Temporal logics for multi-agent systems

Nicolas Markey
LSV – ENS Cachan

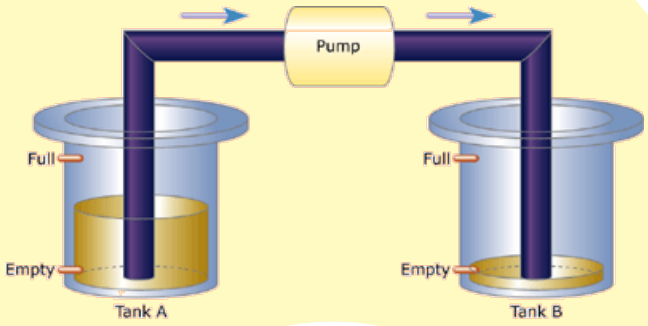
(based on joint works with Thomas Brihaye,
Arnaud Da Costa-Lopes, François Laroussinie
Patricia Bouyer, Patrick Gardy)

Centre Fédéré en Vérification

Brussels, January 29, 2016

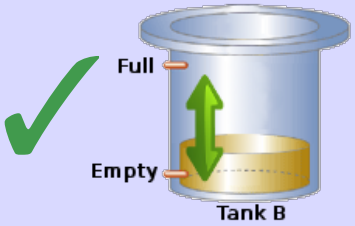
Model checking and synthesis

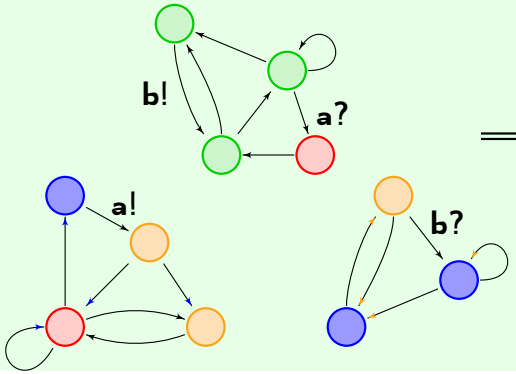
system:



[http://www.embedded.com]

property

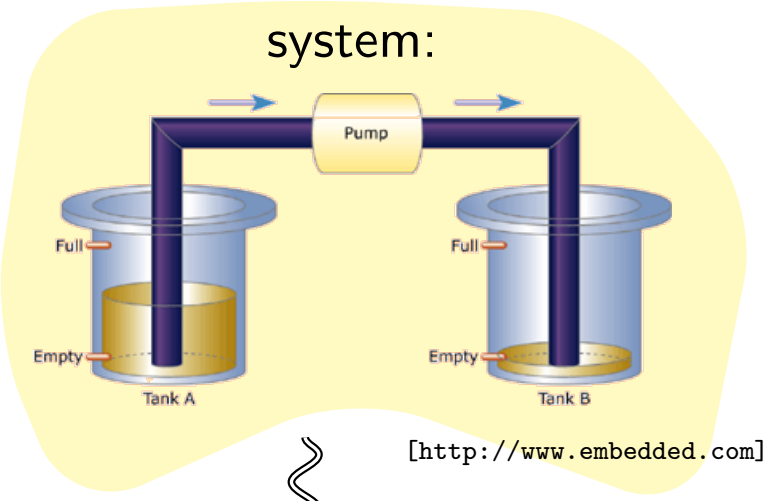


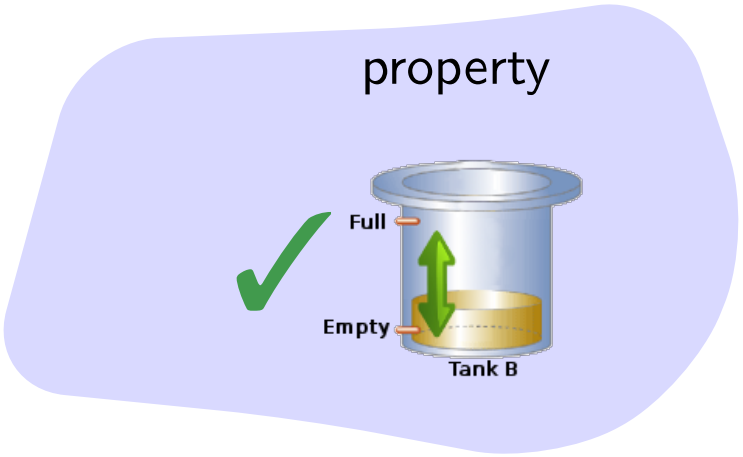


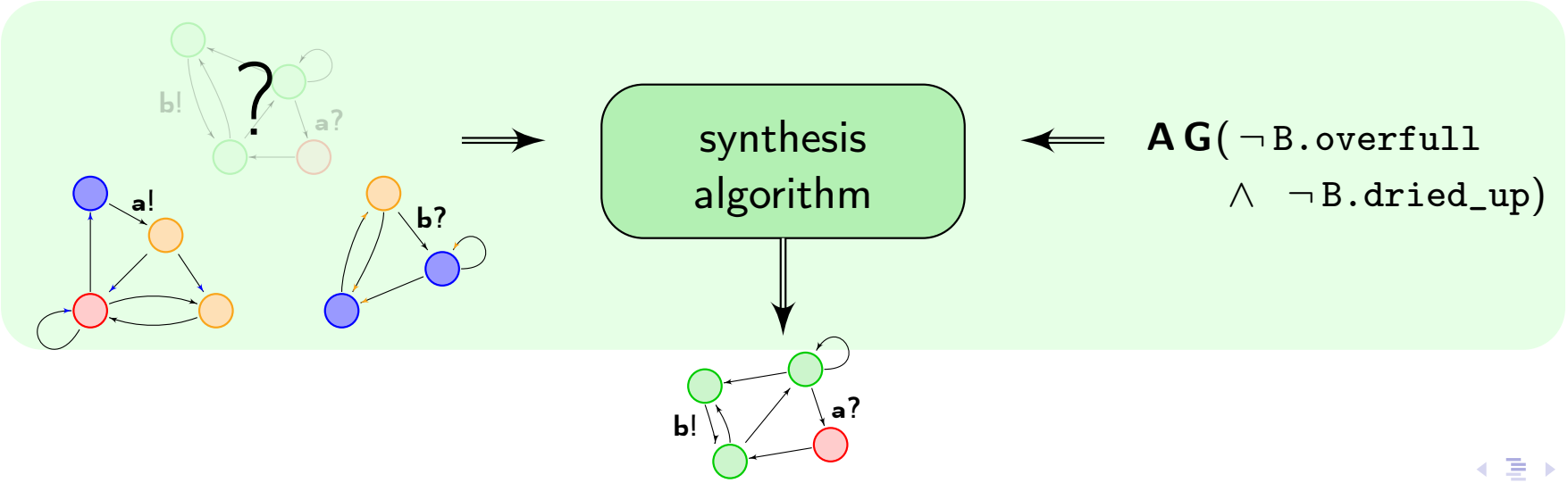
$\Longrightarrow$ model-checking algorithm $\Longleftarrow$ $\mathbf{A}\,\mathbf{G}(\neg\,\texttt{B.overfull} \wedge \neg\,\texttt{B.dried\_up})$

yes/no

Model checking and synthesis







Outline of the presentation

1. Introduction
2. Basics of CTL and ATL
   - expressing properties of reactive systems
   - efficient verification algorithms
3. ATL with strategy contexts
   - specifying properties of complex interacting systems
   - expressive power of ATL_{sc}
   - translation into Quantified CTL (QCTL)
   - algorithms for ATL_{sc}
4. Strategy Logic
5. Conclusions and future works

Outline of the presentation

Computation-Tree Logic (CTL)

- **atomic propositions:**  ,  , ...

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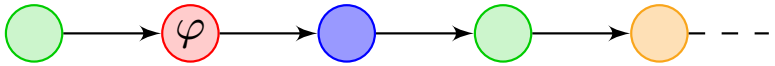
- **temporal modalities:**

$\mathbf{X}\,\varphi$ 

"next φ"

$\varphi\ \mathbf{U}\ \psi$ 

"φ until ψ"

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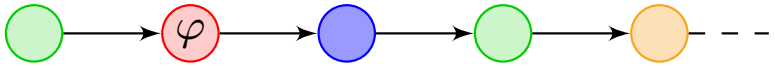
$\mathbf{X}\,\varphi$ 

"next φ"

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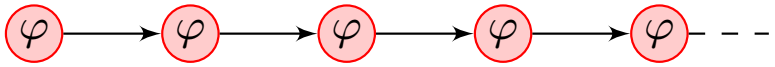
$\texttt{true}\ \mathbf{U}\ \varphi \equiv \mathbf{F}\,\varphi$ 

"eventually φ"

$\neg\,\mathbf{F}\,\neg\varphi \equiv \mathbf{G}\,\varphi$ 

"always φ"

Computation-Tree Logic (CTL)

- **atomic propositions:** ◯, ◯, ...
- **boolean combinators:** $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, ...
- **temporal modalities:**

$\mathbf{X}\,\varphi$ — "next φ"

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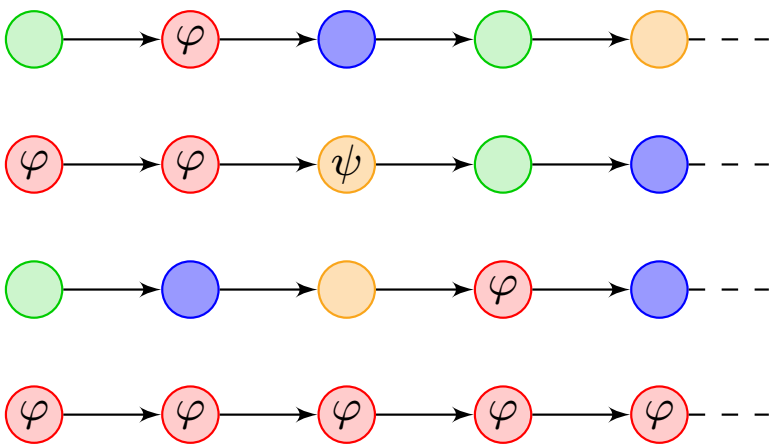
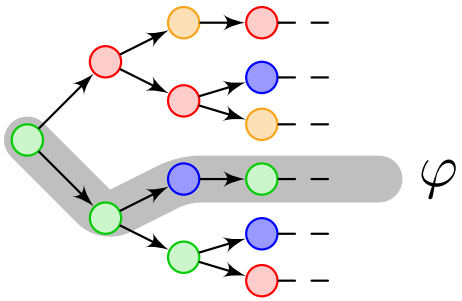
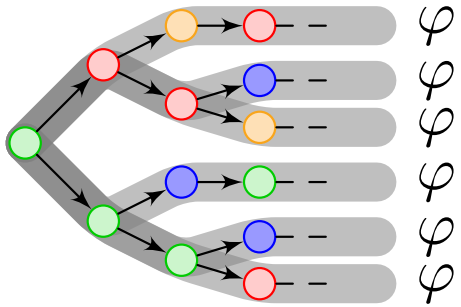


- **path quantifiers:**

$\mathbf{E}\varphi$



$\mathbf{A}\varphi$



Examples of CTL formulas

In CTL, each temporal modality is in the immediate scope of a path quantifier.

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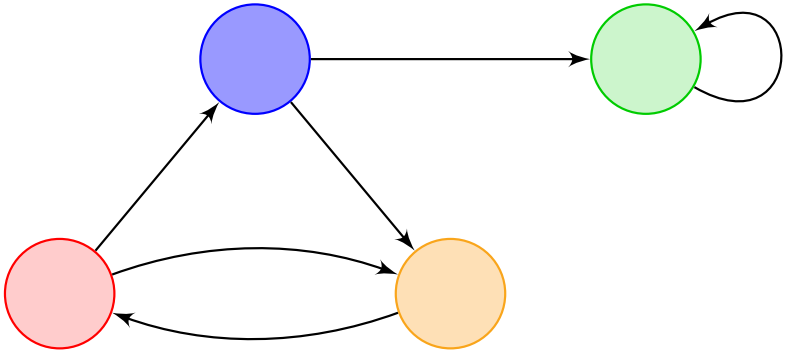
E F   is reachable



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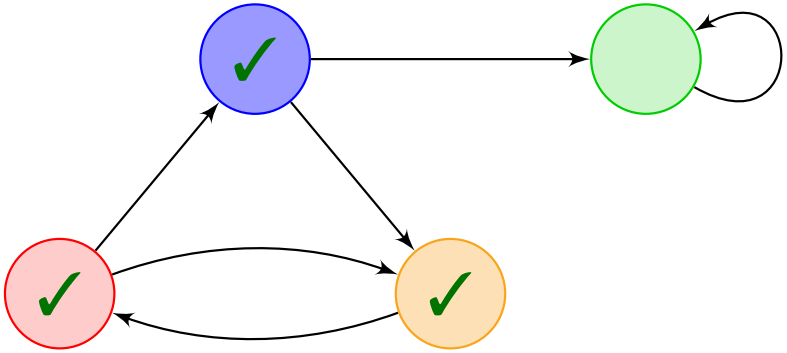

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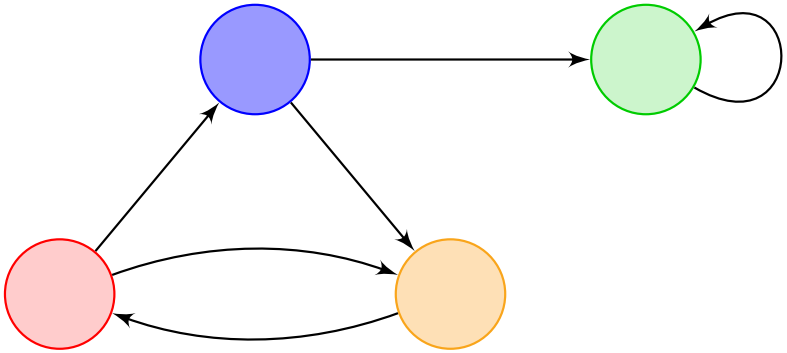

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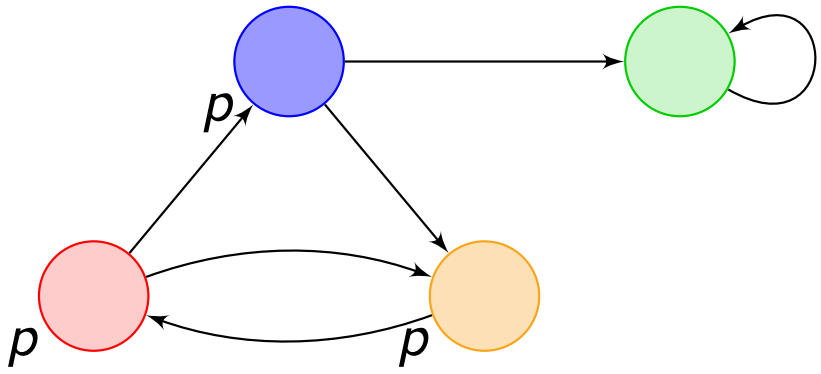

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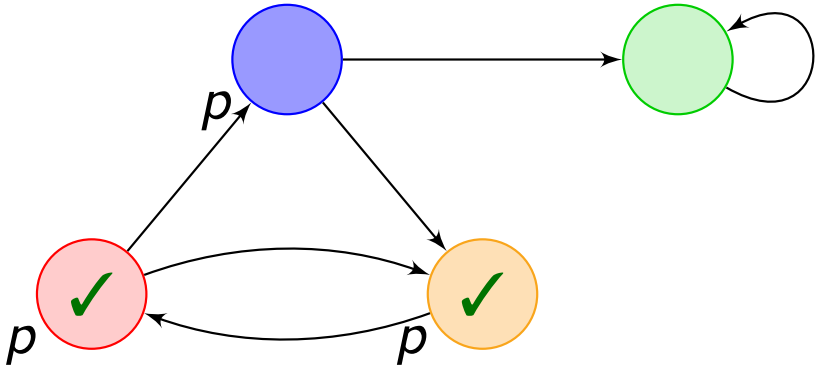

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Theorem ([KVW94])

CTL model checking on product structures is PSPACE-complete.

