Temporal logics for multi-agent systems

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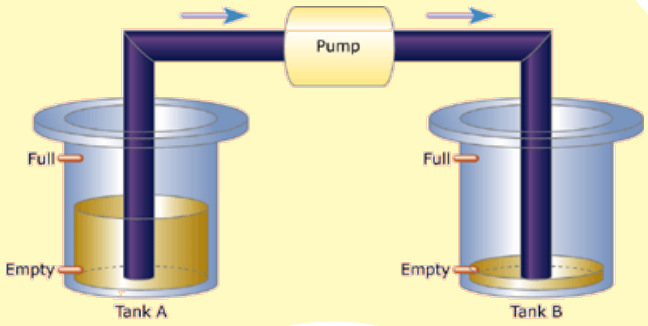
CNRS – IRISA (Rennes, France)

(based on joint works with Patricia Bouyer, Patrick Gardy, and François Laroussinie)

MFCS'17 – August 23, 2017

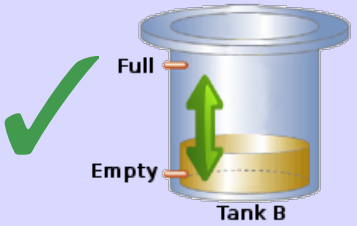
Model checking and synthesis

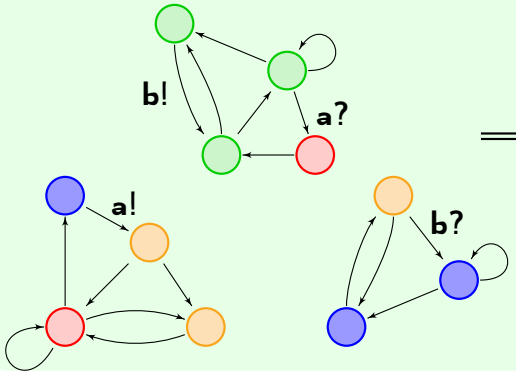
system:



[http://www.embedded.com]

property:





$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ $\mathbf{A}\,\mathbf{G}(\neg\,\texttt{B.overfull} \wedge \neg\,\texttt{B.dried\_up})$

yes/no

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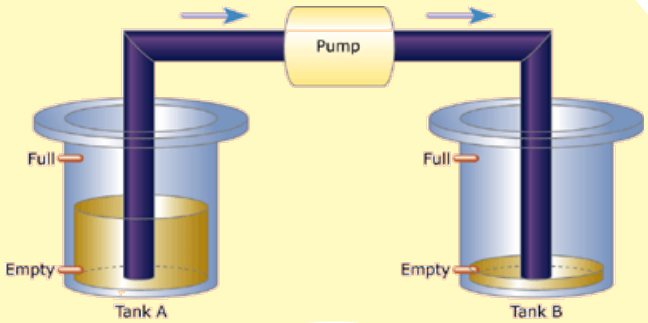

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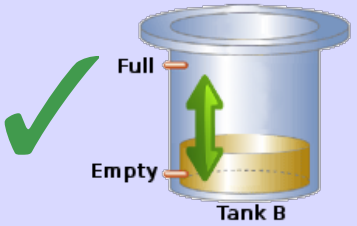

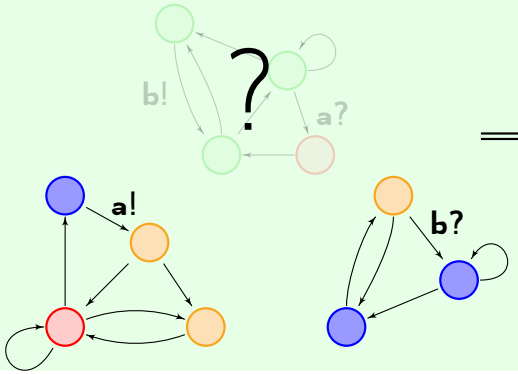

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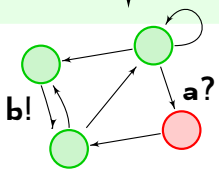

Computation-Tree Logic (CTL)

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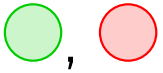
- **atomic propositions:**  , ...
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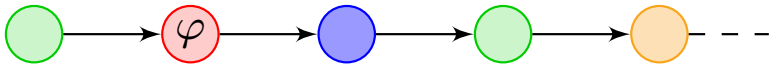
$\mathbf{X}\,\varphi$ 

"next φ"

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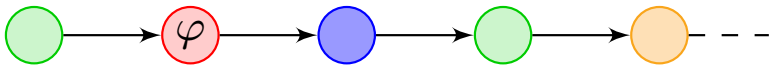

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$\texttt{true}\ \mathbf{U}\ \varphi \equiv \mathbf{F}\,\varphi$

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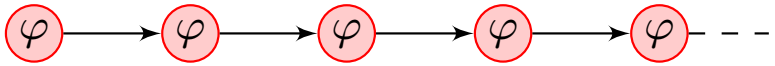
$\neg\ \mathbf{F}\ \neg\varphi \equiv \mathbf{G}\,\varphi$



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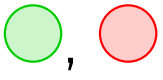
- **atomic propositions:**  , ...
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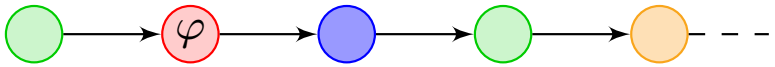
$\mathbf{X}\,\varphi$  "next φ"

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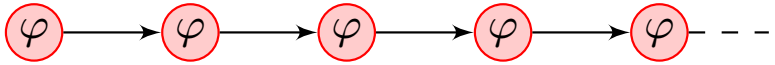
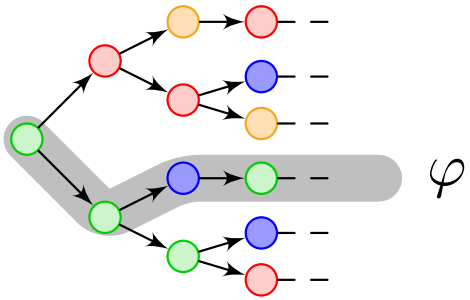
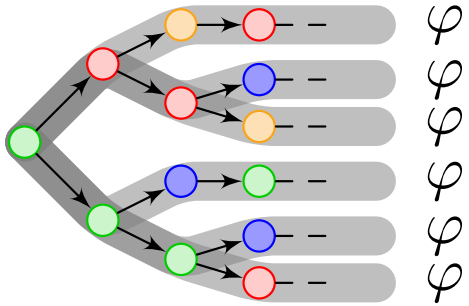
$\neg\,\mathbf{F}\,\neg\varphi \equiv \mathbf{G}\,\varphi$  "always φ"

- **path quantifiers:**

$\mathbf{E}\varphi$ 

$\mathbf{A}\varphi$ 

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- **temporal modalities**

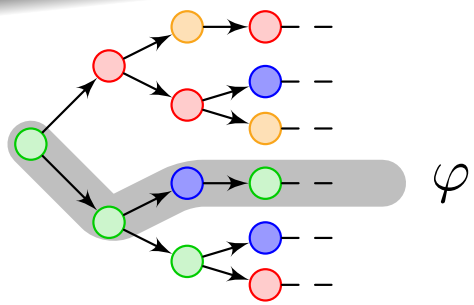
true U

¬ F

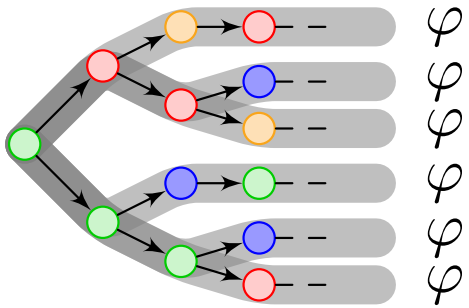
Examples

- **safety**: **AG** `safe`
  only `safe` states will be reached
- **fairness**: **AG AF** `fair`
  `fair` states will be visited infinitely many times
- **request-response**: **AG**(`request` ⇒ **EF** `grant`)
  any `request` can eventually be granted

Eφ



Aφ



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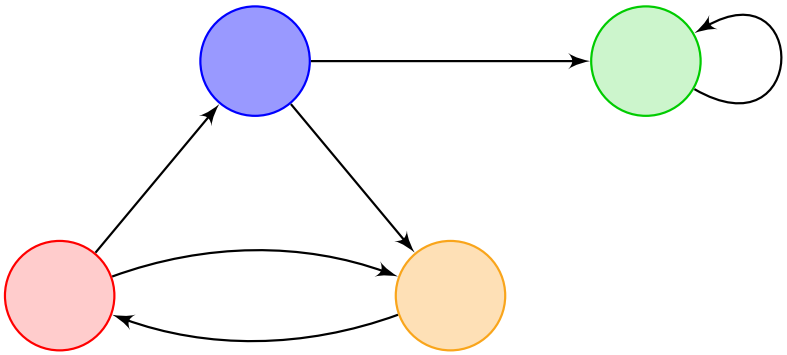

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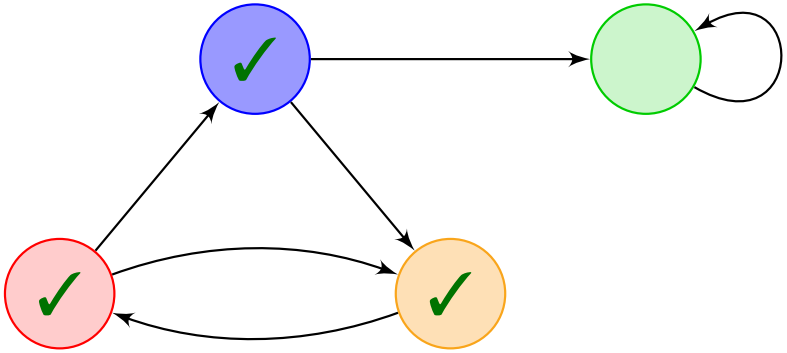

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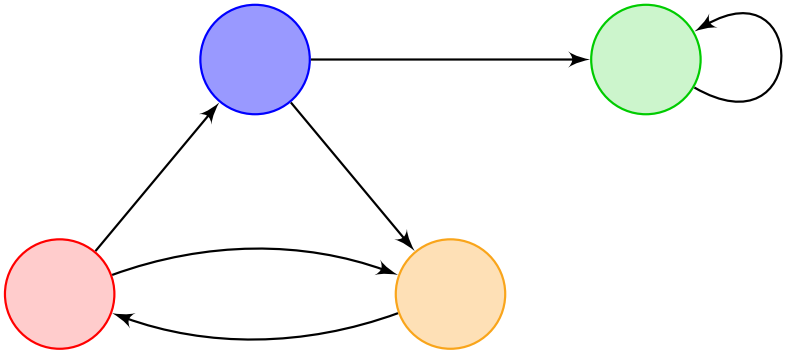

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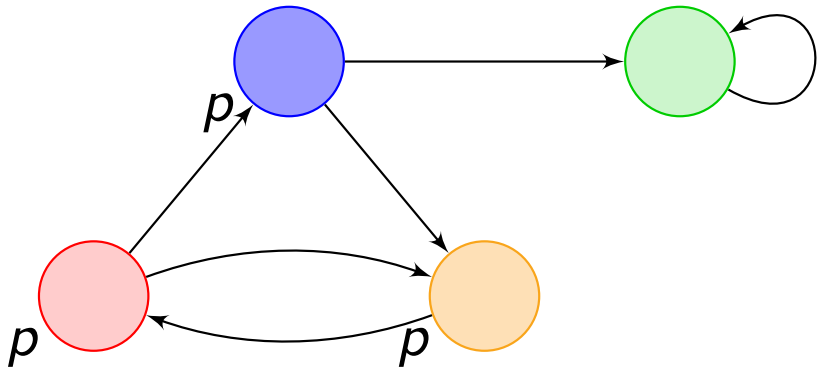

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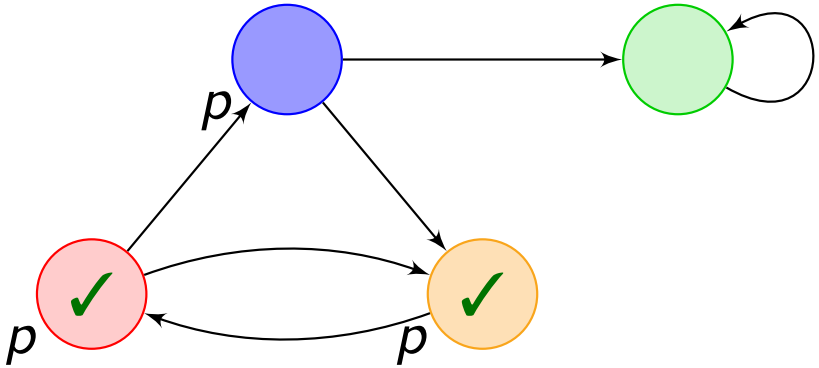

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Theorem ([CE81,QS82])

CTL model checking is PTIME*-complete.*

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Remark ([KVW94])

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Outline of the presentation

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Games on graphs

Concurrent games

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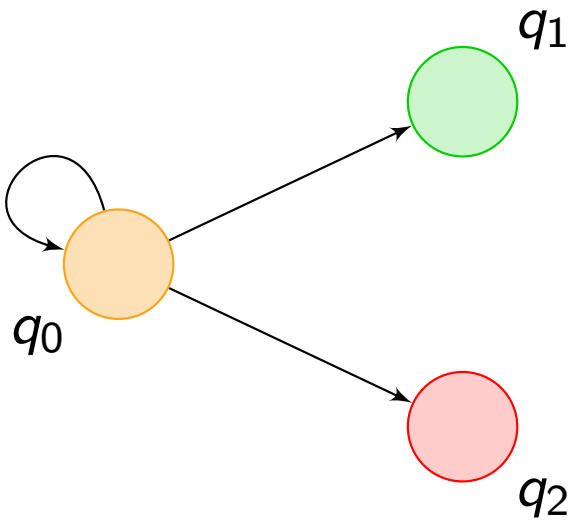
- a transition system;



