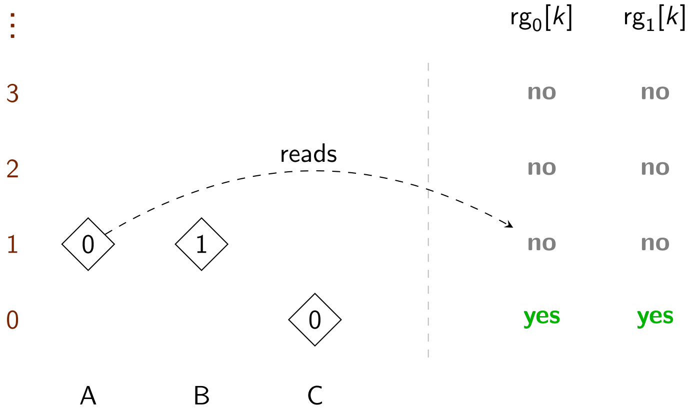

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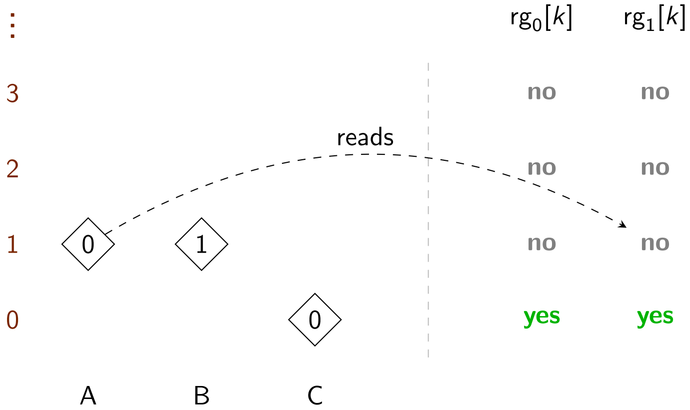

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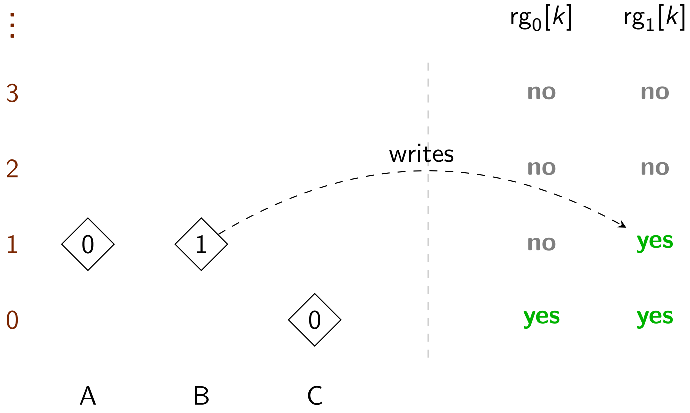

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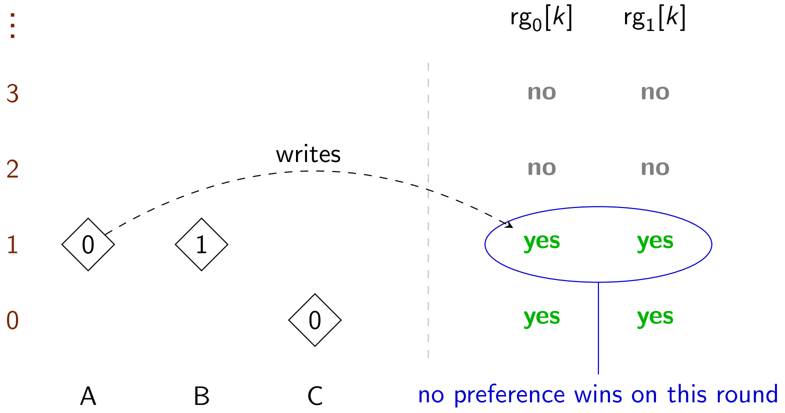

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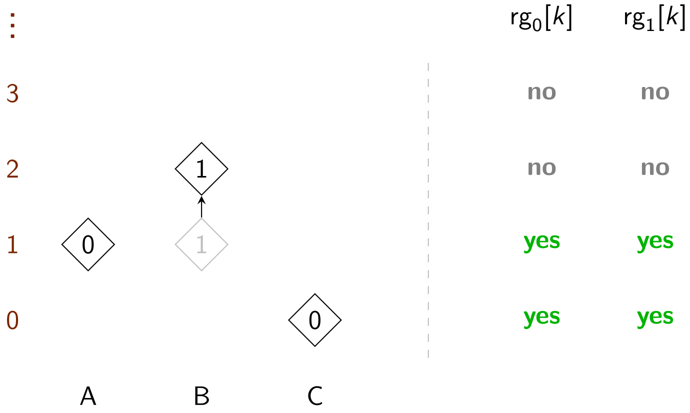

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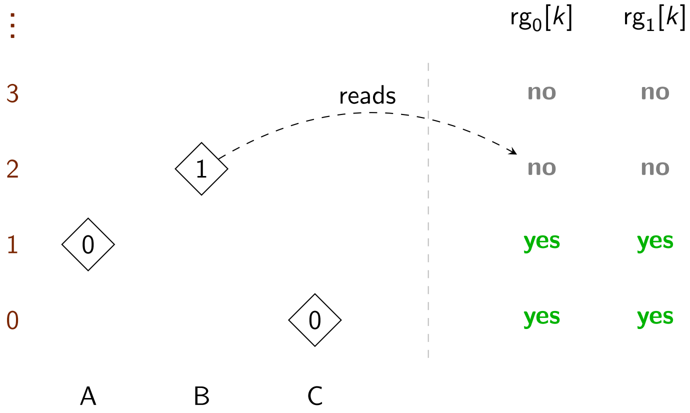

