

Hardware implementation of the arithmetic of fields of characteristic 2 and 3

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Introduction

- ▶ Curve-based cryptography relies on finite fields
 - Among them: small characteristic fields (2 and 3)
 - ★ Example: pairings on supersingular elliptic curves
- ▶ Need for hardware implementations
 - Embedded systems (RFID, smart card, sensors, ...)
 - High-performance cryptographic coprocessor (bank servers, ...)
 - ★ No native support in CPUs (Partial support in Intel AVX instruction set)

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- ▶ Reconfigurable hardware (FPGA)
 - great prototyping platform
 - flexibility when increasing security is needed

Introduction

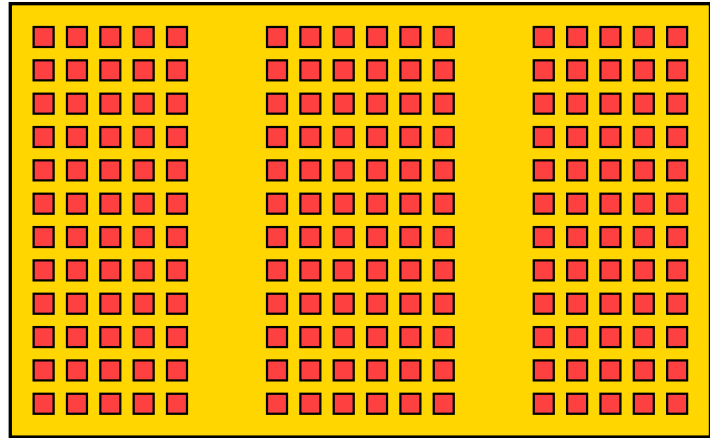
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 - flexibility when increasing security is needed
- ▶ Join work with:
 - Jean-Luc Beuchat, LCIS, University of Tsukuba, Japan.
 - Jérémie Detrey, CAMEL project-team, INRIA Nancy Grand-Est, France.

FPGA architecture

- ▶ FPGAs are composed of:

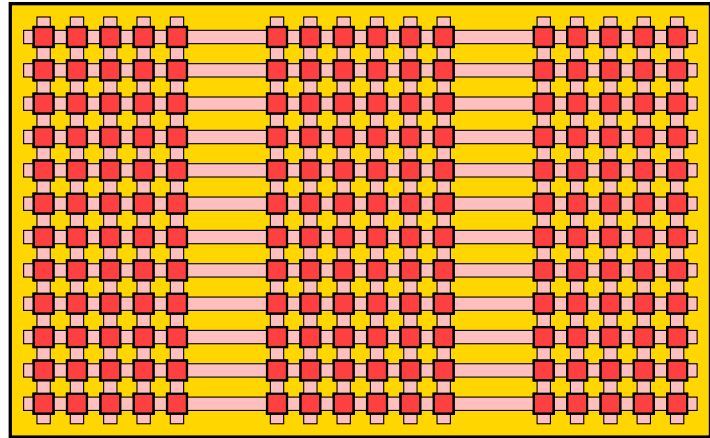
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 - programmable logic cells



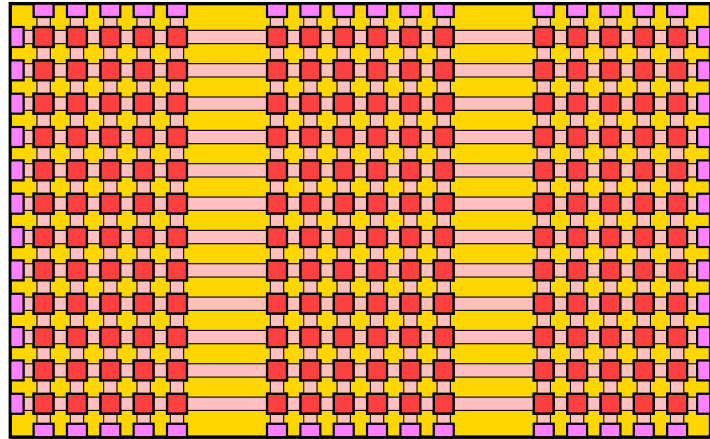
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 - a configurable routing matrix



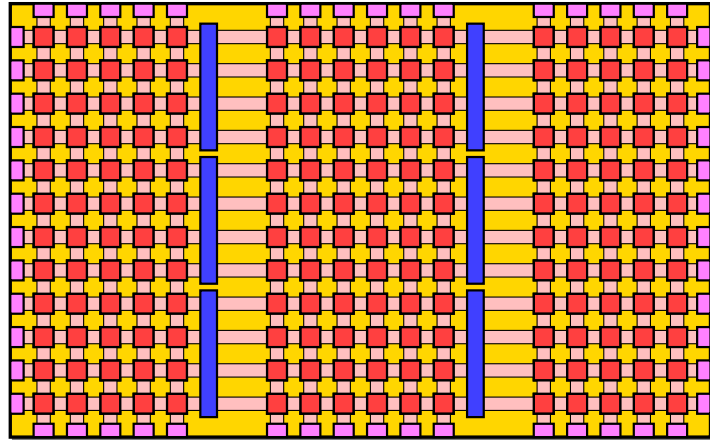
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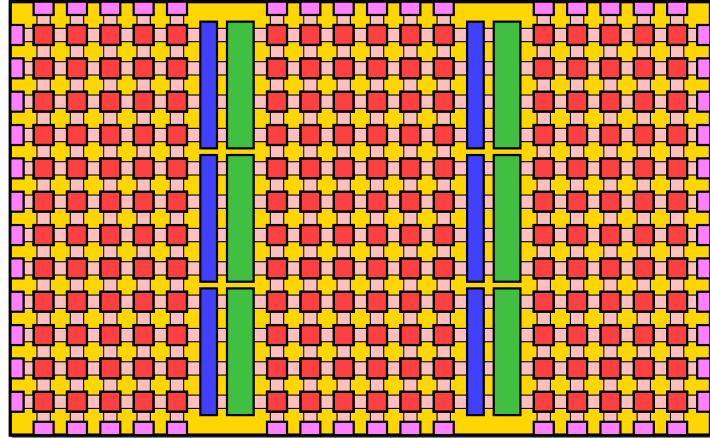
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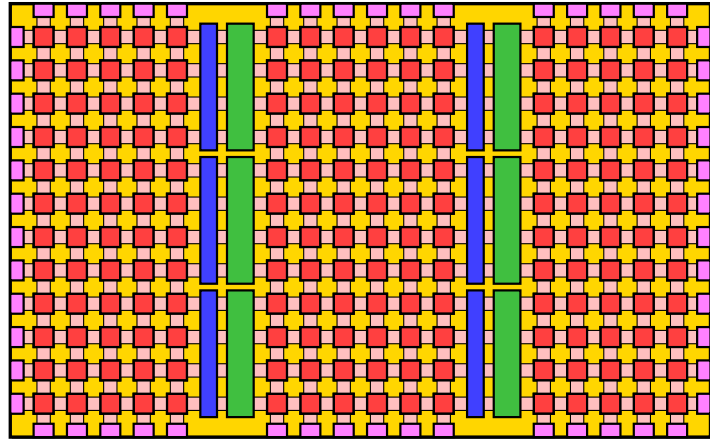
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 - small embedded multipliers