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[KVW94] Kupferman, Vardi, Wolper. An automata-theoretic approach to branching-time... CAV, 1994.

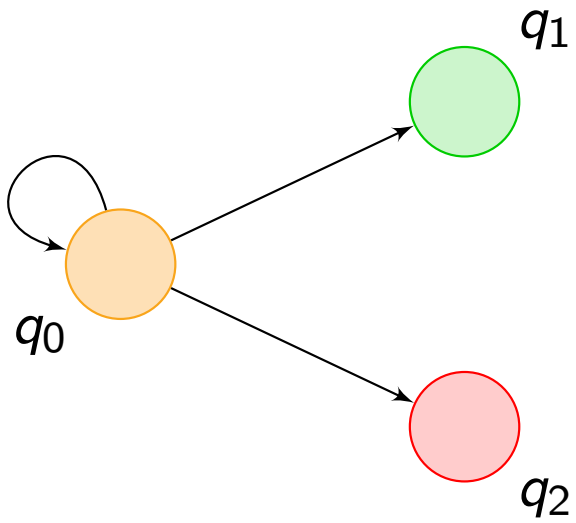
Reasoning about open systems

Reasoning about open systems

Concurrent games

A concurrent game is made of

- a transition system;



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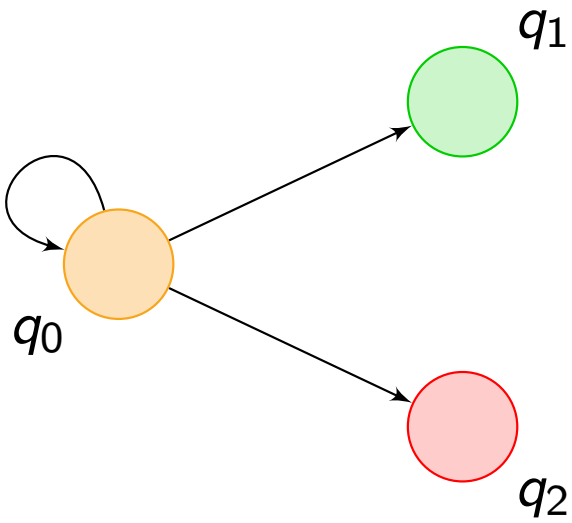

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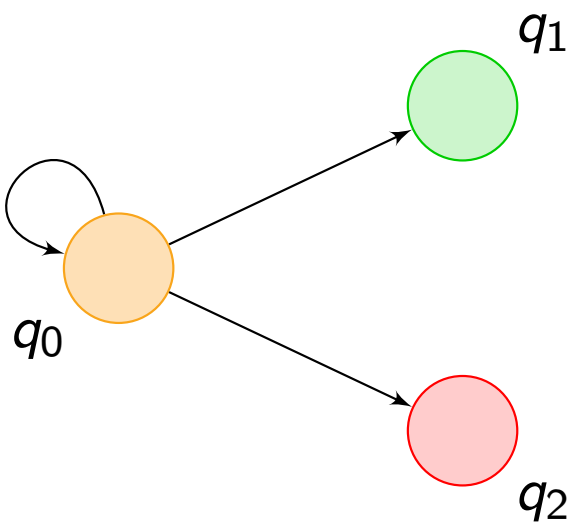

		player 1		
				
player 2		q_0	q_2	q_1
		q_1	q_0	q_2
		q_2	q_1	q_0

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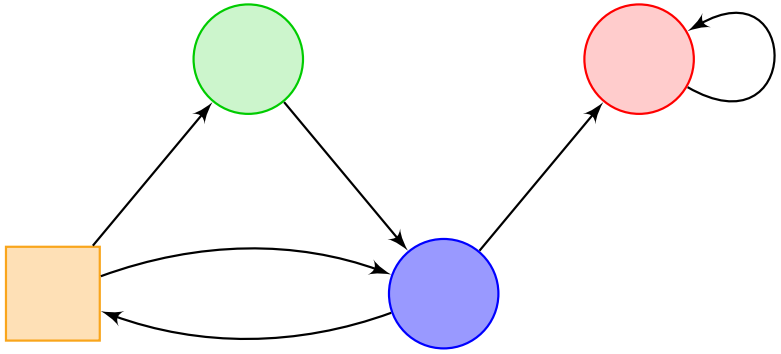
Turn-based games

A turn-based game is a game where only one agent plays at a time.



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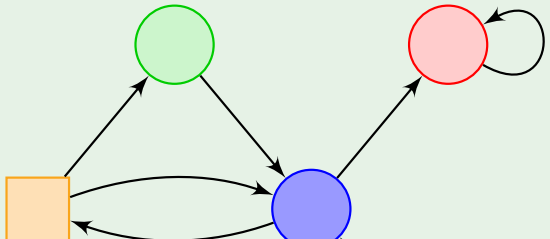

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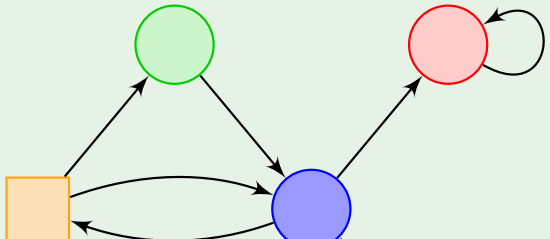
Strategy for player



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Strategy for player

alternately go to and

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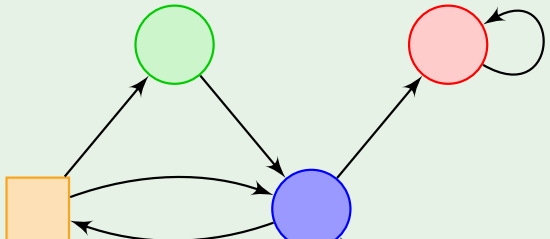

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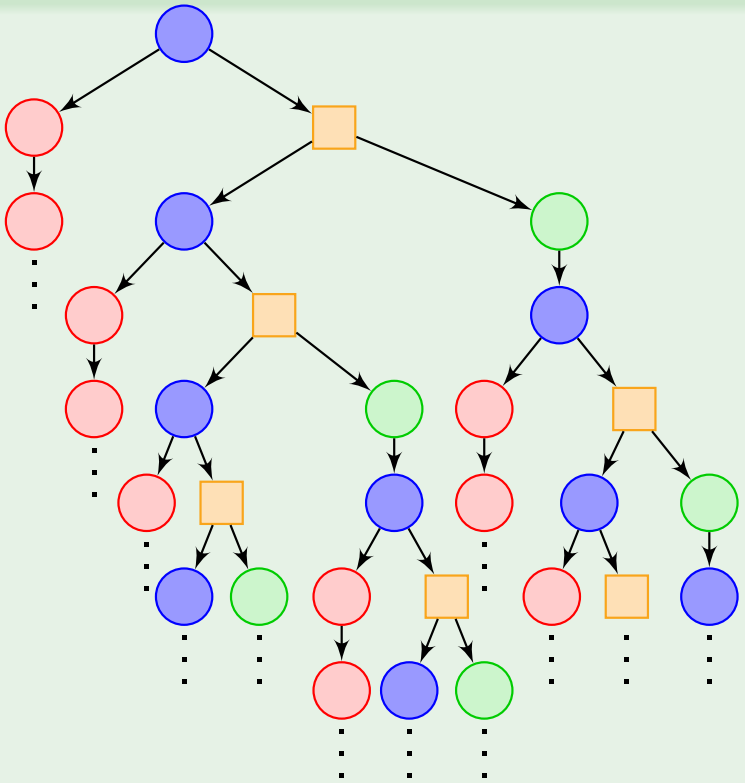

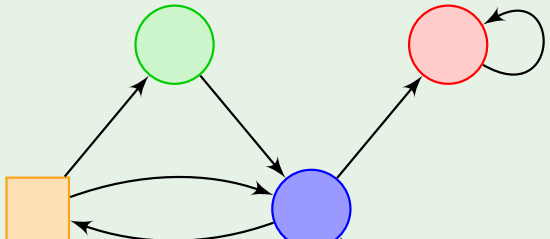

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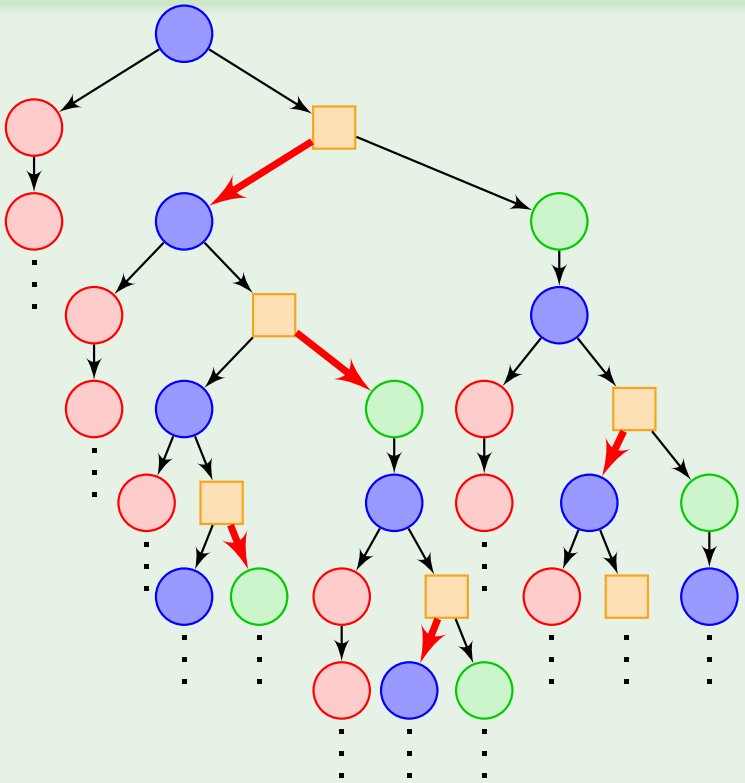

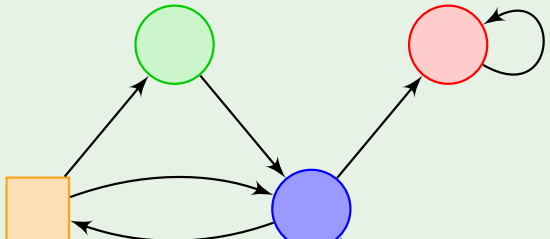

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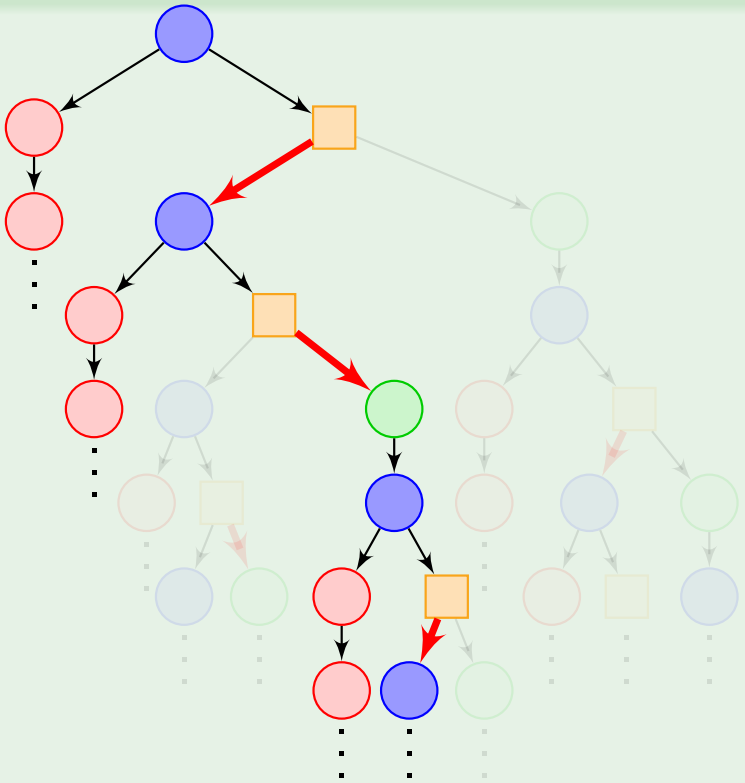

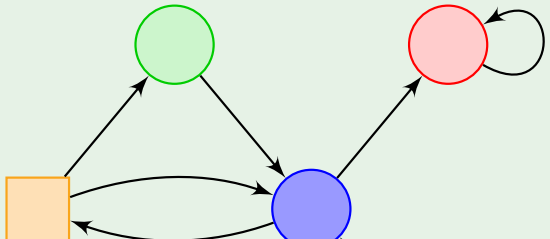

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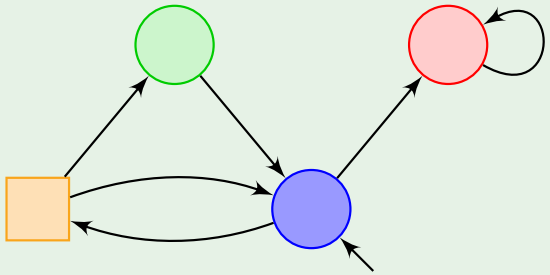
Memoryless strategy for player



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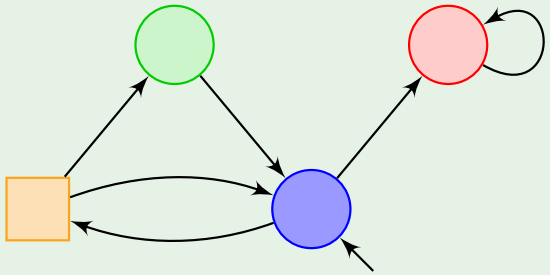

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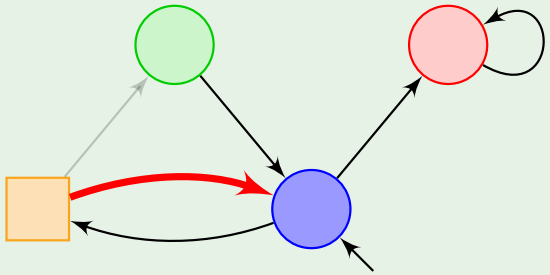

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Memoryless strategy for player ◻

