CINVESTAV-IPN, November 2009

Fast hardware accelerator for the Tate pairing based on a fully parallel Karatsuba multiplier

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Eiji Okamoto	LCIS, University of Tsukuba, Japan
Francisco Rodríguez-Henríquez	CINVESTAV-IPN, Mexico City, Mexico

Outline of the talk

- Context
- Reduced Tate pairing
- Non-reduced Tate pairing
- Fully parallel Karatsuba multiplier
- Final Exponentiation
- Results & Conclusion
- Appendix

Pairing-based cryptography

- Origin of pairings in cryptography
  - attack against some elliptic curves
    - Menezes–Okamoto–Vanstone, 1993
    - Frey–Rück, 1994

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- Constructive properties
  - One-round three party key exchange
    - Joux, 2000
  - short digital signature
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  - identity-based encryption
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  - ...
- Standardization in progress
  - ISO/IEC 14888-3
  - IEEE P1363.3

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  - \+ easier arithmetic on the curve
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- Small characteristic
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Pairing is a bilinear map

- $\mathbb{G}_1 = \langle P \rangle$: additively-written cyclic group of prime order $\#\mathbb{G}_1 = \ell$
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- $\mathbb{G}_2$: multiplicatively-written cyclic group of order $\#\mathbb{G}_2 = \#\mathbb{G}_1 = \ell$
- $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$ is a bilinear pairing iff:
  - non-degeneracy: $\hat{e}(P, P) \neq 1_{\mathbb{G}_2}$
  - bilinearity:
    - $\hat{e}(Q_1 + Q_2, R) = \hat{e}(Q_1, R) \cdot \hat{e}(Q_2, R)$
    - $\hat{e}(Q, R_1 + R_2) = \hat{e}(Q, R_1) \cdot \hat{e}(Q, R_2)$
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- Important property for cryptographic applications:

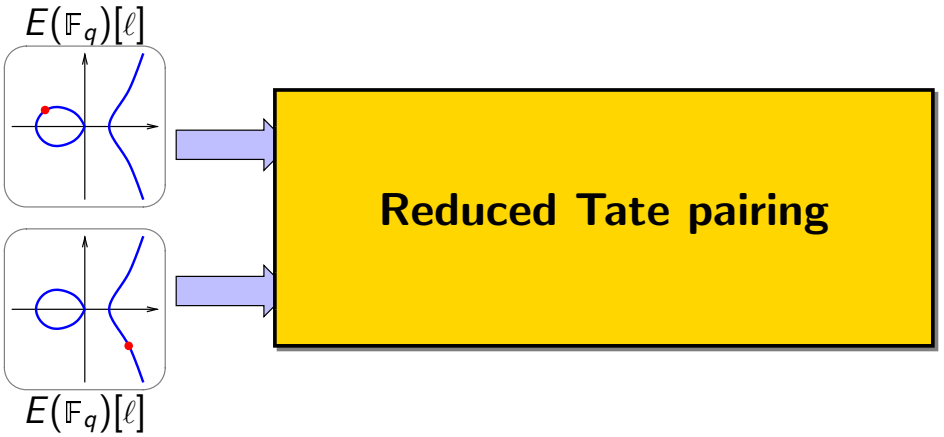
$$\hat{e}(k_1 P, k_2 P) = \hat{e}(k_2 P, k_1 P) = \hat{e}(P, P)^{k_1 k_2}$$

Combining secrets without having to reveal them!

Pairing over elliptic curve

Reduced Tate pairing

Pairing over elliptic curve



- Input: two points P and Q in $\mathbb{G}_1 = E(\mathbb{F}_q)[\ell]$, where:
  - $q = p$, 2^m or 3^m
  - E is an elliptic curve over $\mathbb{F}_q$
  - ℓ is a large prime factor of $\#E(\mathbb{F}_q)$
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- Output: an ℓ-th root of unity
  - $\mathbb{G}_2 = \mu_\ell = \left\{U \in \overline{\mathbb{F}_q}^\times | U^\ell = 1\right\}$
  - k is the embedding degree: the smallest integer such that $\mu_\ell \subseteq \mathbb{F}_{q^k}^\times$

Security considerations

$$\hat{e} : E(\mathbb{F}_q)[\ell] \times E(\mathbb{F}_q)[\ell] \to \mu_\ell \subseteq \mathbb{F}_{q^k}^\times$$

- Security should be enough in $\mathbb{G}_1$ and in $\mathbb{G}_2$
- Supersingular curves: k is bounded
- Some typical cases:

Base field $(\mathbb{F}_q)$	$\mathbb{F}_{2^m}$	$\mathbb{F}_{3^m}$	$\mathbb{F}_p$
Embedding degree (k)	4	6	2
Lower security $(\sim 2^{64})$	$m = 239$	$m = 97$	$\lvert p\rvert \sim 320$
Medium security $(\sim 2^{80})$	$m = 373$	$m = 163$	$\lvert p\rvert \sim 500$
Higher security $(\sim 2^{128})$	$m = 1103$	$m = 503$	$\lvert p\rvert \sim 1400$

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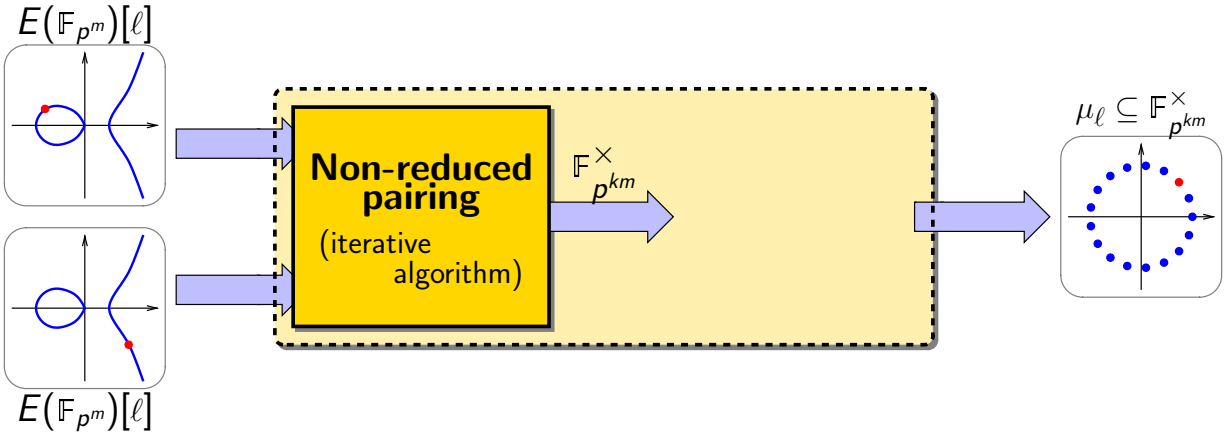
- $\mathbb{G}_2 = \mu_\ell \subseteq \mathbb{F}_{q^k}^\times$ is the bottleneck
- Low characteristic ($p = 2$ or 3) because of higher embedding degree

