Groupe de travail Arénaire — January 6, 2011

Compact hardware for computing the Tate pairing over 128-bit-security supersingular curves

Nicolas Estibals

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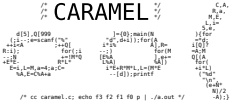

Outline of the talk

- Context
- Pairings-friendly curves with 128 bits of security
- Taking advantage of the tower field
- Performance results
- Conclusion

Pairing-based cryptography

- Origin of pairings in cryptography
  - attack against some elliptic curves
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- Standardization in progress
  - ISO/IEC 14888-3
  - IEEE P1363.3

Hardware accelerators for pairing computation

- Pairings are (almost) everywhere!
  - wide range of targets and applications
    - ⋆ low-resource environment (embedded systems, smart card, . . .)
    - ⋆ high-performance computation (bank server, . . .)
  - non-trivial to compute
    - ⋆ complex mathematical structure
    - ⋆ finite field arithmetic
    - ⋆ substantial amount of computation
- Needs in hardware implementation
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- Previous work on FPGA implementations
  - low-security pairings
  - most are performance-oriented designs
- Our goal:
  - AES-128 equivalent security
  - compact accelerator

Tate pairing

- Bilinear pairing:

$$e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$$

Tate pairing

- E elliptic curve over $\mathbb{F}_q$
- ℓ large prime dividing $\#E(\mathbb{F}_q)$
  - in general, $\ell \approx \#E(\mathbb{F}_q)$
  - Hasse's bound : $|\#E(\mathbb{F}_q) - (q+1)| \leqslant 2\sqrt{q}$
  - thus, $\ell \approx q$
- $\mathbb{F}_q$-rational ℓ-torsion of E: $E(\mathbb{F}_q)[\ell] = \{P \in E(\mathbb{F}_q) \mid [\ell]P = \mathcal{O}\}$
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- Set of ℓ-th root of unity: $\mu_\ell = \{u \in \mathbb{F}_{q^k}^* \mid u^\ell = 1\}$
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Compute the Tate pairing

$$e : E(\mathbb{F}_q)[\ell] \times E(\mathbb{F}_{q^k})[\ell] \rightarrow \mu_\ell \subset \mathbb{F}_{q^k}^*$$

- Miller's algorithm
- Iterative algorithm
- Complexity:
  - number of iteration proportional to definition field size
  - a multiplication over $\mathbb{F}_{q^k}$ at each iteration

General attacks

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- k acts as a cursor to balance the complexity of the two attacks
- $k = 12$: optimal for the 128-bit security level

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Supersingular elliptic curves Vs. Barreto–Naehrig curves

- Definitions:

$$E/\mathbb{F}_3: \quad y^2 = x^3 - x + b, b \neq 0$$
$$E/\mathbb{F}_2: \quad y^2 + y = x^3 + x + b$$

- Supersingular curve

- Definition:

$$E/\mathbb{F}_p: \quad y^2 = x^3 + b, b \neq 0,$$
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  $\mathbb{F}_p$ with p a 256-bit prime.

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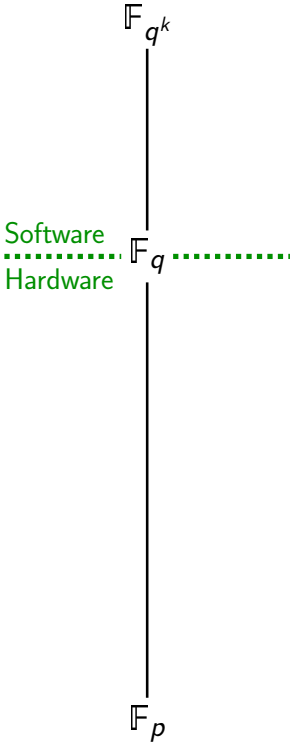

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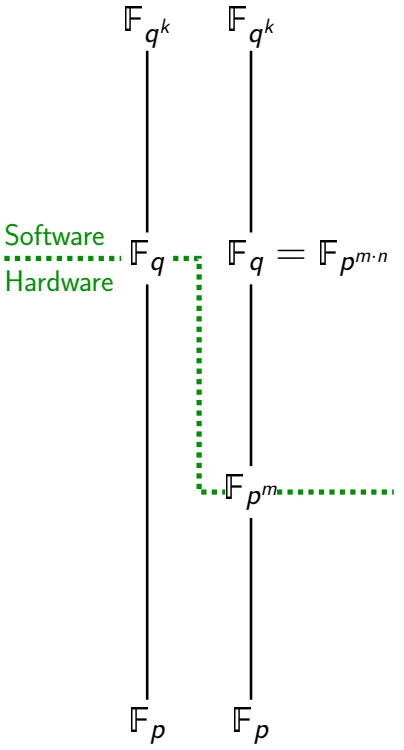

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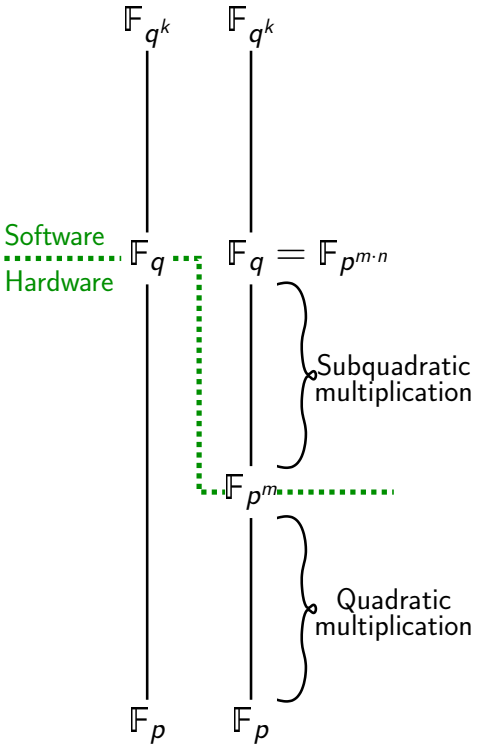

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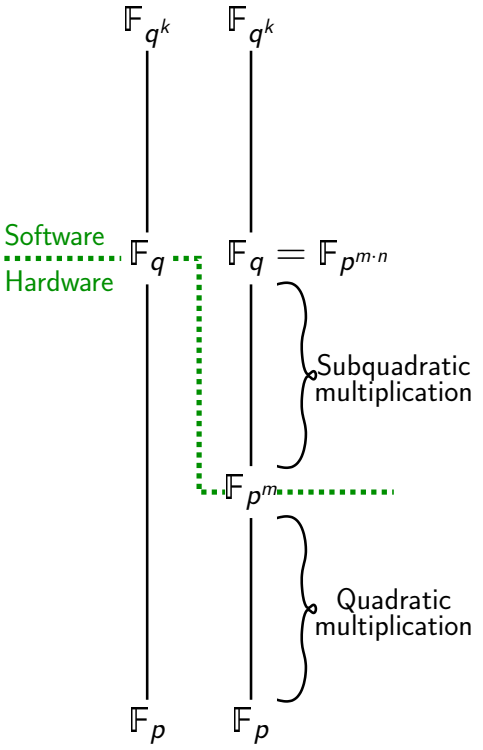