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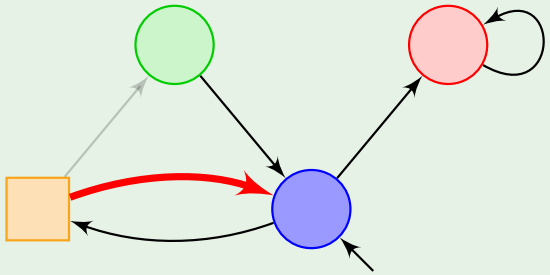

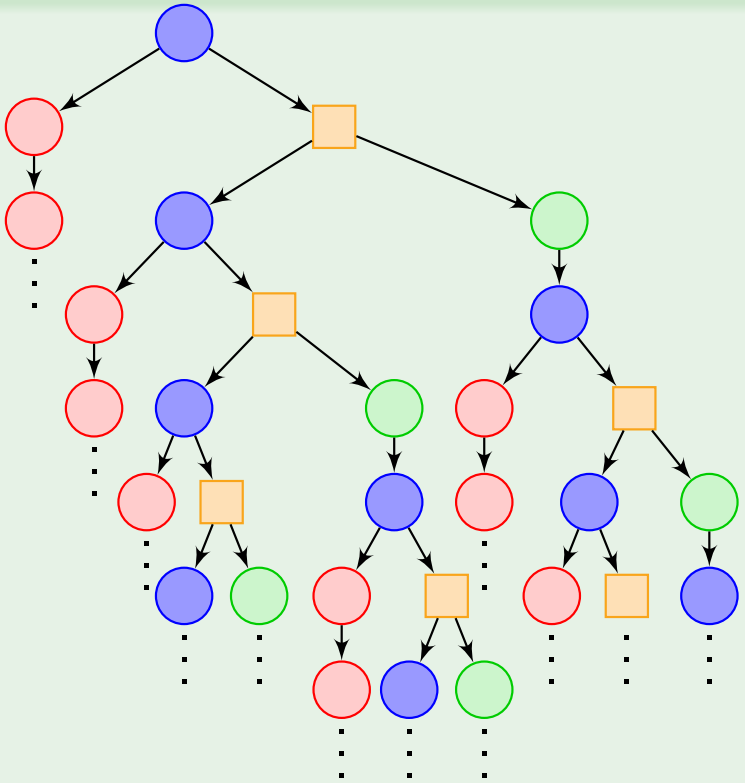

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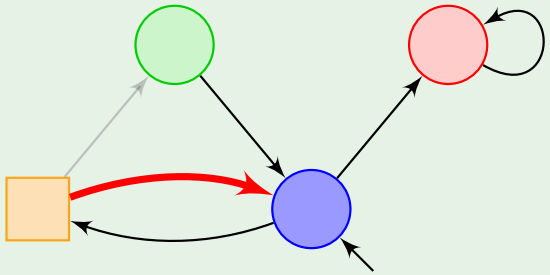

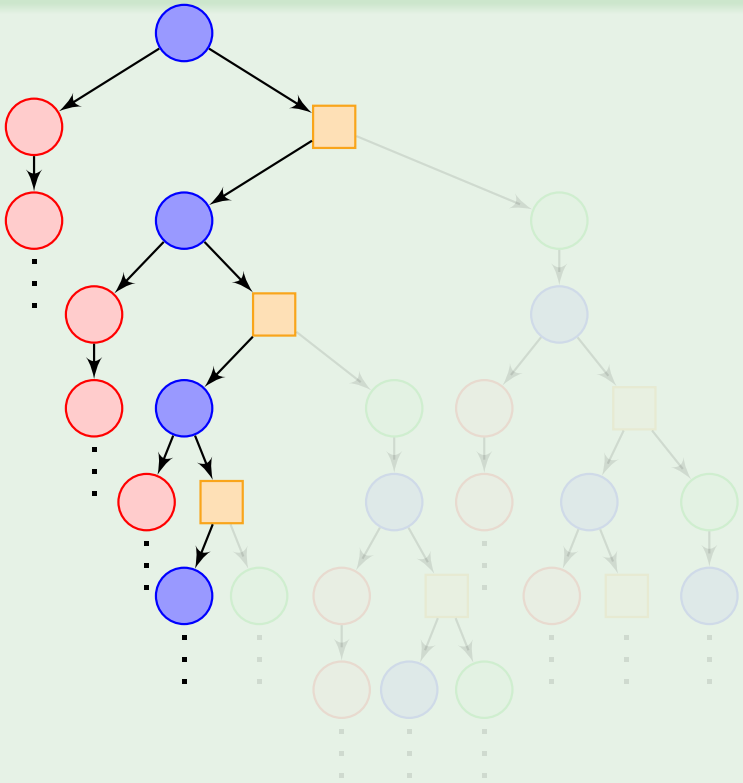

Temporal logics for games: ATL

ATL extends CTL with strategy quantifiers

$\langle\langle A \rangle\rangle \varphi$ expresses that A has a strategy to enforce φ.

[AHK02] Alur, Henzinger, Kupferman. Alternating-time Temporal Logic. J. ACM, 2002.

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Semantics of $\langle\langle A \rangle\rangle\, \varphi$

Existential quantification (over strategies) implicitly includes a universal quantification (over outcomes):

$$\mathcal{G}, \bigcirc \models \langle\langle A \rangle\rangle\, \varphi \quad \iff \quad \exists \sigma_A.\ \forall \pi \in \mathrm{Out}(\bigcirc, \sigma_A).\ \pi \models \varphi.$$

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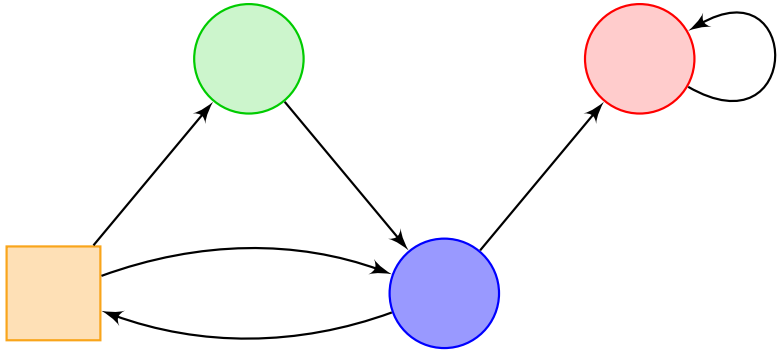

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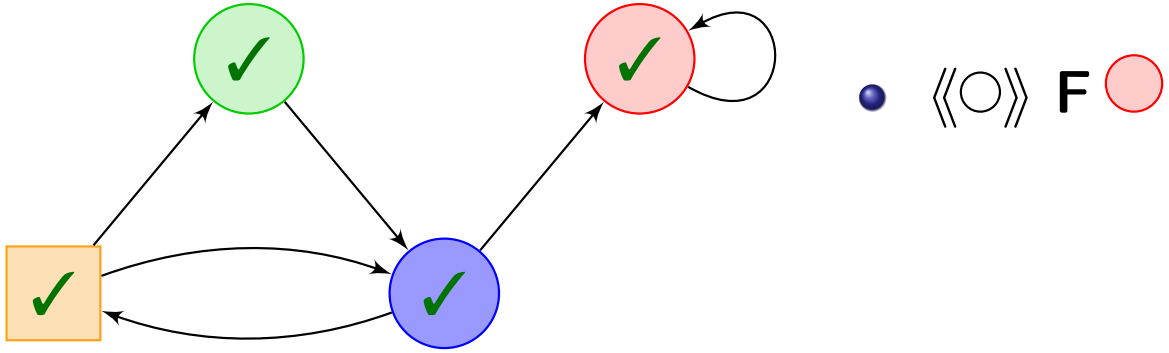

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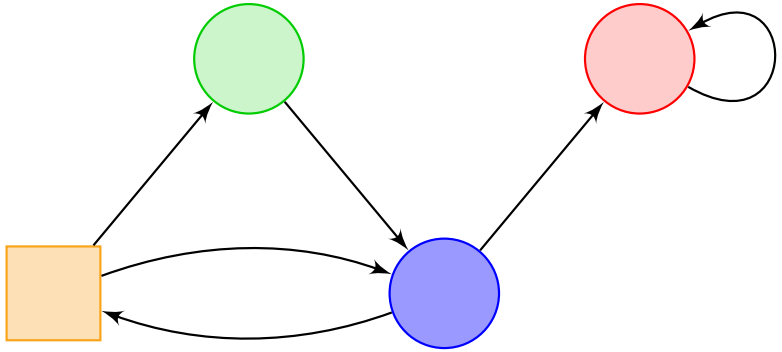

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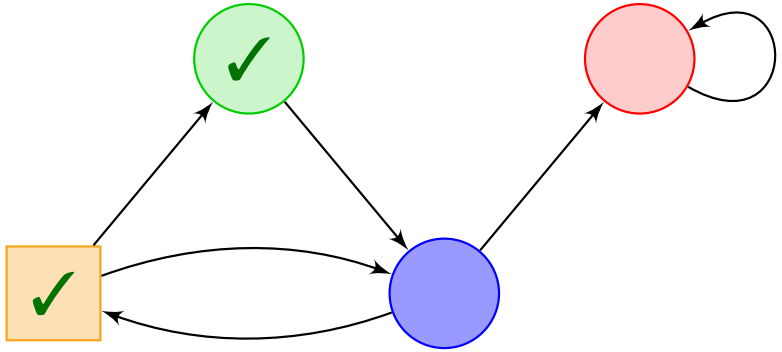

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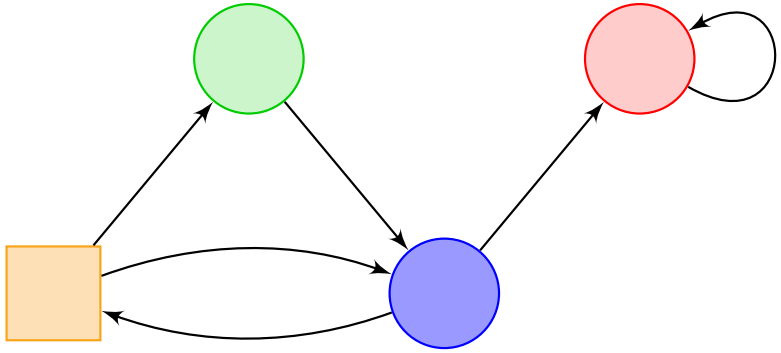

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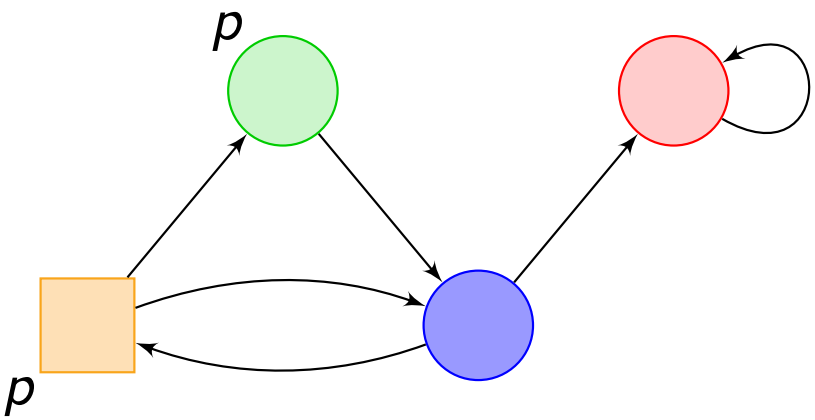

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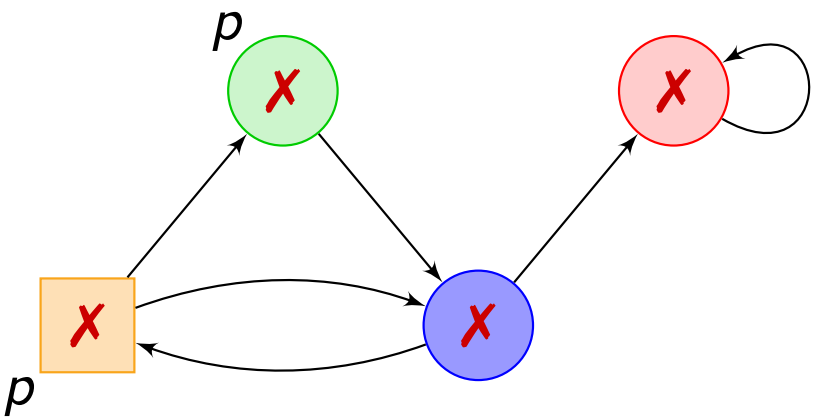

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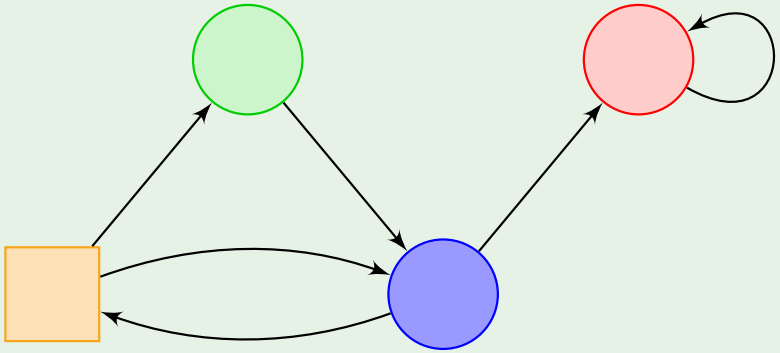

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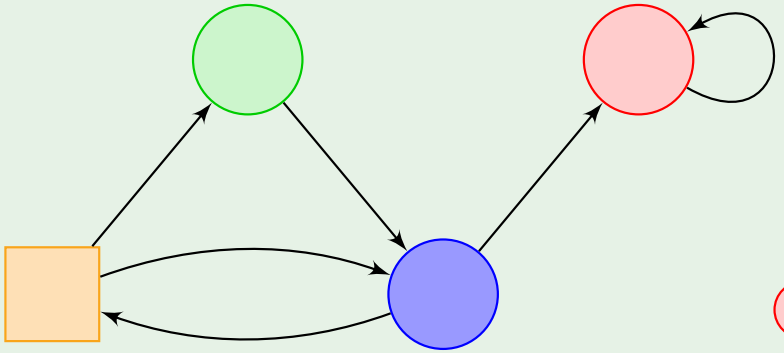