Reduced Tate pairing

Reduced Tate pairing

- Two very different steps:

Reduced Tate pairing



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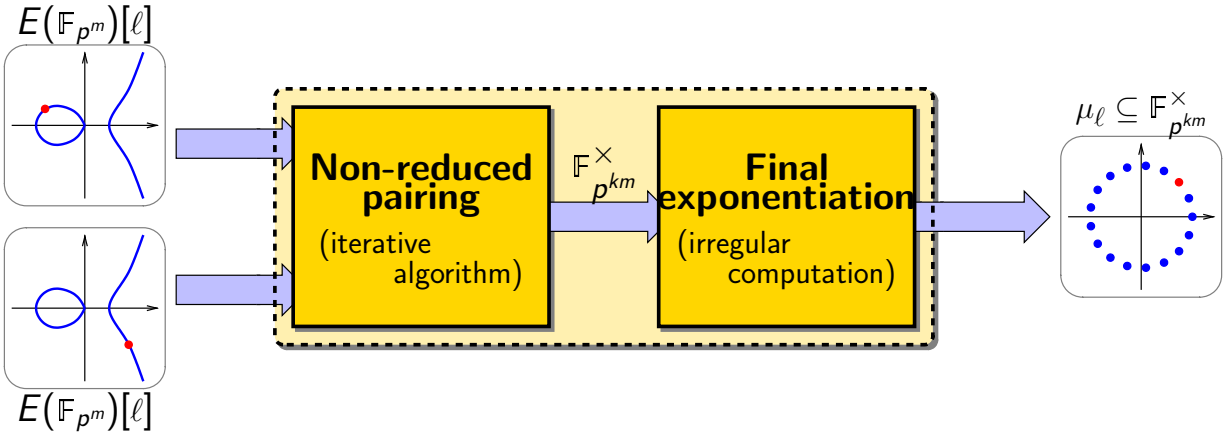

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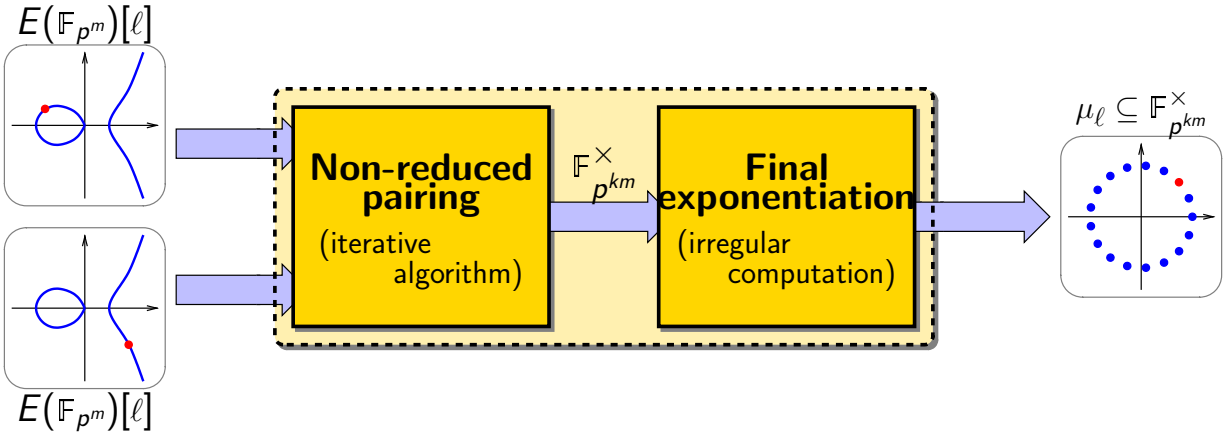

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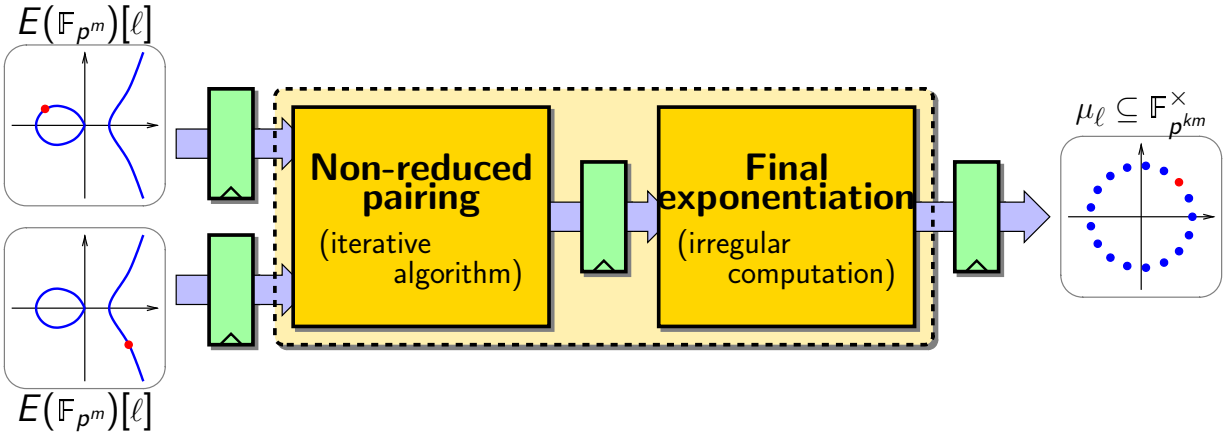

- Two very different steps:
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- Idea: use two separate coprocessors
  - pipeline the two computations
  - balance the latencies