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- Problem:
  - field with composite extension degree
  - allows some additional attacks

Weil Descent-based attacks

- We now consider:

$$E(\mathbb{F}_{p^{m \cdot n}})[\ell] \text{ with } m \text{ prime and } n \text{ small}$$

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- Static Diffie–Hellman problem
  - leakage when reusing private key (*e.g.* ElGamal encryption)
  - Granger's attack: complexity in $\tilde{O}((p^m)^{1-\frac{1}{n+1}})$
  - revoke key after a certain amount of use is an effective workaround

Suitable curves for 128-bit security level

			Cost of the attacks (bits)			
p^m	n	$\log_2 \ell$	Pollard's ρ	FFS		
3^{503}	1	697	342	132		
3^{97}	5	338	163	130		
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2^{367}	3	698	342	**128**	489	275
2^{227}	5	733	359	**129**	363	189
2^{163}	7	753	370	**129**	279	142
2^{127}	9	487	236	**130**	225	114
2^{103}	11	922	454	**129**	187	94
2^{89}	13	1044	515	164	**130**	82
2^{73}	15	492	239	136	**127**	68

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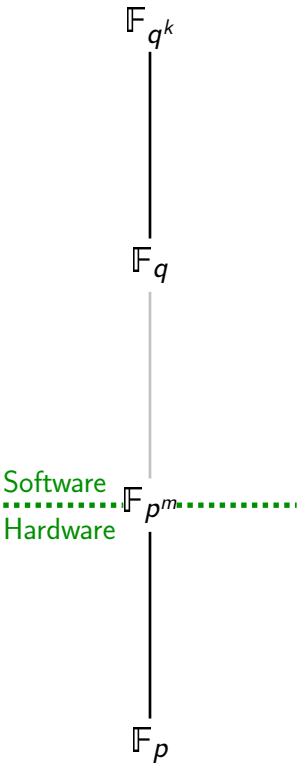

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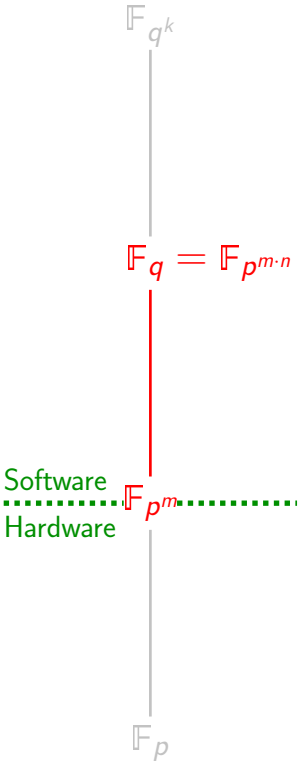

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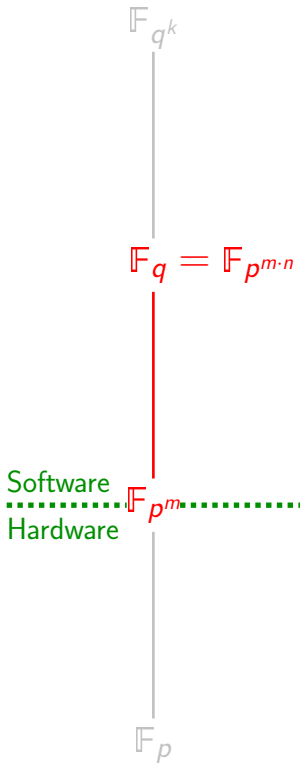

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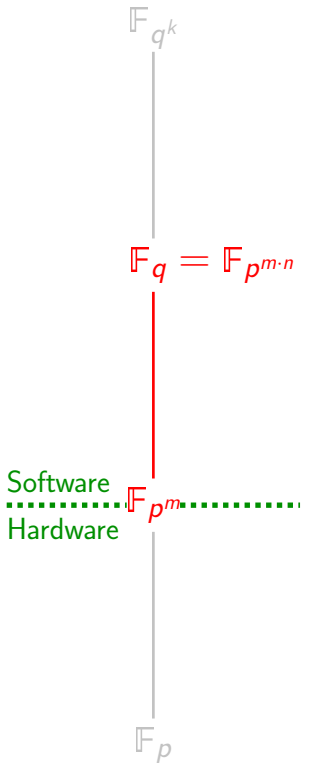

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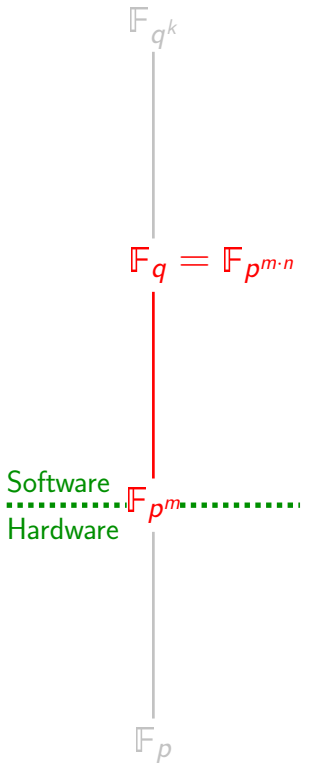

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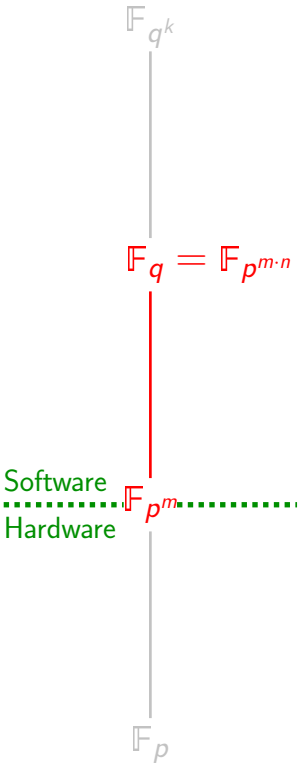
- Inversion
  - Itoh & Tsujii algorithm (Fermat's little theorem)
  - only need additions, multiplications and Frobenius in $\mathbb{F}_{p^m}$