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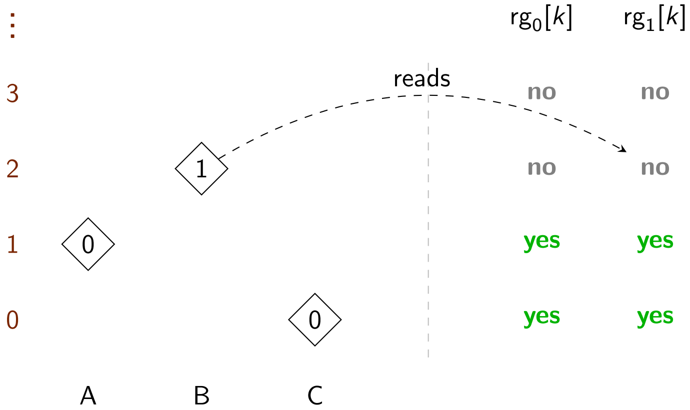

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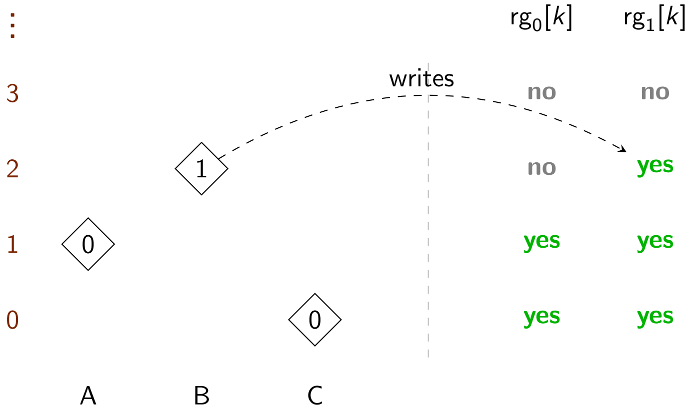

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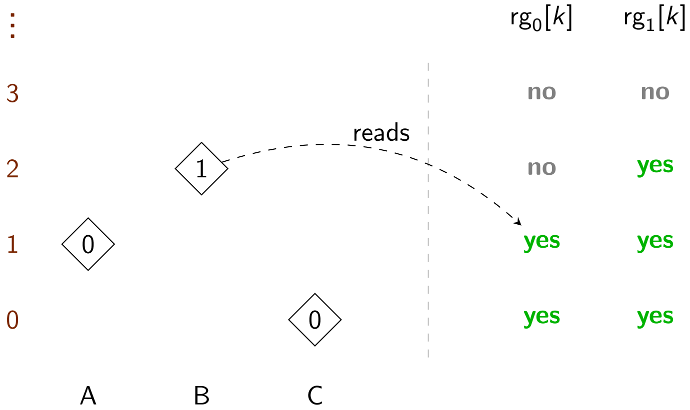

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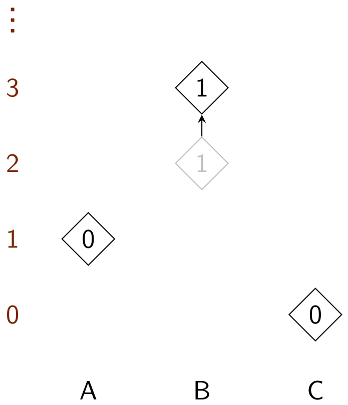

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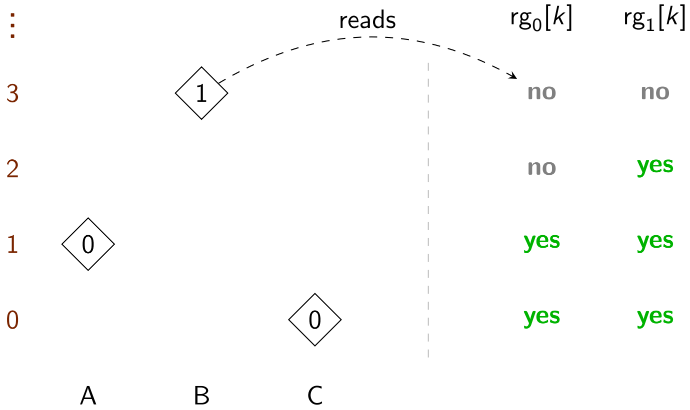

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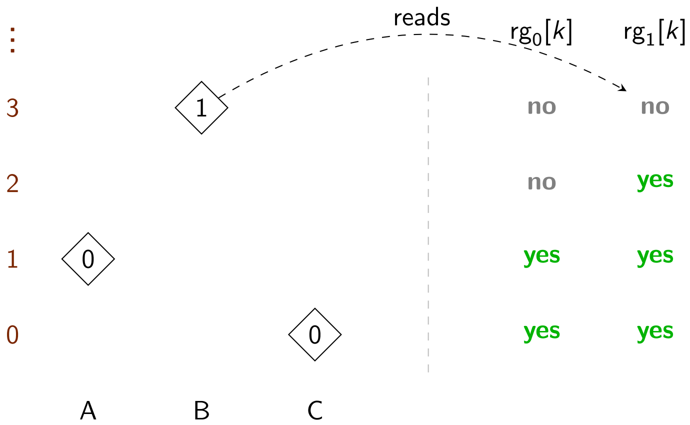