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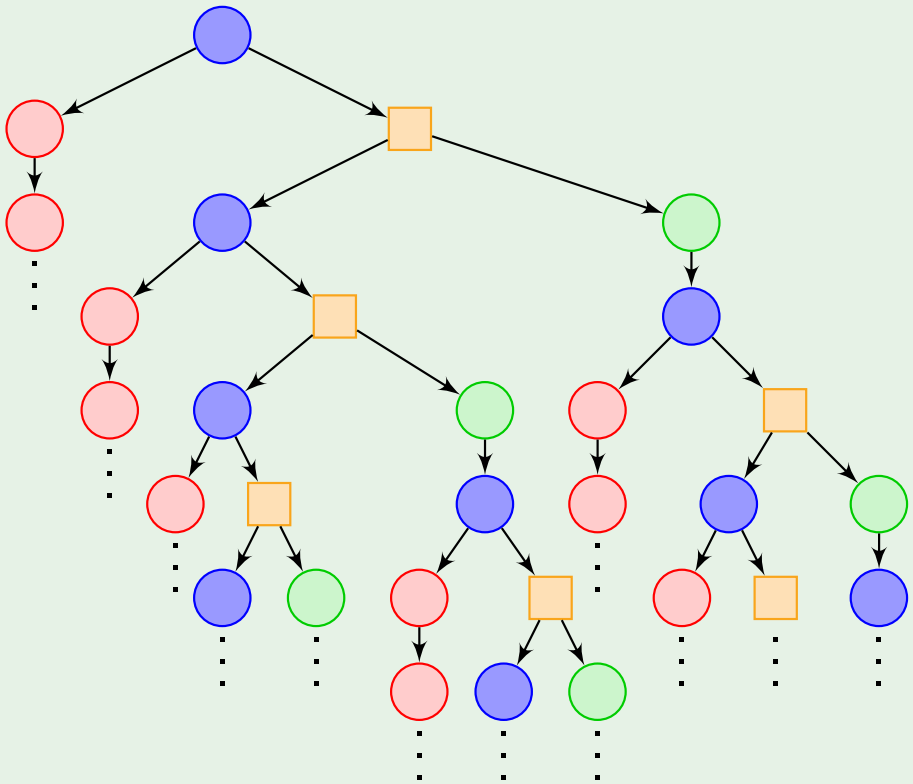

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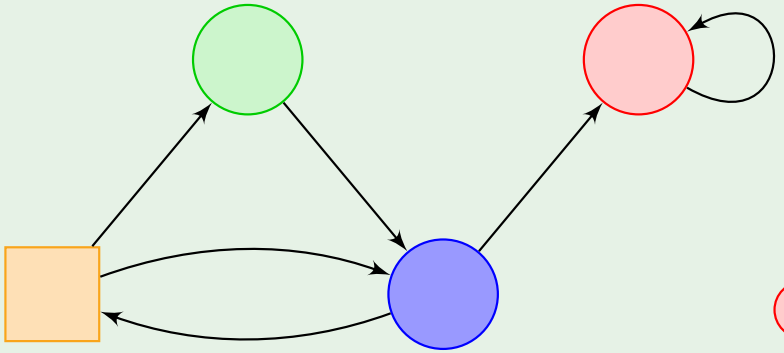

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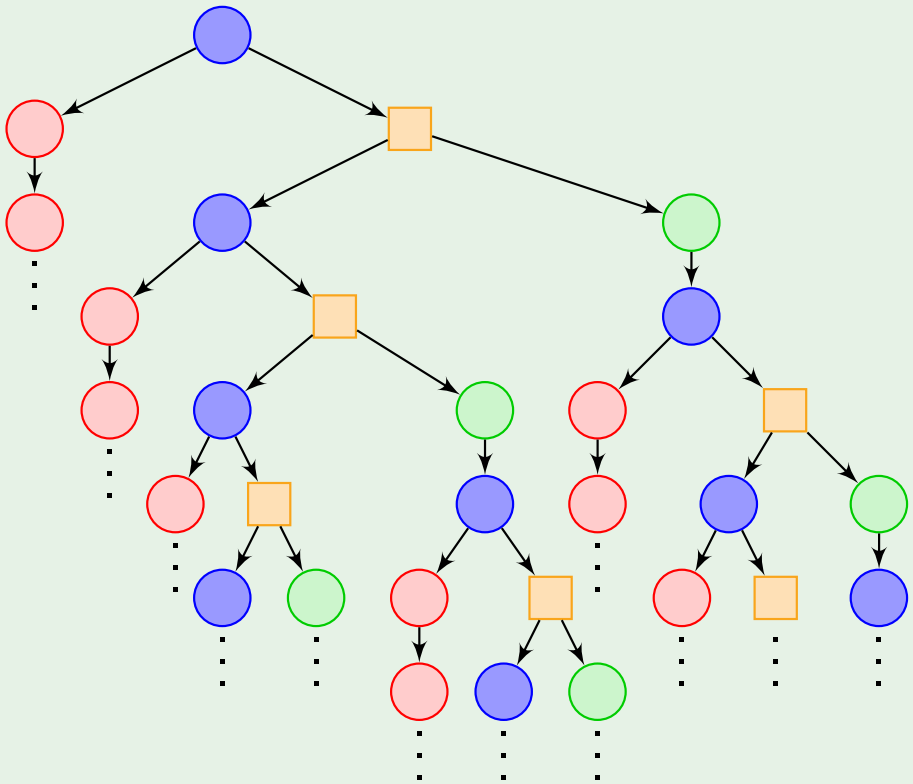

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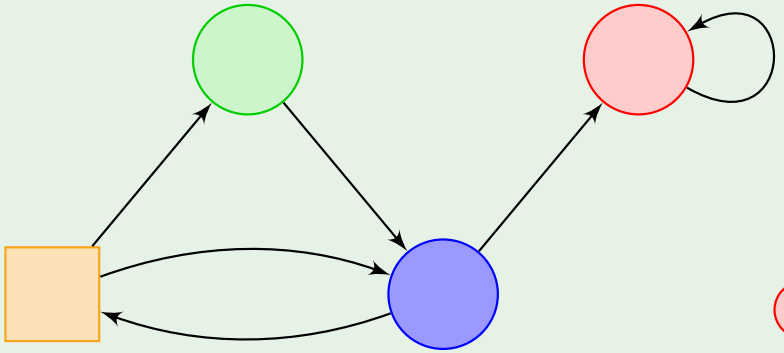

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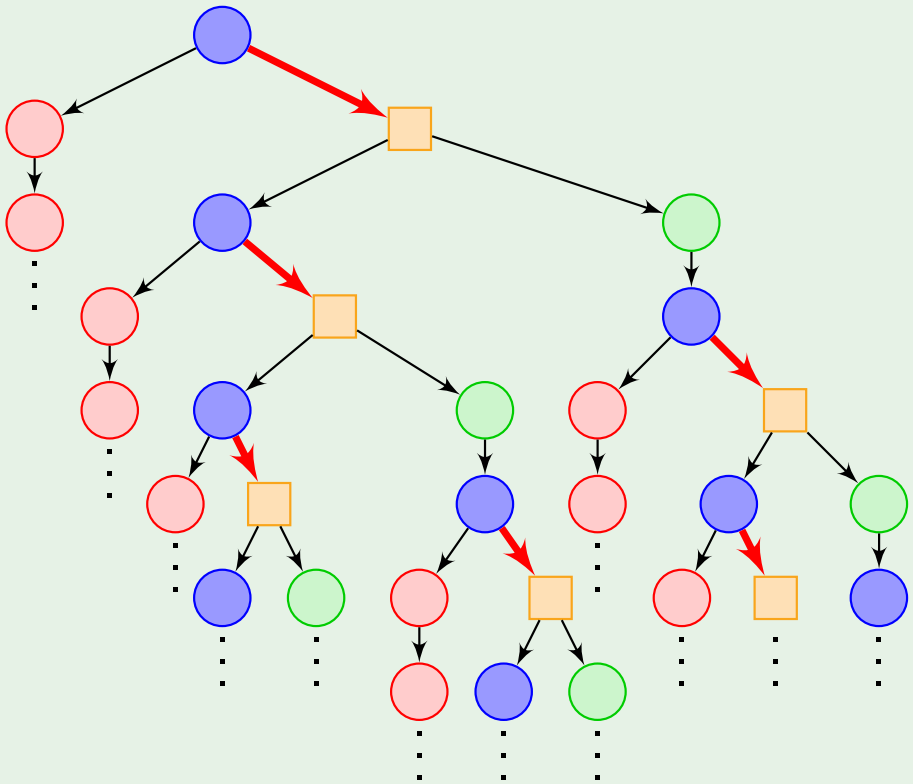

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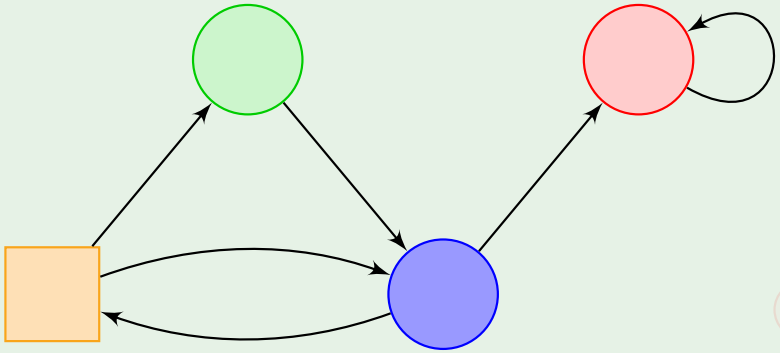

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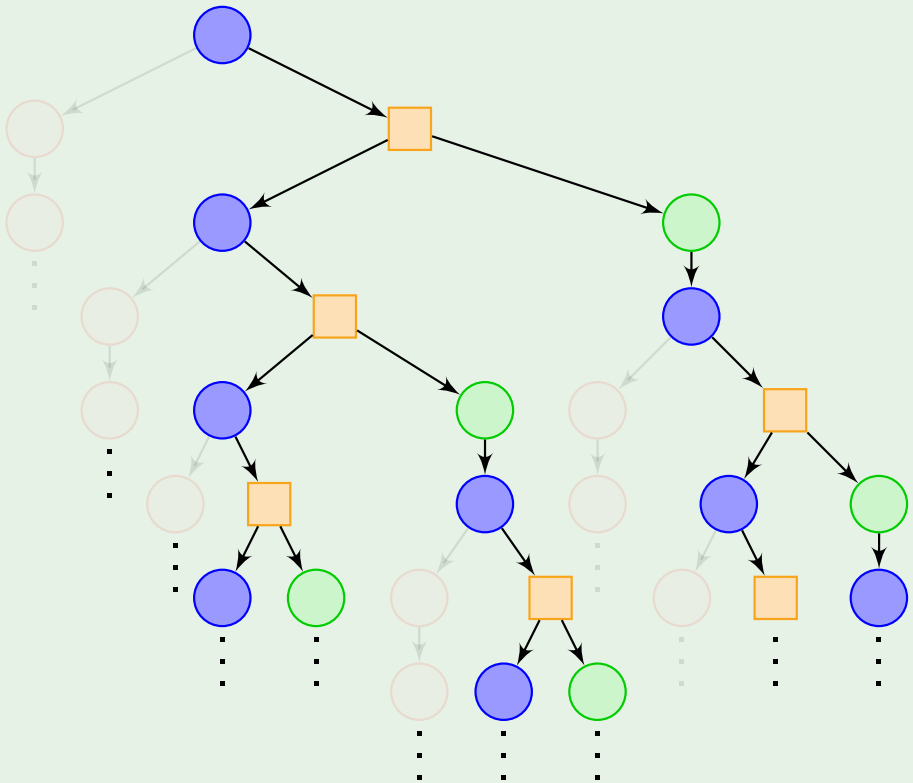

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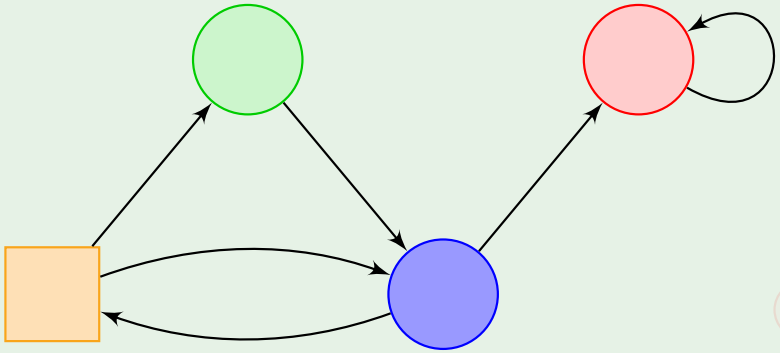

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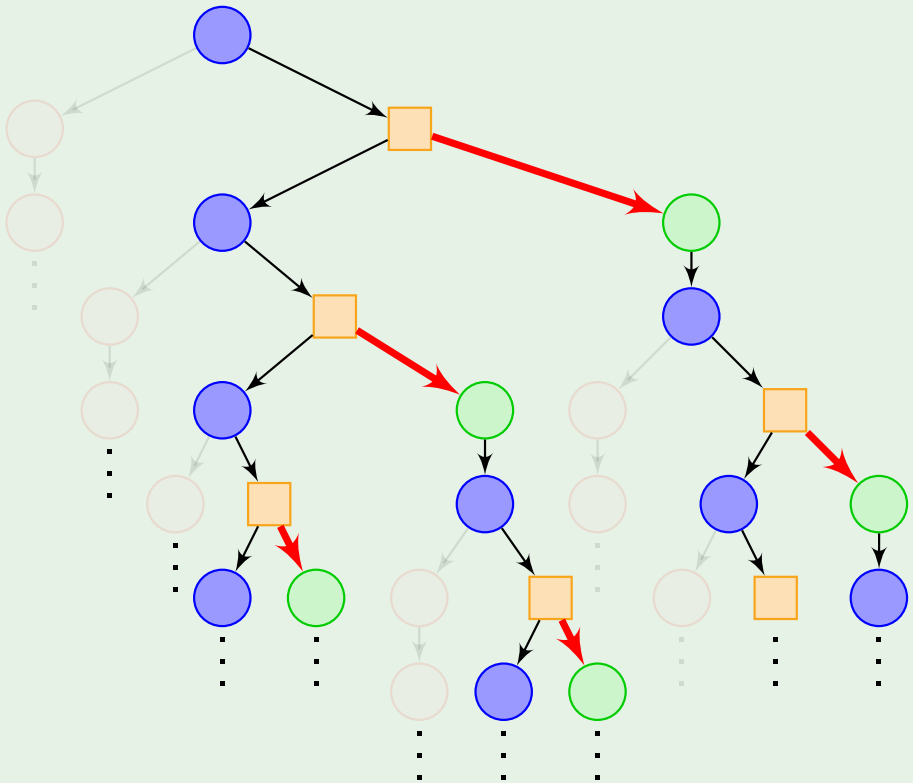

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ATL_{sc} has new strategy quantifiers:

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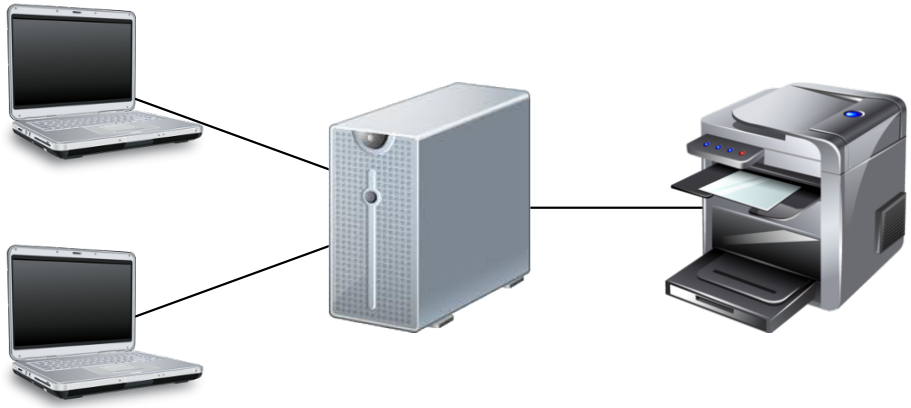
$\rightsquigarrow$ newly selected strategies added to the context:

$$\sigma_A \circ \sigma_B : \quad \begin{array}{ll} a \mapsto \sigma_A(a) & \text{if } a \in A \setminus B \\ b \mapsto \sigma_B(b) & \text{if } b \in B \setminus A \\ c \mapsto \sigma_A(c) & \text{if } c \in B \cap A \end{array}$$