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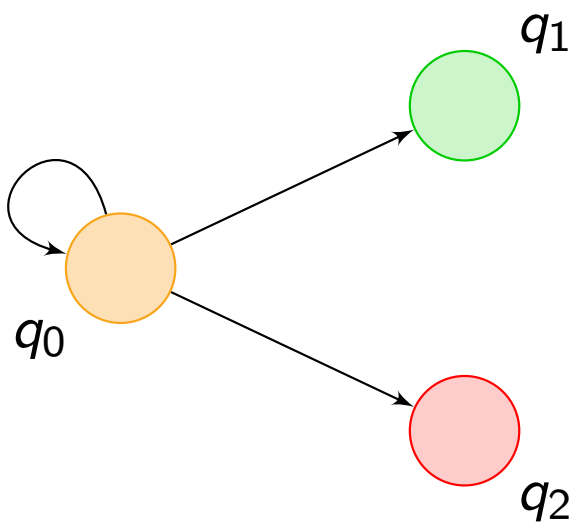

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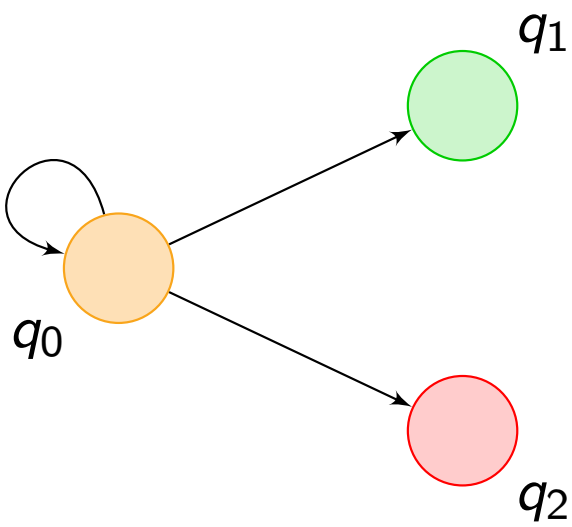

		player 1		
				
player 2		q_0	q_2	q_1
		q_1	q_0	q_2
		q_2	q_1	q_0

Games on graphs

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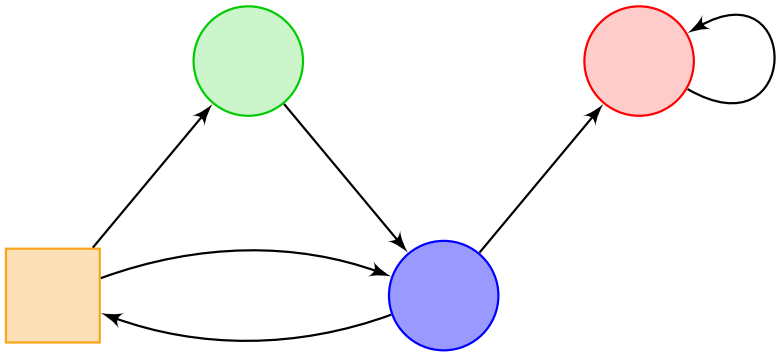
Turn-based games

A turn-based game is a game where only one agent plays at a time.



Reasoning about open systems

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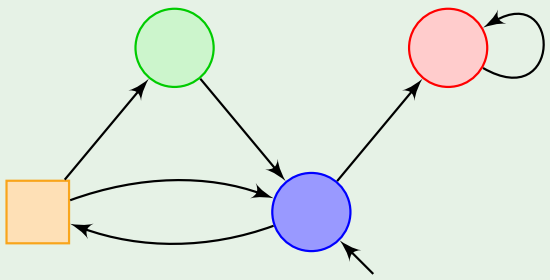

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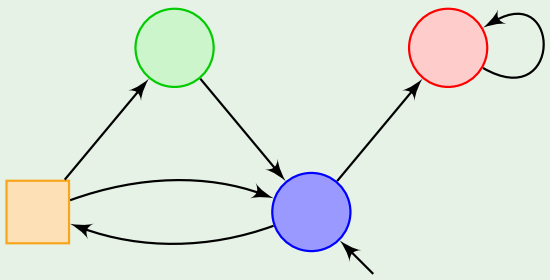

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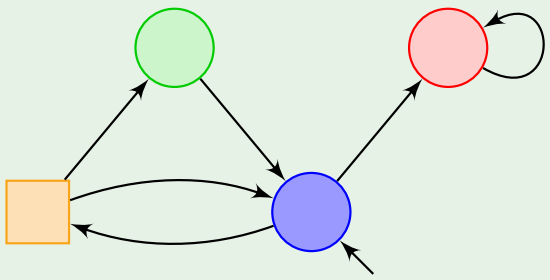

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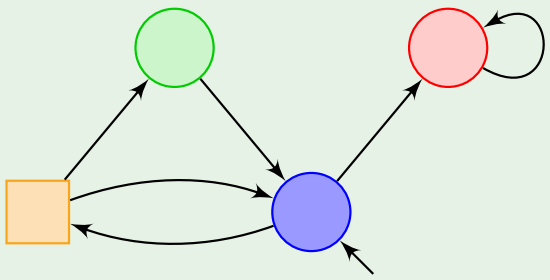

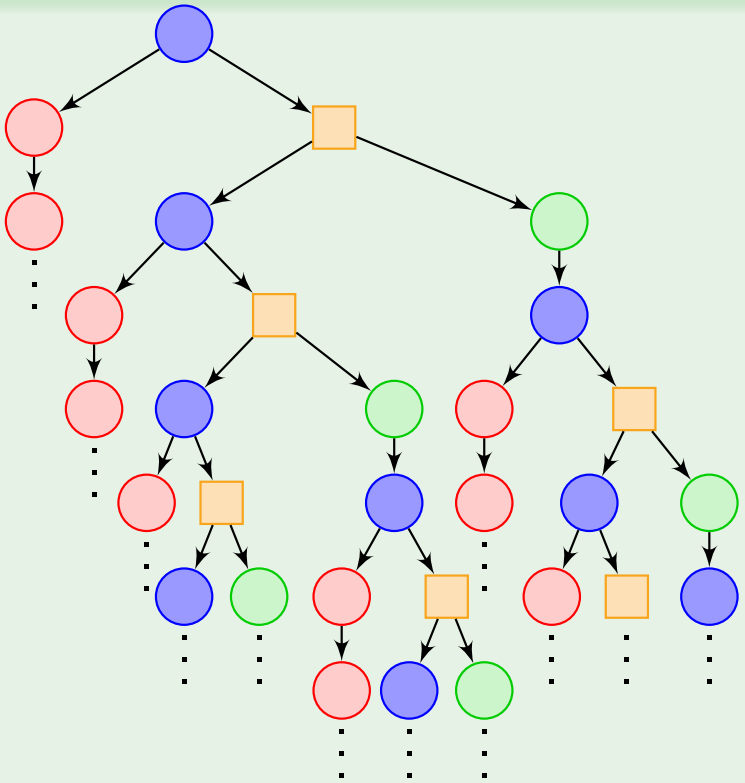

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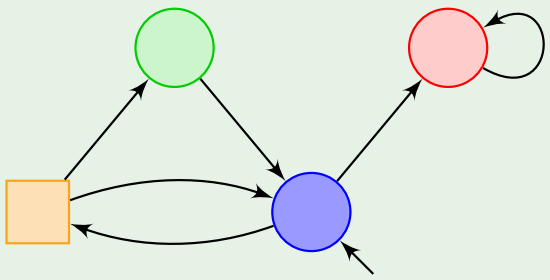

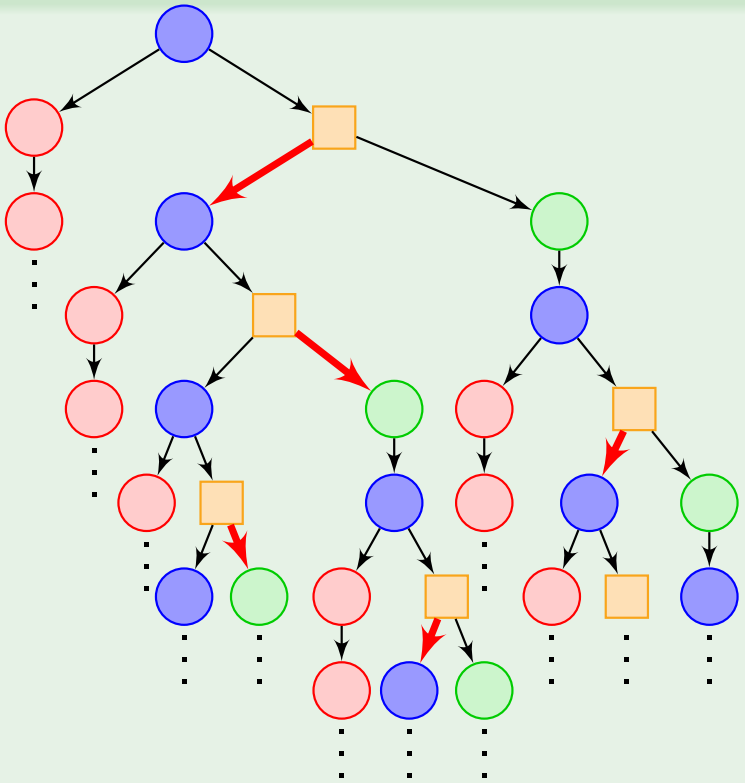

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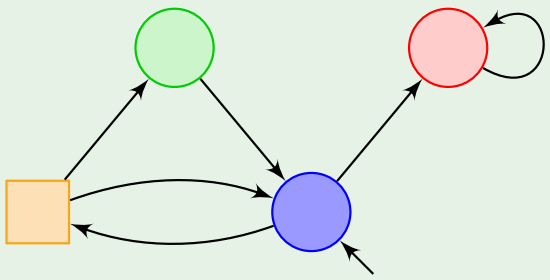

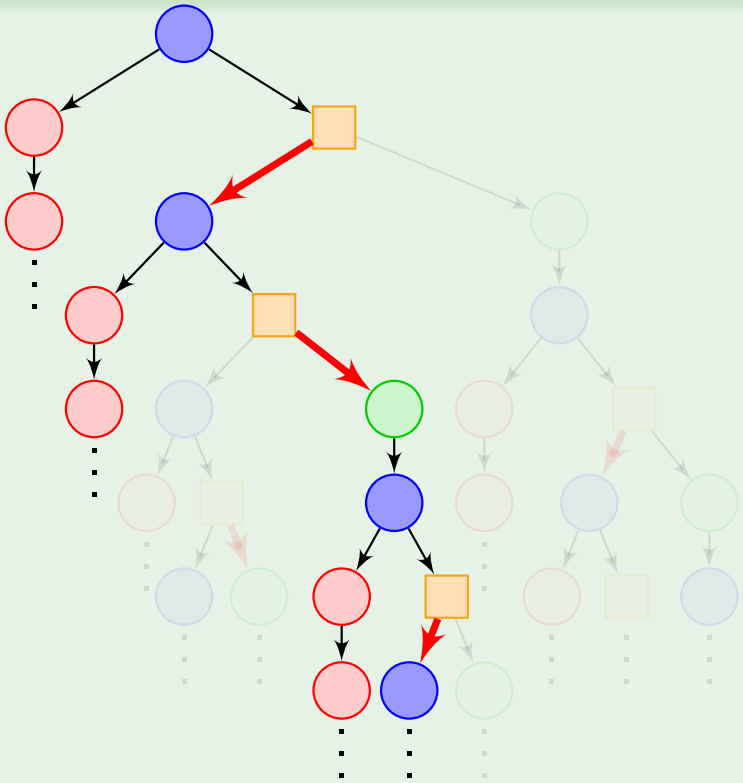

Temporal logics for games: ATL [AHK02]

ATL extends CTL with strategy quantifiers

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ.

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Semantics of $\langle\langle A \rangle\rangle\, \varphi$

Existential quantification (over strategies) implicitly includes a universal quantification (over outcomes):

$$\mathcal{G}, \bigcirc \models \langle\langle A \rangle\rangle\, \varphi \quad \iff \quad \exists \sigma_A.\ \forall \pi \in \mathsf{Out}(\bigcirc, \sigma_A).\ \pi \models \varphi.$$

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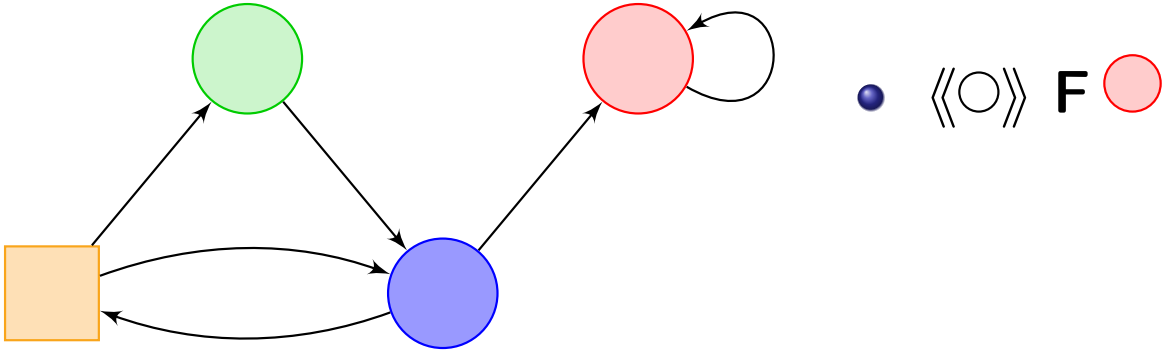

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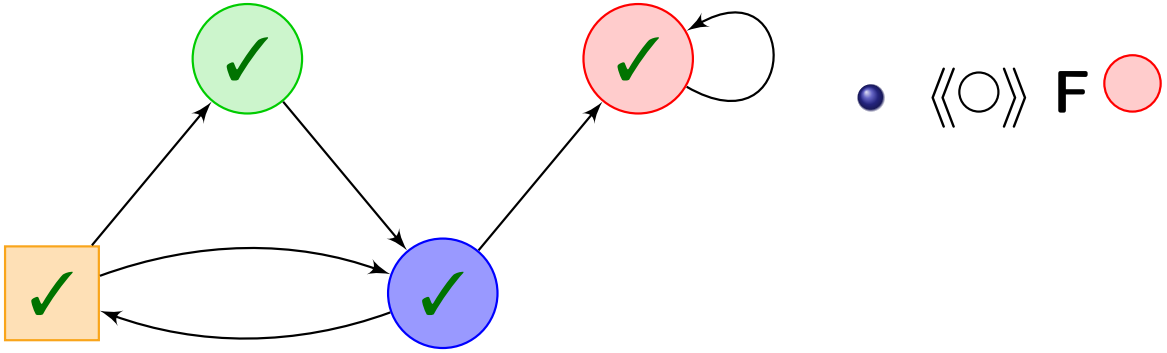