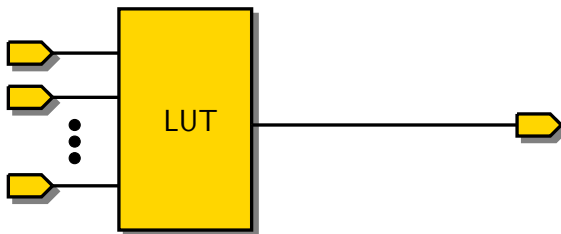
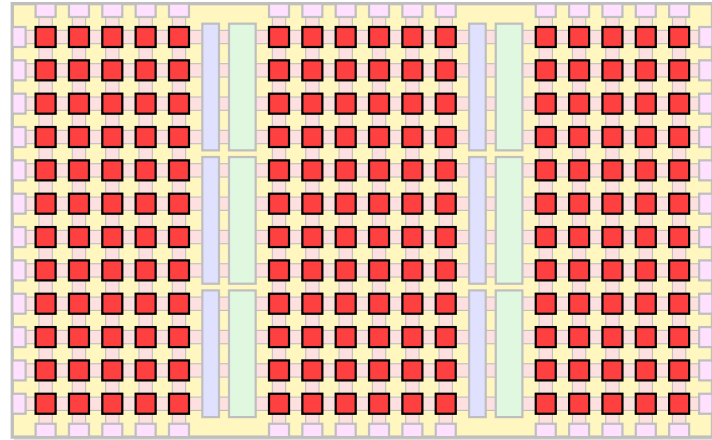
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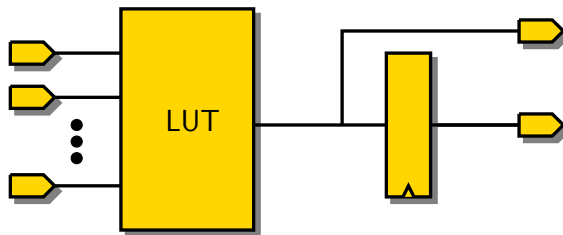
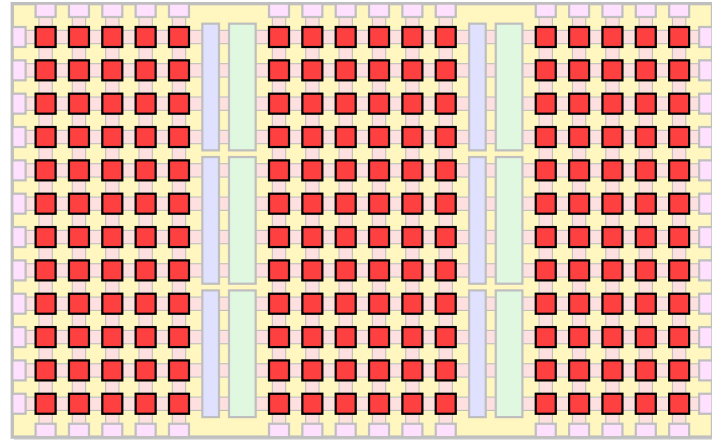
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- ▶ Inside a logic cell:
 - connections to the routing matrix
 - programmable lookup-tables
 - ★ 4 inputs, 1 output
 - ★ 6 inputs, 1 output
 - ★ 6 inputs, 2 outputs

FPGA architecture

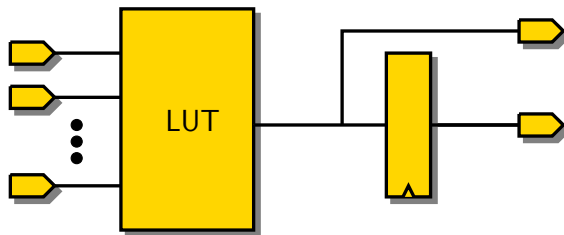
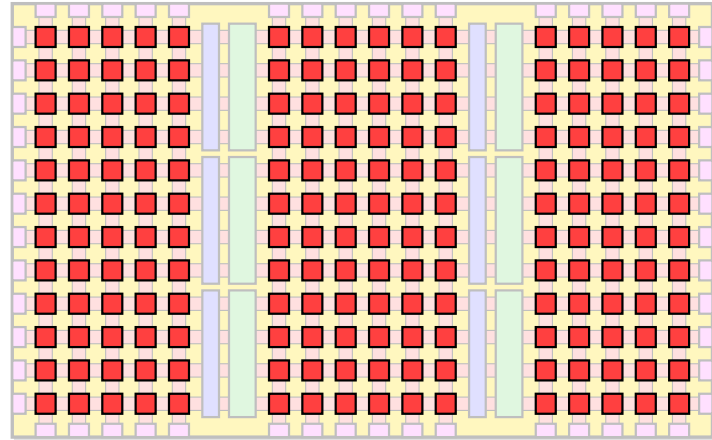
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 - optional registers
 - ★ free pipelining
 - more logic for fast carry-propagation

Outline of the talk

- ▶ Small characteristic finite fields
- ▶ Multiplication algorithms and hardware implementation
- ▶ A finite field coprocessor
- ▶ Finite fields of composite extension degree
- ▶ Conclusion

Representation of the elements

- ▶ Small characteristic: $p = 2$ or 3
- ▶ Polynomial basis:
 - $\mathbb{F}_{p^m} = \mathbb{F}_p[x]/(f(x))$
 - $f(x)$ irreducible polynomial of degree m
 - \mathbb{F}_{p^m} represented by $\mathbb{F}_p[x]^{\leq(m-1)}$
 - Reduction modulo $f(x)$ possibly required

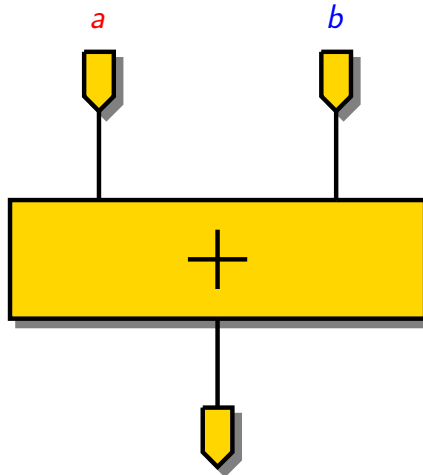
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- ▶ Operations in the field
 - Addition
 - Frobenius automorphism: $(\cdot)^p$
 - Multiplication
 - Inversion
 - ★ Itoh & Tsujii algorithm (Fermat's little theorem)
 - ★ or Extended Euclidean algorithm

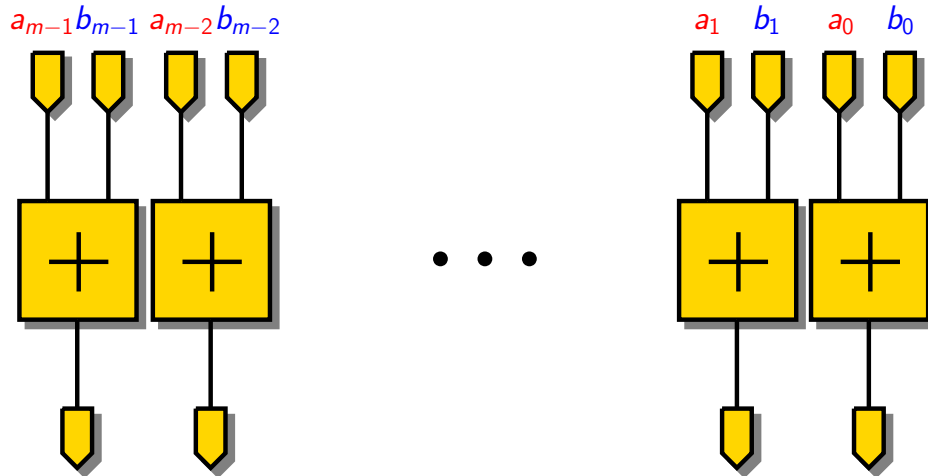
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Hardware implementation of addition