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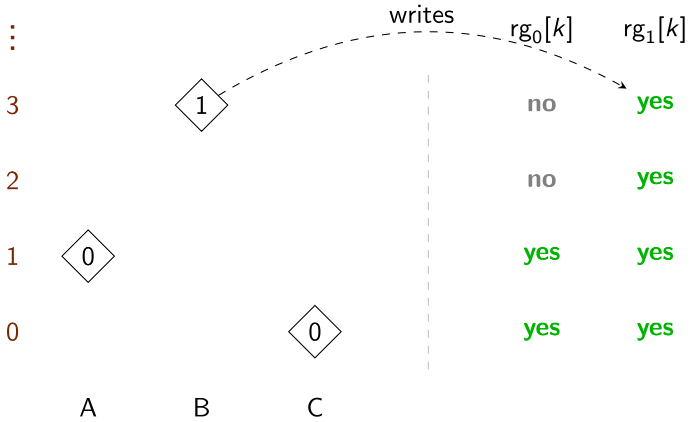

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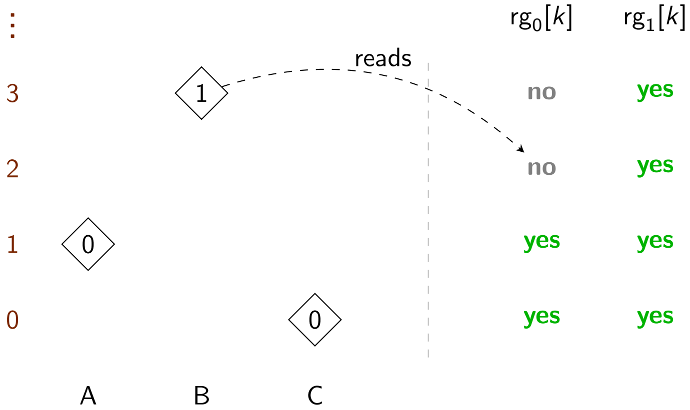

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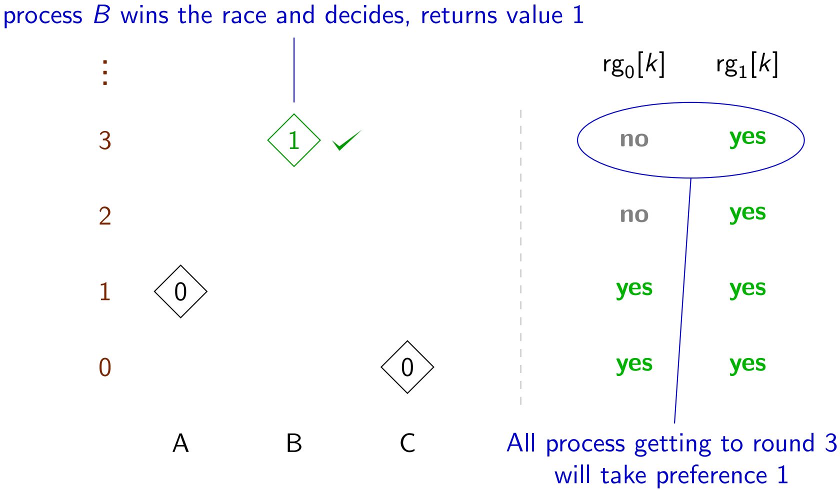

A model: round-based register protocols

Inspired by models for shared-memory systems without rounds[2,3].

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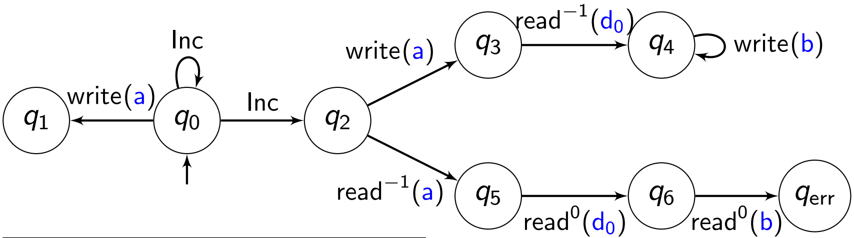

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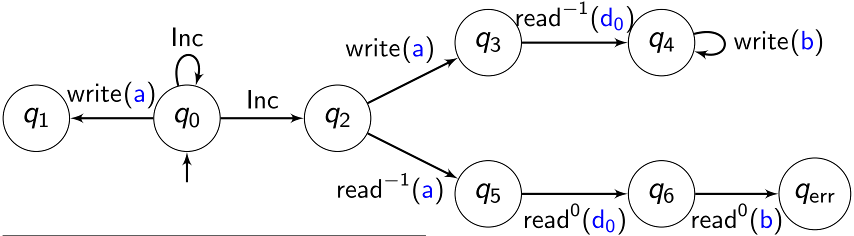