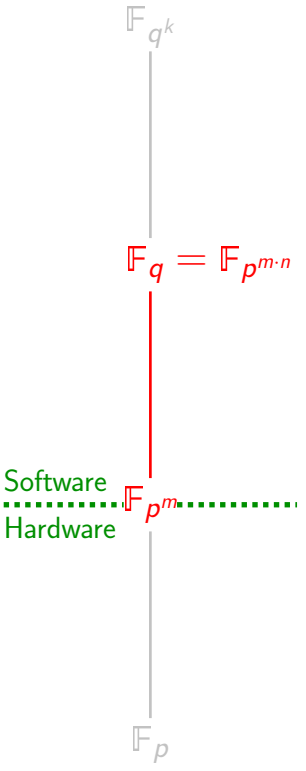
Multiplication in the extension field

- Bottleneck of pairing computation



Multiplication in the extension field

- Bottleneck of pairing computation



- Our test cases:
  - multiplication in $\mathbb{F}_{3^{97 \cdot 5}}$
    - ⋆ using "schoolbook" algorithm
    - ⋆ 25 products in $\mathbb{F}_{3^{97}}$
    - ⋆ 24 additions in $\mathbb{F}_{3^{97}}$
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    - ⋆ 49 products in $\mathbb{F}_{2^{163}}$
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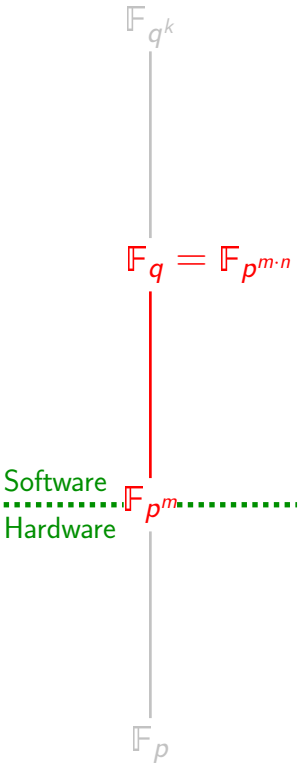

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  - Karatsuba
  - Karatsuba with Montgomery's trick
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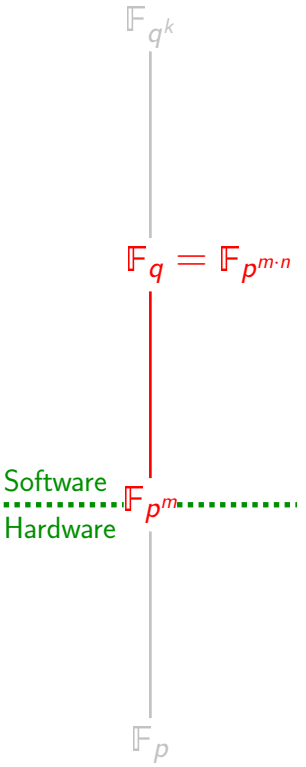

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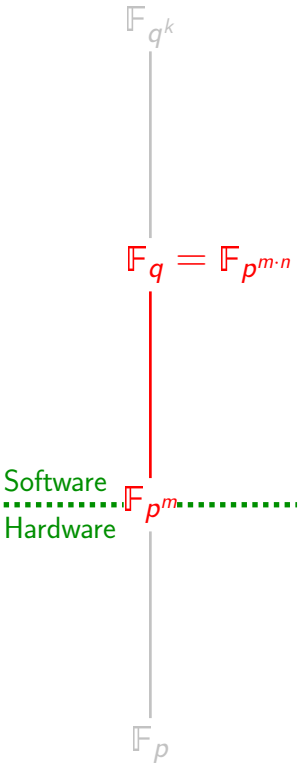

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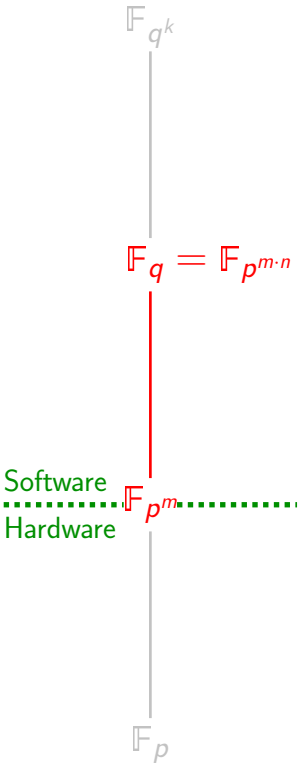

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- Choice of algorithm depends on relative cost of multiplication and addition

Count of operations

- Full pairing computation over $E(\mathbb{F}_{3^{97\cdot 5}})$

	$\times$	$+$	$(.)^3$	$1/.$
$\mathbb{F}_{3^{97\cdot 5}}$	3104	13127	4123	1

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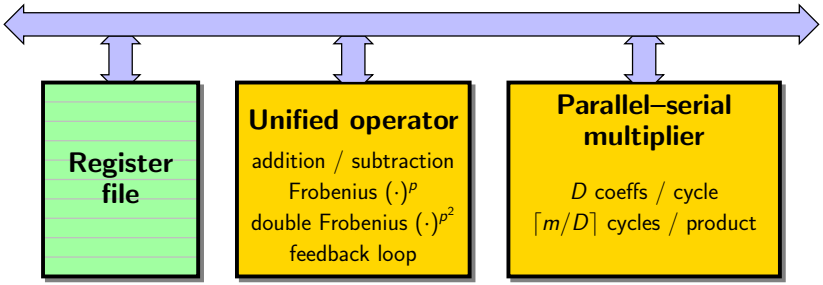
	$\times$	$+$	$(.)^2$	$1/.$
$\mathbb{F}_{2^{163\cdot 7}}$	4019	12605	7424	1
$\mathbb{F}_{2^{163}}$	88509	448361	52782	–

Outline of the talk

- Context
- Pairings-friendly curves with 128 bits of security
- Taking advantage of the tower field
- Performance results
- Conclusion

Experimental setup

- Finite field coprocessors



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  - Post-place-and-route timing and area estimations

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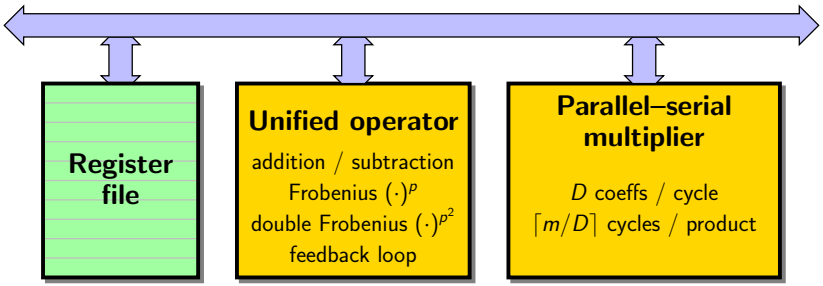