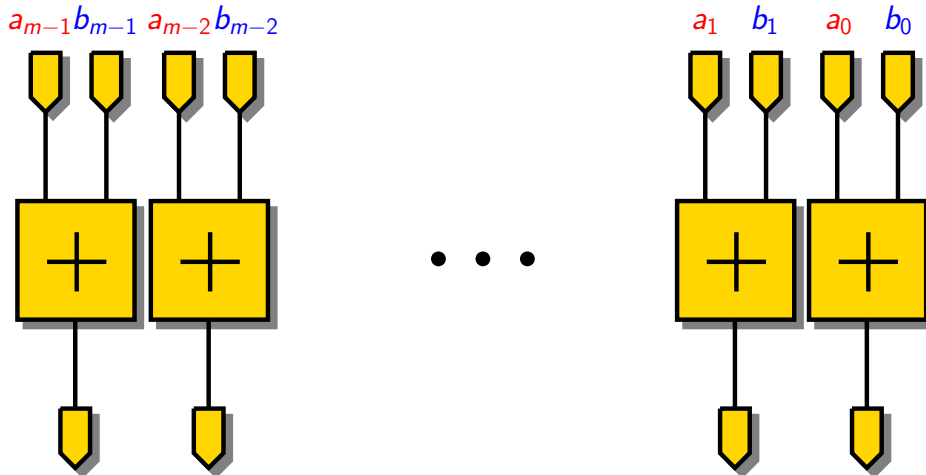


Hardware implementation of addition



► Add coefficient-wise

Hardware implementation of addition



► Add **coefficient-wise**

► Representation of coefficient

- \mathbb{F}_2 : two values \rightarrow one wire, addition is the XOR boolean operator
- \mathbb{F}_3 : three values \rightarrow **borrow-save** representation on two wires
 - ★ Opposite of a coefficient: swap the two wires

Computation of the Frobenius automorphism

$$a^p \equiv (a_{m-1}x^{m-1} + \cdots + a_1x + a_0)^p \pmod{f(x)}$$

- ▶ Raising to the p -th power

Computation of the Frobenius automorphism

$$a^p \equiv a_{m-1}x^{p \cdot (m-1)} + \dots + a_1x^p + a_0 \pmod{f(x)}$$

- ▶ Raising to the p -th power
- ▶ Linear operation
 - Since $\binom{p}{i} \equiv 0 \pmod{p}$ when $i \neq 0$, non-linear terms disappear
- ▶ Need reduction
 - reduce each $x^{p \cdot i}$
 - linear combination of the coefficients

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 - tri- or pentanomials
 - each coefficient of the results is the sum of few coefficients

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 - tri- or pentanomials
 - each coefficient of the results is the sum of few coefficients
- ▶ Hardware implementation
 - Selection of coefficient is free (just wiring!)
 - Same cost as for a few additions
 - Depending on LUTs' size, one LUT per coefficient might be enough

Itoh & Tsujii algorithm

► Fermat's little theorem

$$a^{-1} \equiv a^{p^m-2} \pmod{f}$$

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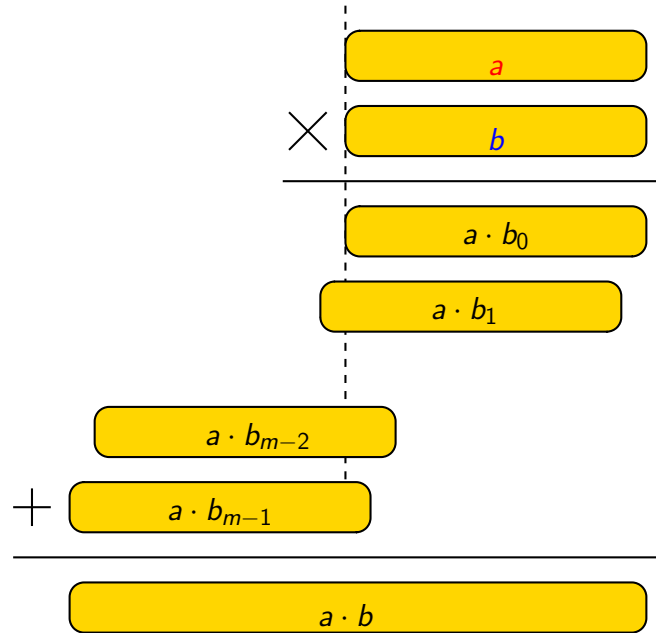
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 - few multiplications
 - an inversion in $\mathbb{F}_p \rightarrow$ identity for $p = 2$ or 3
- ▶ No need for supplementary hardware

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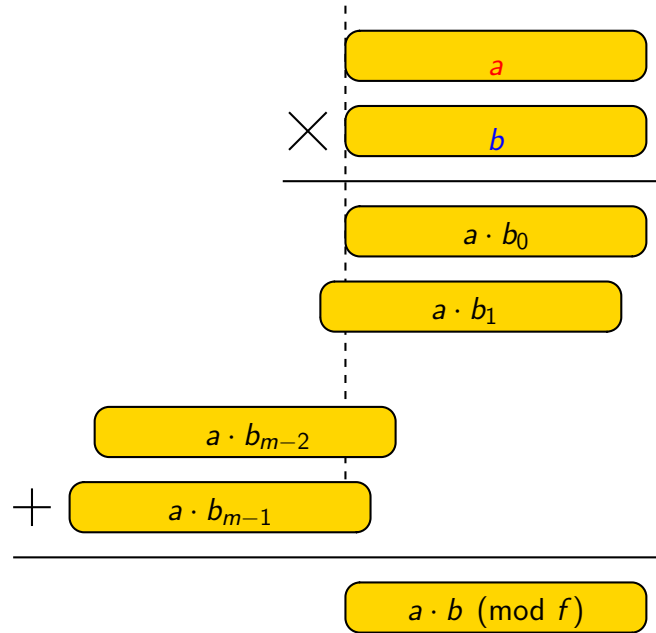
Naive algorithm