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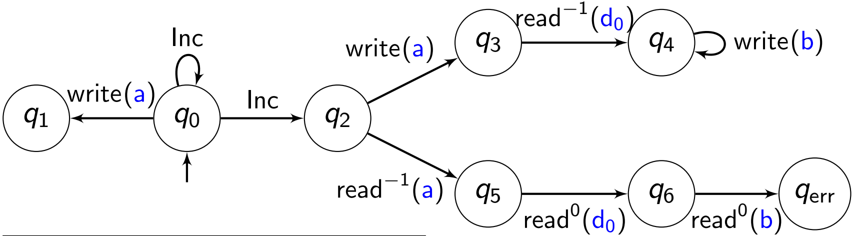

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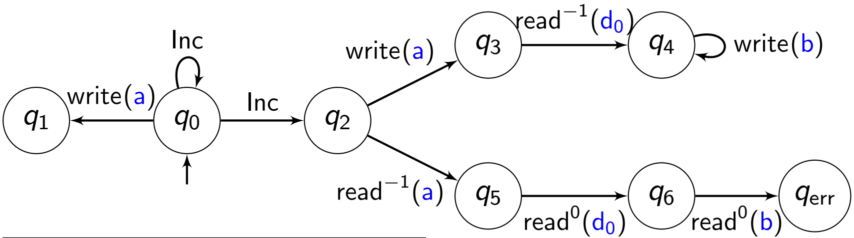
- A fixed number d of registers per round (the total number of registers is hence unbounded)



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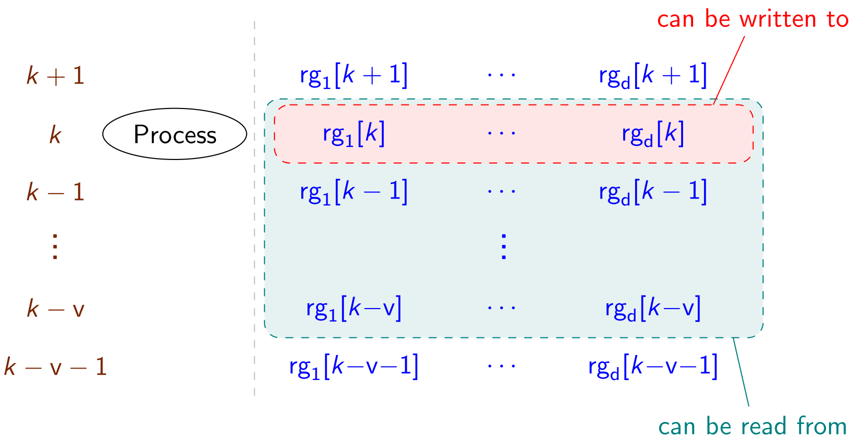
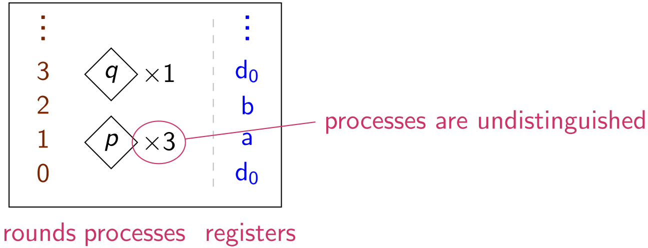
A limited visilibity range



v given in unary

Semantics of the model

From now on, let $d = 1$: one register per round.



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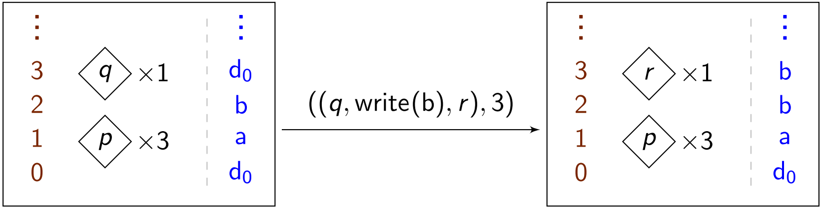

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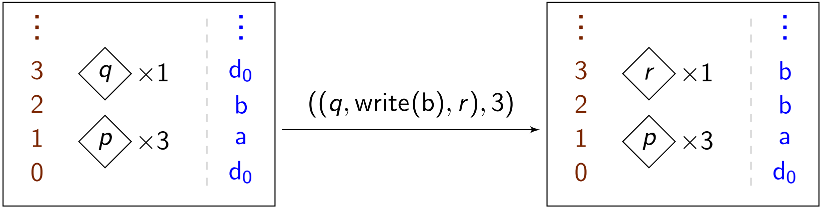
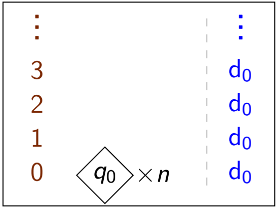


Initial configuration of size n:



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Agreement and Validity of Aspnes' consensus algorithm can be encoded as safety properties.