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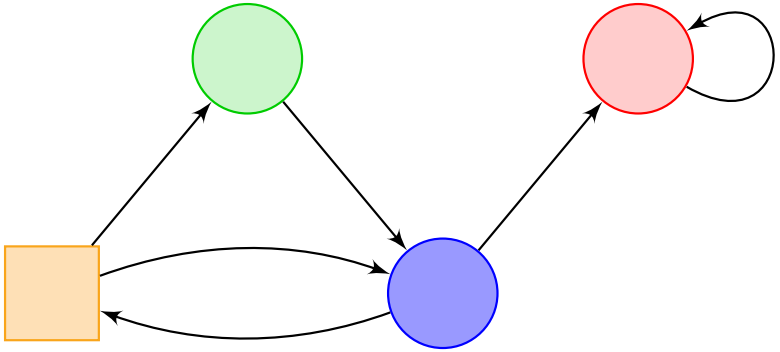

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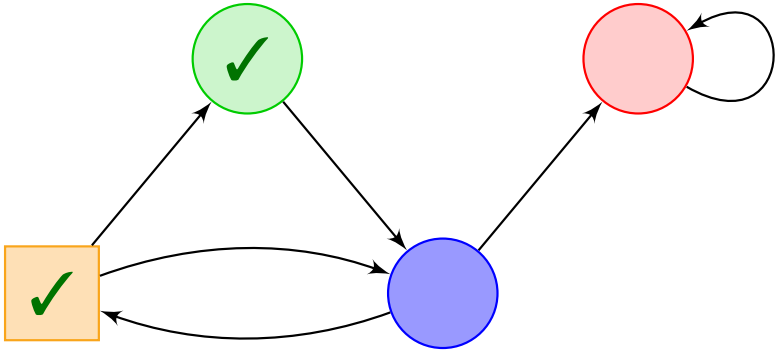

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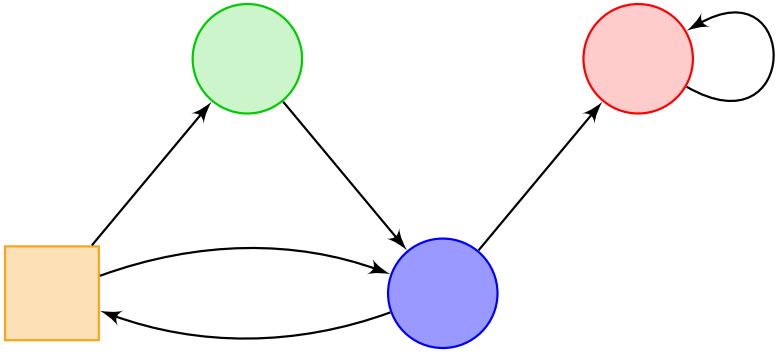

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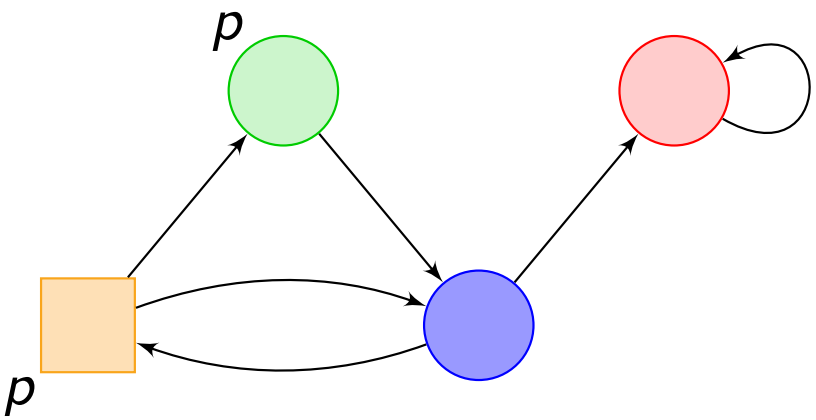

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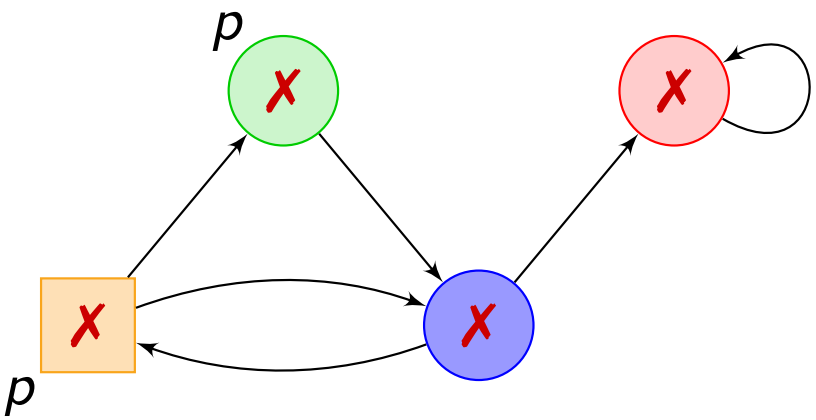

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Model checking ATL is PTIME*-complete.*

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Remark ([LMO08])

In PTIME only if the transition table is given explicitly (size $|\mathsf{Moves}|^{|\mathsf{Agt}|}$)

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ATL with strategy contexts [DLM10]

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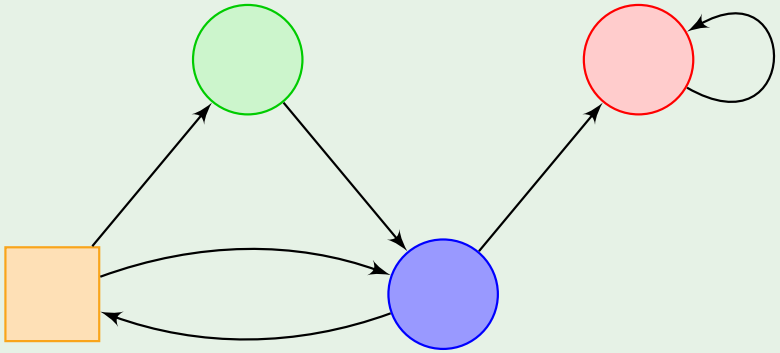

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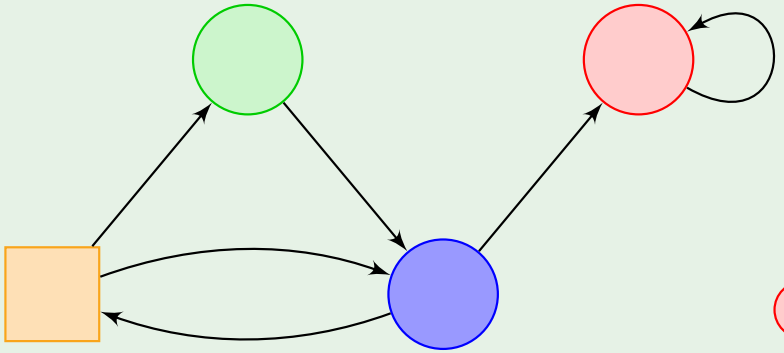

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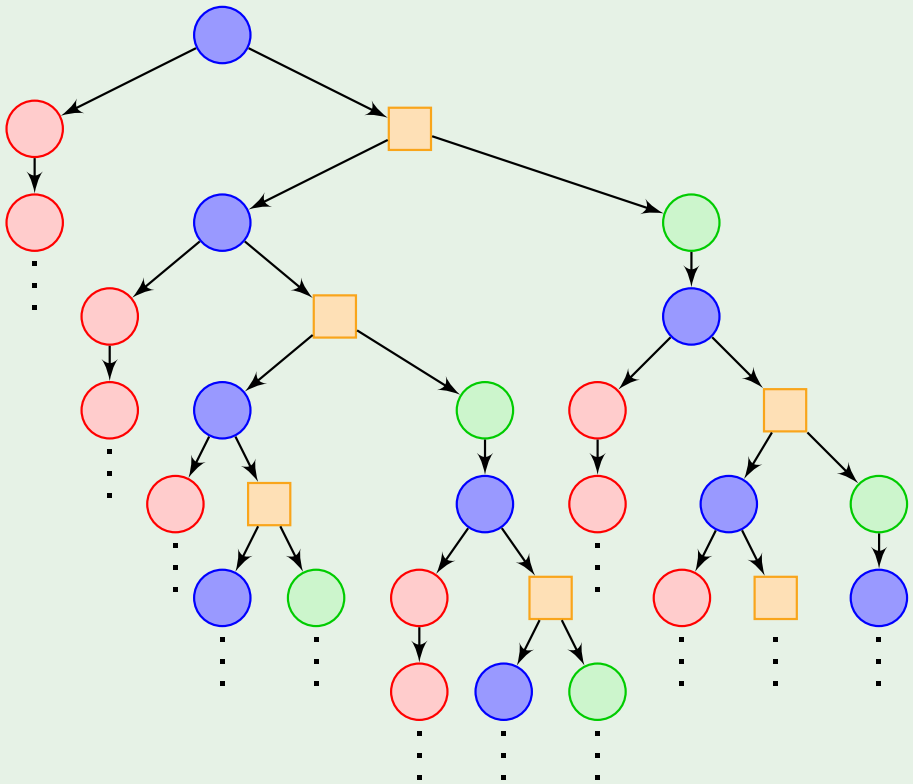

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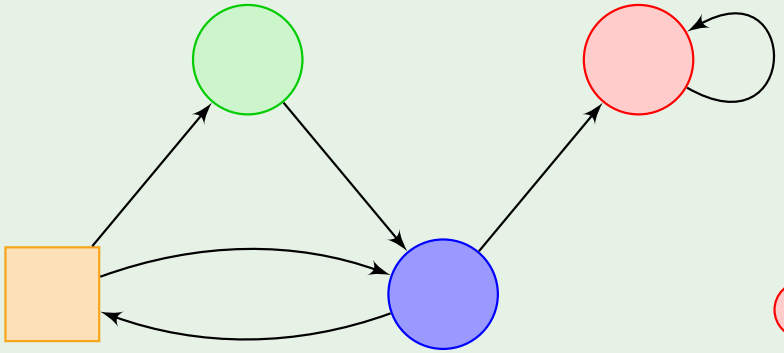

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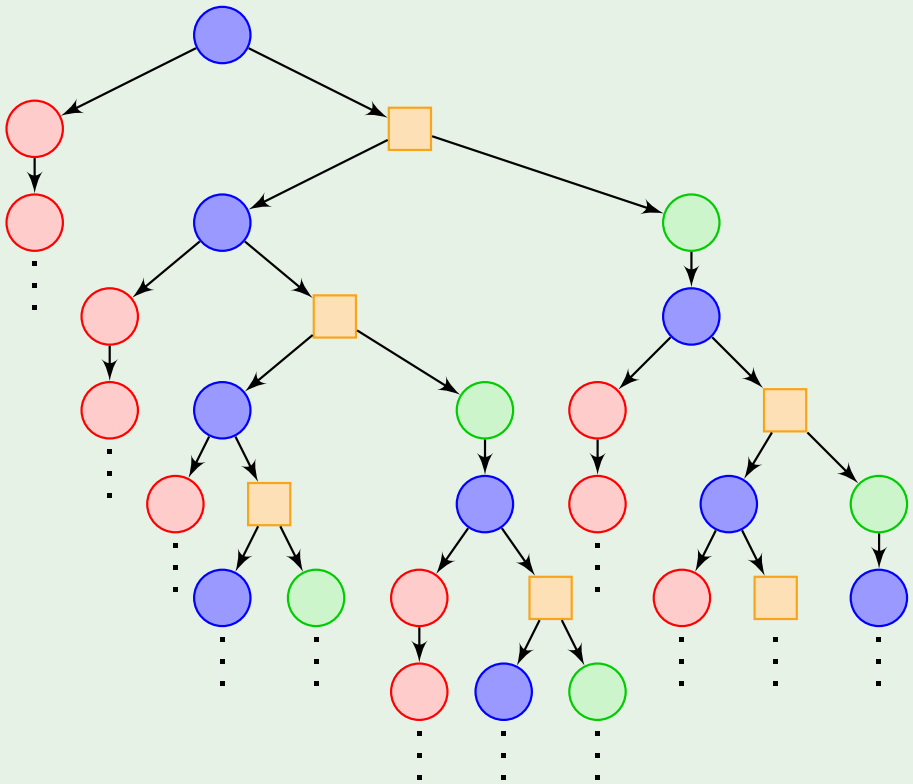

- Player ○ in ● always plays to □.