- ▶ **Schoolbook** algorithm
 - express each partial product
 - add them



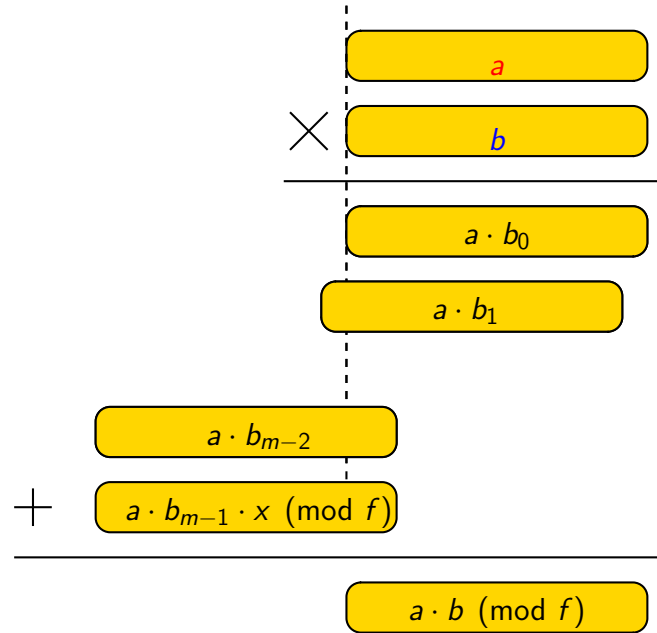
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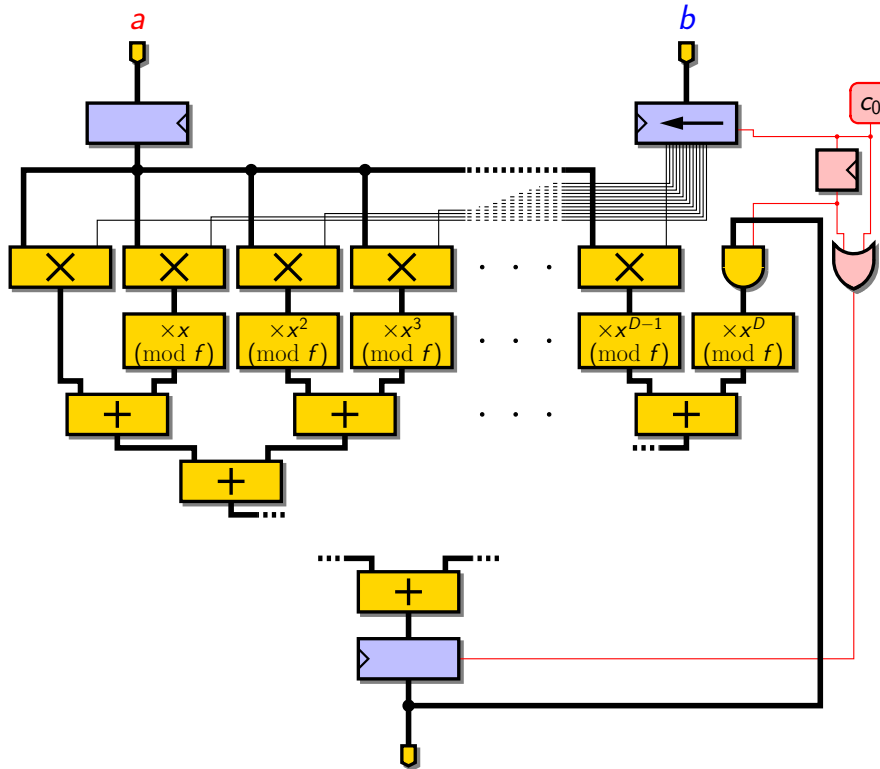
Naive algorithm

- ▶ **Schoolbook** algorithm
 - express each partial product
 - add them
- ▶ Reduction modulo f needed
- ▶ **Reduce** each partial product sequentially



Parallel-serial multiplier

- ▶ Operand a is treated in parallel
- ▶ Operand b is treated D coefficients per cycle
- ▶ Need $\lceil m/D \rceil$ cycles to complete the product



Karatsuba algorithm

A

B

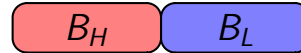
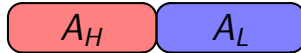
$A \cdot B$

Karatsuba algorithm

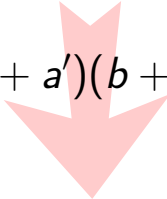


$$A_H B_H X^{2n} + (A_H B_L + A_L B_H) X^n + A_L B_L$$

Karatsuba algorithm

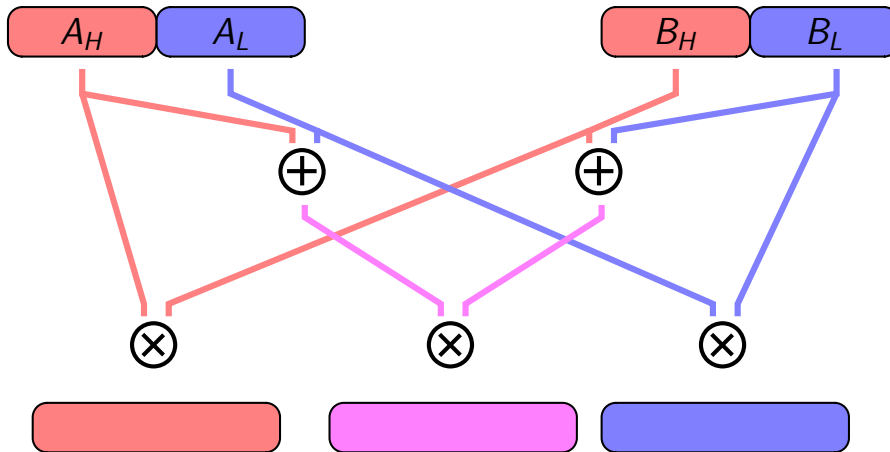


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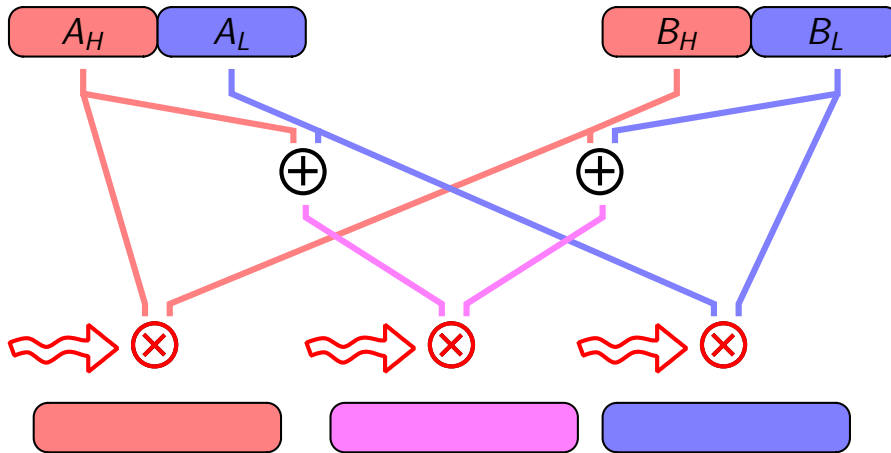
$$ab' + a'b = (a + a')(b + b') - ab - a'b'$$