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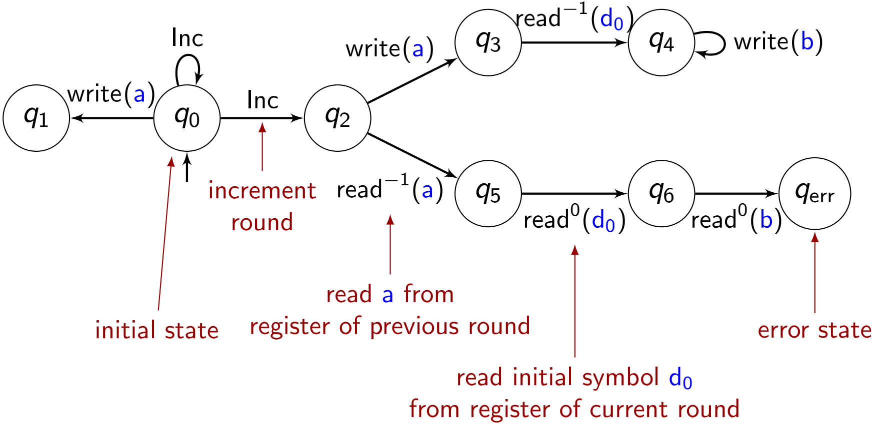

v = 1 (processes can read one round back)

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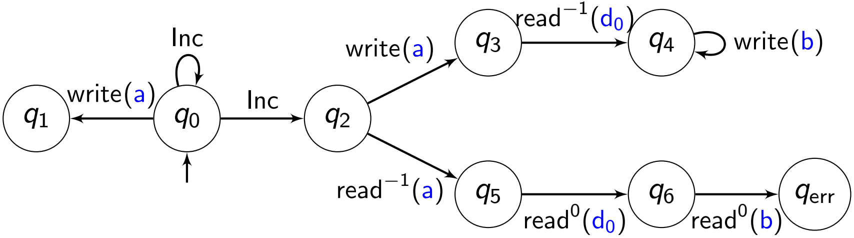
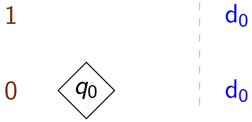


State q_4 can be covered from the initial configuration with one process:



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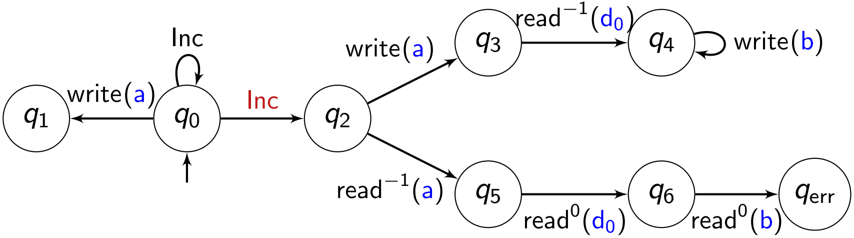

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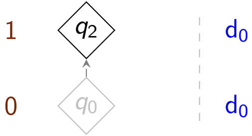

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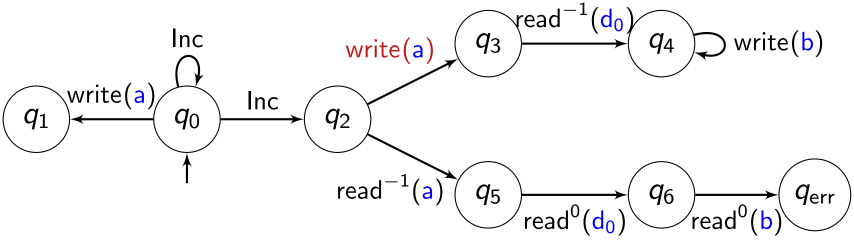

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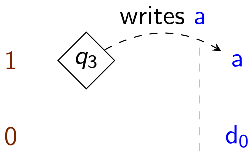

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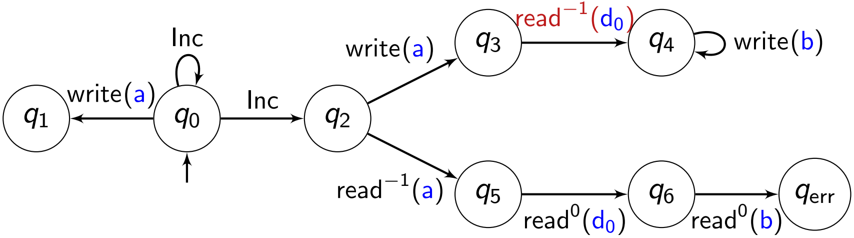

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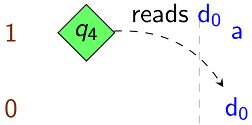

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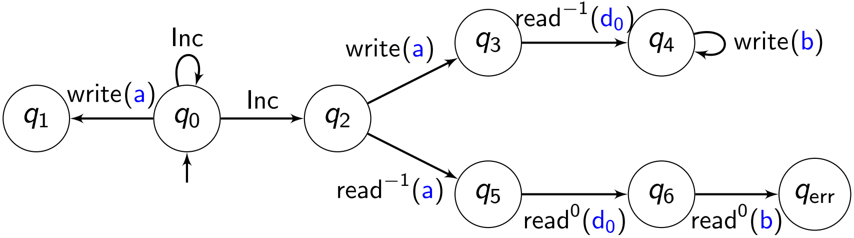

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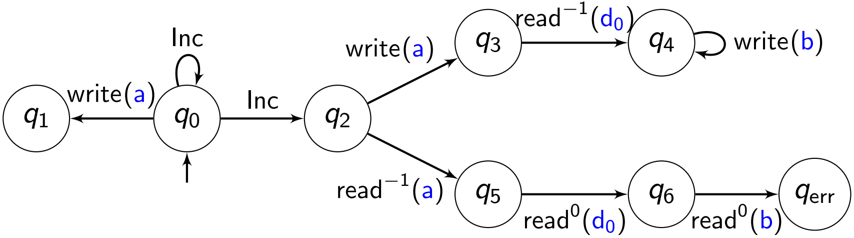