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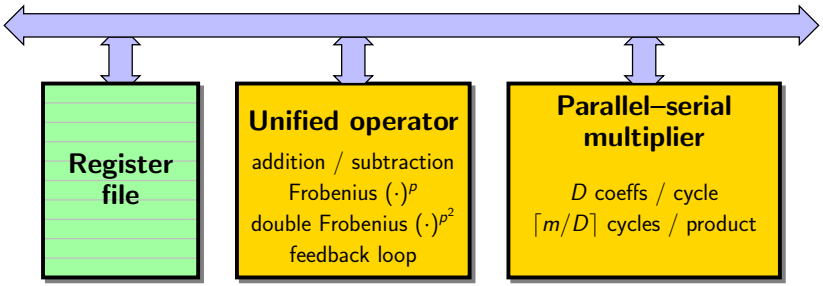

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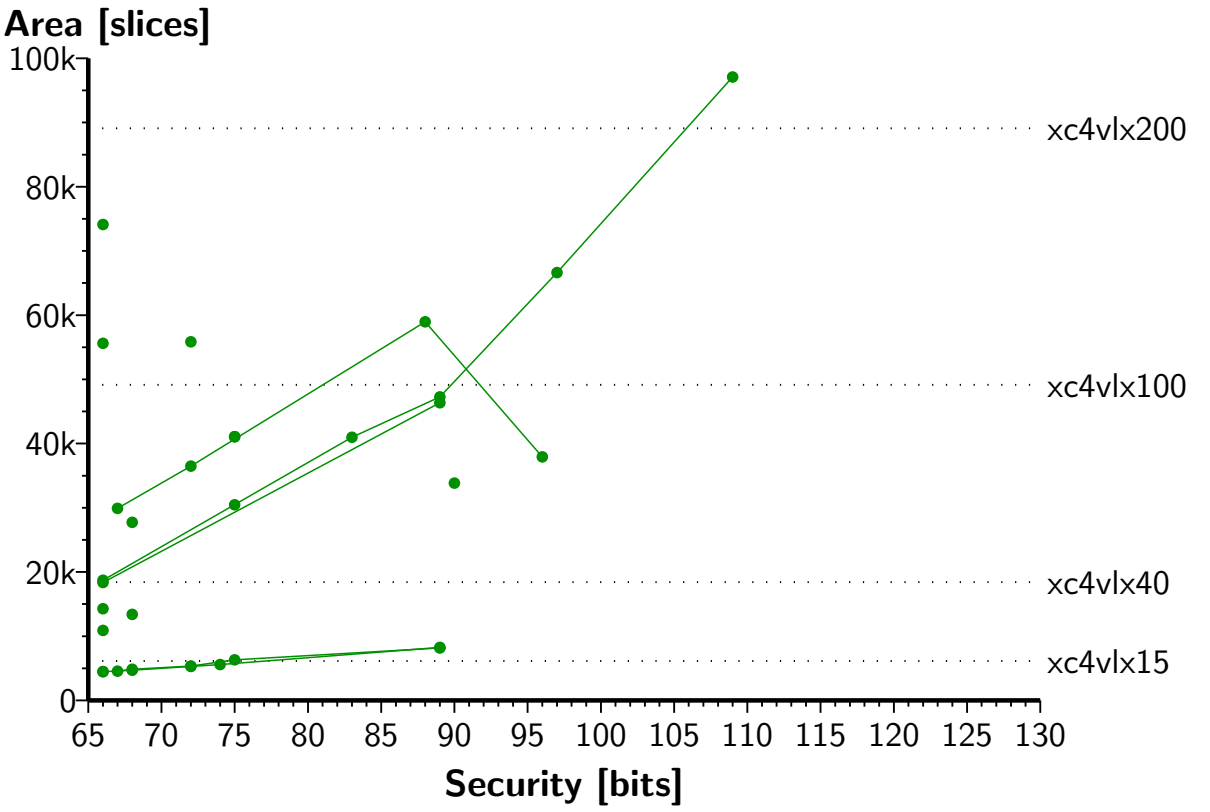
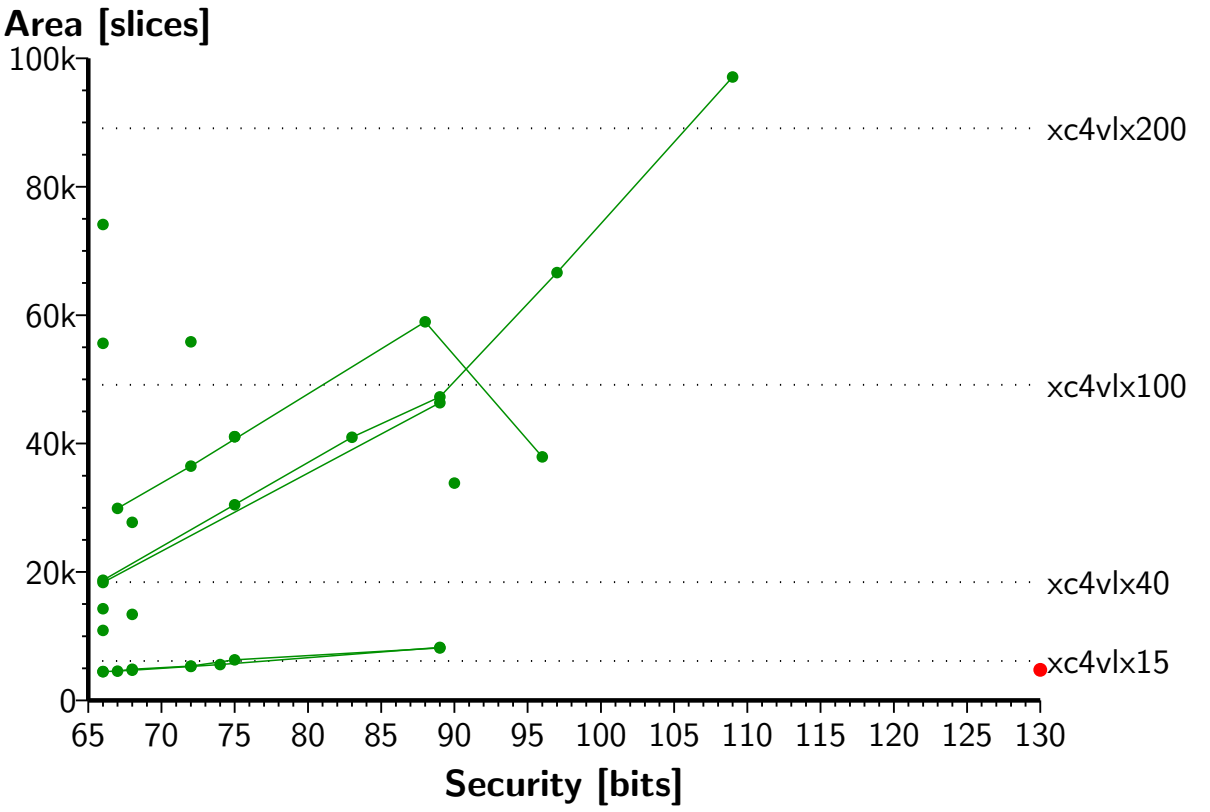
- Need of a code factorization method
  - arithmetic of $\mathbb{F}_{p^{m \cdot n}}$ is microcoded