$$A_H B_H X^{2n} + ((A_H + A_L)(B_H + B_L) - A_H B_H - A_L B_L) X^n + A_L B_L$$

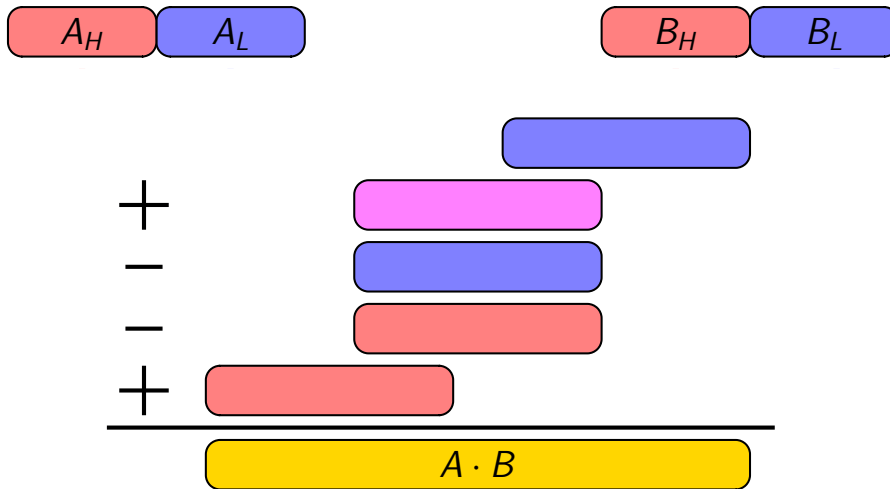
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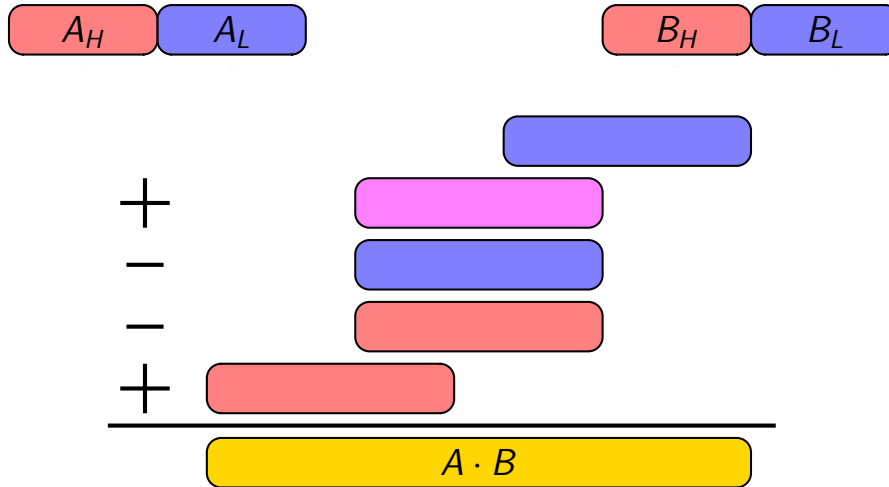
Karatsuba algorithm



Karatsuba algorithm



Karatsuba algorithm



► 3-way Karatsuba

- split operands in three parts
- only 6 subproducts needed

Odd-even split for Karatsuba multiplication

A

B

$A \cdot B$

Odd-even split for Karatsuba multiplication



$$(A_O B_O X^2 + A_E B_E) + X(A_O B_E + A_E B_O)$$

Odd-even split for Karatsuba multiplication

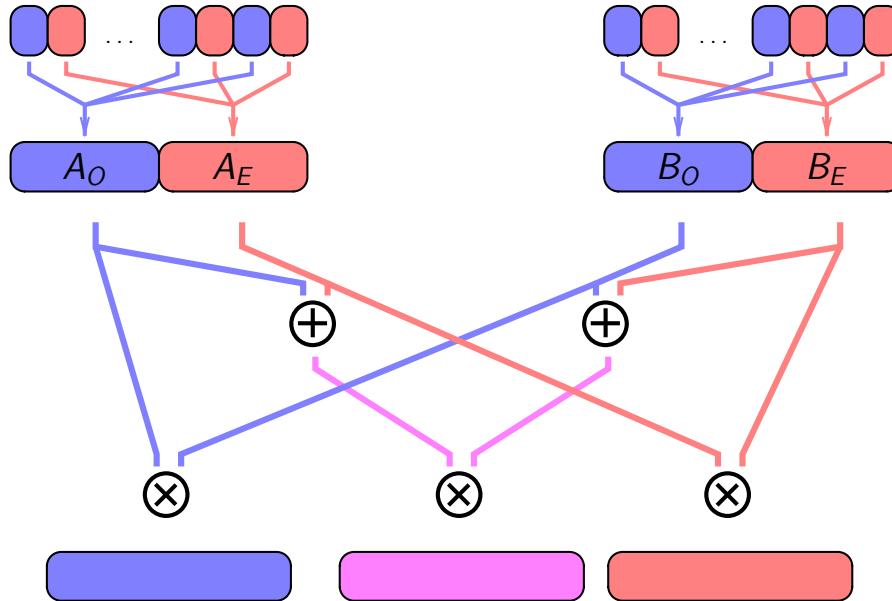


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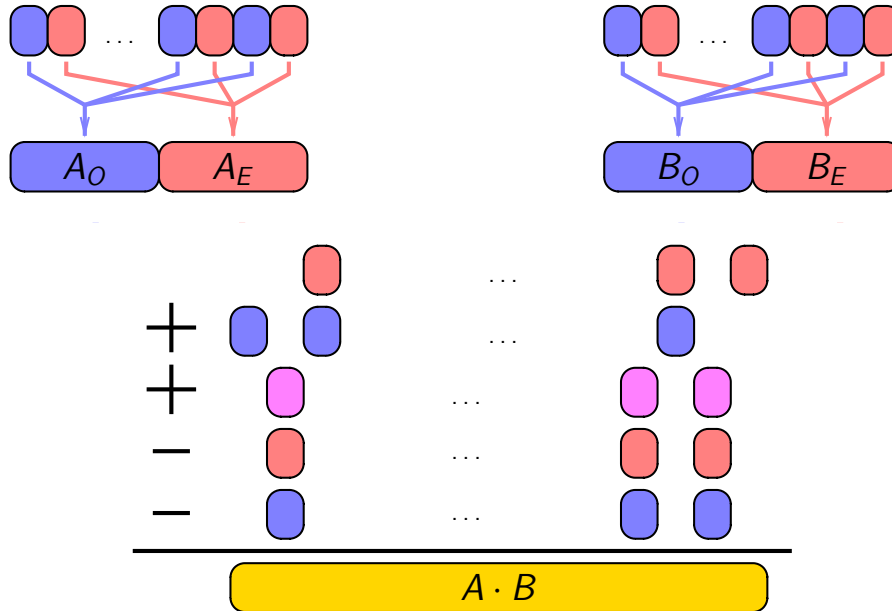
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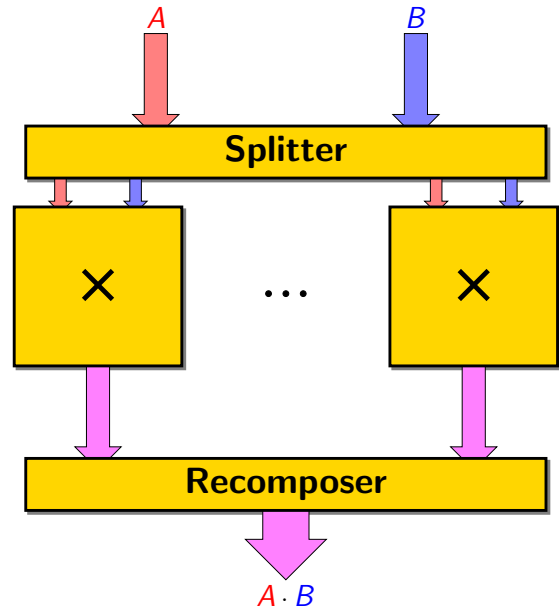


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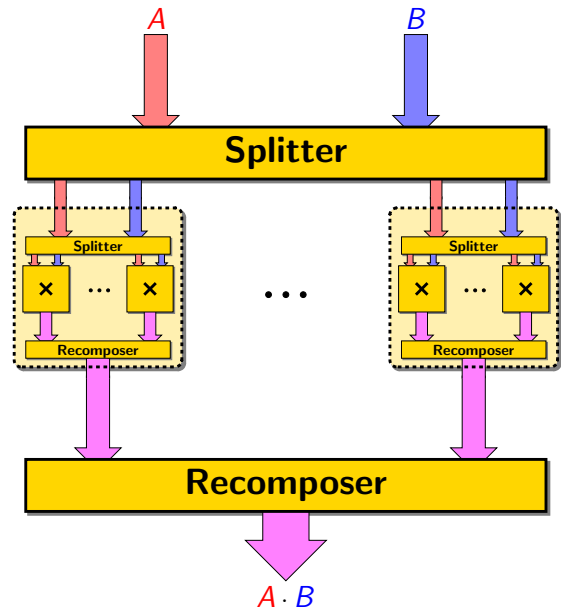
Fully parallel pipelined Karatsuba multiplier