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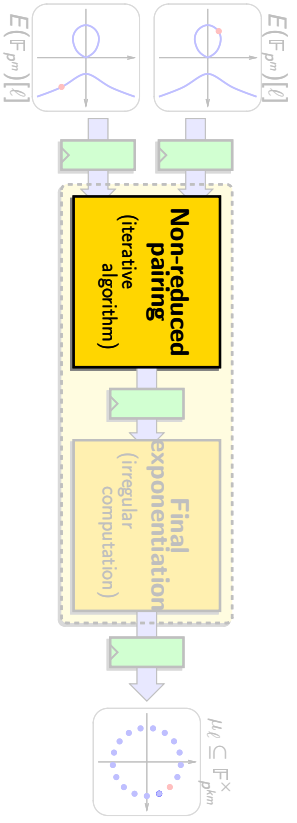

A closer look on Miller's loop (char. 3)

- η_T pairing: shorter loop

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

end for

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- Based on Miller's algorithm:

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

$$x_P \leftarrow \sqrt[3]{x_P} \quad ; \quad y_P \leftarrow \sqrt[3]{y_P}$$
$$x_Q \leftarrow x_Q^3 \quad ; \quad y_Q \leftarrow y_Q^3$$

$$t \leftarrow x_P + x_Q \qquad u \leftarrow y_P y_Q$$
$$S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$$

$$R \leftarrow R \cdot S$$

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- Fully parallel, pipelined multiplier over $\mathbb{F}_{3^m}$

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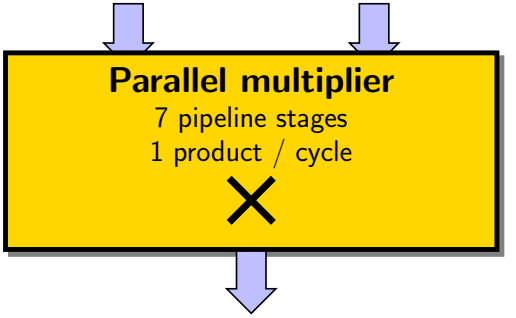
  - $15\times$ and $29+$ over $\mathbb{F}_{3^m}$ (Beuchat *et al.*, ARITH 18)
- Objective: keep the multiplier pipeline busy
  - 7-stages pipeline
  - one product per cycle
  - 17 cycles per iteration

Coprocessor for the non-reduced pairing (char. 3)



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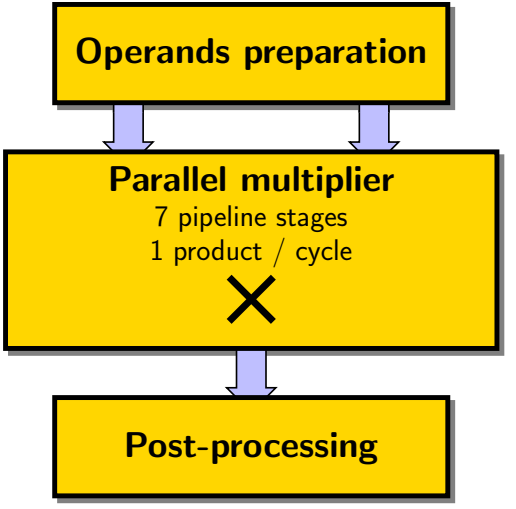

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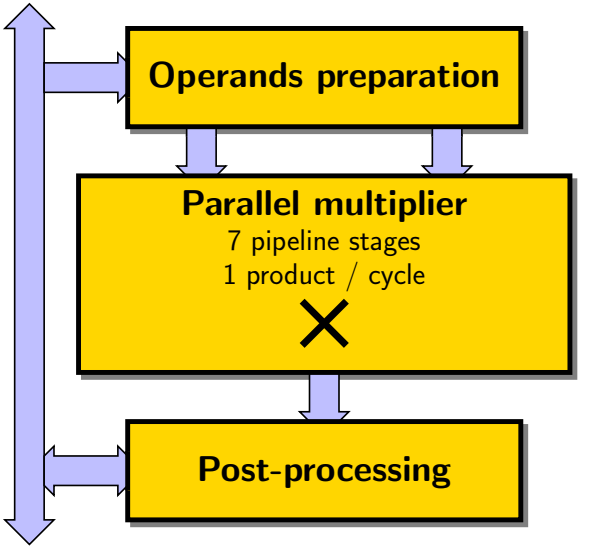

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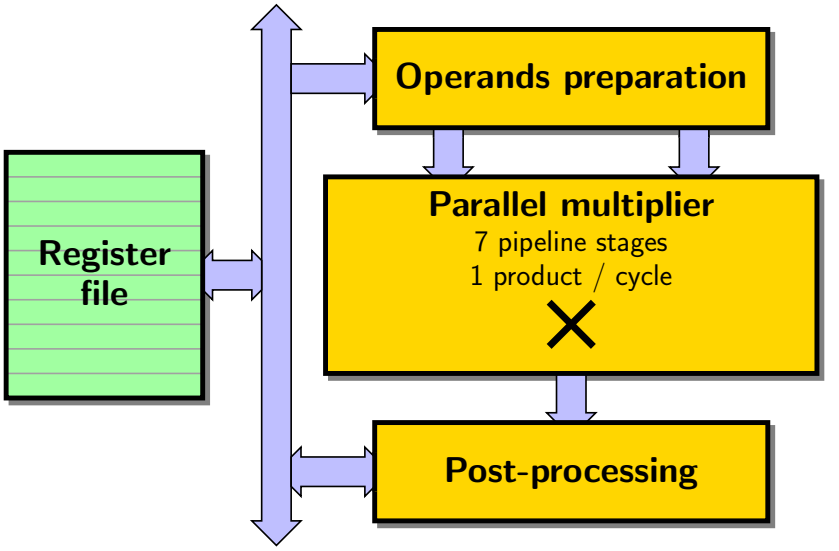