What ATL_{sc} can express

- Client-server interactions for accessing a shared resource:

$$\langle\cdot \mathrm{Server} \cdot\rangle\ \mathbf{G} \left[\begin{array}{l} \bigwedge_{c \in \mathrm{Clients}} \langle\cdot c \cdot\rangle\ \mathbf{F}\ \mathrm{access}_c \\ \wedge \\ \neg \bigwedge_{c \neq c'} \mathrm{access}_c \wedge \mathrm{access}_{c'} \end{array} \right]$$



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Theorem

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$$\langle\langle A \rangle\rangle\, \varphi \equiv (\!|\overline{\emptyset}|\!)\ \langle\cdot A \cdot\rangle\, \hat{\varphi}$$

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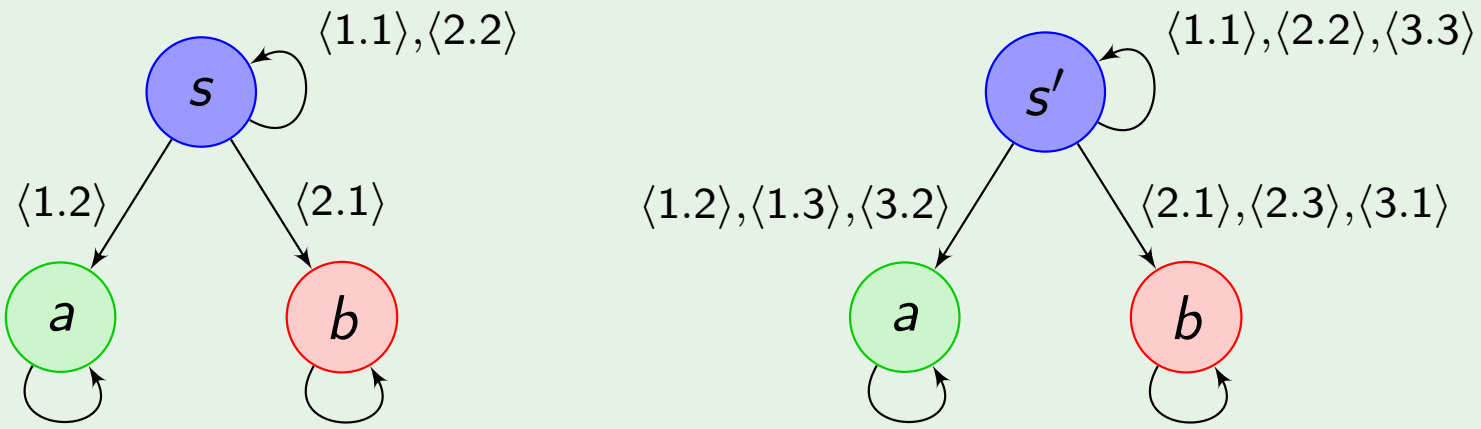
Proof

$\langle\cdot 1\cdot\rangle\,(\,\langle\cdot 2\cdot\rangle\ \mathbf{X}\ a \wedge\ \langle\cdot 2\cdot\rangle\ \mathbf{X}\ b)$ is only true in the second game.
But ATL cannot distinguish between these two games.



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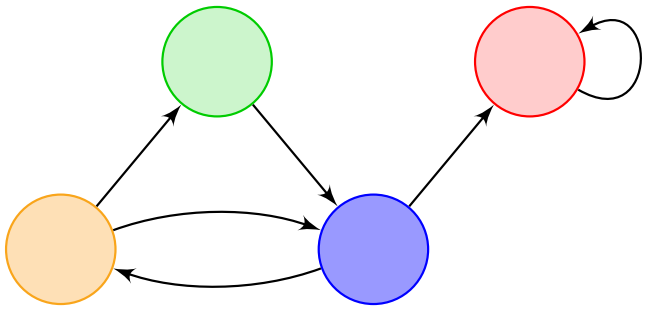

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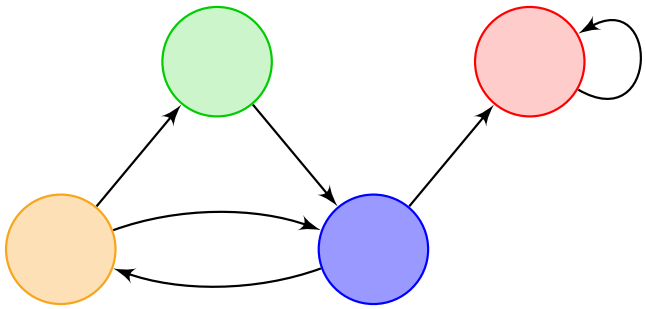

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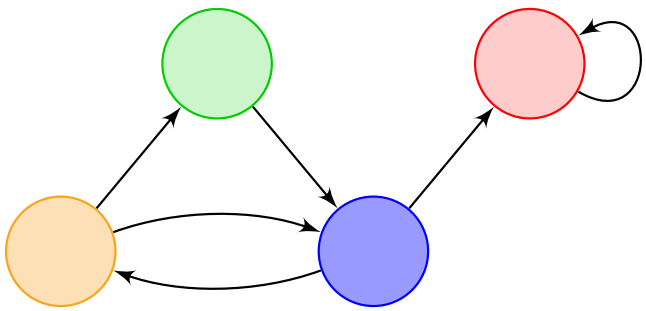

$\rightsquigarrow$ true if we label the Kripke structure;

$\rightsquigarrow$ false if we label the computation tree;

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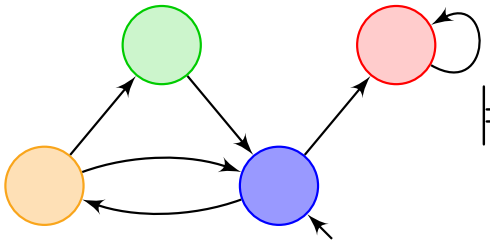
Semantics of QCTL

- structure semantics:



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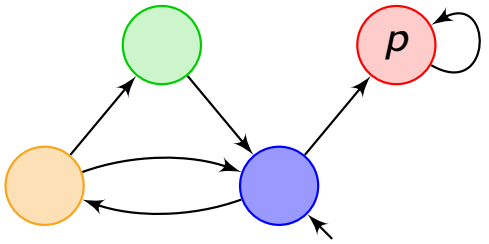

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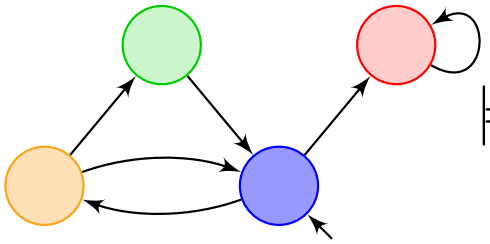

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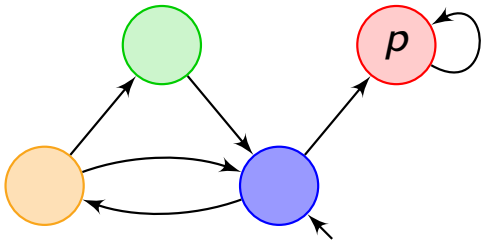
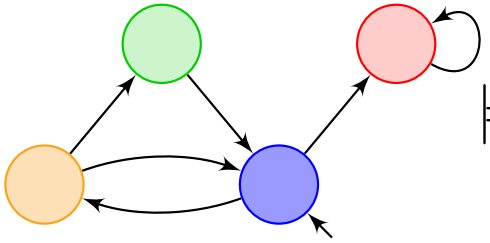


$\models \varphi$

- tree semantics:



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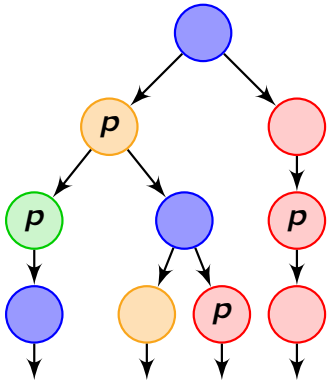

$\models \varphi$

Expressiveness of QCTL

- QCTL can "count":

$$\mathbf{E}\,\mathbf{X}_1\,\varphi \equiv \mathbf{E}\,\mathbf{X}\,\varphi \wedge \forall p.\ [\mathbf{E}\,\mathbf{X}(p \wedge \varphi) \Rightarrow \mathbf{A}\,\mathbf{X}(\varphi \Rightarrow p)]$$
$$\mathbf{E}\,\mathbf{X}_2\,\varphi \equiv \exists q.\ [\mathbf{E}\,\mathbf{X}_1(\varphi \wedge q) \wedge \mathbf{E}\,\mathbf{X}_1(\varphi \wedge \neg q)]$$

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Expressiveness of QCTL

- QCTL can "count":

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Theorem

QCTL, QCTL* and MSO are equally expressive (under both semantics).

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QCTL with structure semantics

Theorem

Model checking QCTL for the structure semantics is PSPACE-*complete.*

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Membership: labelling algorithm.

Iteratively
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Hardness:
QBF is a special case (without even using temporal modalities).

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QCTL satisfiability for the structure semantics is undecidable.

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- *Model checking QCTL with k quantifiers in the tree semantics is* k-EXPTIME-*complete.*
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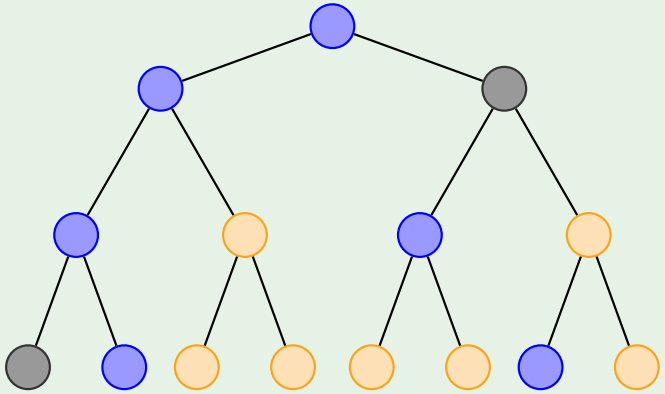

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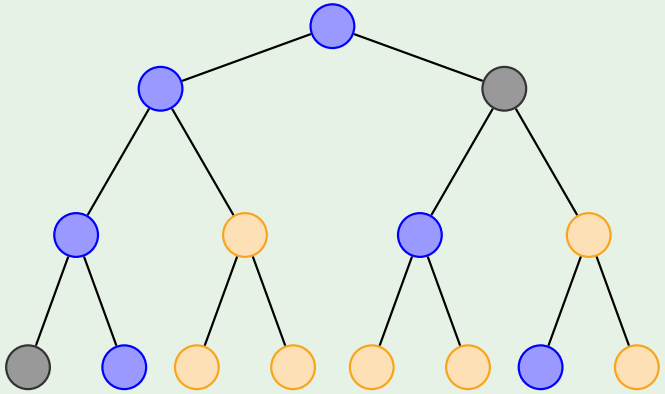

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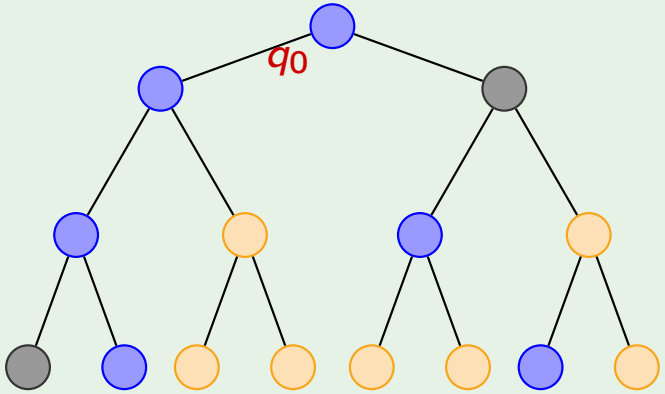

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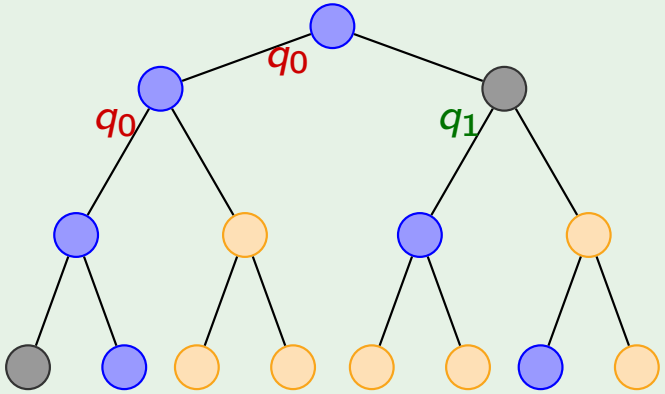

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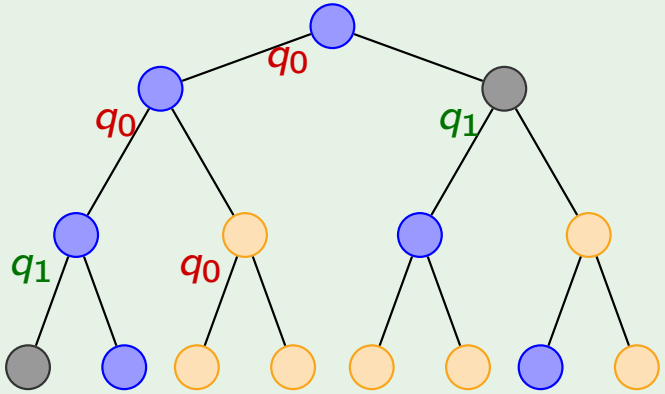

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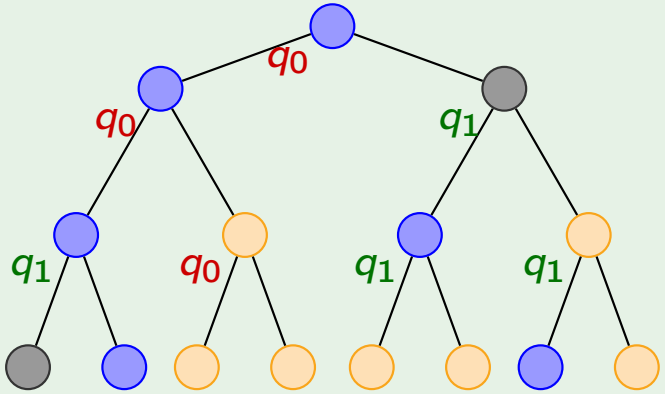