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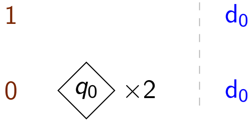

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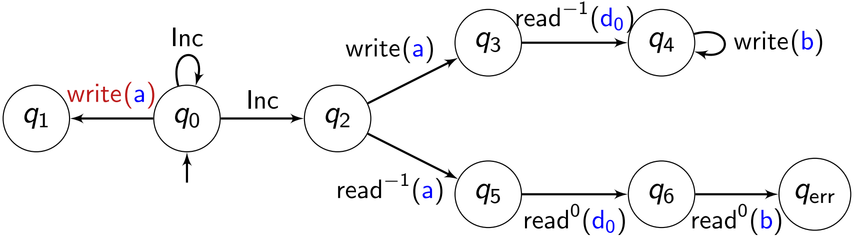

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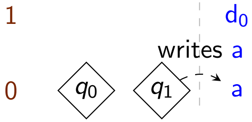

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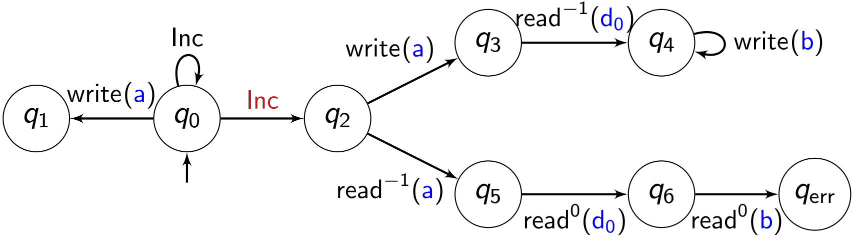

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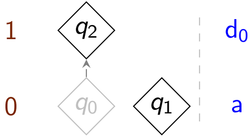

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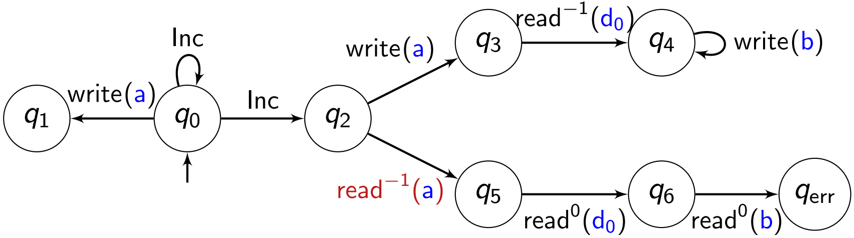

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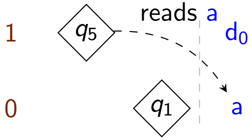

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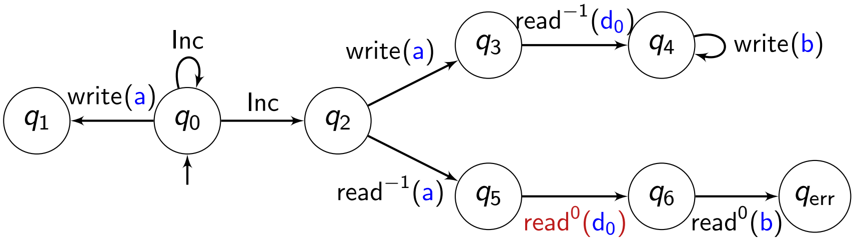

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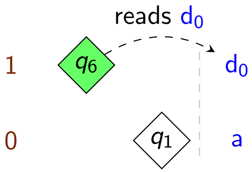
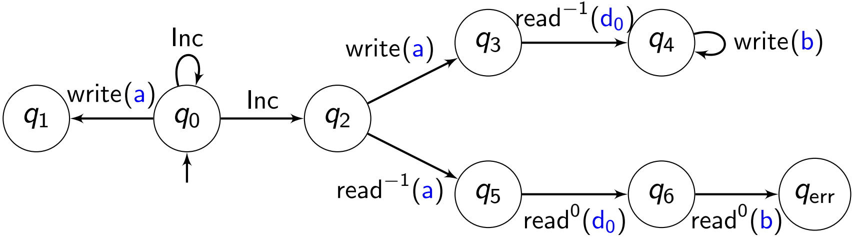


A small example



Claim: the system is safe.

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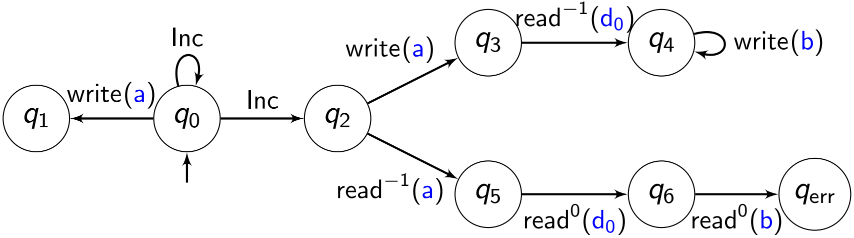

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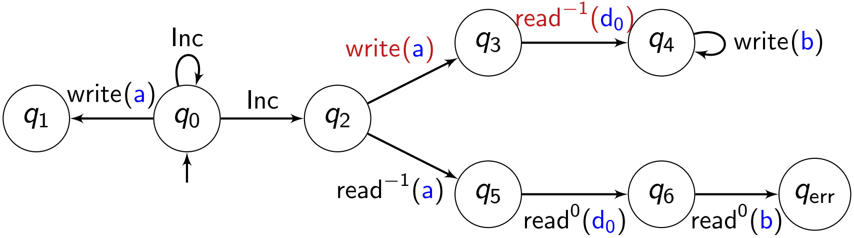