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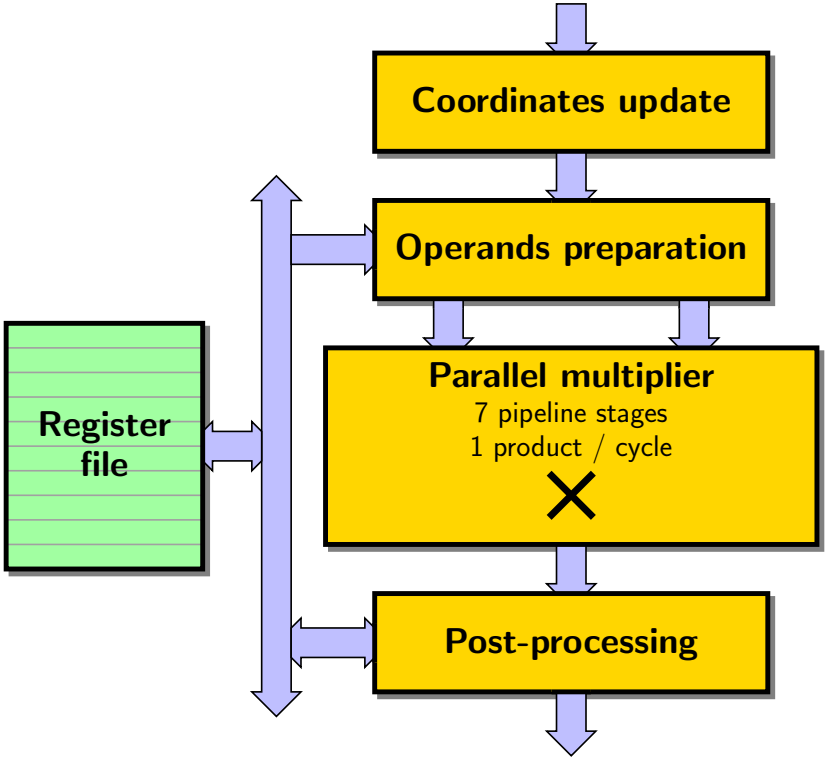
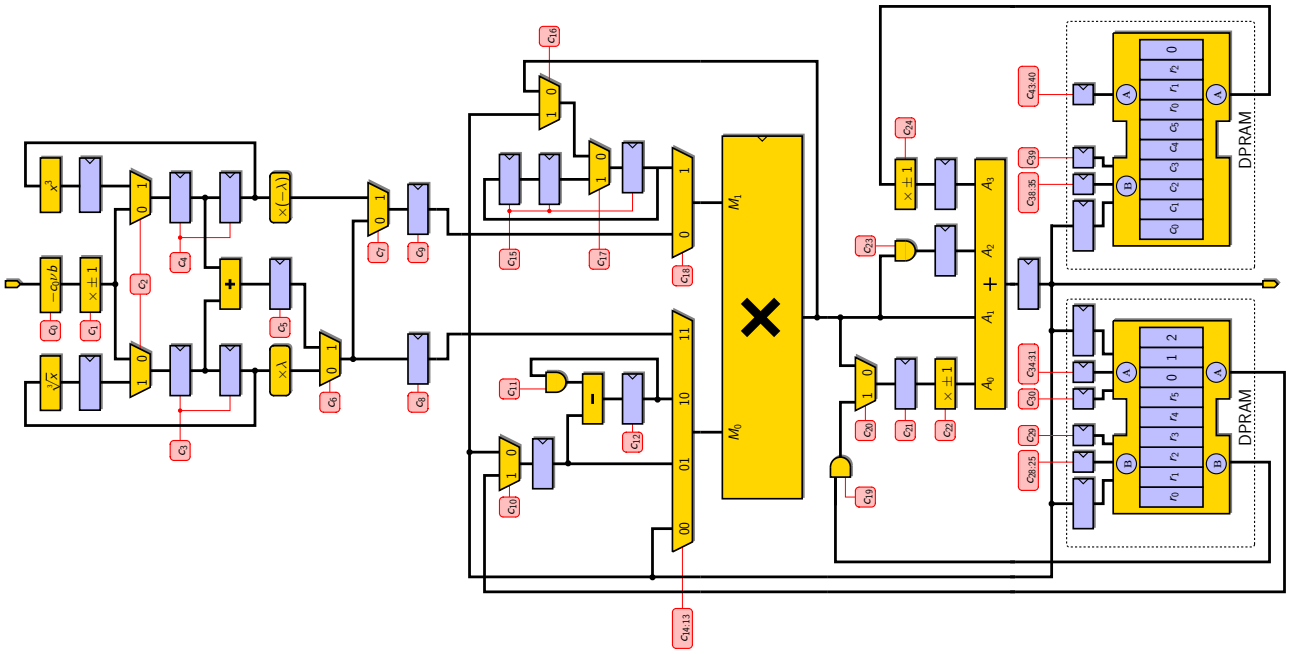


Detailed architecture of the coprocessor (char. 3)



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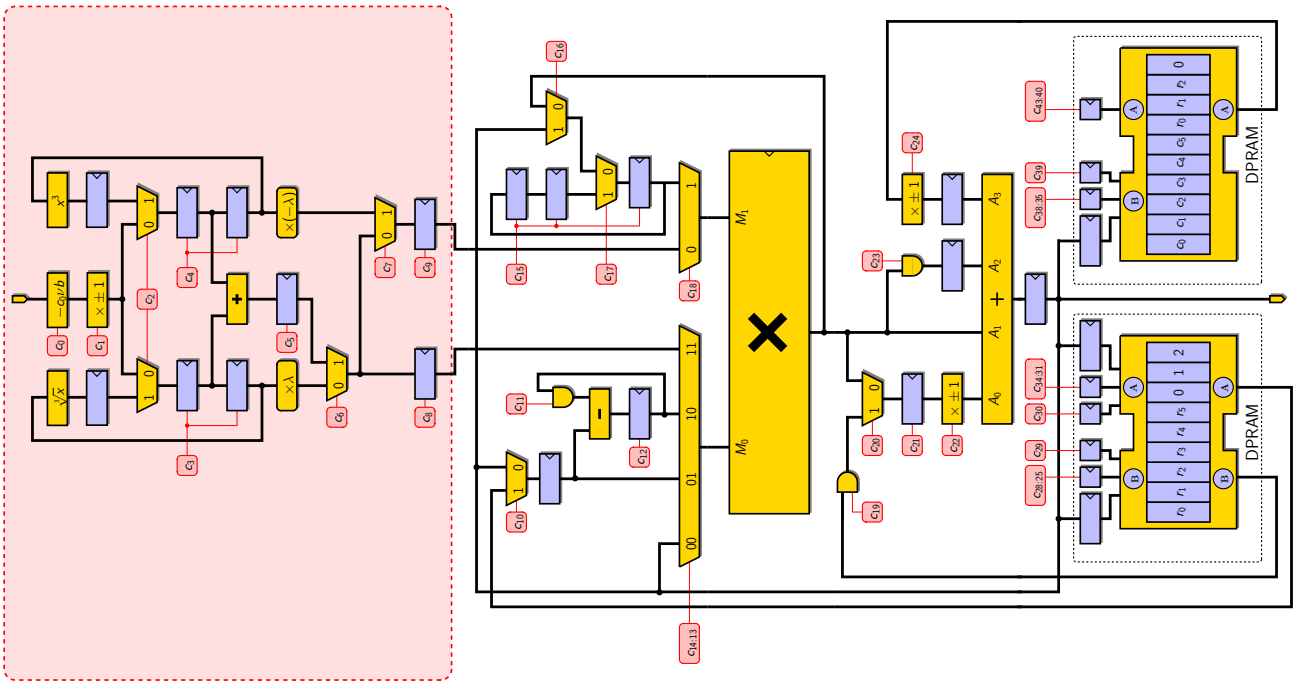