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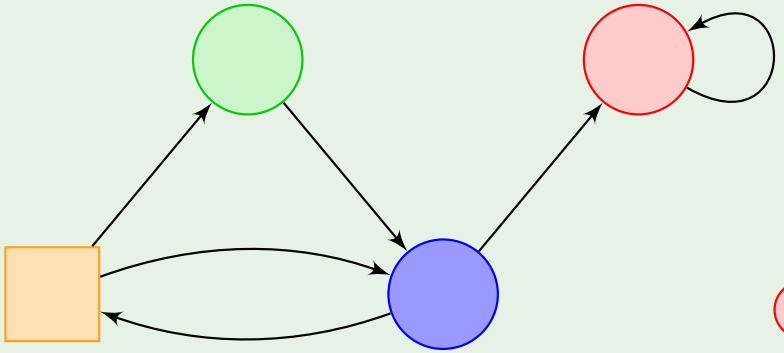

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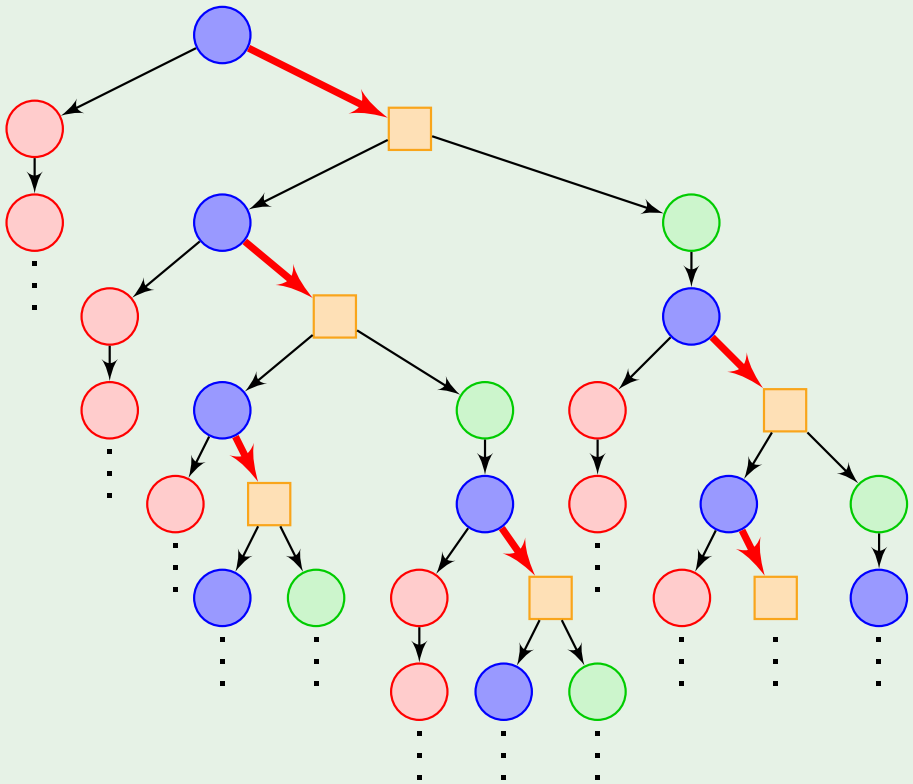

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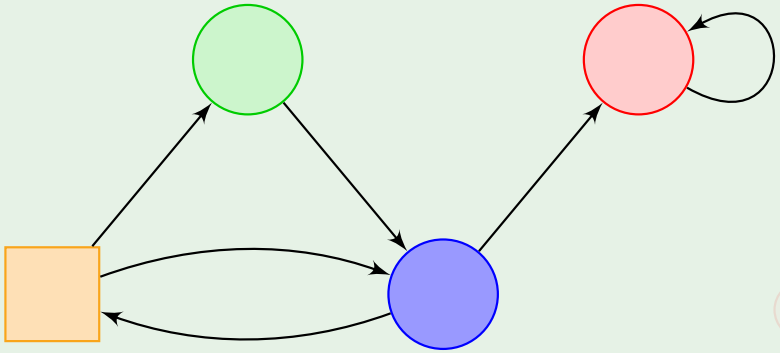

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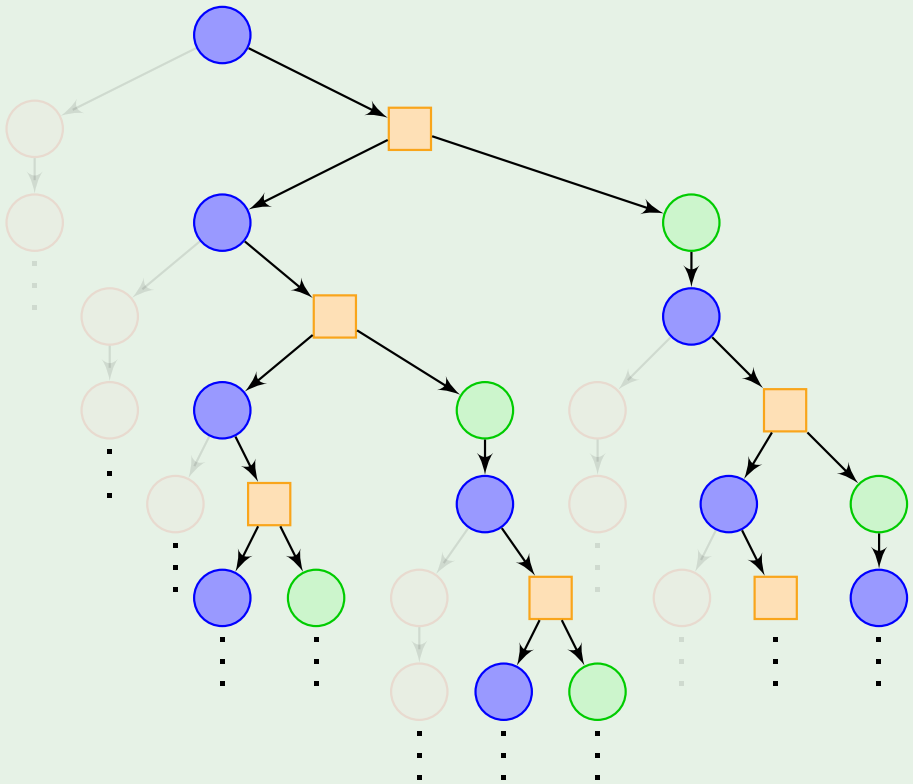

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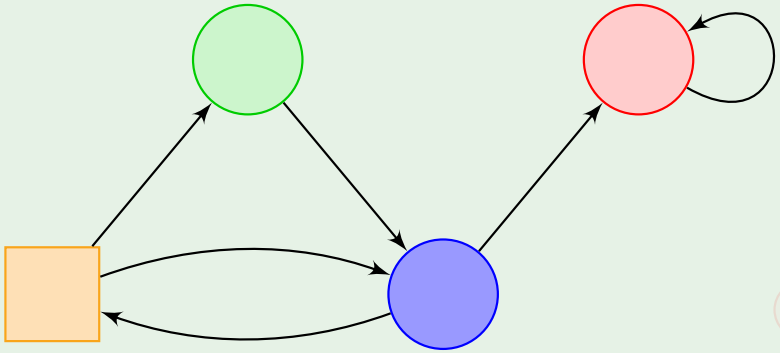

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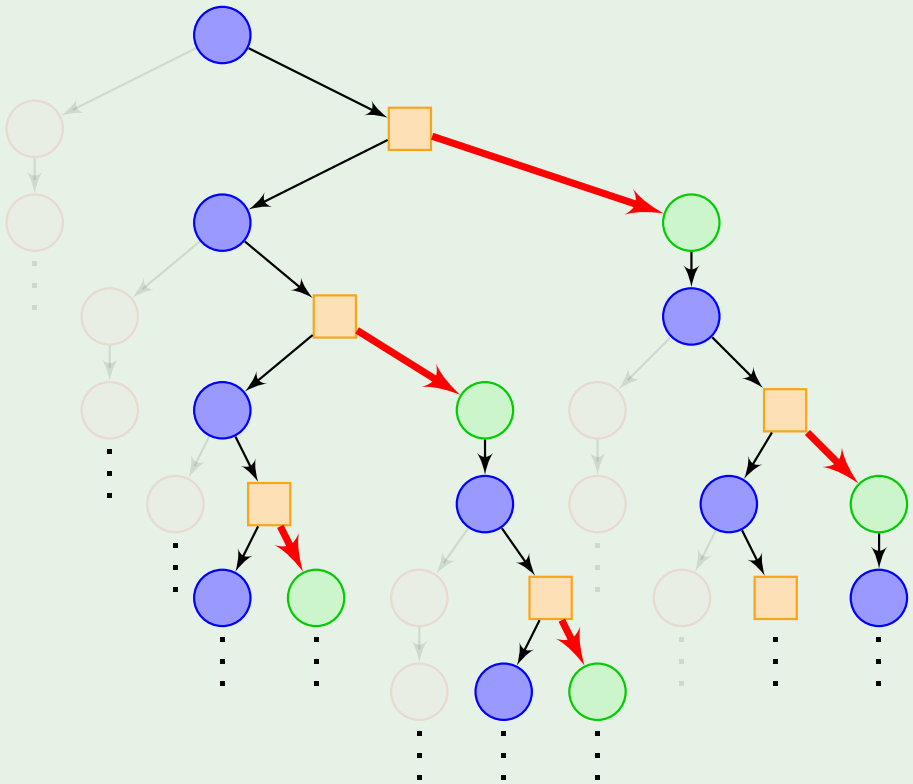

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Definition

ATL_{sc} has a new strategy quantifier:

- $\langle\cdot A\cdot\rangle\,\varphi$ is similar to $\langle\langle A\rangle\rangle\,\varphi$ but assigns the corresponding strategy to A for evaluating φ;

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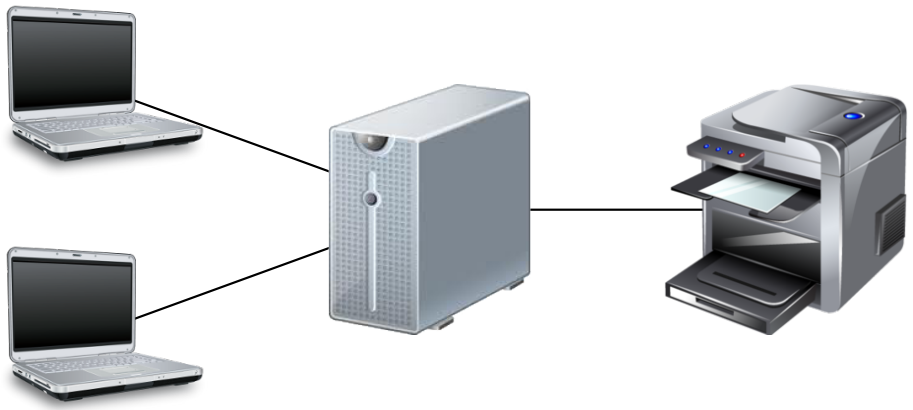
Expressiveness of ATL_{sc}

- Client-server interactions for accessing a shared resource:

$$\langle\cdot\mathrm{Server}\cdot\rangle\ \mathbf{G}\left[\begin{array}{l} \bigwedge_{c\in\mathrm{Clients}} \langle\cdot c\cdot\rangle\ \mathbf{F}\ \mathrm{access}_c \\ \wedge \\ \neg \bigwedge_{c\neq c'} \mathrm{access}_c \wedge \mathrm{access}_{c'} \end{array}\right]$$



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Outline of the presentation

Quantified CTL [ES84,Kup95,Fre01,DLM12]

QCTL extends CTL with propositional quantifiers

$\exists p.\ \varphi$ means that there exists a labelling of the model with p under which φ holds.

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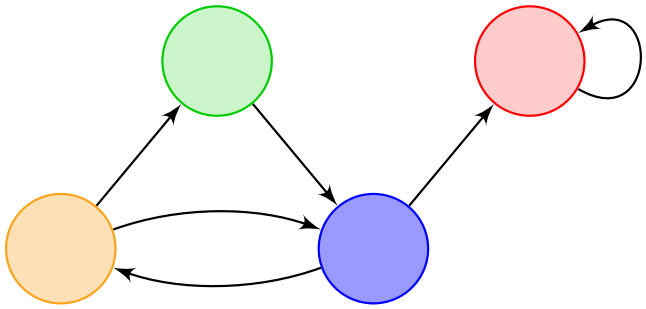

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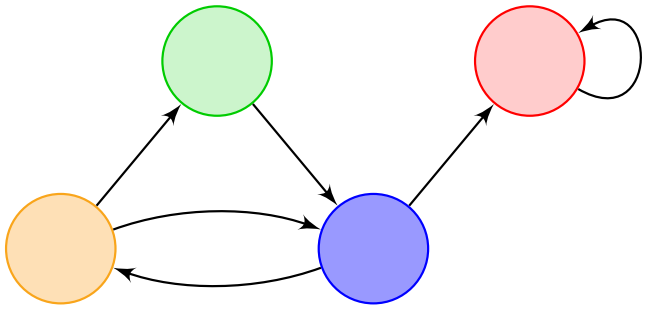

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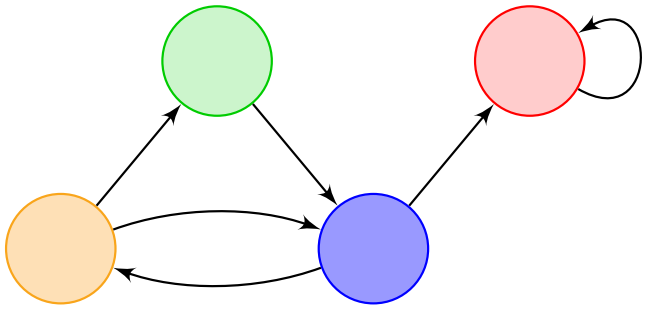

$\rightsquigarrow$ true if we label the Kripke structure;

$\rightsquigarrow$ false if we label the computation tree;

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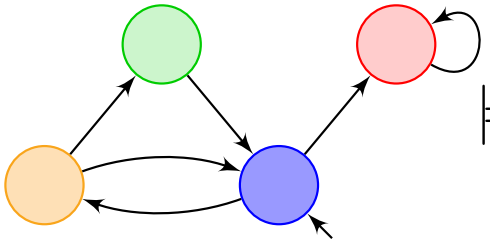
Semantics of QCTL

- structure semantics:



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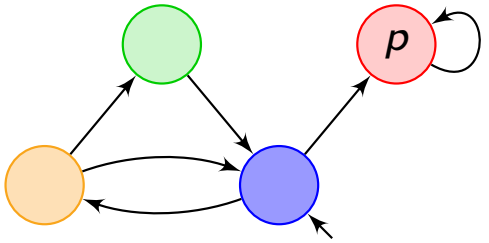

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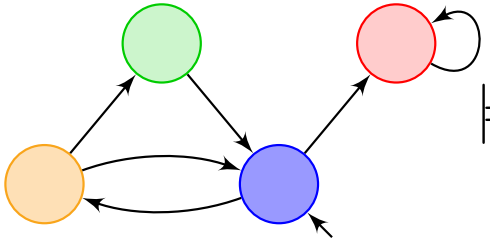

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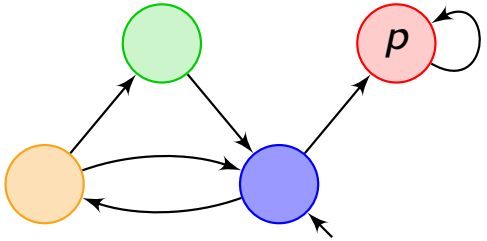

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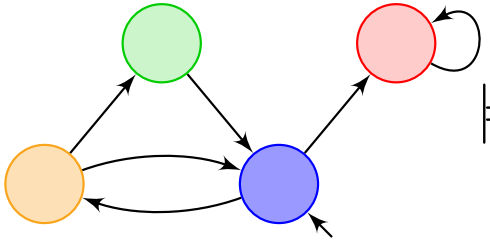

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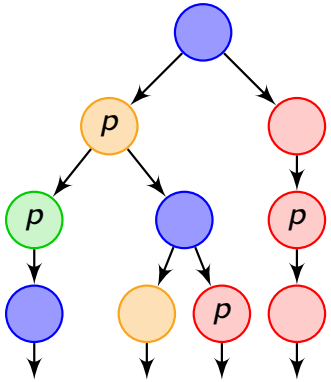

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Expressiveness of QCTL

- QCTL can "count":

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- QCTL can express (least or greatest) fixpoints:

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Theorem

QCTL and MSO are equally expressive.