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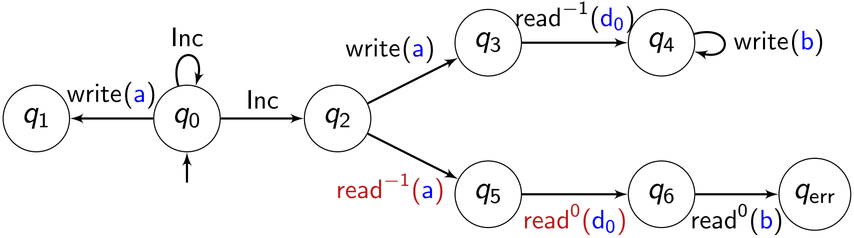

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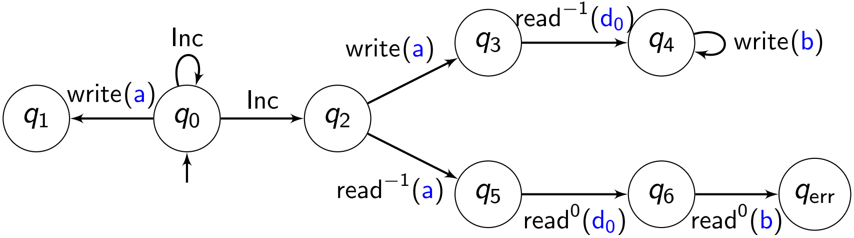

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This is the only source of “incompatibility”!

Main contribution

Parameterized safety in round-based register protocols is PSPACE-complete.

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Exponential lower bounds

In order to reach an error state, one might need at least:

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The safety problem is PSPACE-hard.

By reduction from Quantified Boolean Formula.

PSPACE-membership

Theorem

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- Rely on a sliding window along the rounds

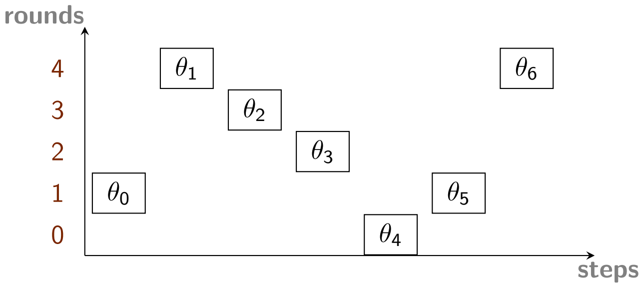
A visual display for executions

Execution: $\sigma_0 \xrightarrow{\theta_0} \sigma_1 \xrightarrow{\theta_1} \sigma_2 \xrightarrow{\theta_2} \sigma_3 \xrightarrow{\theta_3} \sigma_4 \xrightarrow{\theta_4} \sigma_5 \xrightarrow{\theta_5} \sigma_6 \xrightarrow{\theta_6} \sigma_7$

$\Downarrow$

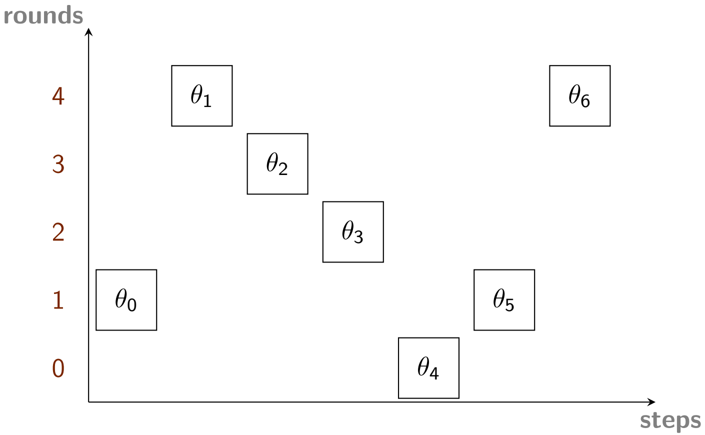
moves:	$\boxed{\theta_0}$	$\boxed{\theta_1}$	$\boxed{\theta_2}$	$\boxed{\theta_3}$	$\boxed{\theta_4}$	$\boxed{\theta_5}$	$\boxed{\theta_6}$
rounds:	1	4	3	2	0	1	4