Area


Area [slices]
100k
80k
60k
40k
20k
0
xc4vlx200
xc4vlx100
xc4vlx40
xc4vlx15
65
70
75
80
85
90
95
100
105
110
115
120
125
130
Security [bits]

Area


Area [slices]
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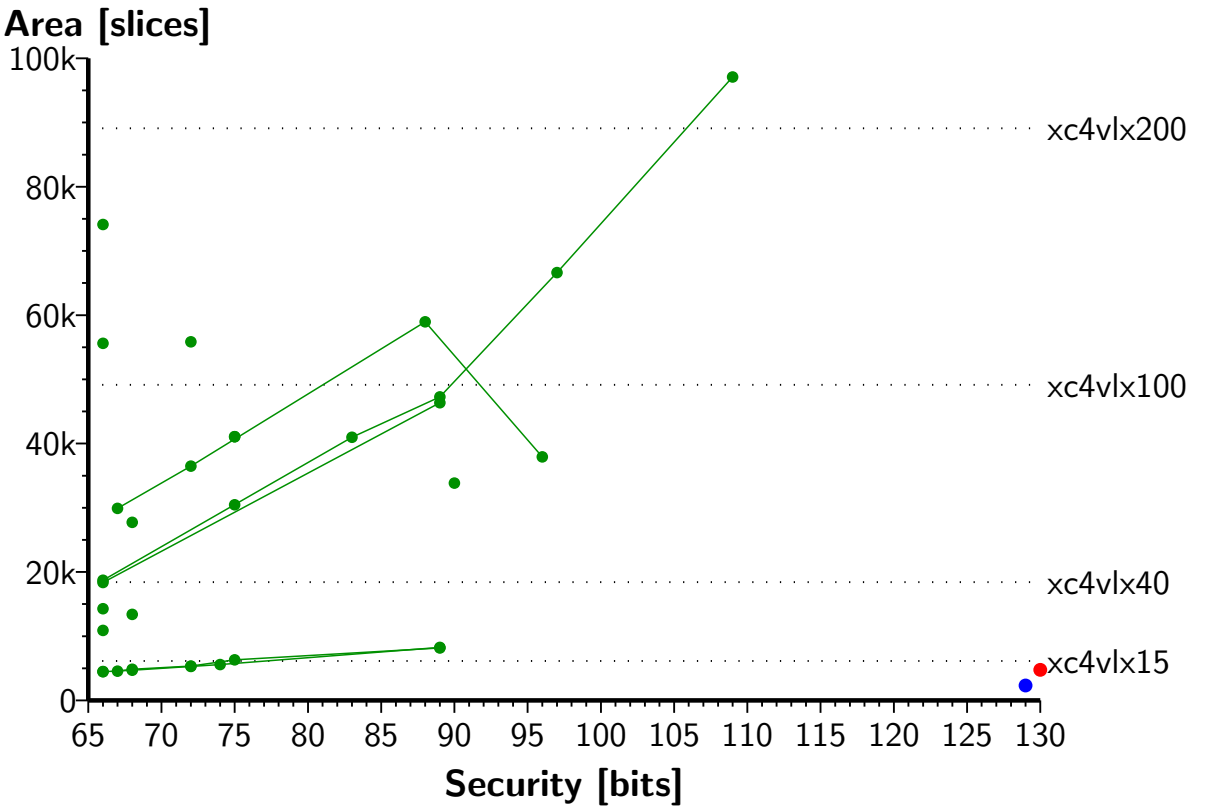
Security [bits]