- ▶ Karatsuba-like algorithm:
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 - recompose the result
- ▶ Fully parallel evaluation of the subproducts



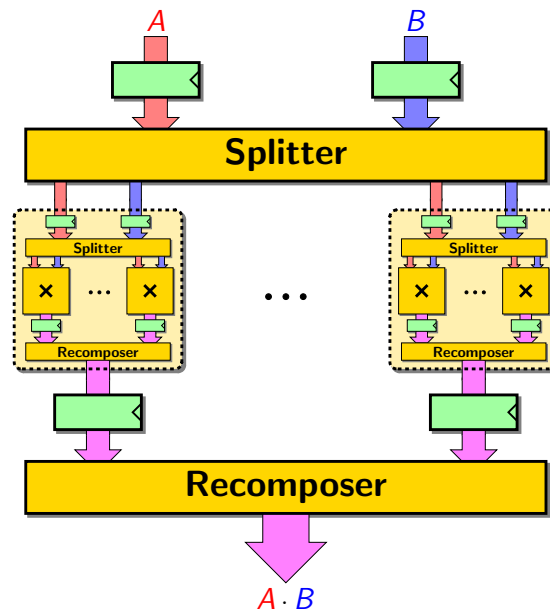
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- ▶ Recursive scheme
 - eventually use different multiplication algorithms
 - end with the quadratic paper-and-pencil algorithm



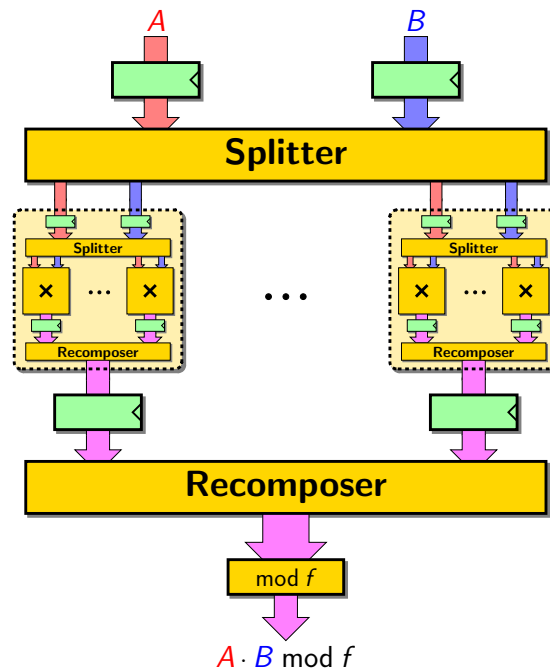
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 - with the help of optional registers
 - cut the critical path
 - increase the frequency



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- ▶ Final reduction modulo f



An example of multipliers over $\mathbb{F}_{3^{239}}$

▶ $\mathbb{F}_{3^{239}} = \mathbb{F}_3[X]/(X^{239} - X^5 + 1) \rightarrow \sim 380\text{-bit field}$

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| Polynomial size | Used algorithm |
|-----------------|-------------------------------------|
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- ▶ Choose to have 7 pipeline stages
- ▶ Post-place-and-route estimation for Xilinx Virtex-II Pro
 - \sim 50000 slices (2 LUTs $4 \rightarrow 1$ per slice)
 - 200 MHz
 - computes $200 \cdot 10^6$ products per second

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 - ~ 50000 slices (2 LUTs $4 \rightarrow 1$ per slice)
 - 200 MHz
 - computes $200 \cdot 10^6$ products per second
- ▶ Comparison with parallel-serial multiplier with $D = 16$
 - ~ 8700 slices
 - 180 MHz
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 - 200 MHz
 - computes $200 \cdot 10^6$ products per second
 - 4000 products per second and per slice
- ▶ Comparison with parallel-serial multiplier with $D = 16$
 - ~ 8700 slices
 - 180 MHz
 - computes $12 \cdot 10^6$ products per second
 - 1400 products per second and per slice

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Designing a finite field coprocessor

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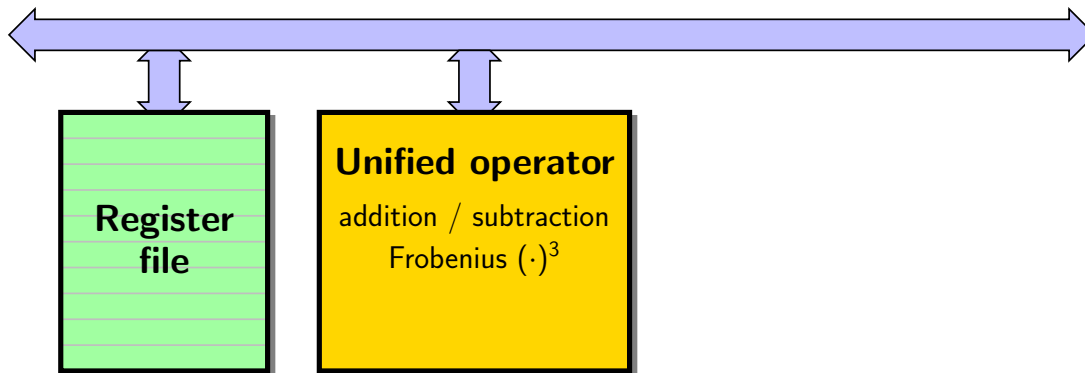
Designing a finite field coprocessor

- ▶ Determine the **specific needs** of operations of your computation
- ▶ **Example:** final exponentiation in pairing computation