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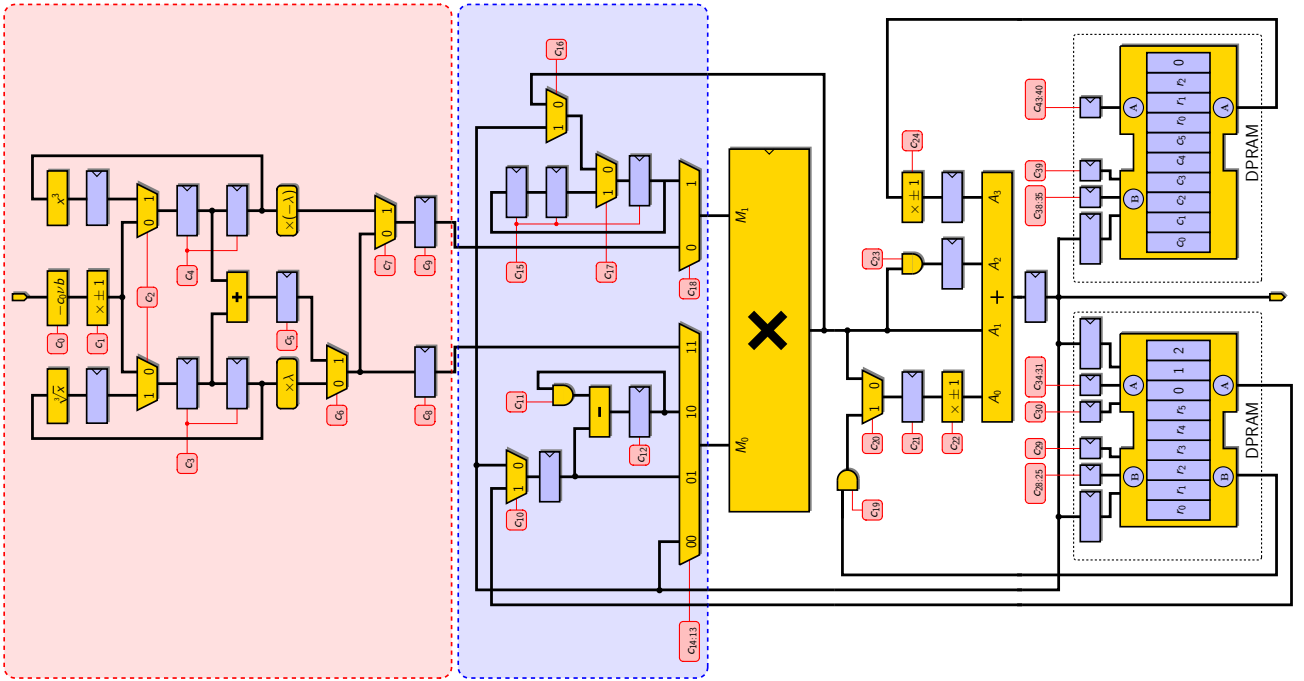

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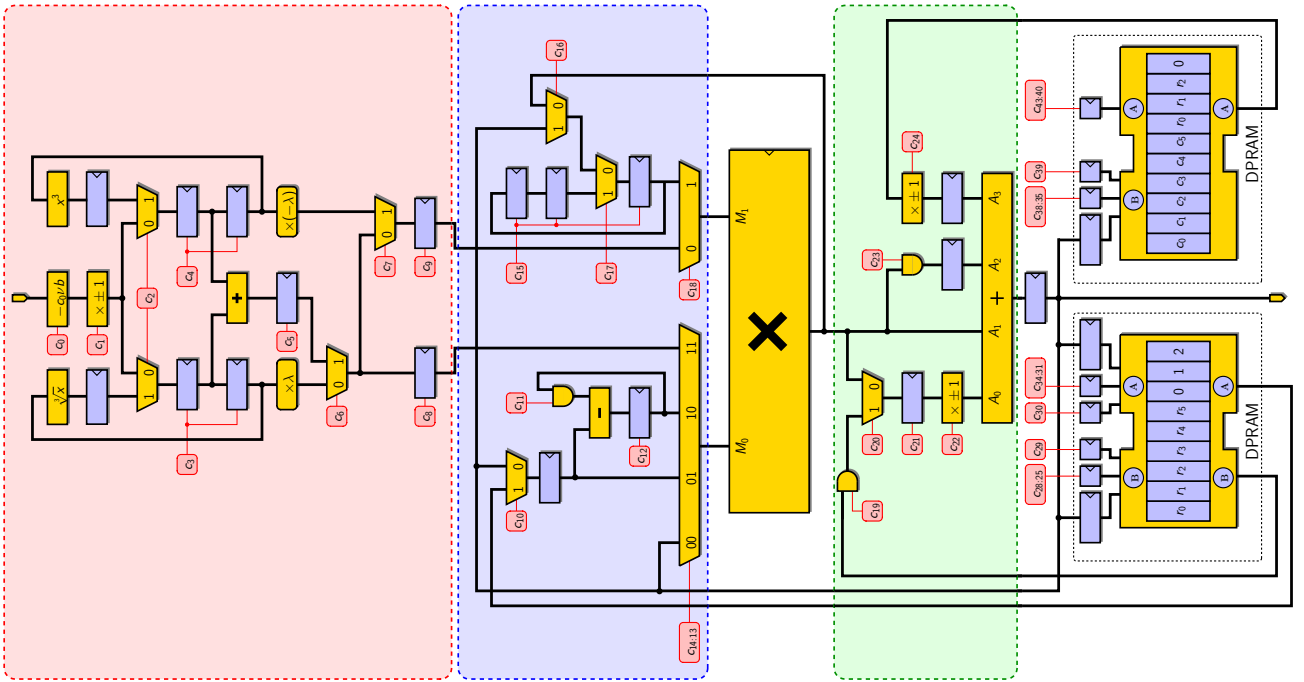