Decision problems for QCTL [LM14]

Theorem

Under the structure semantics:

- *model checking is* PSPACE*-complete;*
- *satisfiability is undecidable.*

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Under the tree semantics: for formulas with k nested quantifiers,

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Model checking QCTL for the structure semantics is PSPACE-*complete.*

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Proof

Membership:

Iteratively
- (nondeterministically) pick a labelling,
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Hardness:

QBF is a special case (without even using temporal modalities).

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Theorem

QCTL satisfiability for the structure semantics is undecidable.

QCTL with tree semantics

Theorem

- *Model checking QCTL with k quantifiers in the tree semantics is k-*EXPTIME*-complete.*
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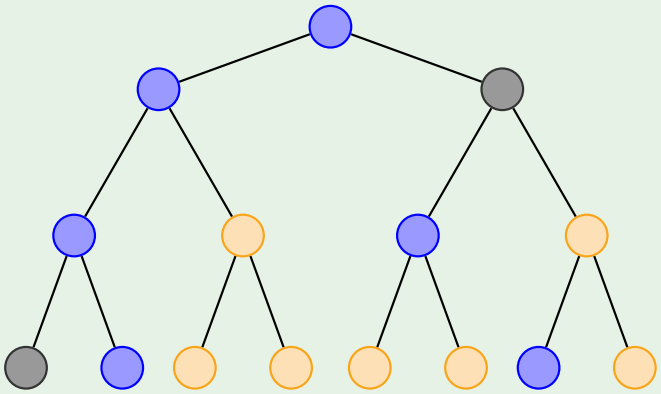

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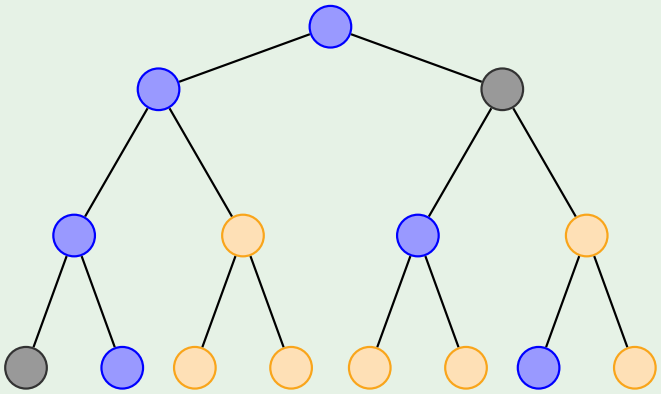

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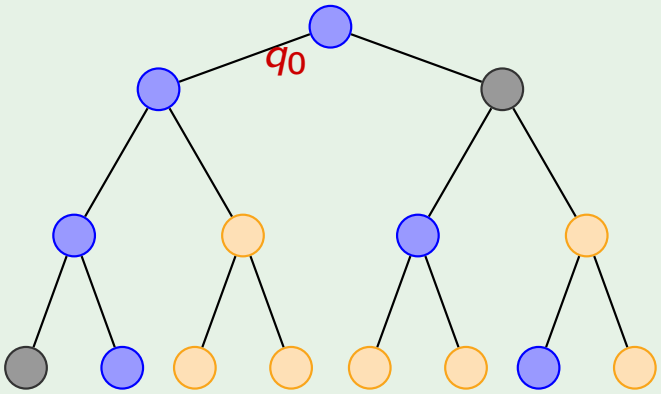

QCTL with tree semantics

Theorem

- *Model checking QCTL with k quantifiers in the tree semantics is k-*EXPTIME*-complete.*
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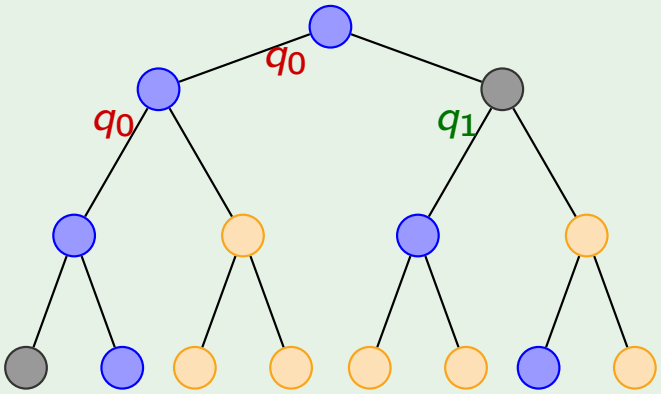

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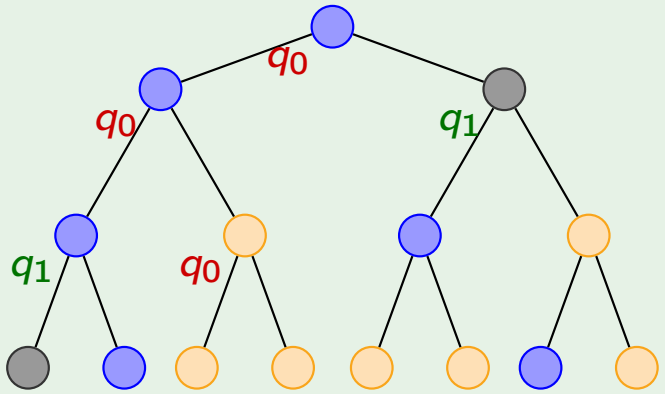

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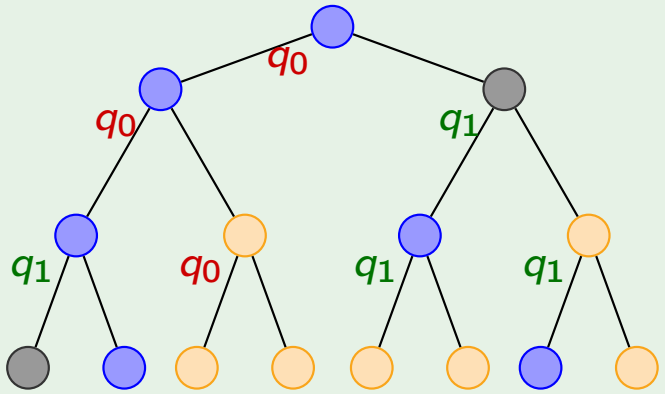

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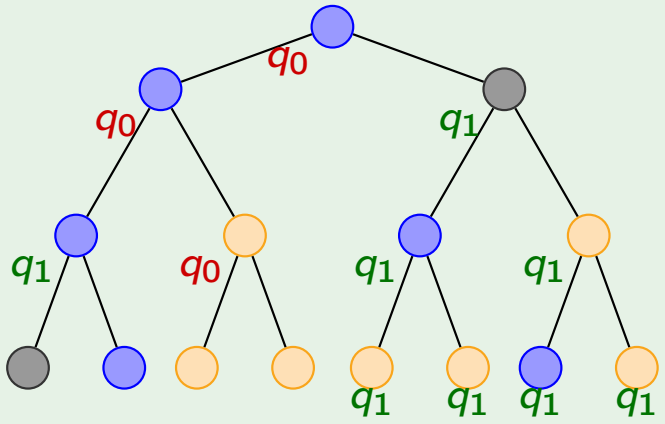

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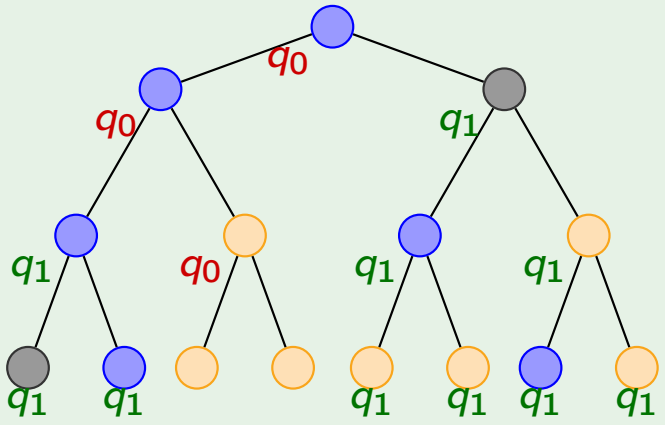

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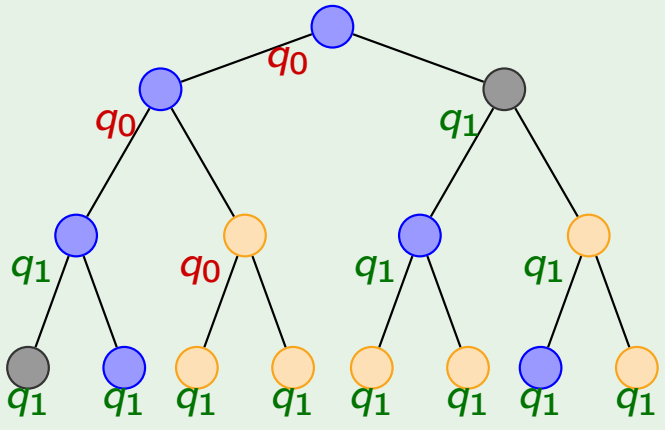

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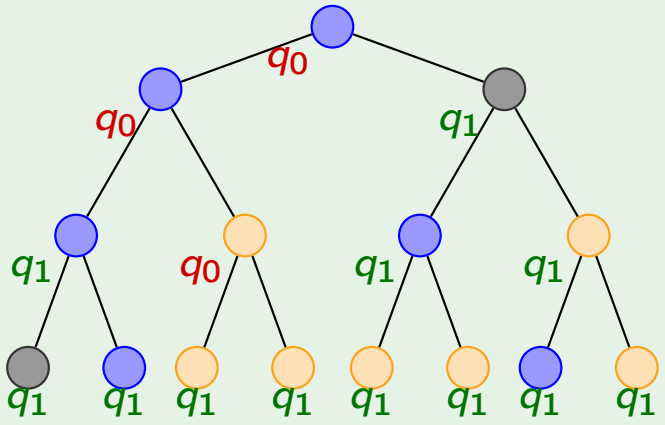

This automaton corresponds to **E** ● **U** ○

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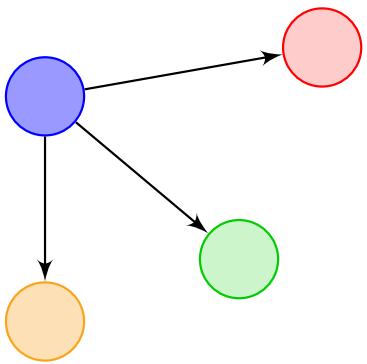
Proof

- polynomial-size automata for CTL;
- quantification is handled by projection, which first requires removing alternation (exponential blowup);
- an automaton equivalent to a QCTL formula can be built inductively;
- emptiness of an alternating parity tree automaton can be decided in exponential time.

Translating ATL_{sc} into QCTL [DLM12]



- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(, A, m_i^A) of states that can be reached from when player A plays m_i^A.

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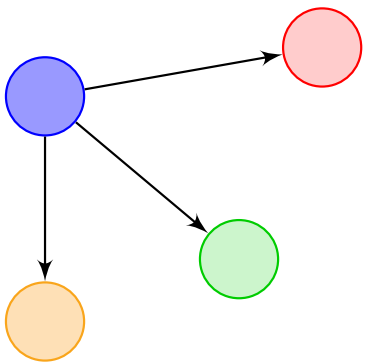

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$\langle \cdot A \cdot \rangle\, \varphi$ can be encoded as follows:

$\exists m_1^A.\ \exists m_2^A \ldots \exists m_n^A.$

- this corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(m_i^A \Leftrightarrow \bigwedge \neg\, m_j^A)$;
- the outcomes all satisfy φ:

$$\mathbf{A}\big[\mathbf{G}(q \wedge m_i^C \Rightarrow \mathbf{X}\,\mathrm{Next}(q, C, m_i^C)) \Rightarrow \varphi\big].$$

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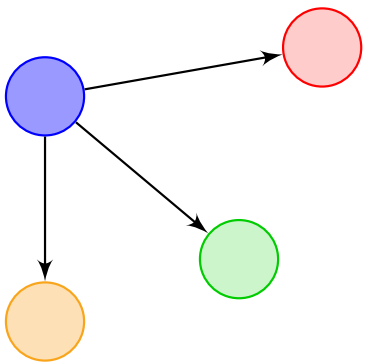

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Corollary

*ATL_{sc} model checking is decidable (*TOWER*-complete).*

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Outline of the presentation

Strategy Logic [CHP07,MMV10]

Strategy logic

Explicit quantification over strategies + strategy assignement:

$$\exists \sigma_A.\ \forall \sigma_B.\mathsf{assign}(A \to \sigma_A, B \to \sigma_B).\ \varphi$$

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SL is at least as expressive as ATL_{sc}.

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SL is at least as expressive as ATL_{sc}.