Area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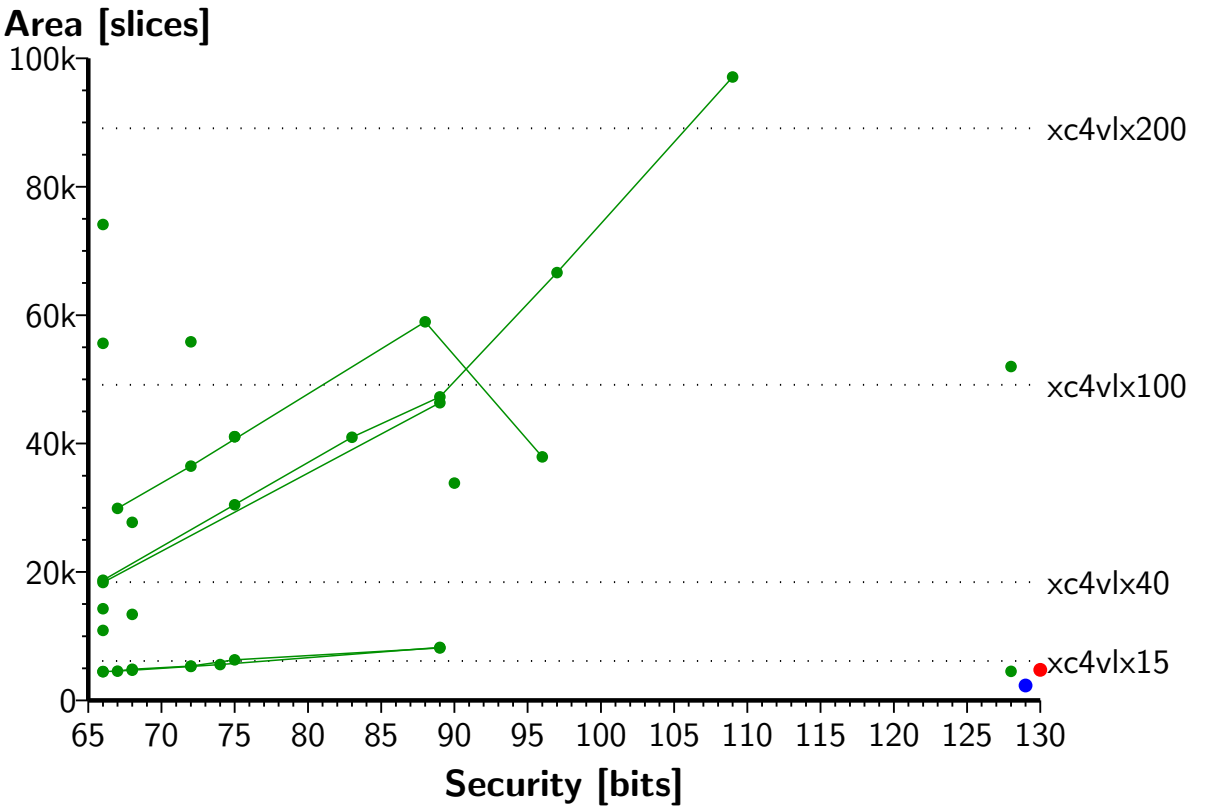
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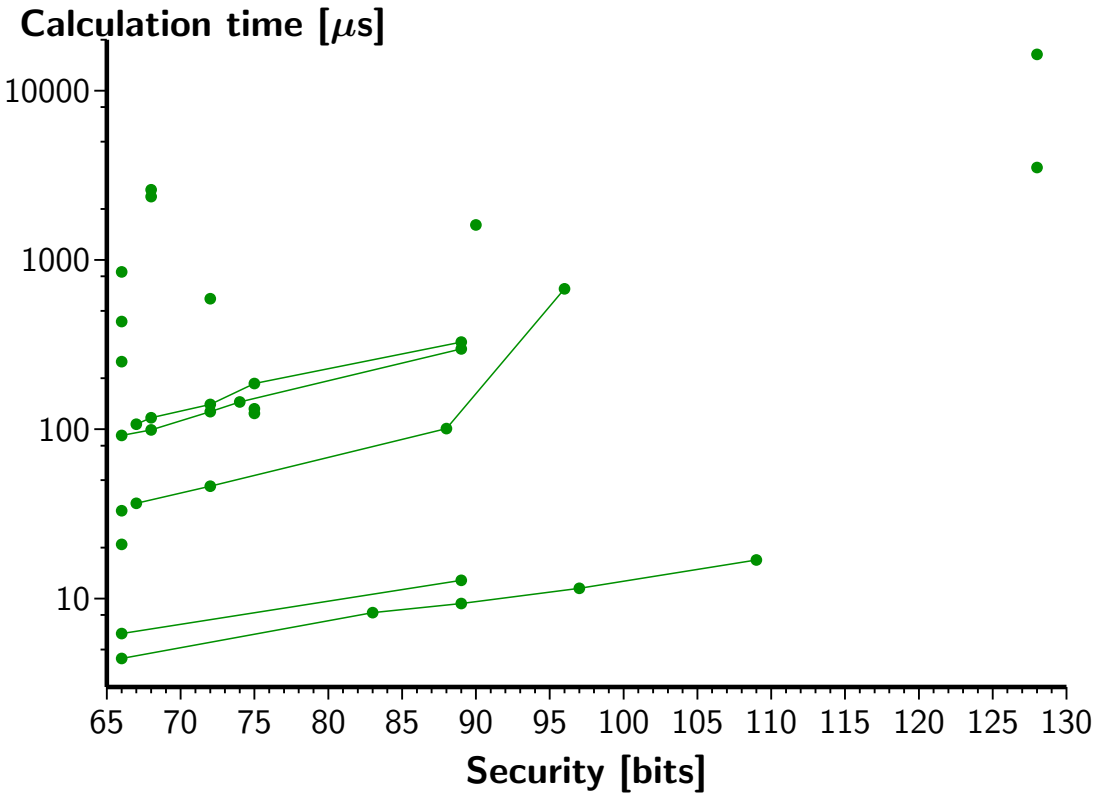
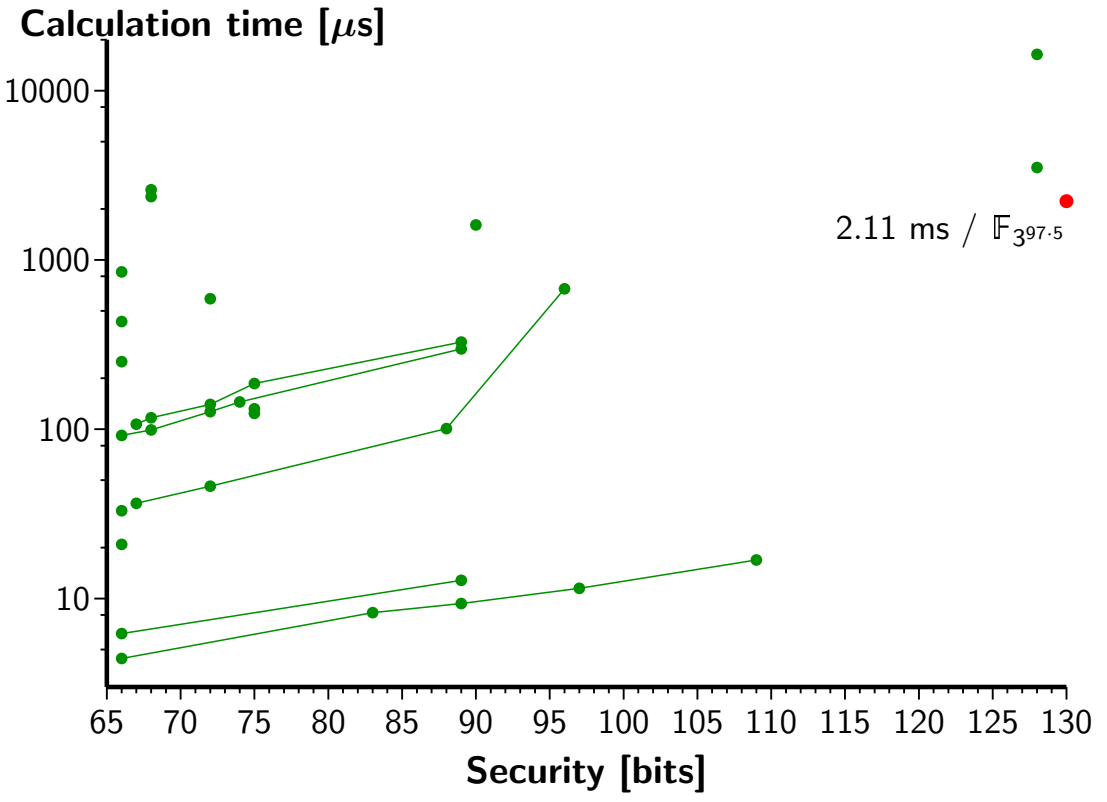
Calculation time