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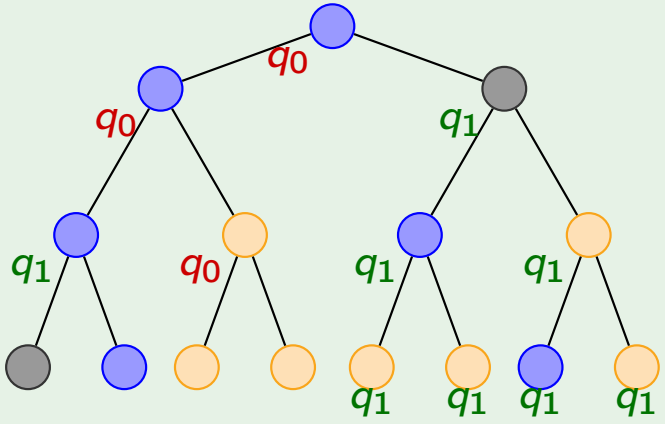

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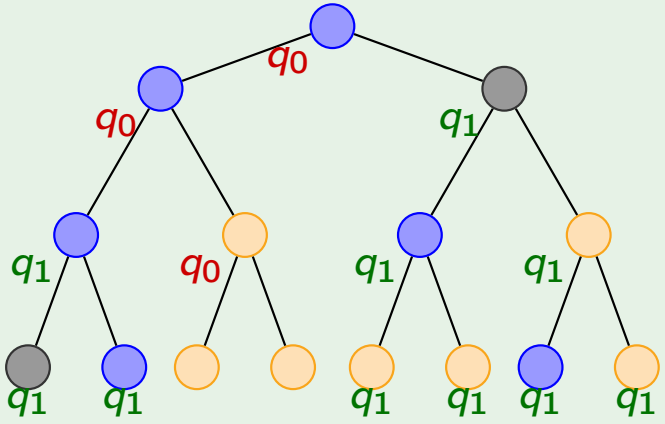

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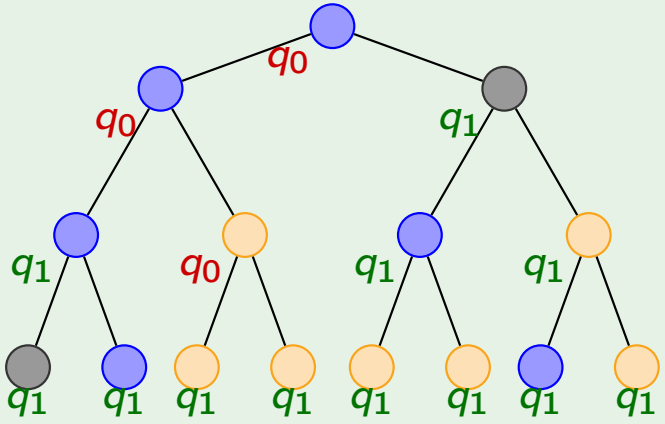

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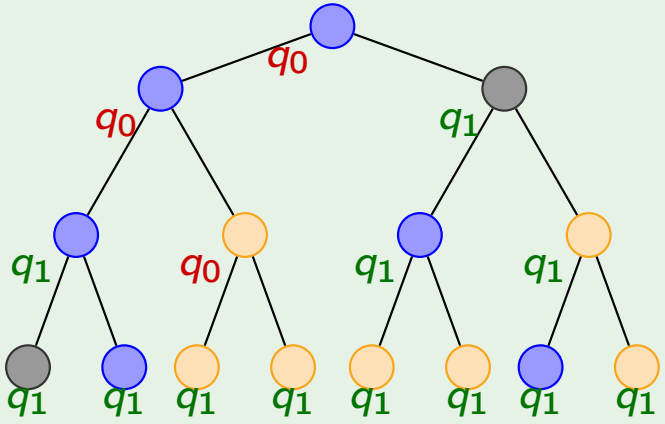

This automaton corresponds to **E** ● **U** ○

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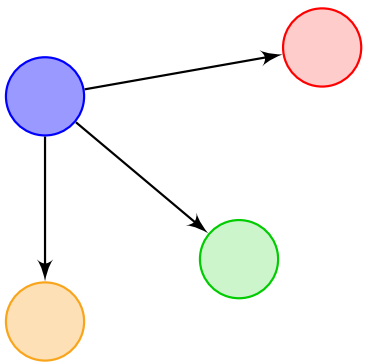
Proof

- polynomial-size tree automata for CTL;
- quantification is handled by projection, which first requires removing alternation (exponential blowup);
- an automaton equivalent to a QCTL formula can be built inductively;
- emptiness of an alternating parity tree automaton can be decided in exponential time.

Translating ATL_{sc} into QCTL



- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(, A, m_i^A) of states that can be reached from when player A plays m_i^A.

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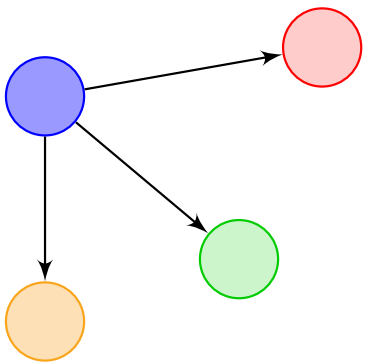

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$\langle \cdot A \cdot \rangle\, \varphi$ can be encoded as follows:

$\exists m_1^A.\ \exists m_2^A \ldots \exists m_n^A.$

- this corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(m_i^A \Leftrightarrow \bigwedge \neg\, m_j^A)$;
- the outcomes all satisfy φ:

$$\mathbf{A}\big[\mathbf{G}(q \wedge m_i^A \Rightarrow \mathbf{X}\,\mathrm{Next}(q, A, m_i^A)) \Rightarrow \varphi\big].$$

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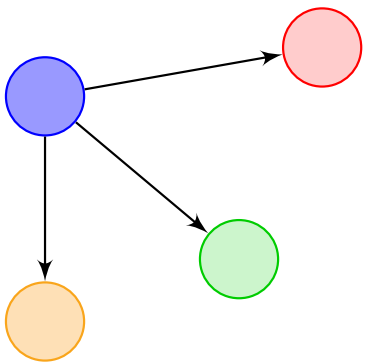

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Corollary

ATL_{sc} model checking is decidable, with non-elementary complexity.

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ATL_{sc}^0 (quantification restricted to memoryless strategies) model checking is PSPACE*-complete.*

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Hardness of model checking ATL_{sc}

Encode QLTL satisfiability

Example: $\Phi = \forall p_1.\ \exists p_2.\ \mathbf{G}(p_2 \iff \mathbf{X}\, p_1).$

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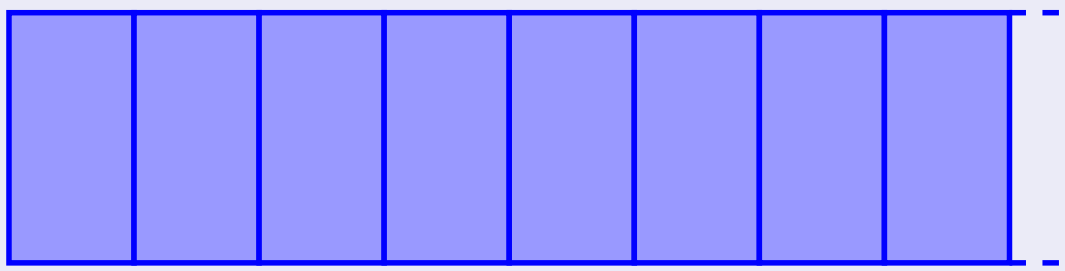

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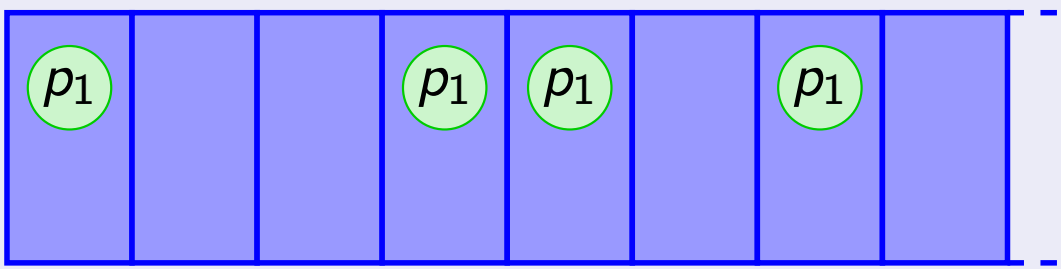

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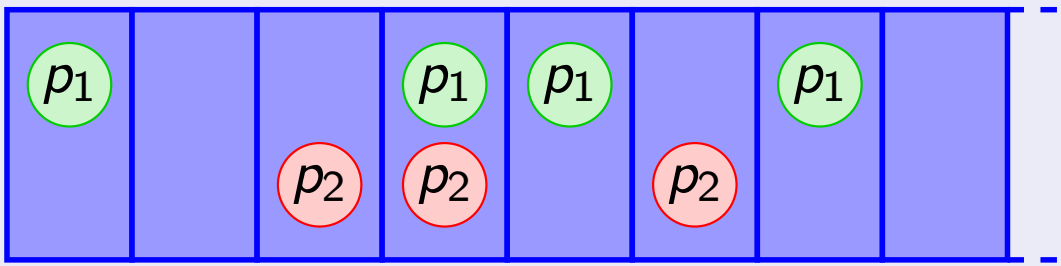

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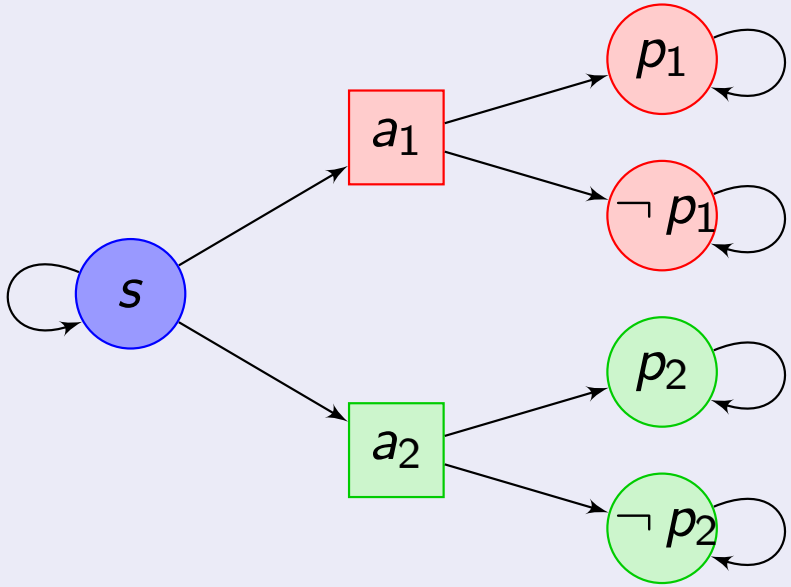

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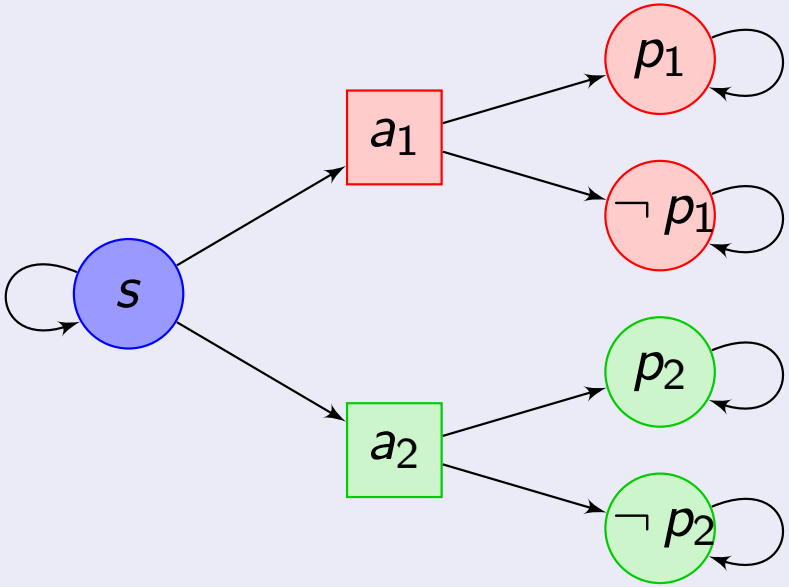

$$[\cdot\ \bigcirc\ \cdot]\ \langle\cdot\ \bigcirc\ \cdot\rangle\ \langle\cdot\ \bigcirc\ \cdot\rangle\ \mathbf{G}(\ s\ )\ \wedge$$

$$\mathbf{G}\Big(\big(\langle\cdot\ \bigcirc\ \cdot\rangle\ \mathbf{X}\,\mathbf{X}\ p_2\big) \iff \big(\mathbf{X}\ \langle\cdot\ \bigcirc\ \cdot\rangle\ \mathbf{X}\,\mathbf{X}\ p_1\big)\Big)$$

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ATL_{sc} satisfiability is undecidable.

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Why?

The translation from ATL_{sc} to QCTL assumes that the game structure is given!

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Satisfiability for turn-based games

Theorem ([LM13])

When restricted to turn-based games, ATL_{sc} satisfiability is decidable.

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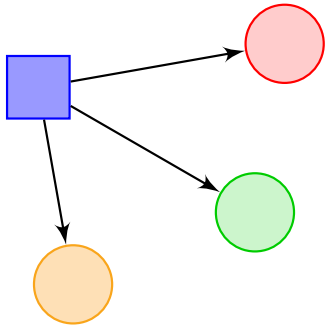

- player $\square$ has moves ●, ● and ●.
- a strategy can be encoded by marking some of the nodes of the tree with proposition mov_A.

$\langle\cdot A\cdot\rangle\,\varphi$ can be encoded as follows:

$\exists \mathrm{mov}_A.$

- it corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(\mathrm{turn}_A \Rightarrow \mathbf{E}\,\mathbf{X}_1\,\mathrm{mov}_A)$;
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Outline of the presentation

Strategy Logic [CHP07,MMV10]

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Explicit quantification and binding of strategies

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Strategy Logic [CHP07,MMV10]

Definition

Strategy Logic (SL) formulas are built using:

- strategy quantifications: $\exists\sigma.\ \psi$;
- strategy bindings: $\mathsf{bind}(A \mapsto \sigma).\ \varphi$;
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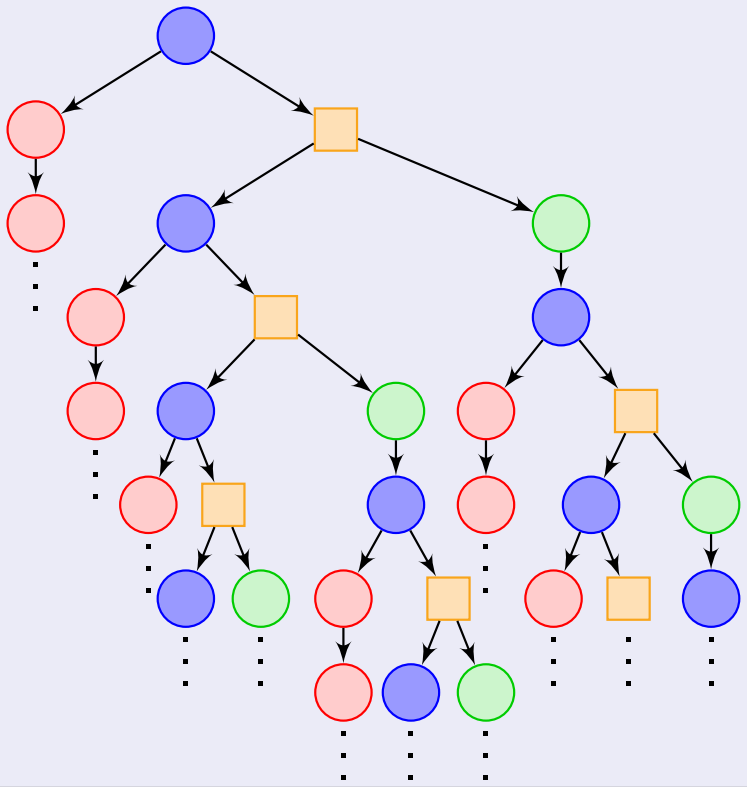
Semantics of SL [BGM16]

What does $\exists\sigma.\ \mathbf{A}\,\mathbf{G}(\mathsf{bind}(A \mapsto \sigma).\ \varphi)$ mean?