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Outline of the presentation

What is the exact semantics? [BGM16]

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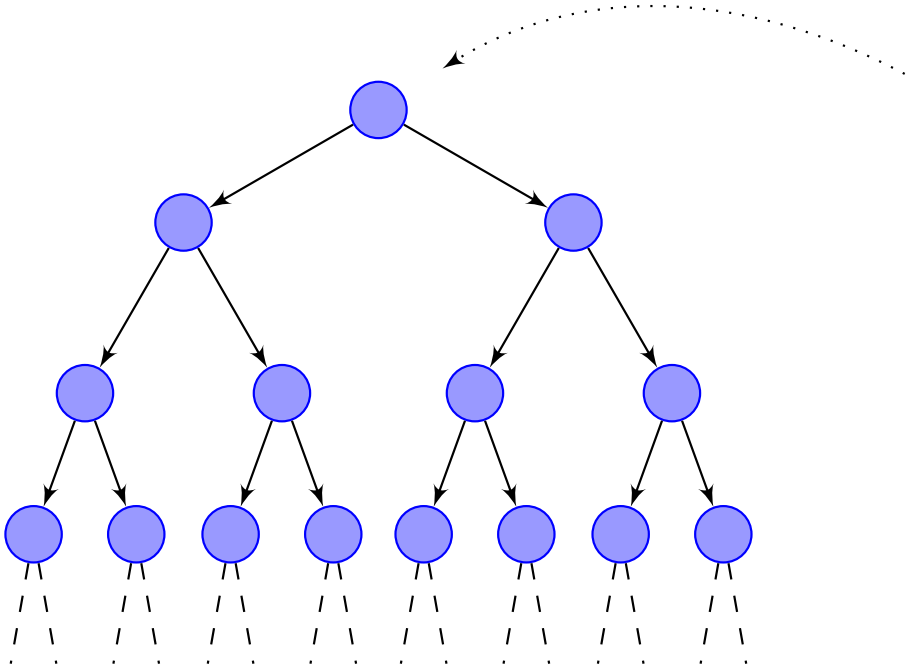

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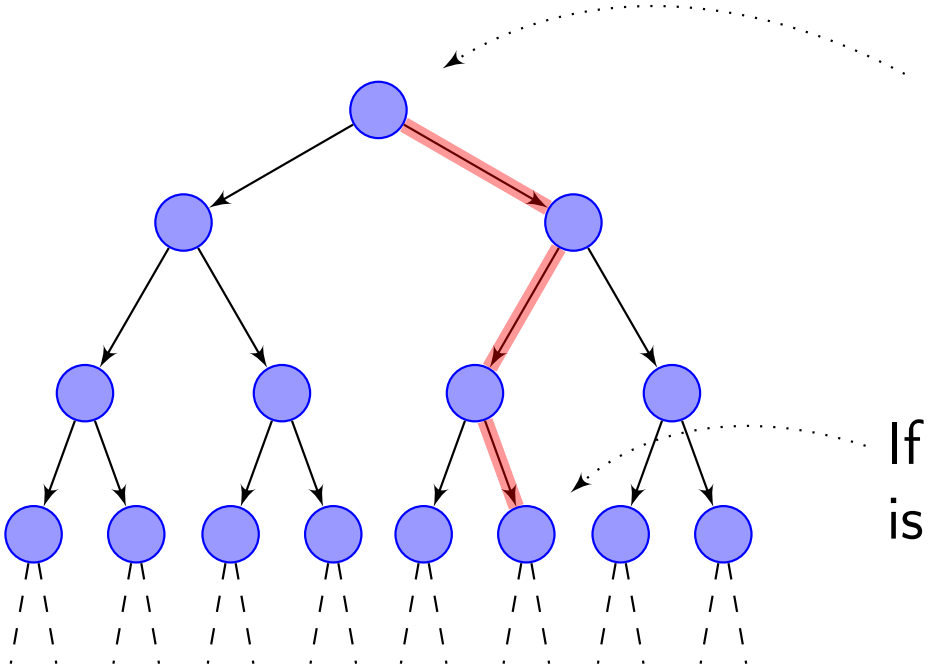

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If strategy σ is assigned here,
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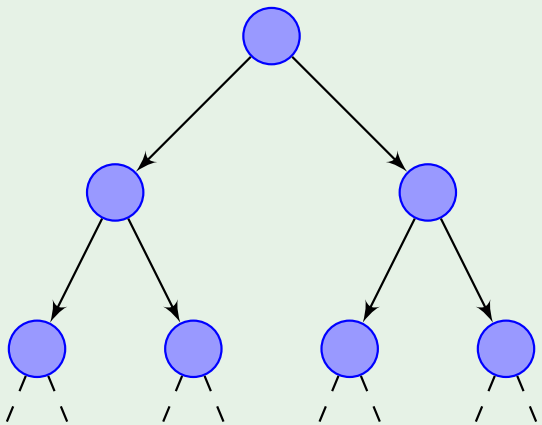

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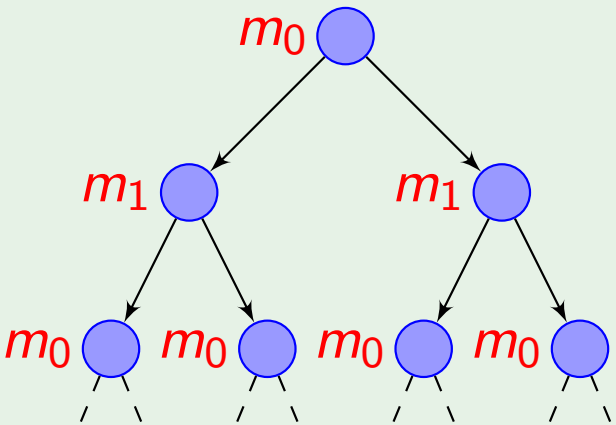

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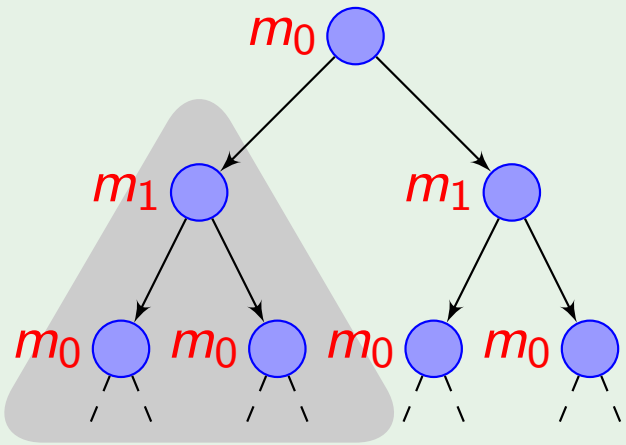

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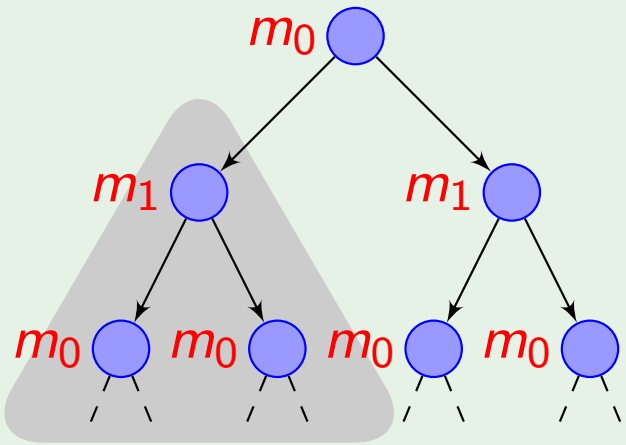

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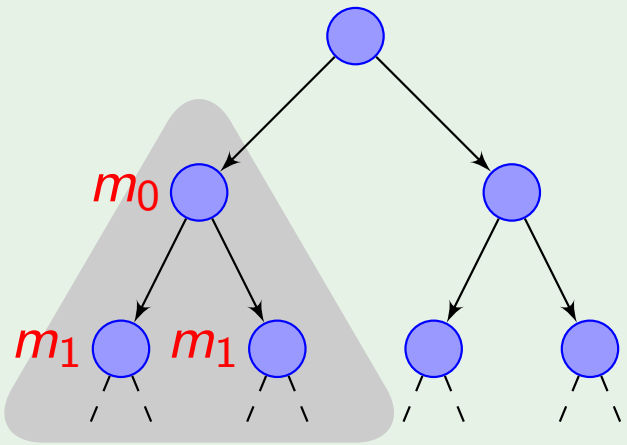

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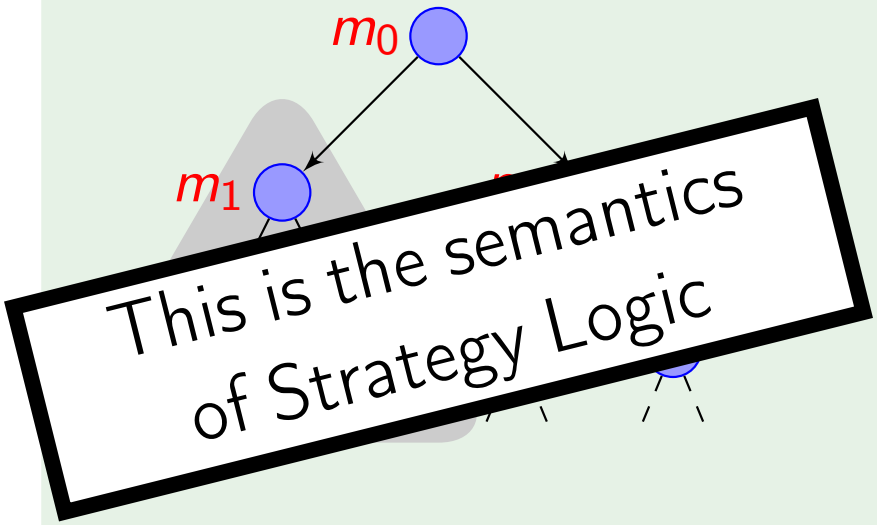

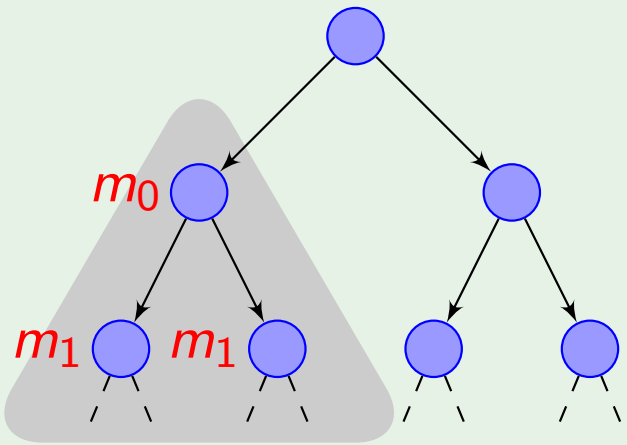

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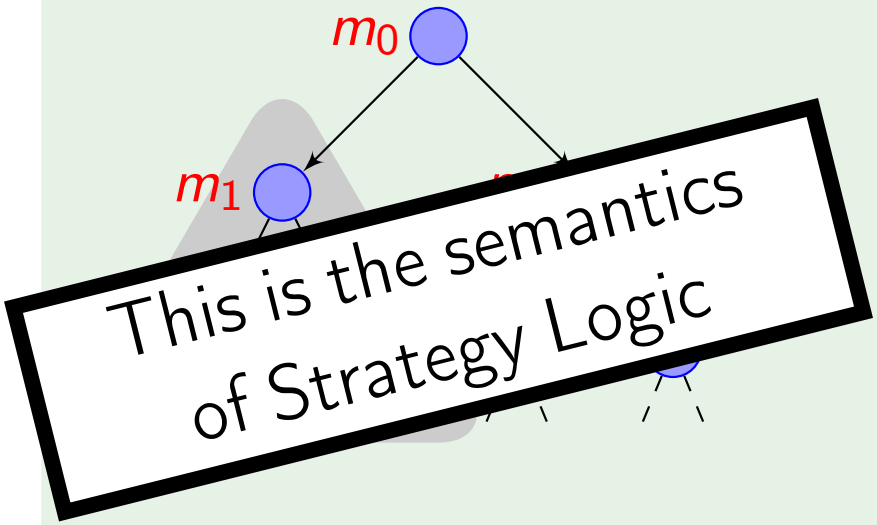

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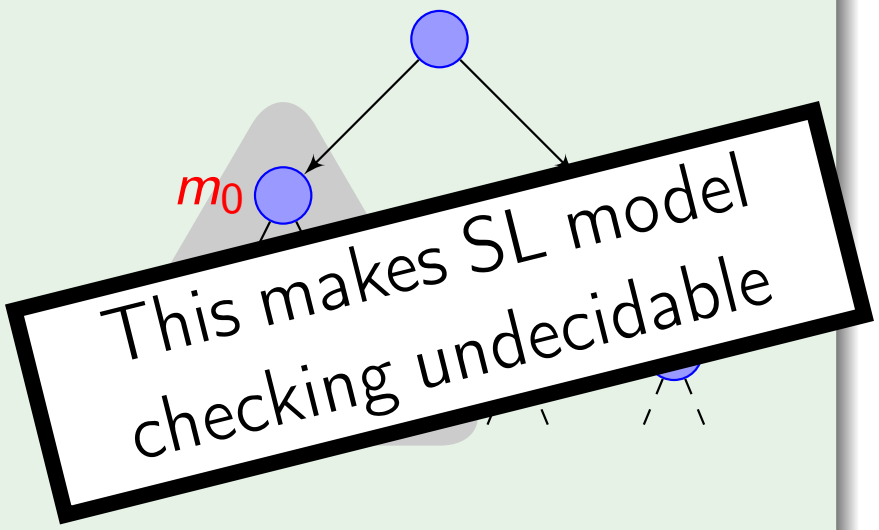

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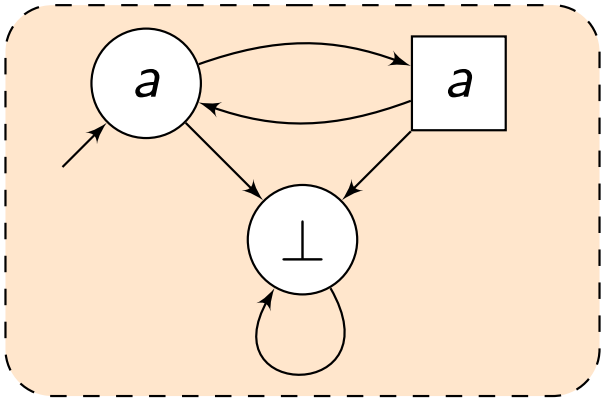
Encoding of a 2-counter machine



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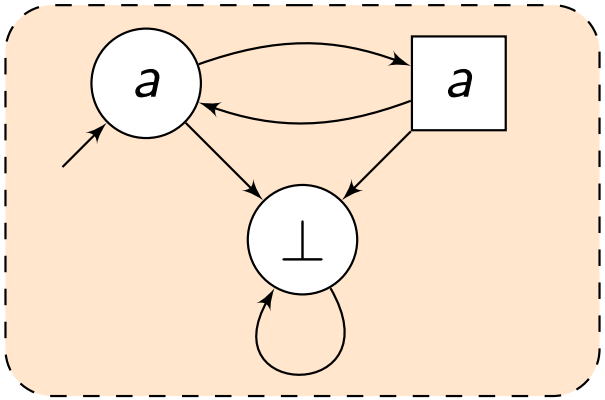

Strategy $\sigma_{\bigcirc}$ encodes counter value k if it plays k times to $\boxed{a}$ before playing to $\perp$ (similarly for $\sigma_{\square}$).