Calculation time



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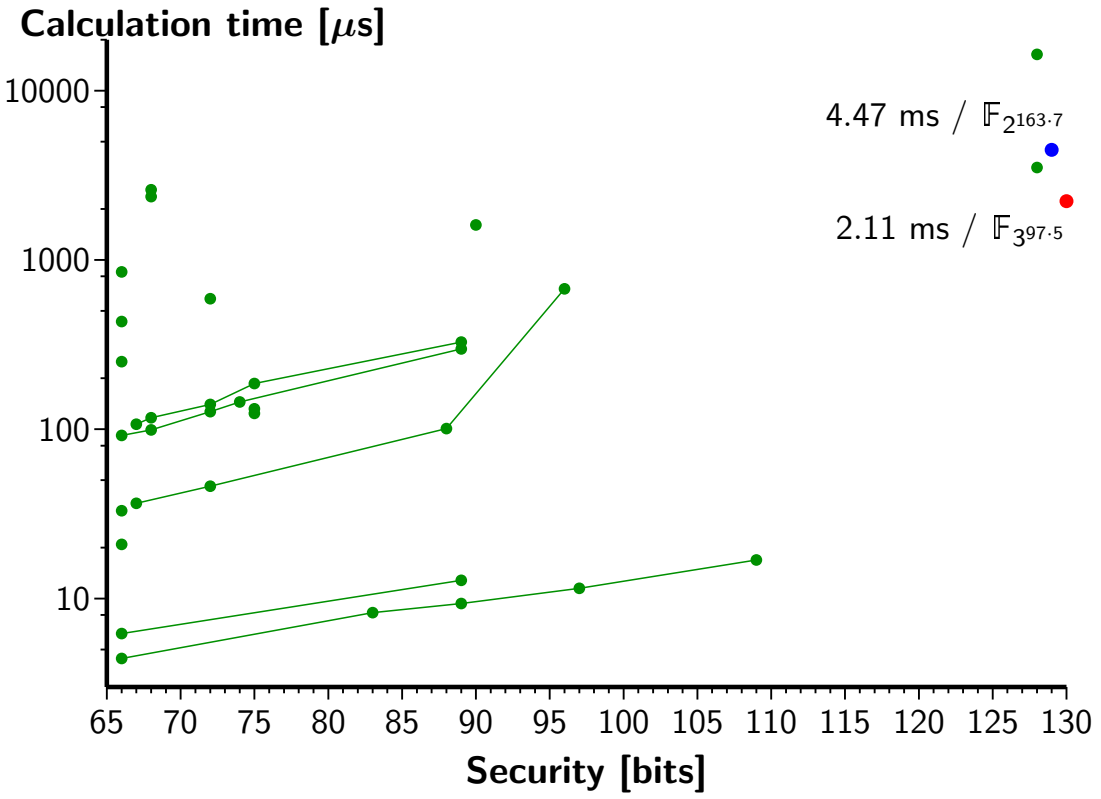
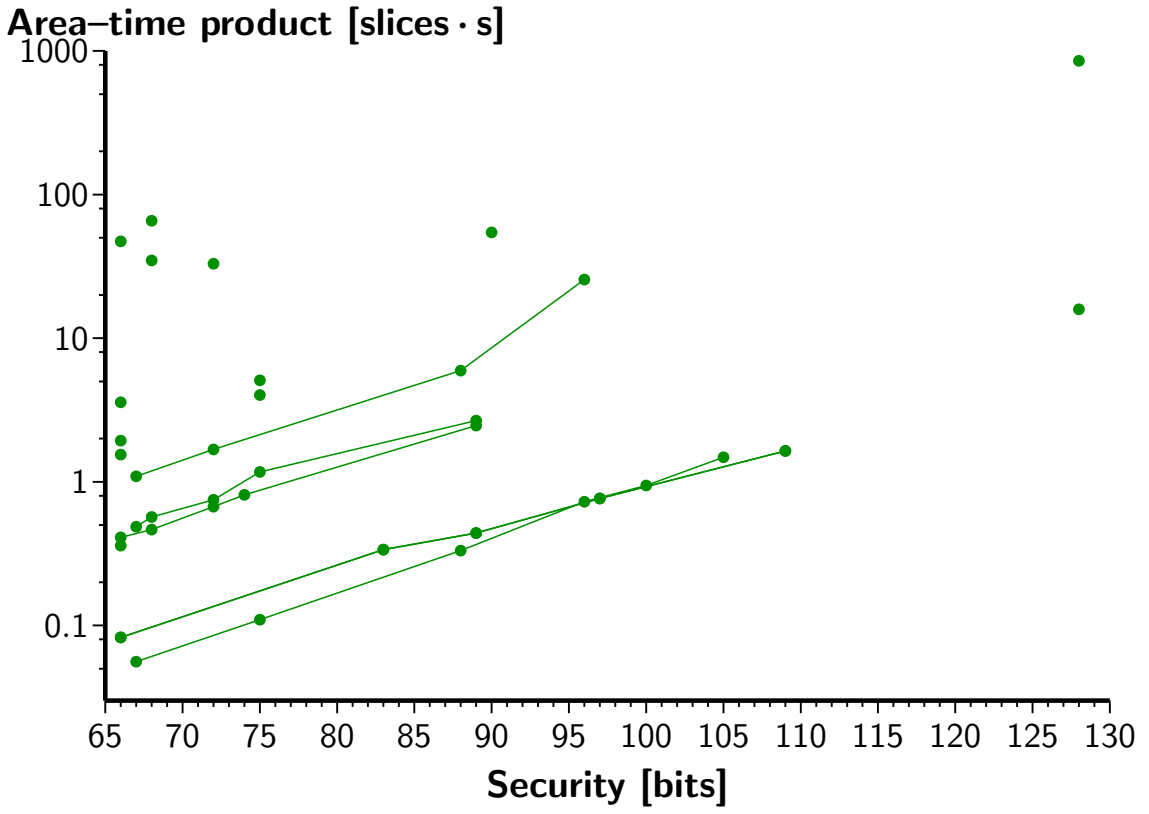