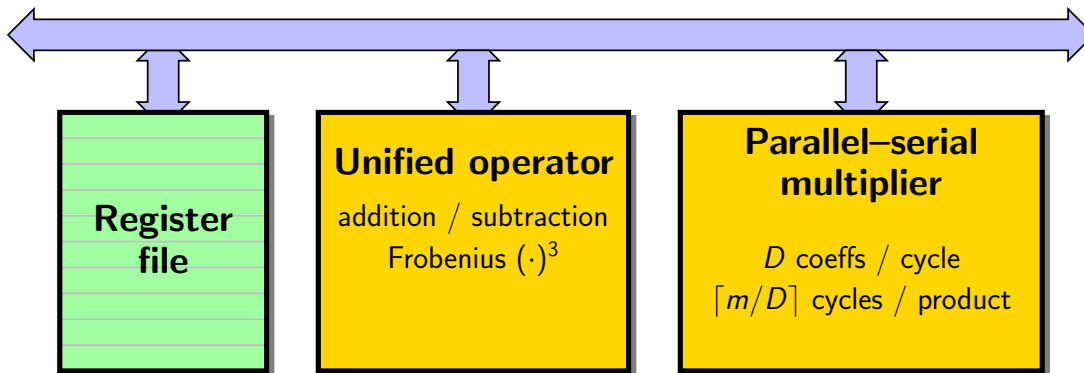
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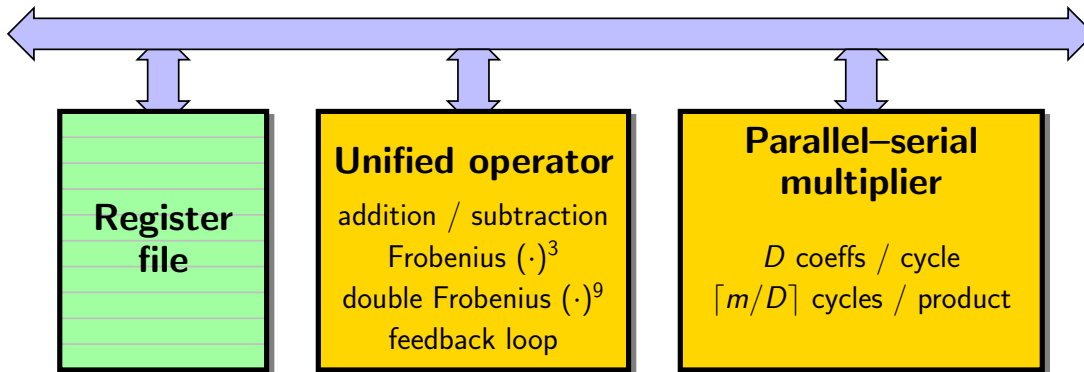
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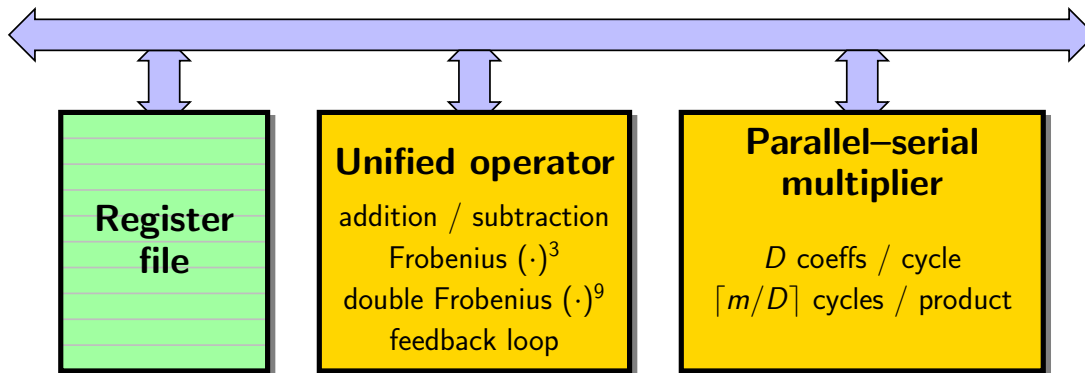
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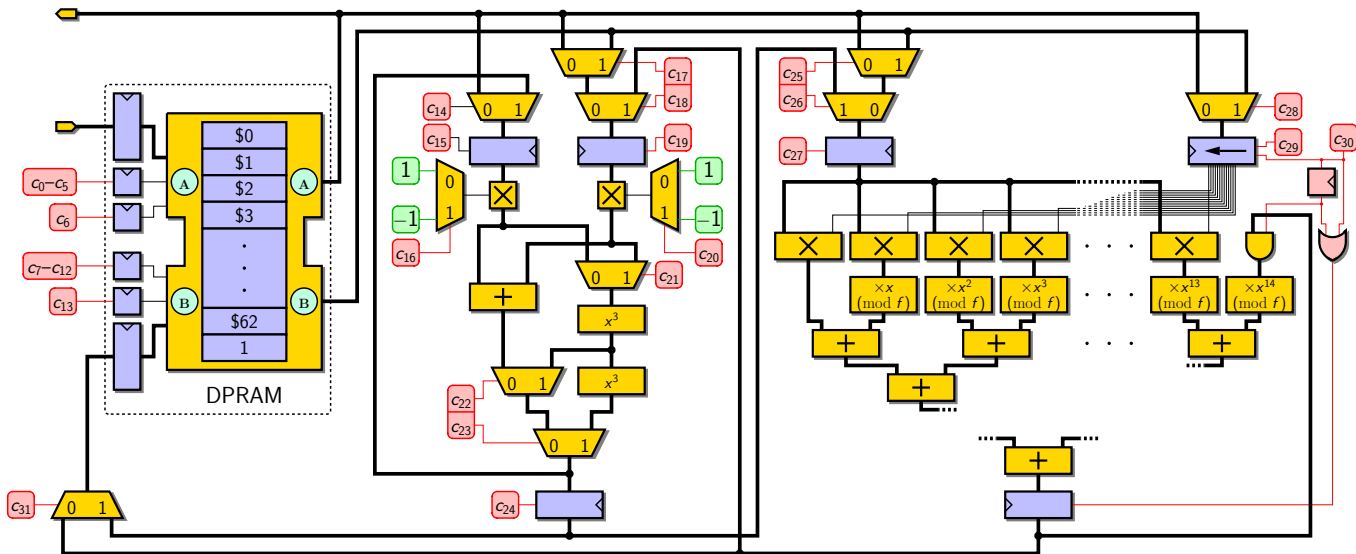


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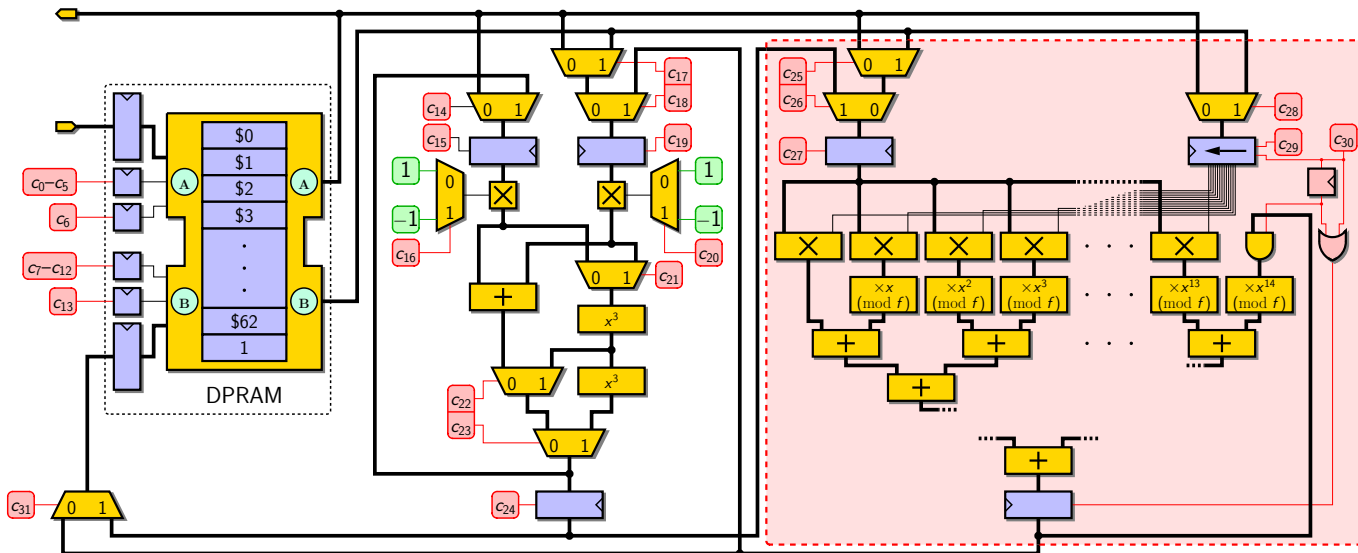
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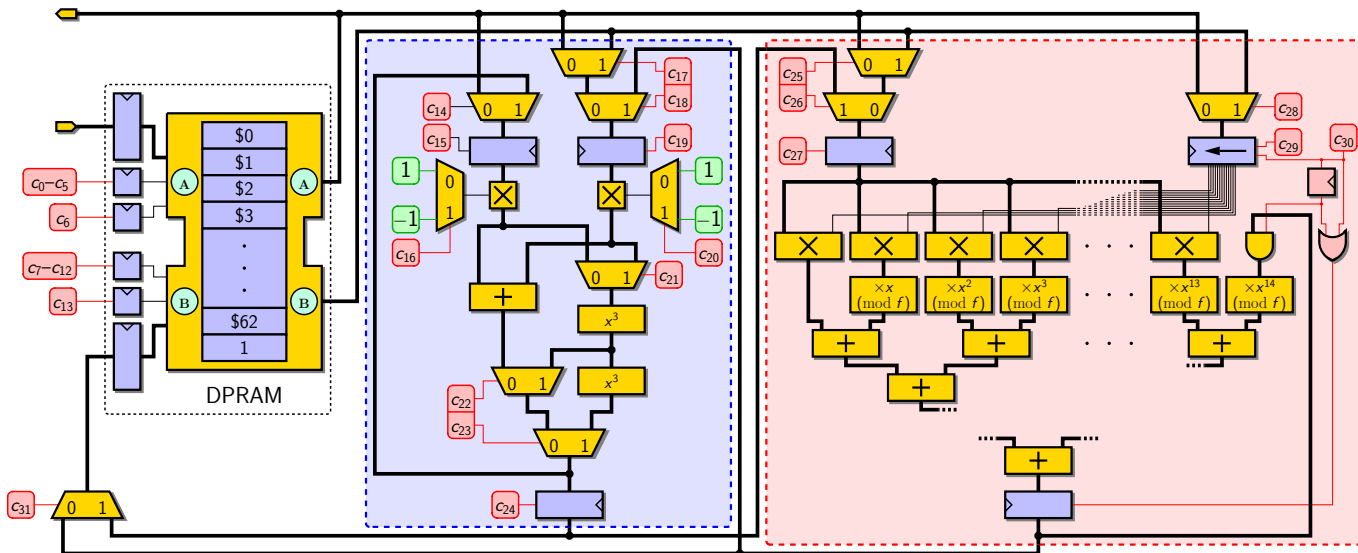
Detailed architecture of the coprocessor (char. 3)