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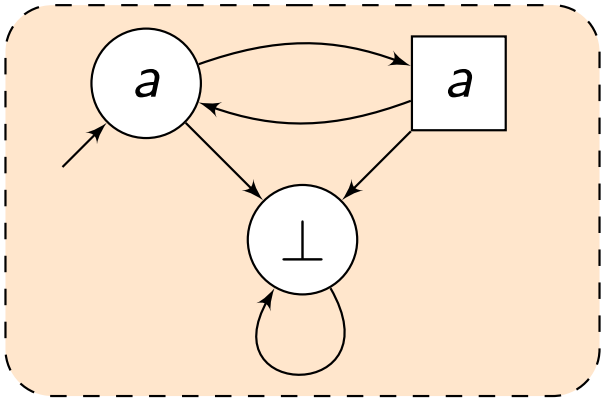

Strategy $\sigma_{\bigcirc}$ encodes counter value k if it plays k times to $\boxed{a}$ before playing to $\bot$ (similarly for $\sigma_{\square}$).

Lemma

Two strategies $\sigma_{\bigcirc}$ and $\sigma_{\square}$ encode the same counter value iff, when entering the game above and applying these strategies, it holds

$$\mathbf{G}(\boxed{a} \Rightarrow \mathbf{X}\,a) \wedge \mathbf{F}\left[(a \wedge \mathbf{X}\,\bot) \wedge \neg\langle\cdot\bigcirc\cdot\rangle\,\neg\,\mathbf{X}\,\mathbf{X}\,\bot\right] \qquad (\varphi_{=})$$

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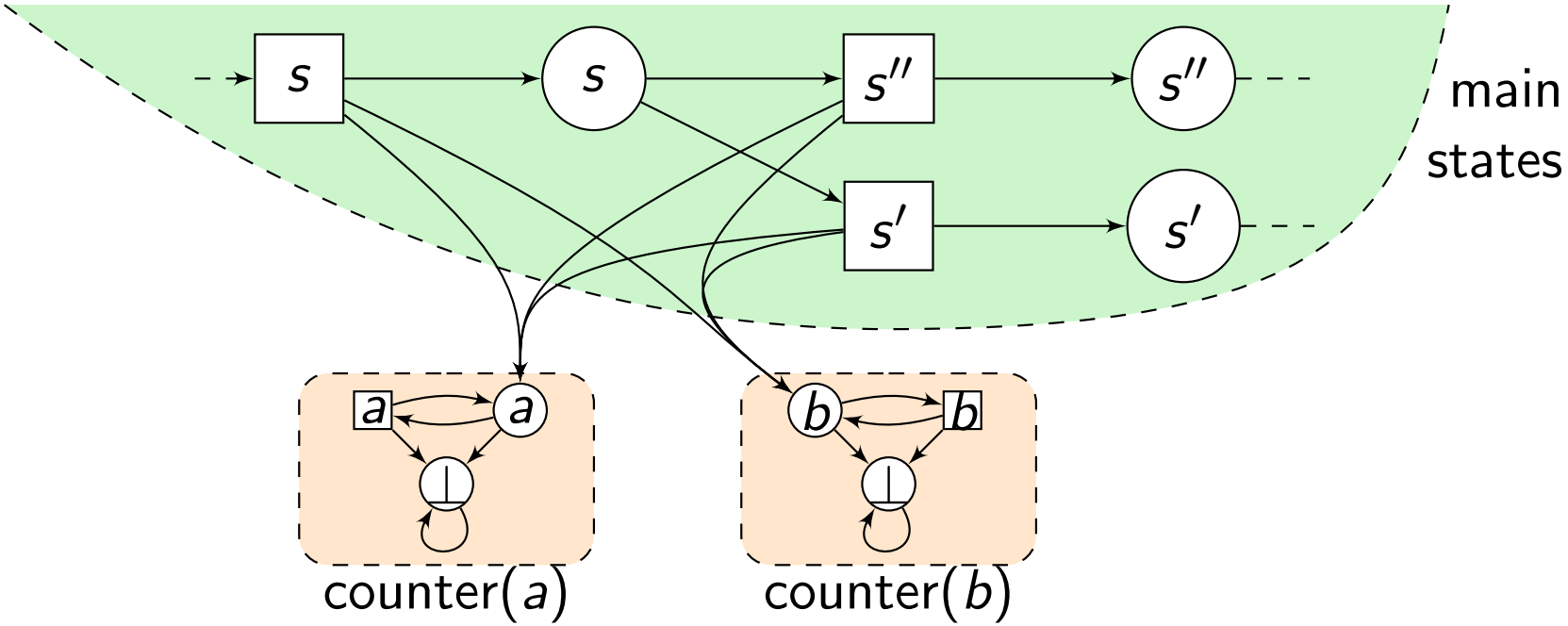

- A strategy for ○ encodes a run of the 2-counter machine;
- At each step, Player □ can check correct values of counters (this is where we use our special semantics).

Strategy dependences (ongoing work) [MMPV11,GBM17]

[MMPV11] Mogavero, Murano, Perelli, Vardi. Reasonning about strategies: ... J. ACM, 2011.
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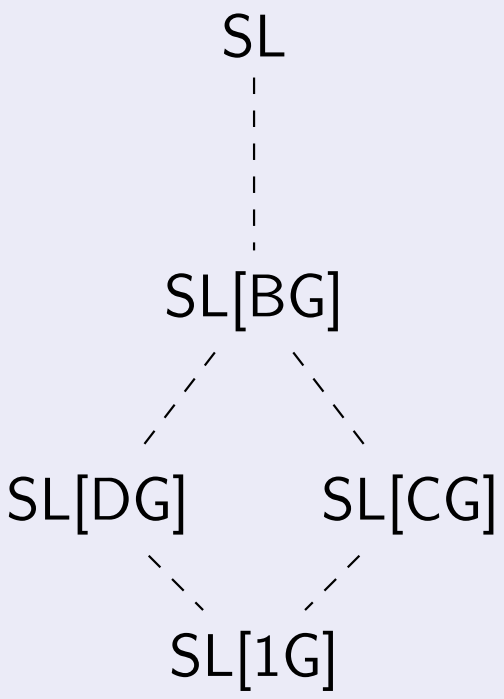

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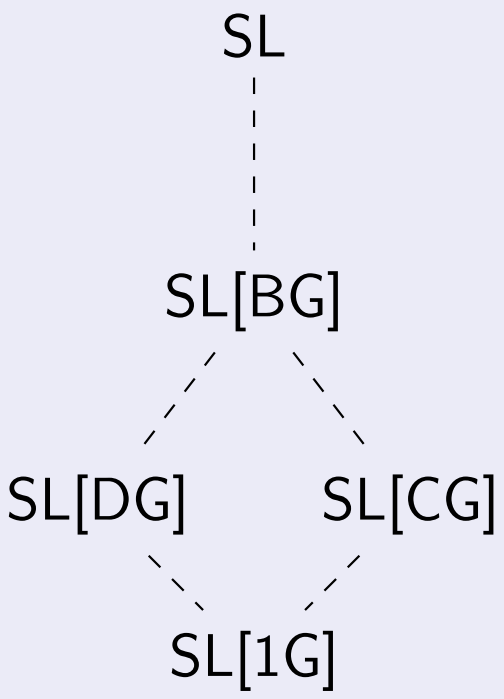

disjunction/conjunction of goals

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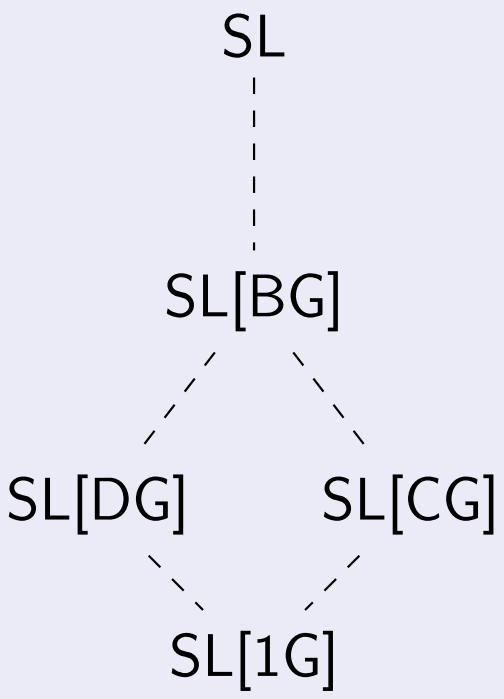

boolean combination of goals

disjunction/conjunction of goals

$$\underbrace{(Q_i\sigma_i).}_{\text{quantifier block}}\ \underbrace{\mathsf{assign}(A_j \mapsto \sigma_i).\ \varphi_{\mathsf{LTL}}}_{\text{goal}}$$

[MMPV11] Mogavero, Murano, Perelli, Vardi. Reasonning about strategies: ... J. ACM, 2011.

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$$\forall \sigma_A \exists \sigma_B.\ \mathsf{assign}(A \mapsto \sigma_A; B \mapsto \sigma_B).\ \varphi$$

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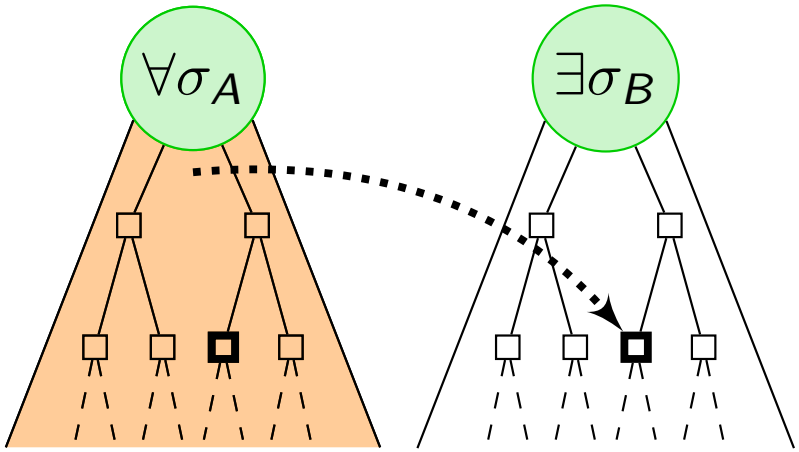

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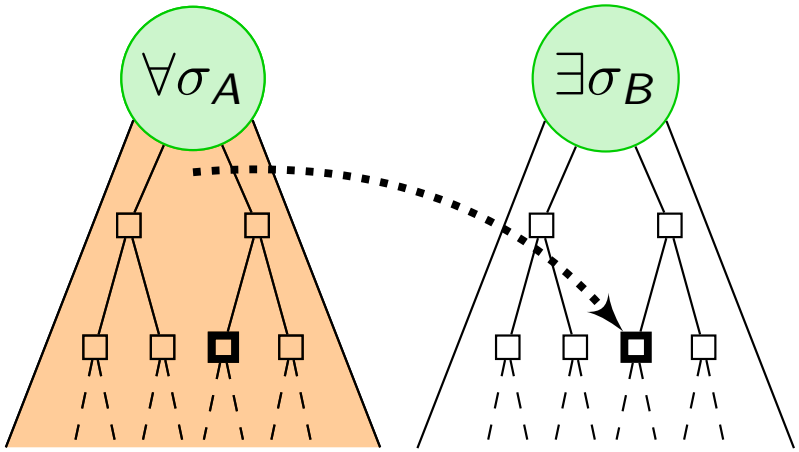

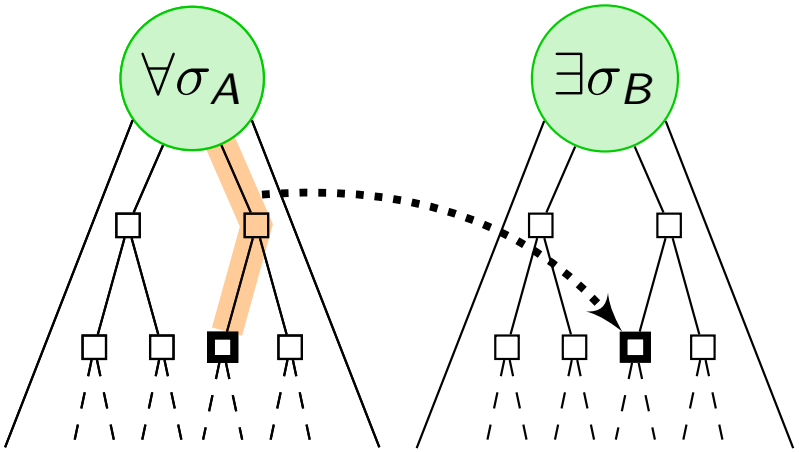

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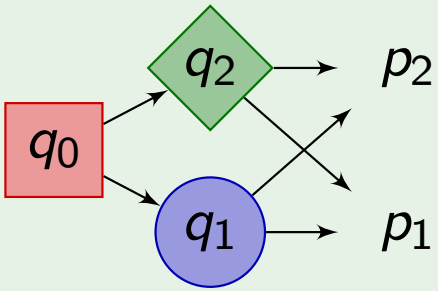
Dependence maps (Skolemization)

Example



$$\varphi = \forall y.\exists z.\forall x_A.\forall x_B.$$

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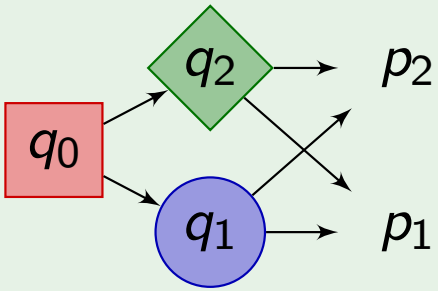

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Dependence maps select $\exists$-strategies from $\forall$-strategies:

$$\begin{array}{rcl} \theta\colon (\mathcal{V}^{\forall} \to \text{Strats}) & \to & (\mathcal{V}^{\exists} \to \text{Strats}) \\ w & \mapsto & \theta(w) \end{array}$$