Area–Time product



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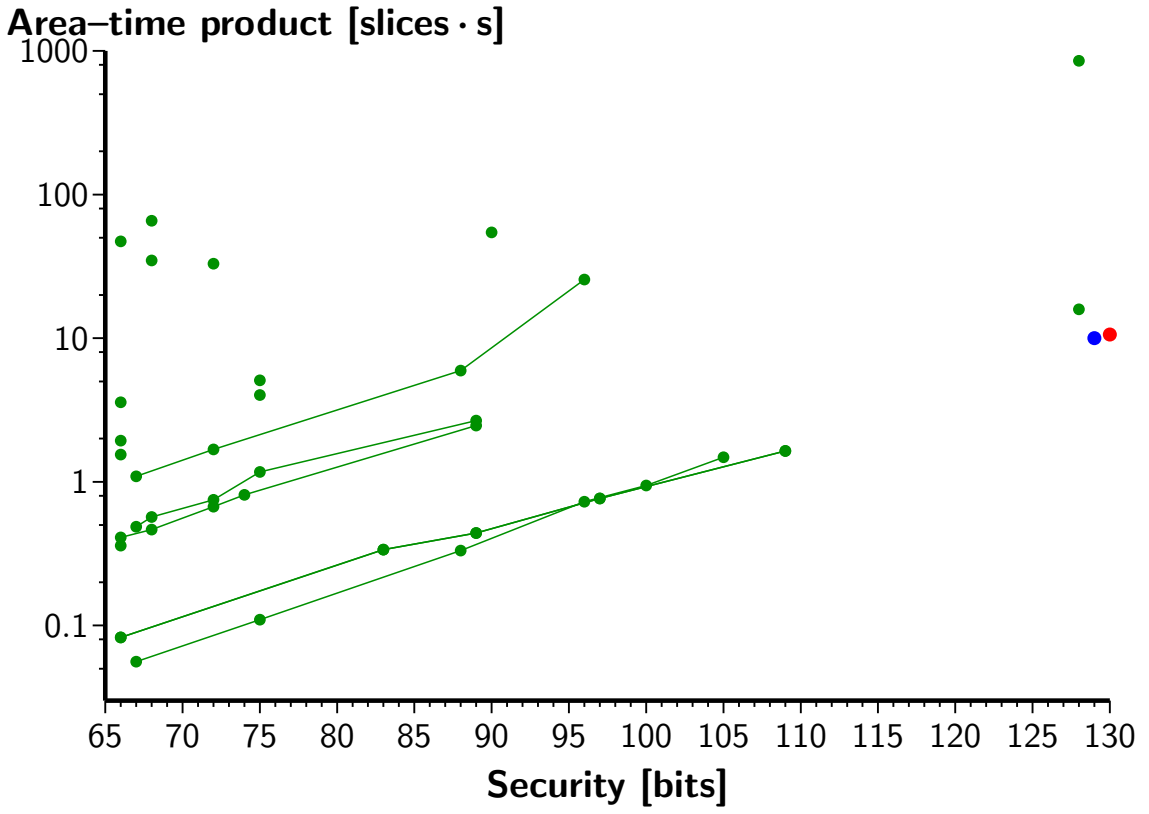

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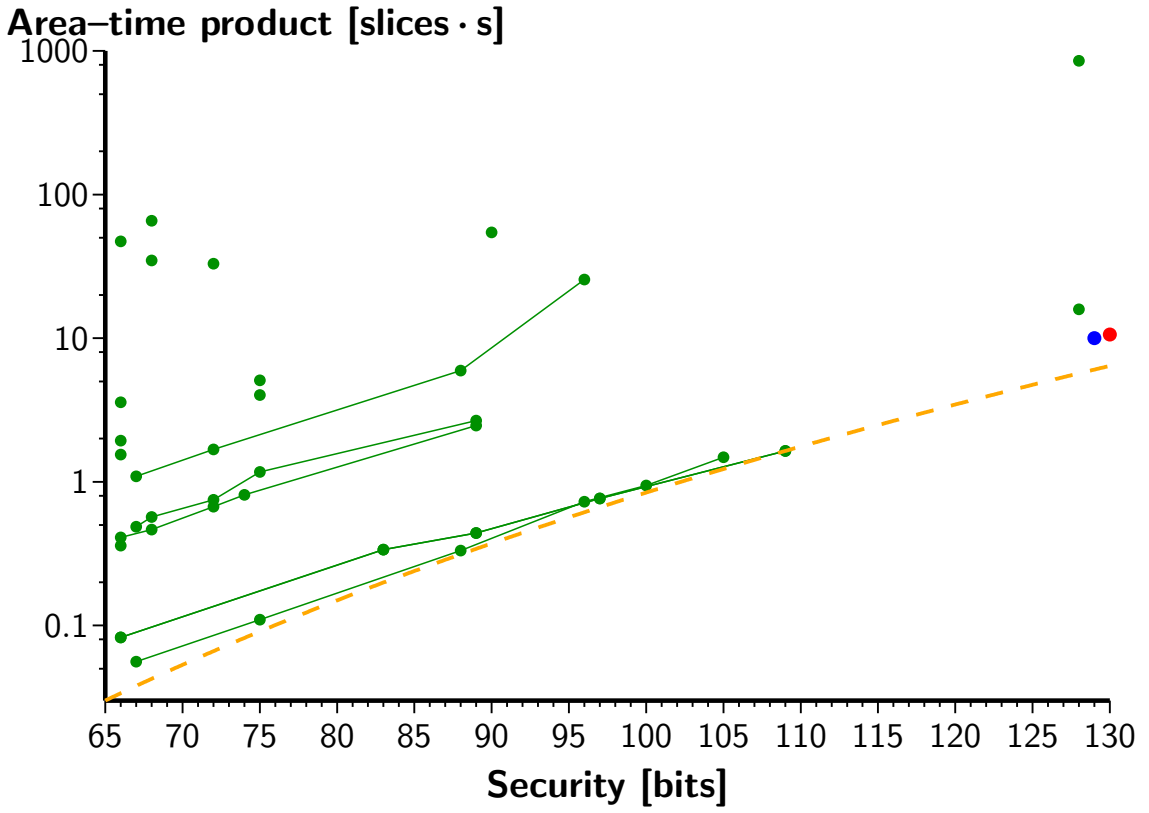

Comparison with ASIC and software

	Supersingular curves	BN-curves
FPGA	2.11 ms (This Work)	52 ms (Ghosh *et al.*, 2010)
ASIC	–	2.91 ms (Fan *et al.*, 2009)
Software (2.4 GHz Intel Core2)	7.59 ms (Beuchat *et al.*, 2009)	0.92 ms (Aranha *et al.*, 2010)

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- Compact, yet reasonably fast, accelerator for pairings with 128 bits of security
  - supersingular elliptic curve
  - low characteristic
  - take advantage of the sub-optimal k to implement efficient field arithmetic
- Implement this pairing on more curves:
  - better understanding of the software/hardware frontier
  - hopefully improve performance
  - try higher security level
  - study genus-2 supersingular curves

Thank you for you attention!

Questions?