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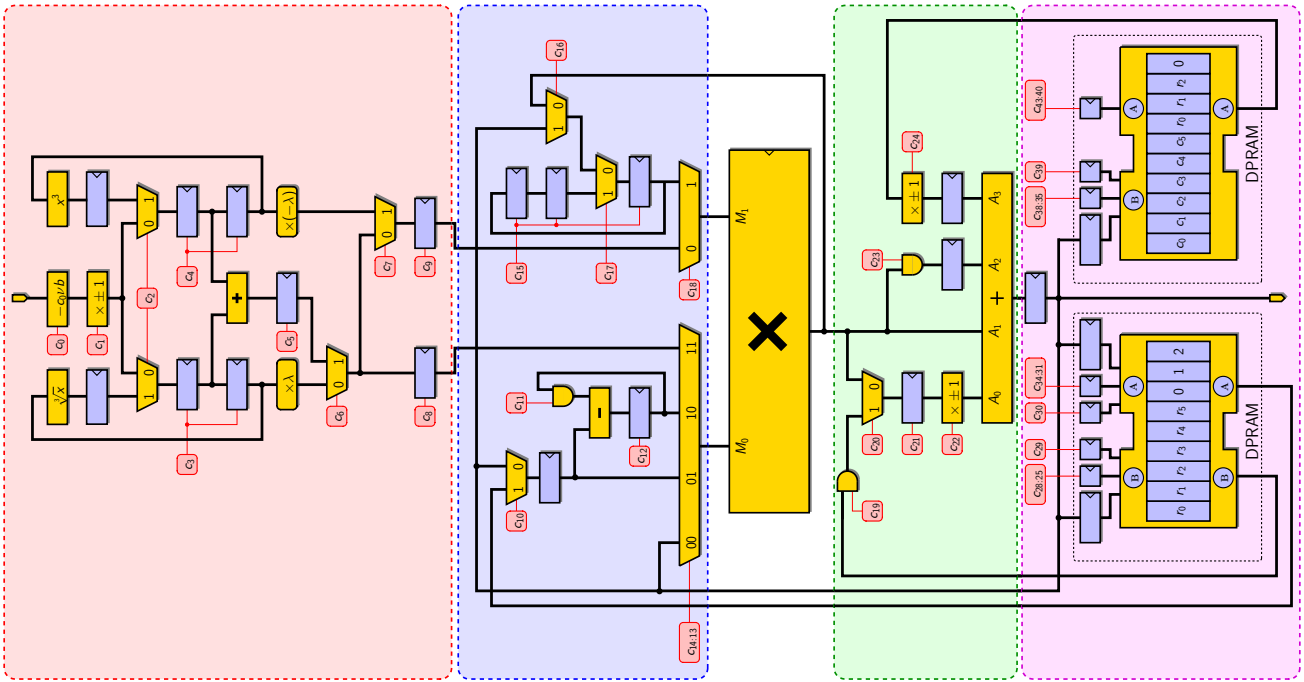
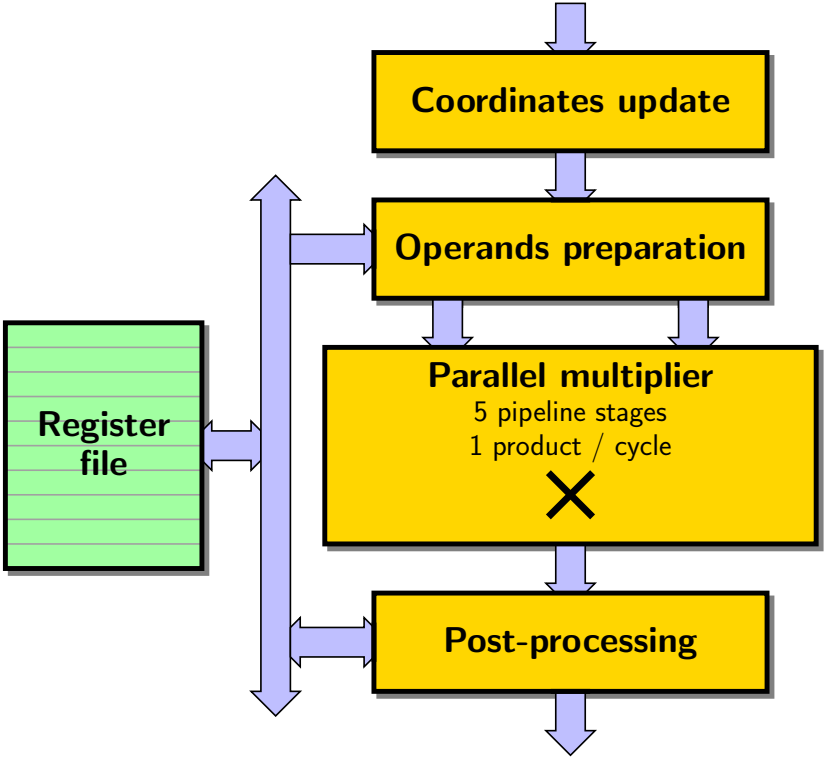


Coprocessor for the non-reduced pairing (char. 2)