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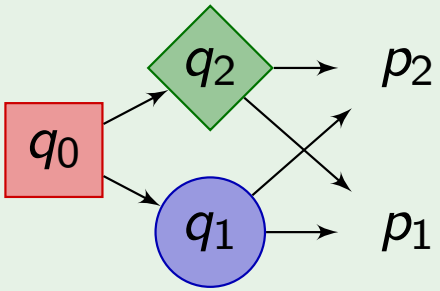

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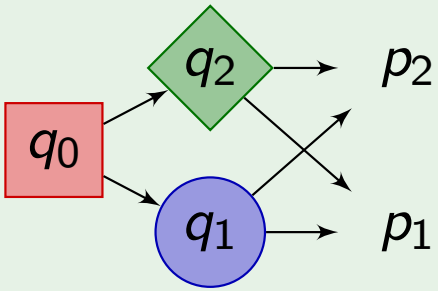

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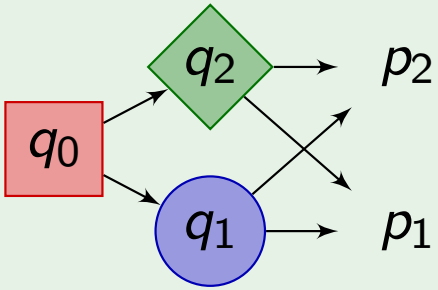

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Dependence maps

- classical SL semantics ($\models^C$):
  - only depend on previously-quantified variables:

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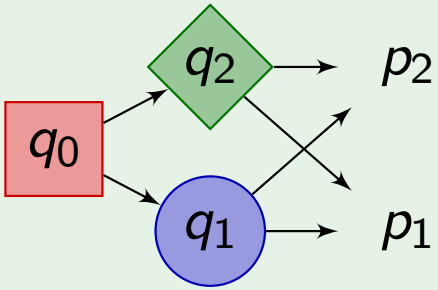

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$$\text{if } w(y) = w'(y), \text{ then } \theta(w)(z) = \theta(w')(z)$$

- elementary semantics ($\models^E$):
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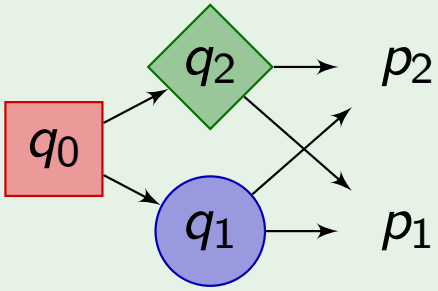
$$\text{for all } n, \text{ if } w(y)(m) = w'(y)(m) \text{ for all } m \preceq n, \text{ then } \theta(w)(z)(n) = \theta(w')(z)(n)$$

Dependence maps (Skolemization)

Example



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Theorem

For any $\varphi \in SL[BG]$:

$$q_0 \models^C \varphi \ \Leftrightarrow\ q_0 \models \varphi \qquad\qquad q_0 \models^E \varphi \ \Rightarrow\ q_0 \models^C \varphi$$

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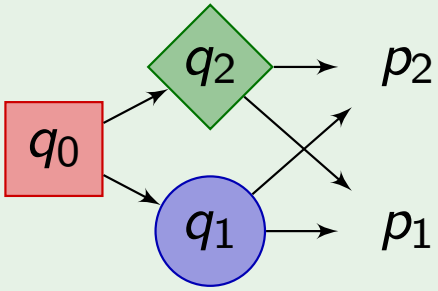

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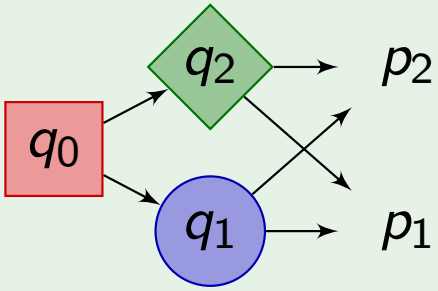

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- $q_0 \not\models^E \varphi$: y only plays in q_1, so z will not depend on y.

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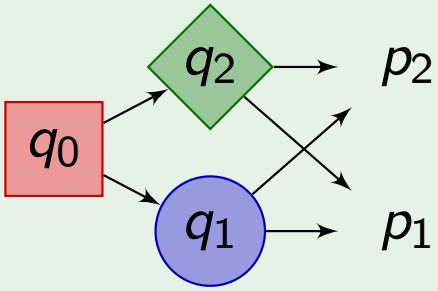

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- $q_0 \not\models^E \neg\varphi$: because otherwise $q_0 \models^C \neg\varphi$...

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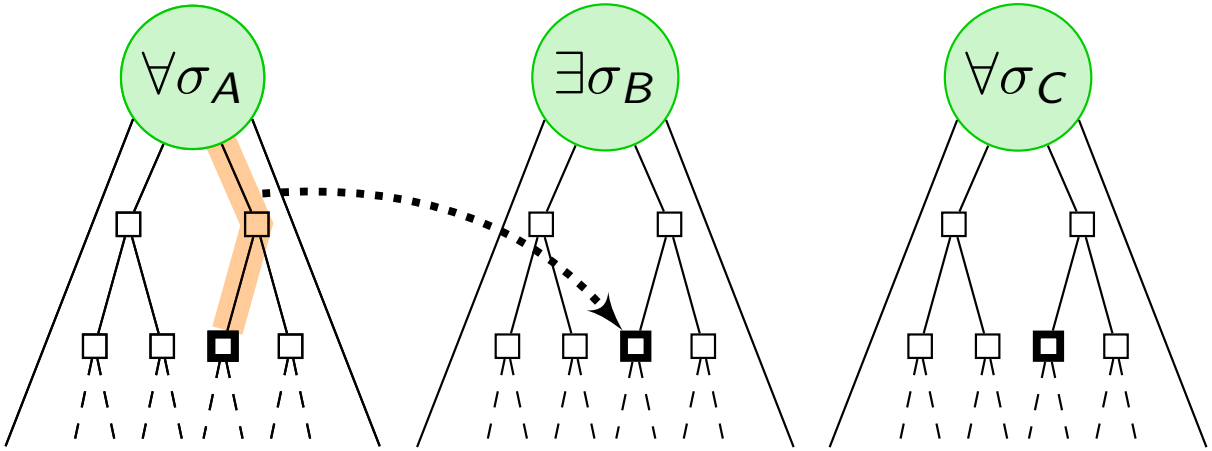
Beyond elementary dependences [GBM17]



Elementary semantics

Strategy σ_B at node n only depends on σ_A along the branch to n.

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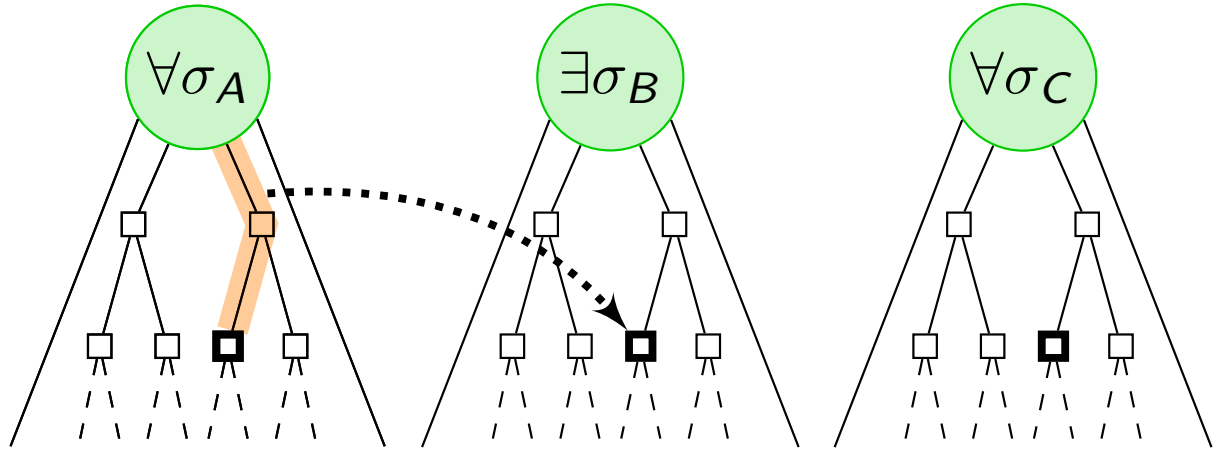

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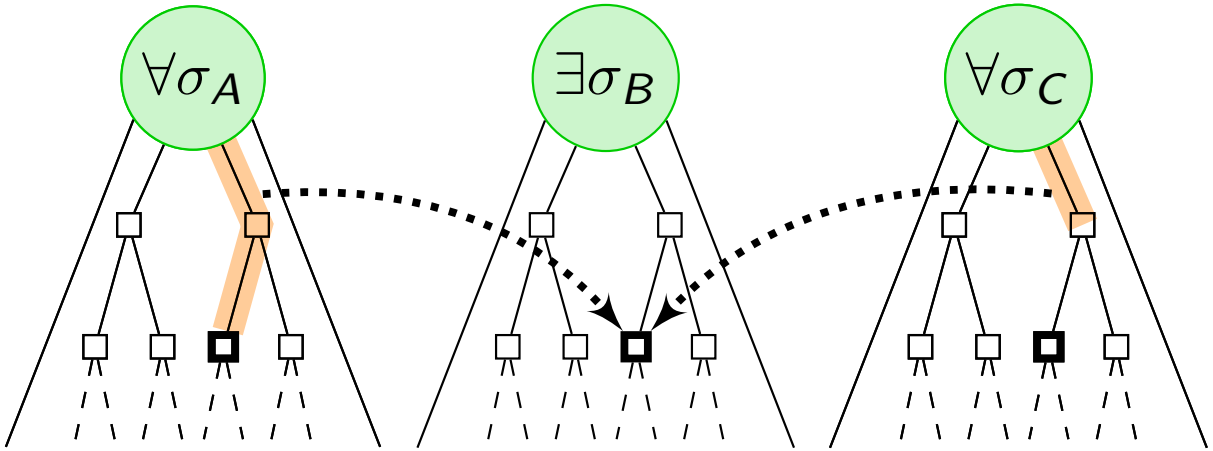

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Strategy σ_B also depends on σ_C along strict prefixes of n.

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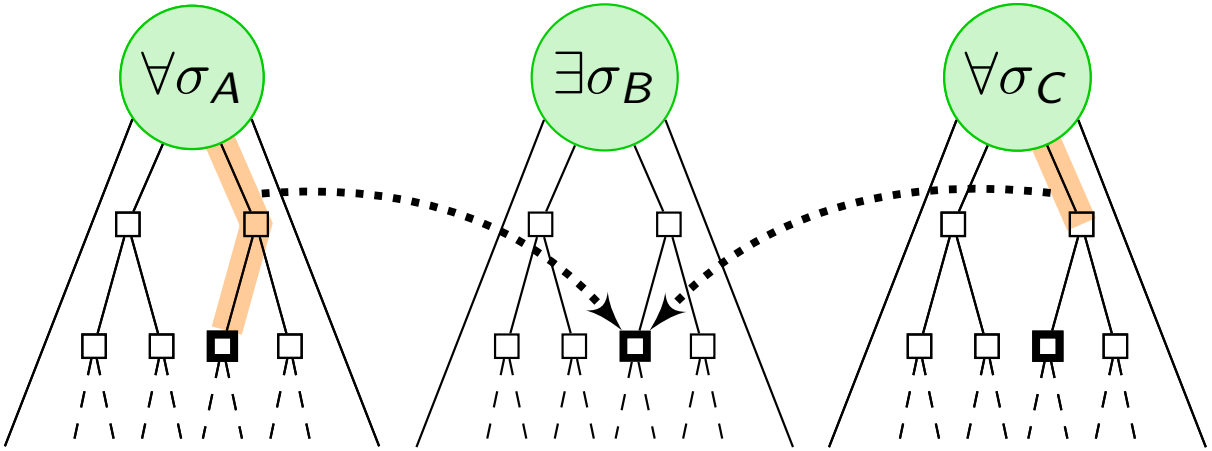

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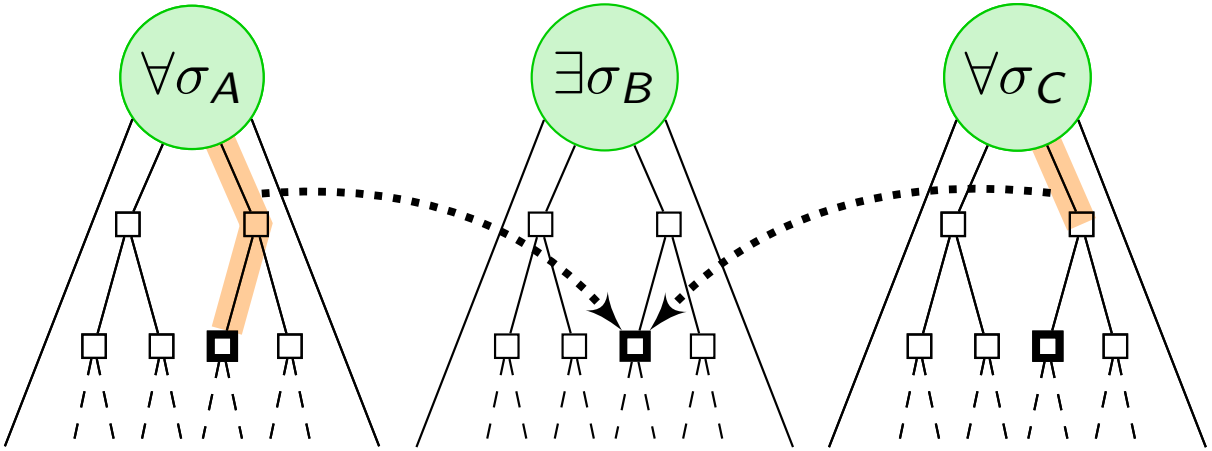

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Theorem

For a large fragment of SL[BG] (containing SL[CG] and SL[DG]),

$$q_0 \models^T \varphi \;\Leftrightarrow\; q_0 \not\models^T \neg\varphi.$$

*Model checking this fragment is 2*EXPTIME*-complete (for $\models^T$).*

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Outline of the presentation

Conclusions and future works

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- Temporal logics have been successful for model checking "closed" systems;
- For multi-agent systems, ATL is not expressive enough;
- But ATL_{sc} and Strategy Logic are too expressive:
  - model checking is very hard;
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Conclusions

- Many open questions:
  - better understand the semantics
  - identify expressive fragments with efficient algorithms
  - extend with stochastic strategies, quantities, partial observation...