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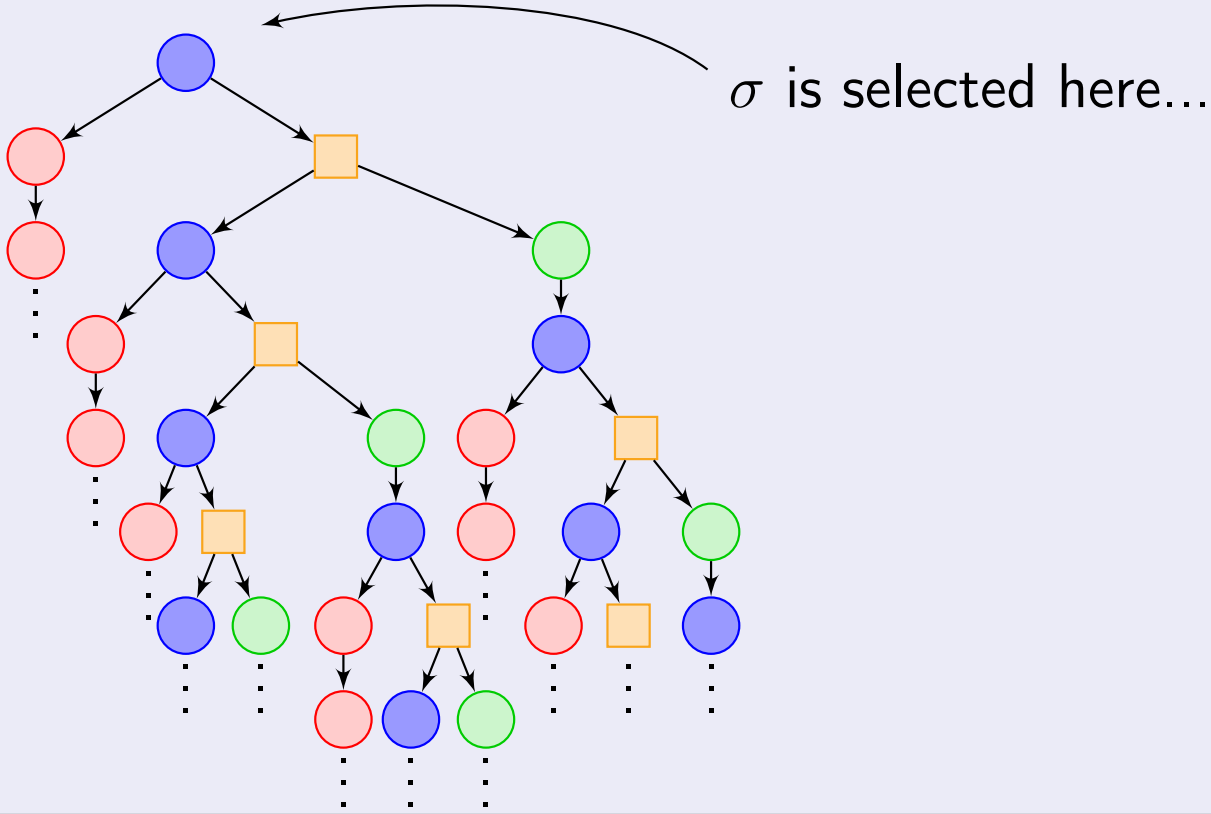

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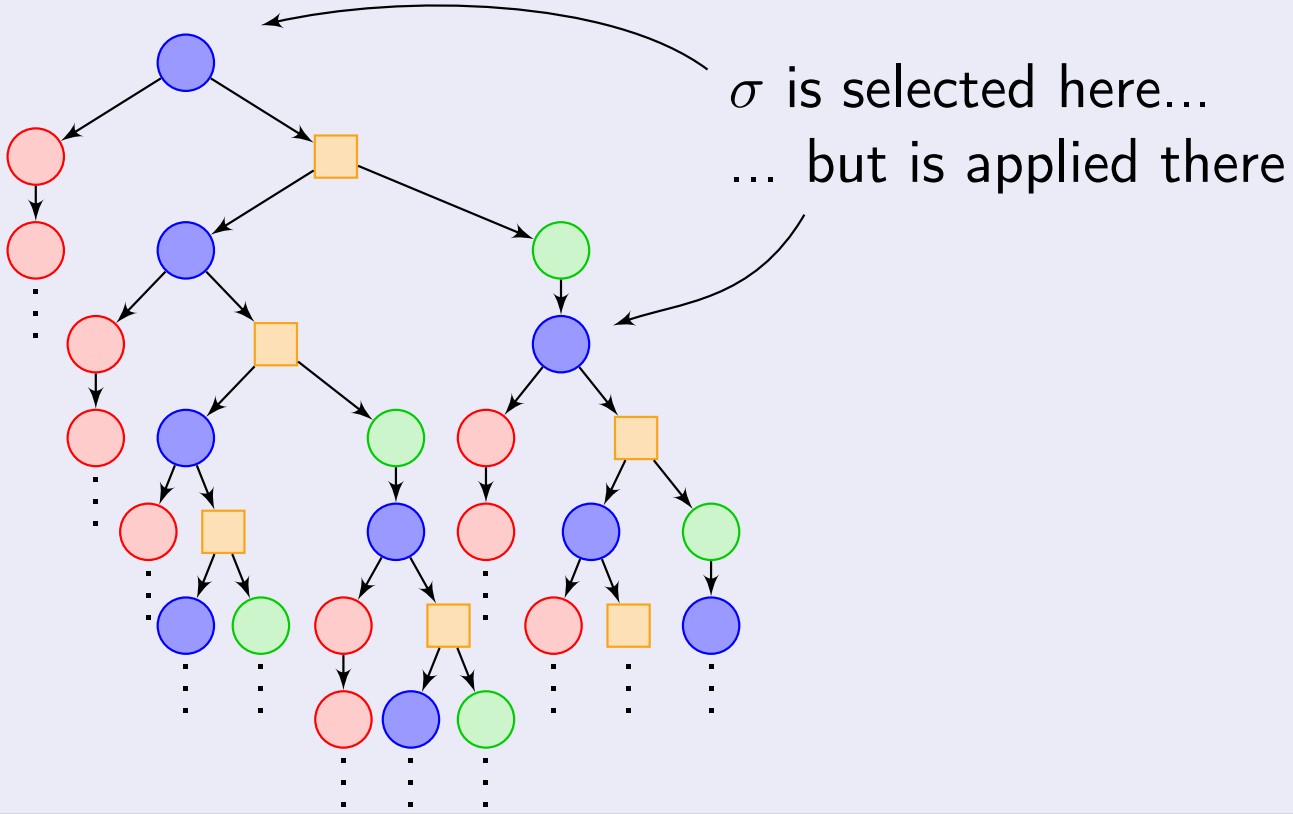

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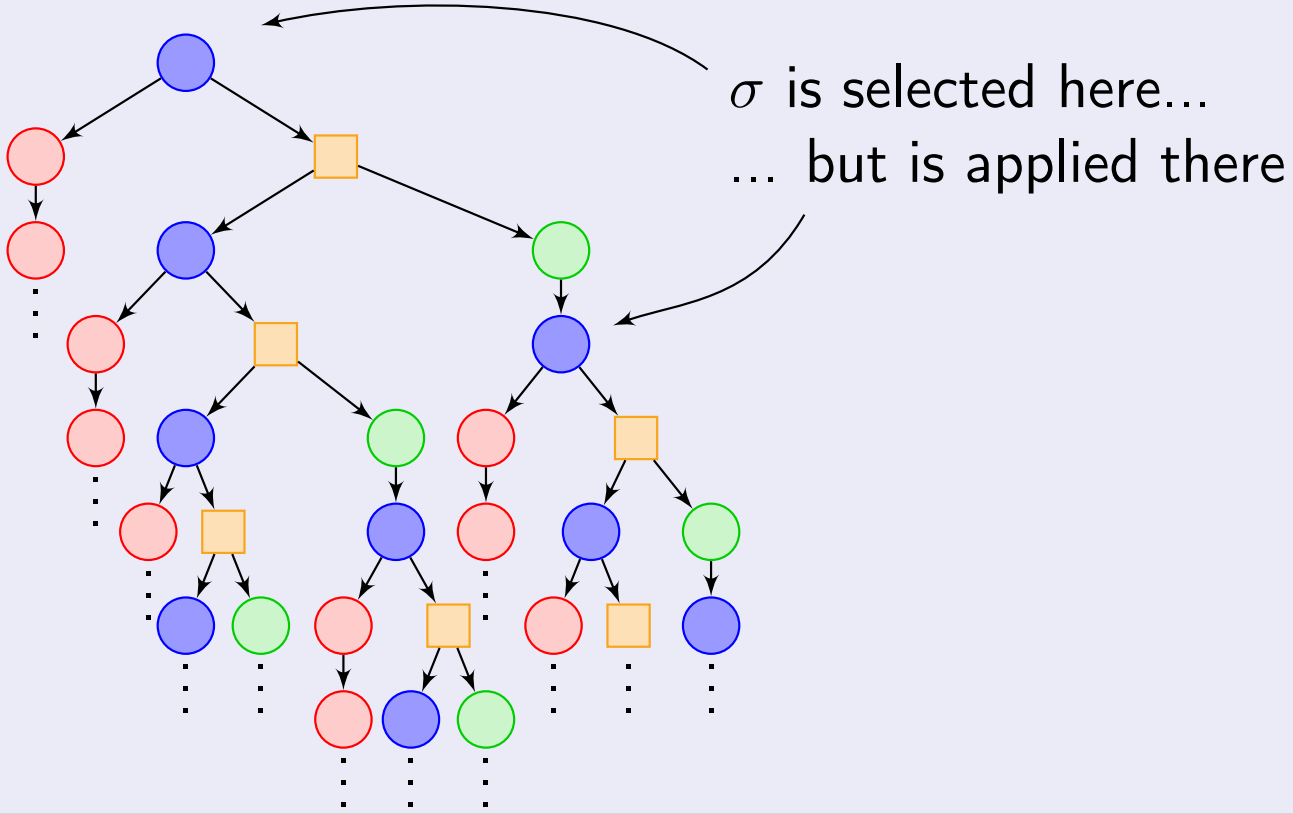

What is the history of σ when a player starts applying it?

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If history starts when selecting strategies...

Theorem

Strategy logic can be translated into QCTL.

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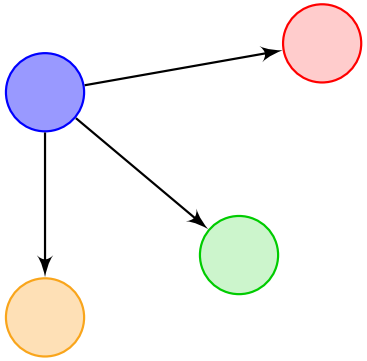

- players has moves m_1, ..., m_n;
- from the transition table, we can compute the set Next(, A, m_i) of states that can be reached from when player A plays m_i.

SL can be translated as follows:

- encoding of $\exists\sigma.\ \psi$:

$$\exists m_1^\sigma\ \exists m_2^\sigma \ldots \exists m_k^\sigma.\ \mathbf{A}\,\mathbf{G}(m_i^\sigma \Leftrightarrow \bigwedge \neg\, m_j^\sigma)$$

- encoding of $\varphi \in$ LTL (under full binding α: Agt $\to$ Strat):

$$\mathbf{A}\big[\mathbf{G}(q \wedge m_i^{\alpha(A)} \Rightarrow \mathbf{X}\,\mathrm{Next}(q, A, m_i^{\alpha(A)})) \Rightarrow \varphi\big]$$

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Theorem ([CHP07,MMV10,DLM12,LM13])

- *Strategy-logic model-checking is decidable.*
- *Strategy-logic satisfiability is decidable when restricted to turn-based games.*

[CHP07] Chatterjee, Henzinger, Piterman. Strategy Logic. CONCUR, 2007.
[MMV10] Mogavero, Murano, Vardi. Reasoning about strategies. FSTTCS, 2010.
[DLM12] Da Costa, Laroussinie, Markey. Quantified CTL: ... CONCUR, 2012.
[LM13] Laroussinie, Markey. Satisfiability of ATL with strategy contexts. Gandalf, 2013.

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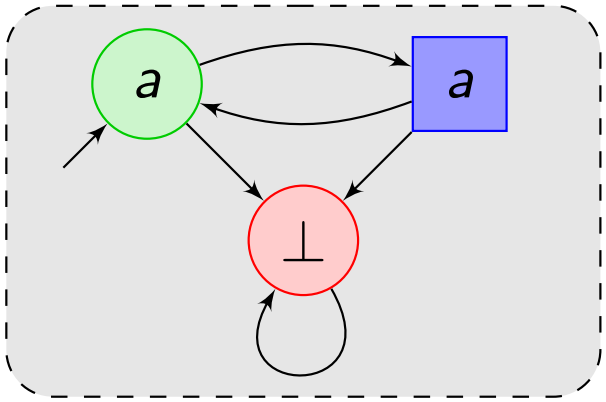

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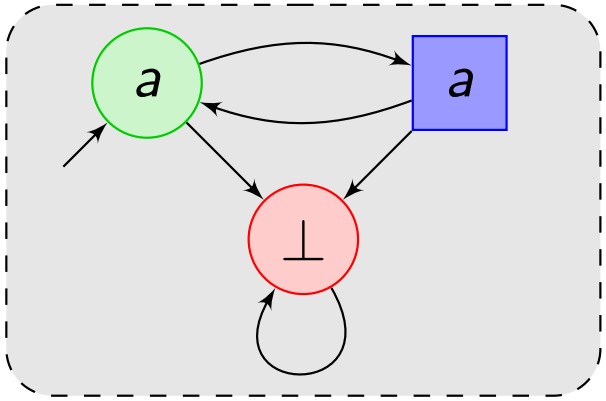

Strategies for ◯ and ▢ characterized by the integer representing the first time they play to ⊥.