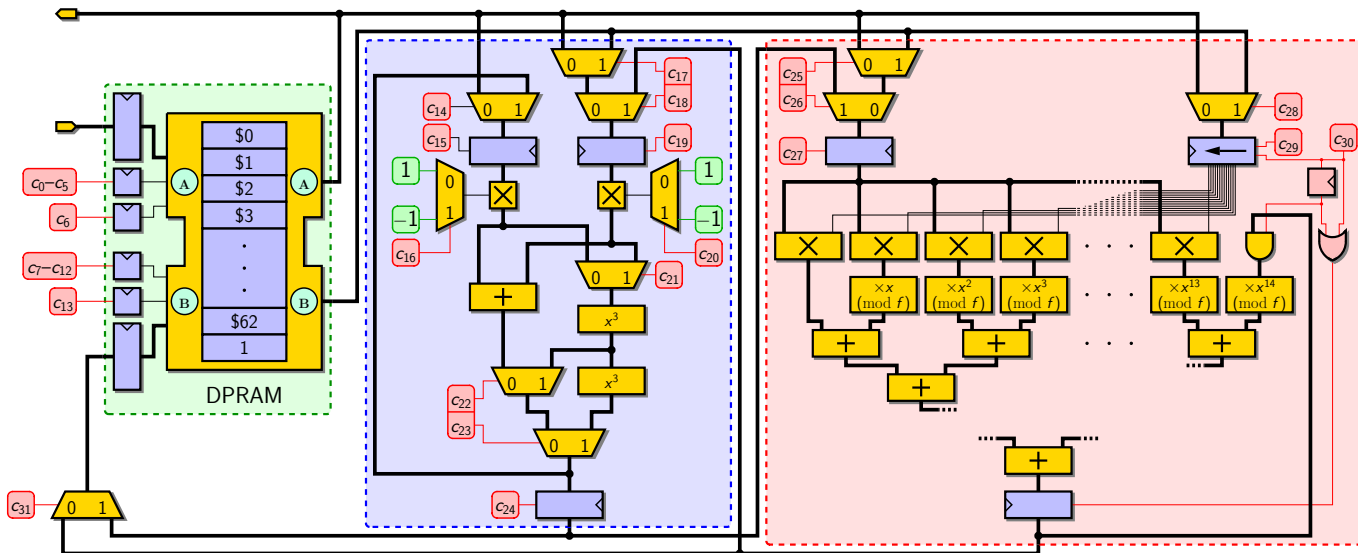
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Outline of the talk

- ▶ Small characteristic finite fields
- ▶ Multiplication algorithms and hardware implementation
- ▶ A finite field coprocessor
- ▶ **Finite fields of composite extension degree**
- ▶ Conclusion

Composite extension degree

- ▶ Needed field might have a **composite extension degree**
- ▶ **Tower field** construction:
 - $\mathbb{F}_{p^{m \cdot n}} = \mathbb{F}_{p^m}[y]/(g(y))$
 - g **irreducible** polynomial of **degree** n

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 - g **irreducible** polynomial of **degree** n
- ▶ Reducing the **datapath**
 - design a **coprocessor** for \mathbb{F}_{p^m}
 - **program** it to implement arithmetic of $\mathbb{F}_{p^{m \cdot n}}$
 - reduce **area** of the design
- ▶ **Operations**
 - same algorithms
 - coefficients are now in \mathbb{F}_{p^m}

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- ▶ Subproducts between field elements, not polynomials
 - no overlapping at reconstruction step
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 - **evaluate** the product **modulo** some irreducible polynomials
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- ▶ **Montgomery's** Karatsuba-like formulae
 - **ad hoc** formulae for degree 4, 5 and 6 polynomials
- ▶ Algorithmic search for **optimal formulae**
 - **Work in progress** with J. Detrey, R. Barbulescu and P. Zimmermann

Choosing multiplication algorithm

- ▶ Evaluate the cost of the different algorithms
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Multiplication in $\mathbb{F}_{3^{m=5}}$

| Algorithm | \times | $+$ | Ratio |
|---|----------|-----|-------|
| Schoolbook | 25 | 24 | 0.96 |
| One-level Karatsuba (Montgomery's trick) | 21 | 29 | 1.38 |
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Multiplication in $\mathbb{F}_{2^{m=7}}$

| Algorithm | × | + | Ratio |
|---|----|----|-------|
| Schoolbook | 49 | 48 | 0.98 |
| One-level Karatsuba (Montgomery's trick) | 40 | 52 | 1.30 |
| Recursive Karatsuba | 25 | 51 | 2.04 |
| Recursive Karatsuba (Montgomery's trick) | 23 | 76 | 3.30 |
| Montgomery's Karatsuba-like | 22 | 84 | 3.818 |
| CRT-based | 22 | 88 | 4.05 |

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Conclusion

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- ▶ Many criterion for optimization
 - **area**: low cost devices
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 - **area-speed tradeoff**: high-throughput application
- ▶ **Multiplication** is the critical operation
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- ▶ Need for algorithms/hardware **codesign**

Thank you for your attention!



Questions?