$\Downarrow$



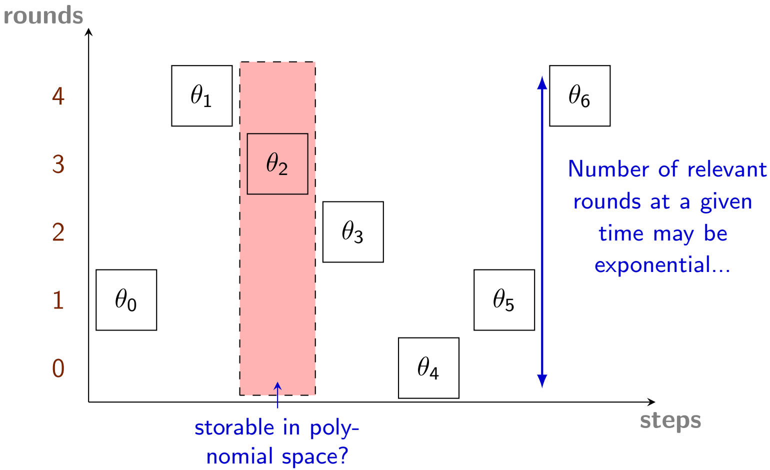
The sliding window

Here v = 1: processes at round k can read from rounds k and $k-1$



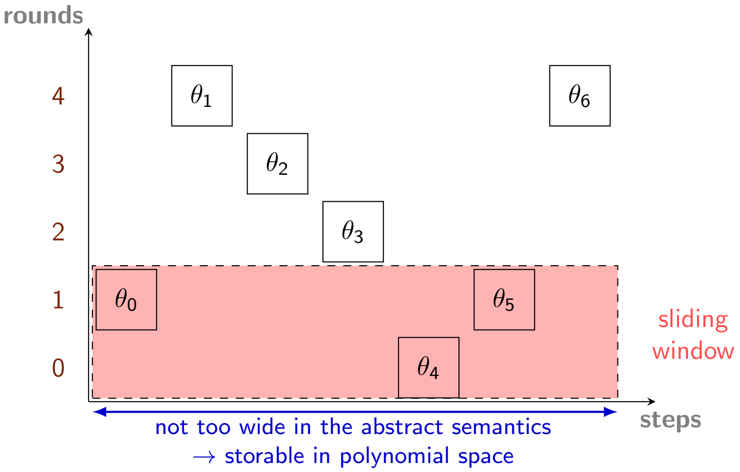
The sliding window

Intuitive idea of proceeding move by move is not working:



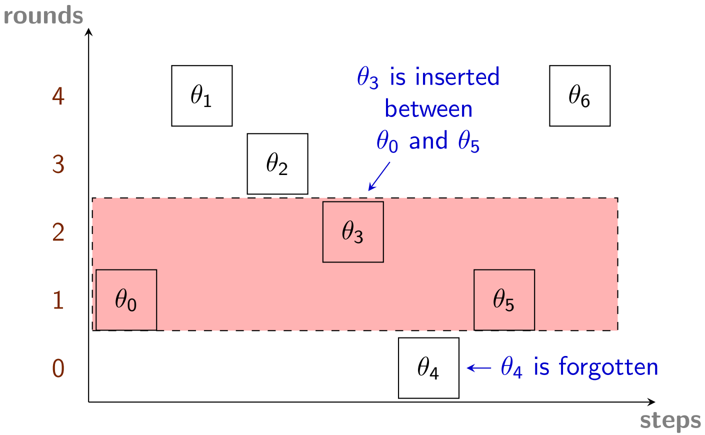
The sliding window

Instead: sliding window along the rounds non-deterministically guessing the execution



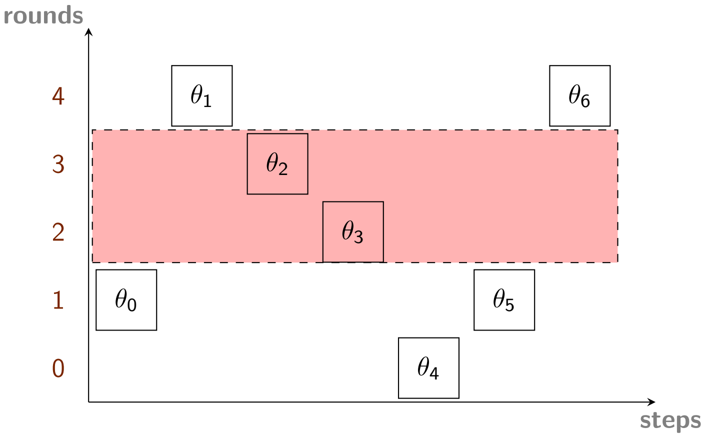
The sliding window

Checking that a move is valid only depends on what happens locally.



The sliding window

And so on...



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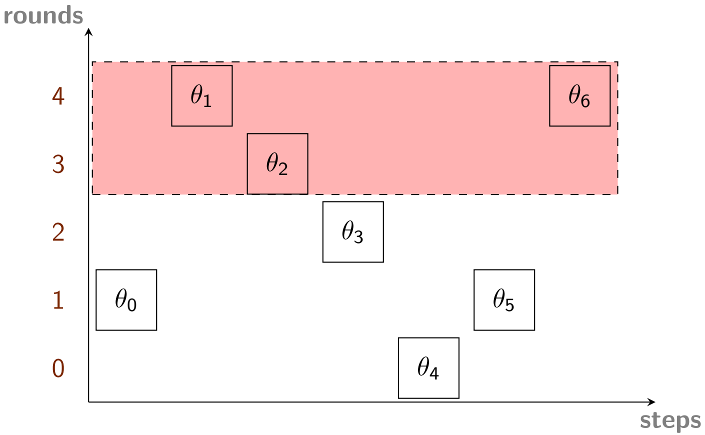

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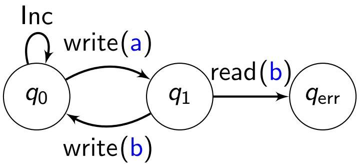
Thank you!

Classical notions of fairness are not satisfactory



q_{err} is reached with probability 1 with a stochastic scheduler with two processes.

Consider the execution with two processes where one process goes to q_1 and back to q_0 on every round, while the other process stays on q_0 forever.

This execution is fair with respect to:

- Fairness on moves: no move is available infinitely often because k increases
- Fairness on transitions: transition from q_1 to q_{err} is never enabled.