- Similar algorithm
  - $\frac{m+1}{2}$ iterations



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- Similar algorithm
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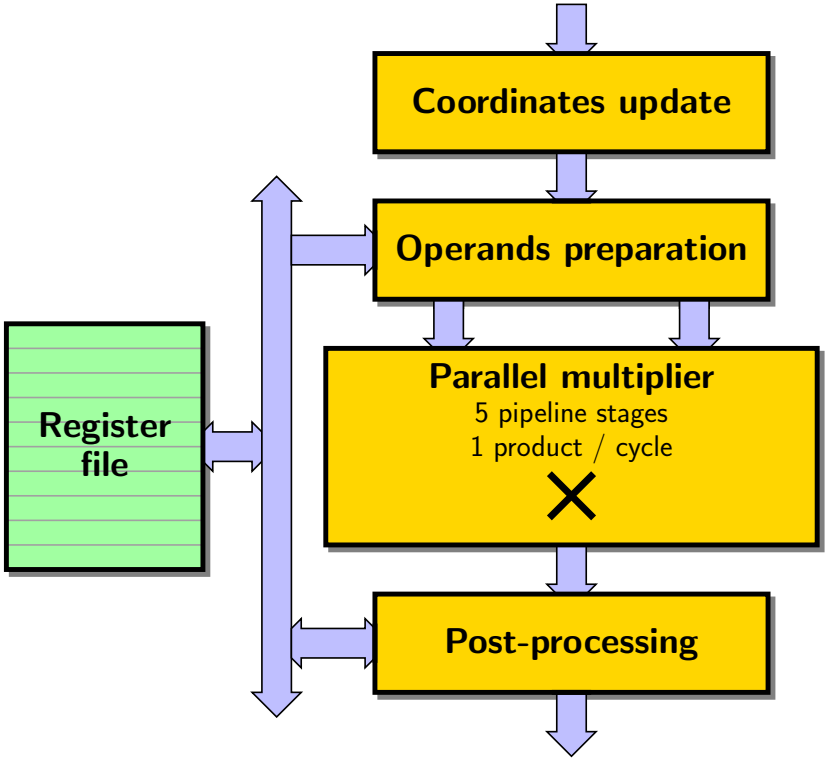
- But some differences:
  - only 6 products over $\mathbb{F}_{2^m}$ per iteration
  - different scheduling
  - 5-stages multiplier pipeline



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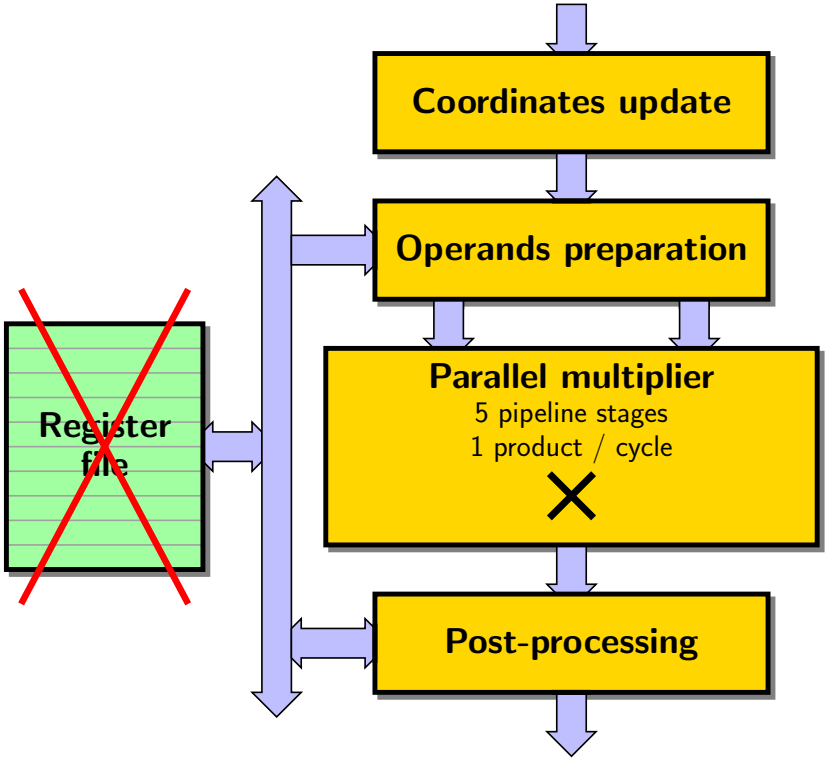
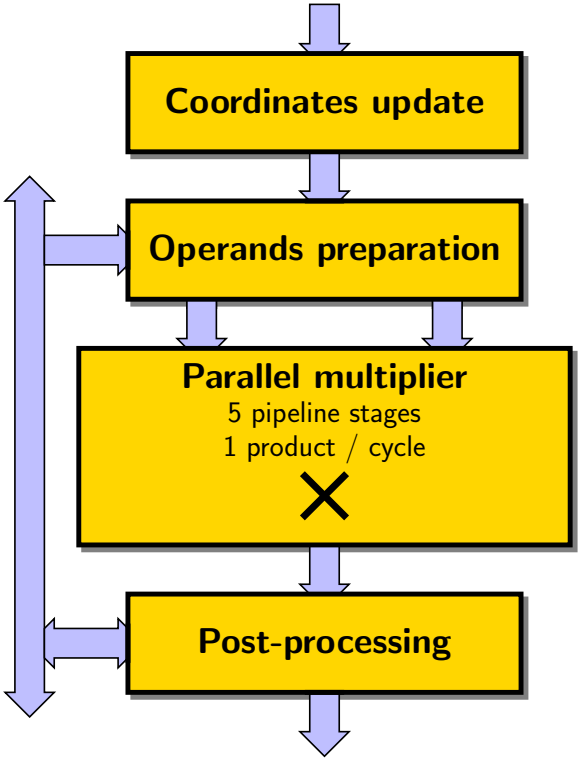
  - $\frac{m+1}{2}$ iterations
- But some differences:
  - only 6 products over $\mathbb{F}_{2^m}$ per iteration
  - different scheduling
  - 5-stages multiplier pipeline
- No need for a register file