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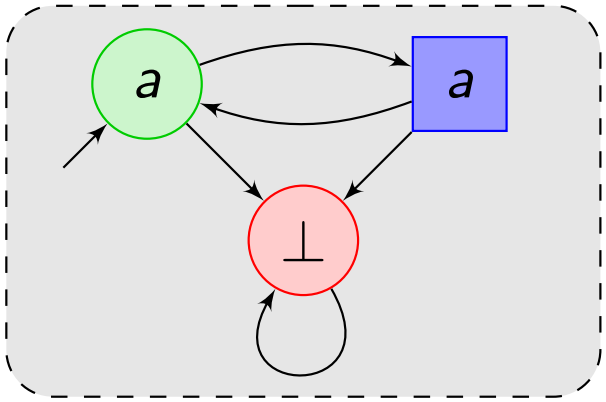

Strategies for ◯ and ▢ characterized by the integer representing the first time they play to ⊥.

Checking that two strategies $\sigma_{\bigcirc}$ and $\sigma_{\square}$ represent the same integer:

$$\mathbf{G}(\boxed{a} \Rightarrow \mathbf{X}\,\textcircled{a}) \wedge \mathbf{F}\left[(\textcircled{a} \wedge \mathbf{X}\,\textcircled{\bot}) \wedge [\cdot\bigcirc\cdot]\,\mathbf{X}\,\mathbf{X}\,\textcircled{\bot}\right] \qquad (\varphi_{=})$$

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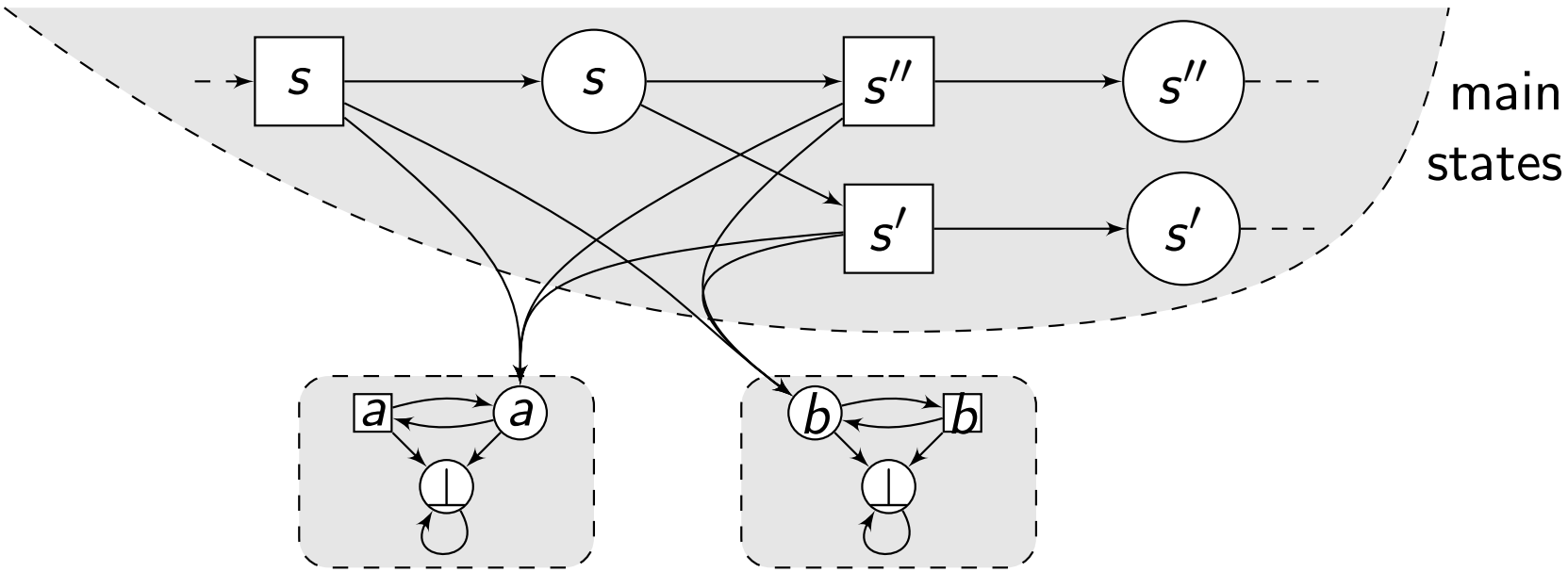

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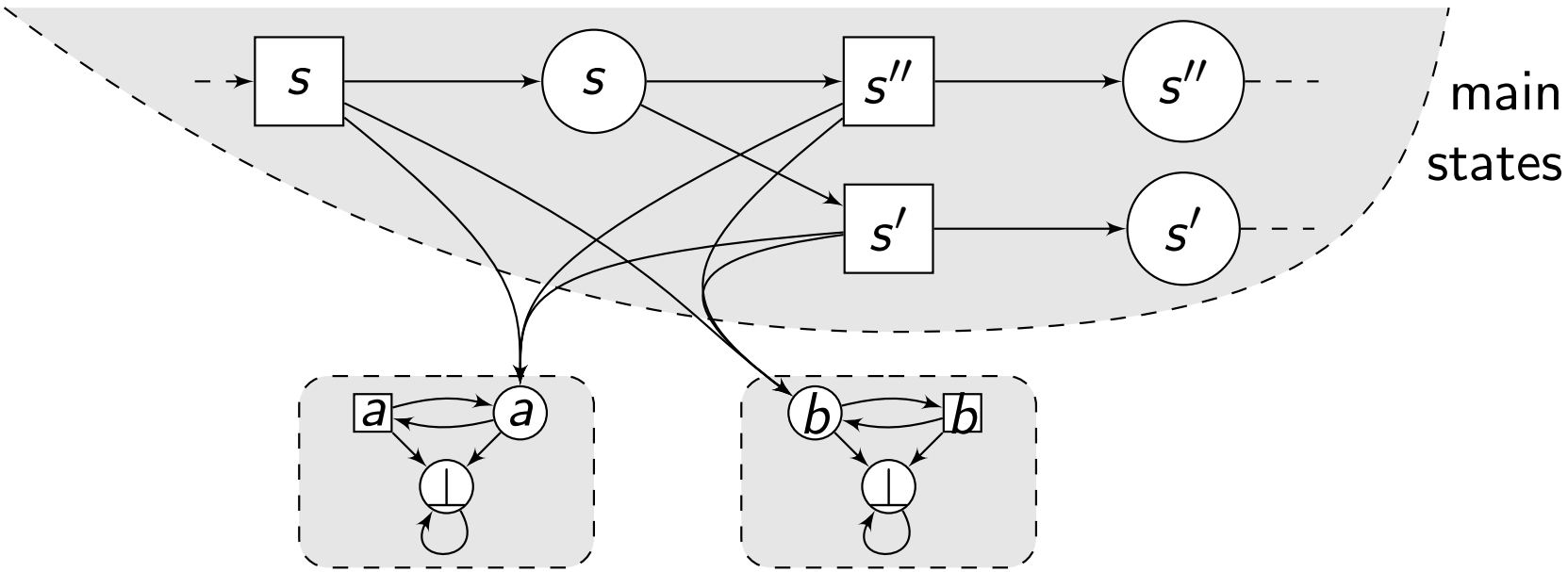
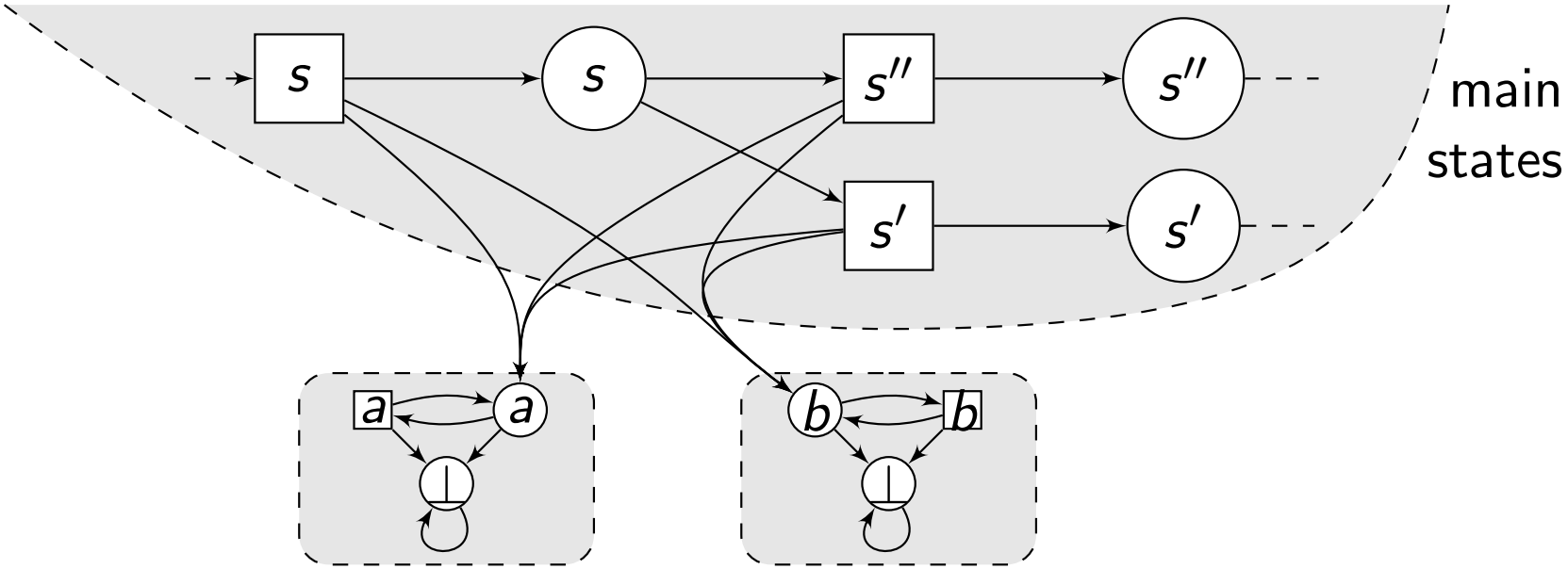


Encode run of a deterministic 2-counter machine $\mathcal{M}$:

- Player ○ plays a strategy that mimics the run of $\mathcal{M}$;
- Player ■ checks validity of simulation.

If history starts when applying strategies...



Encode run of a deterministic 2-counter machine $\mathcal{M}$:

- `s: if c==0 then goto s' else goto s''`

$$[\cdot\blacksquare\cdot]\ \mathbf{G}\left[\bigwedge_{\substack{s \text{ s.t.} \\ \delta(s)=(c,s',s'')}} \boxed{s} \Rightarrow \Big(\big[\langle\cdot\blacksquare\cdot\rangle\,(\mathbf{X}\,c \wedge \mathbf{X}\,\mathbf{X}\,\bot)\big] \Leftrightarrow \big(\langle\cdot\blacksquare\cdot\rangle\ \mathbf{X}\,\mathbf{X}\,\boxed{s'}\big)\Big)\right]$$

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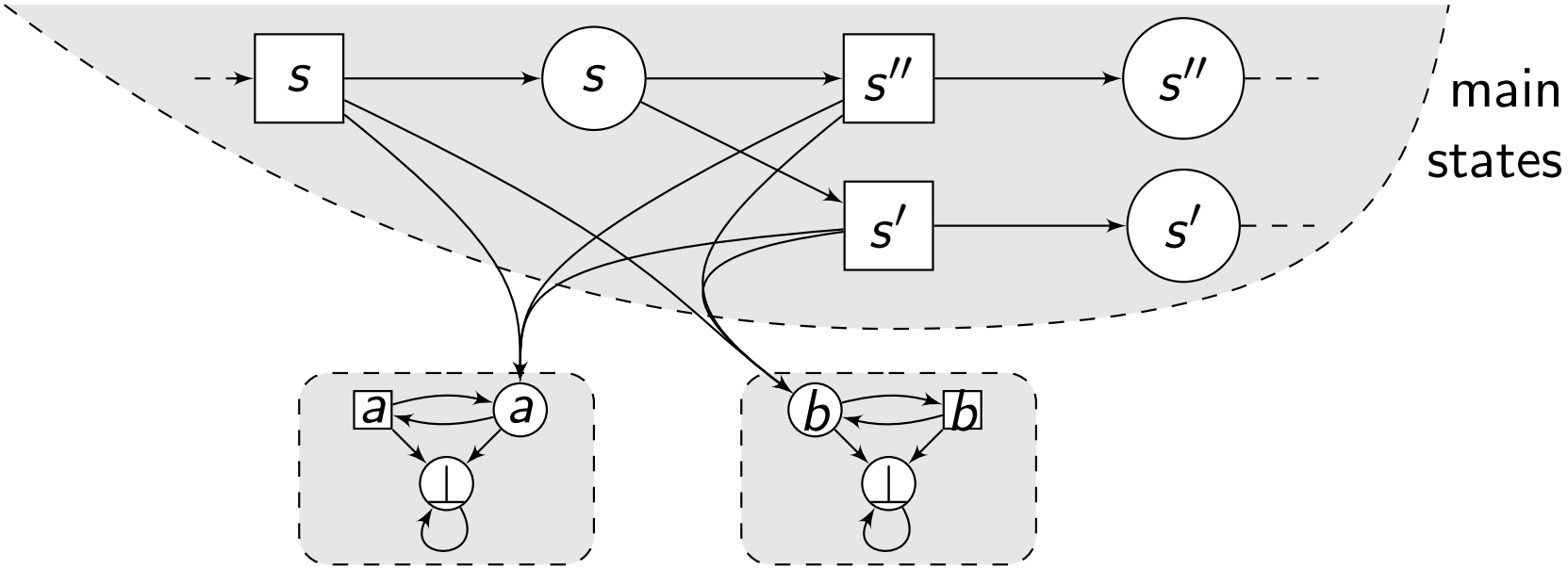

Encode run of a deterministic 2-counter machine $\mathcal{M}$:

- `s:  if c==0 then goto s' else goto s''`
- `s:  c++; goto s'`

$$[\cdot\blacksquare\cdot]\ \mathbf{G}\Big[\boxed{s} \Rightarrow \exists\sigma^{\text{count}}_{\blacksquare}.\ \langle\cdot\blacksquare\cdot\rangle\ \mathbf{X}(\textcircled{c} \wedge \text{bind}(\blacksquare \mapsto \sigma^{\text{count}}_{\blacksquare}).\ \varphi_{=}) \wedge$$
$$\langle\cdot\blacksquare\cdot\rangle\ \mathbf{X}\,\mathbf{X}(\boxed{s'} \wedge \mathbf{X}(\textcircled{c} \wedge \text{bind}(\blacksquare \mapsto \sigma^{\text{count}}_{\blacksquare}).\ \varphi_{+1}))$$

Conclusions and future works

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- ATL_{sc} is a very expressive, yet decidable extension of ATL;
- QCTL is a powerful extension of CTL;
- it is a nice tool to understand temporal logics for games (ATL_{sc}, Strategy Logic, ...);

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Future directions

- Defining and studying symmetric automata for QCTL;
- Defining interesting fragments of those logics;
- Considering partial observation;
- Considering randomised strategies.

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