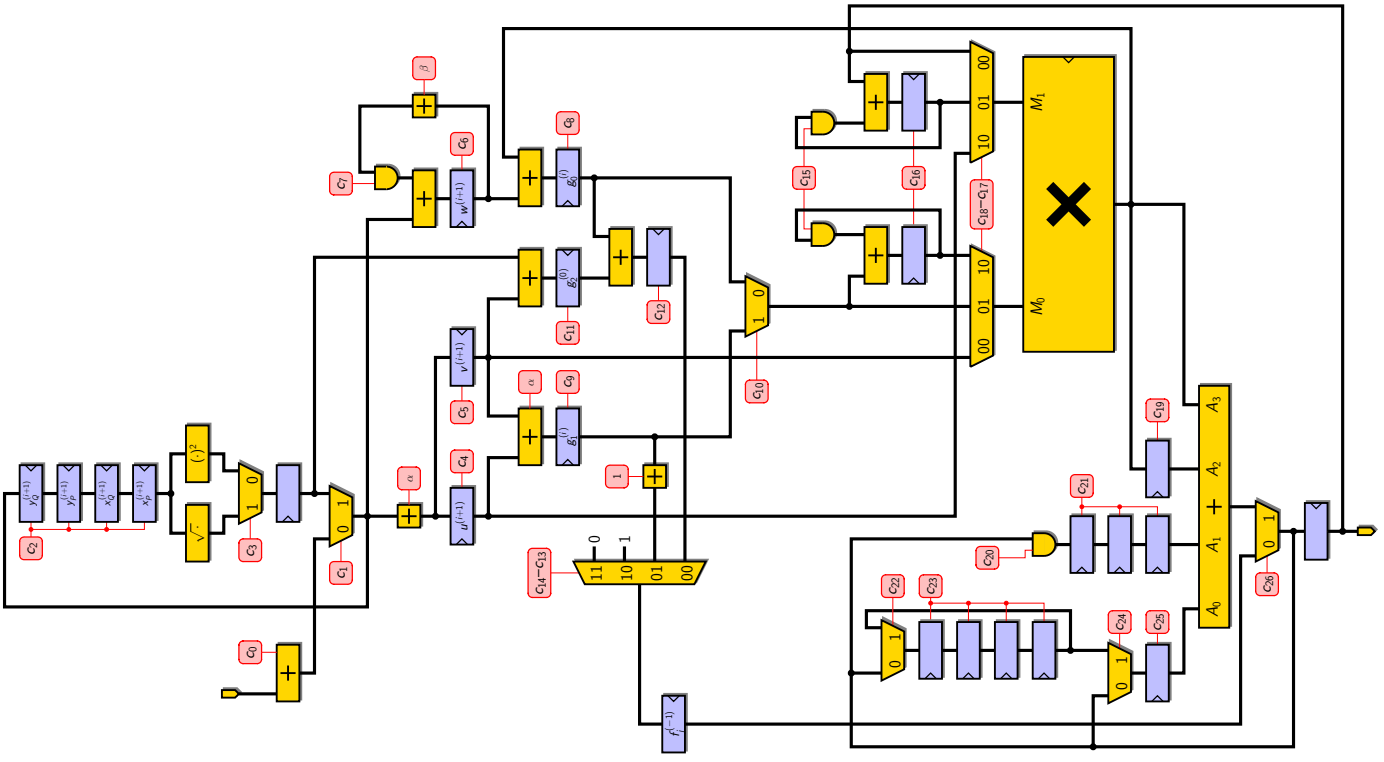


Coprocessor for the non-reduced pairing (char. 2)

- Similar algorithm
  - $\frac{m+1}{2}$ iterations
- But some differences:
  - only 6 products over $\mathbb{F}_{2^m}$ per iteration
  - different scheduling
  - 5-stages multiplier pipeline
- No need for a register file
  - all data in shift register
  - more complex architecture



Detailed architecture of the coprocessor (char. 2)



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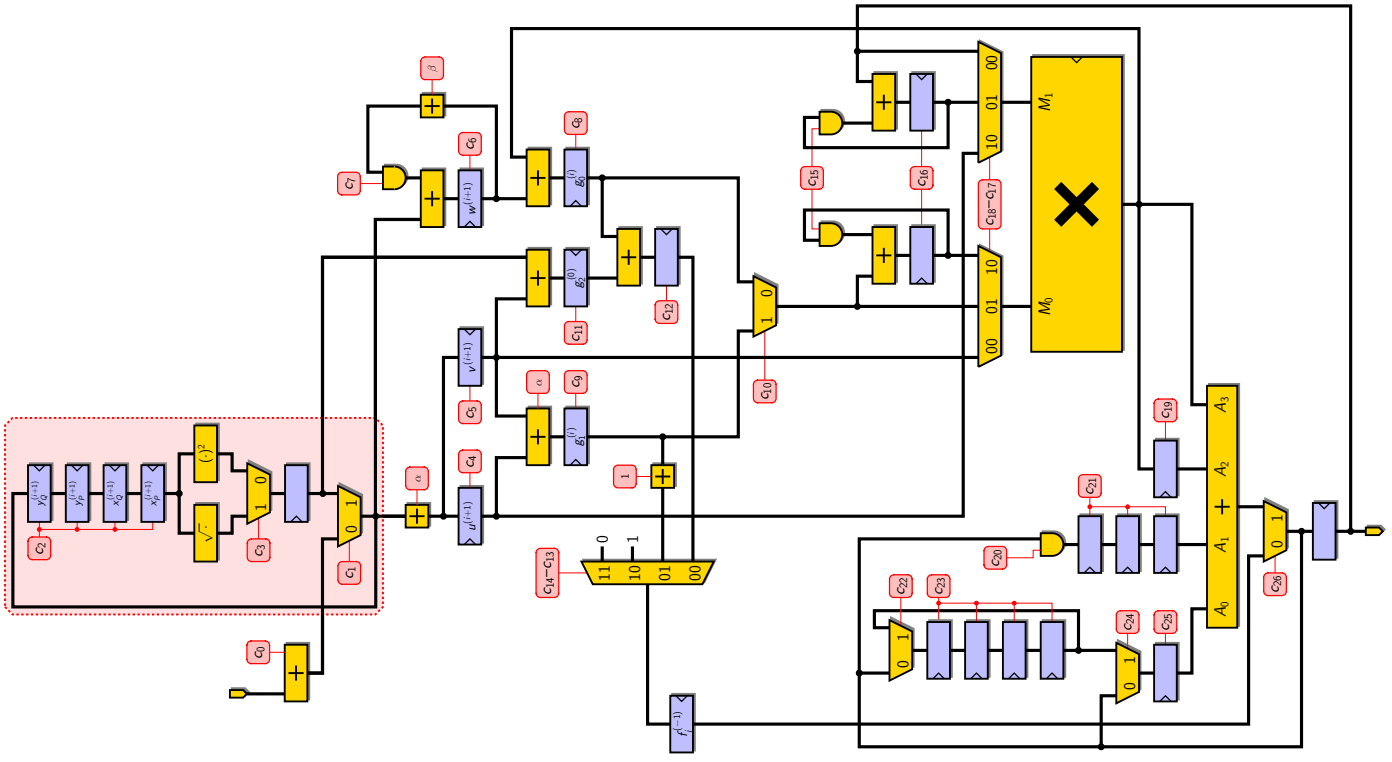

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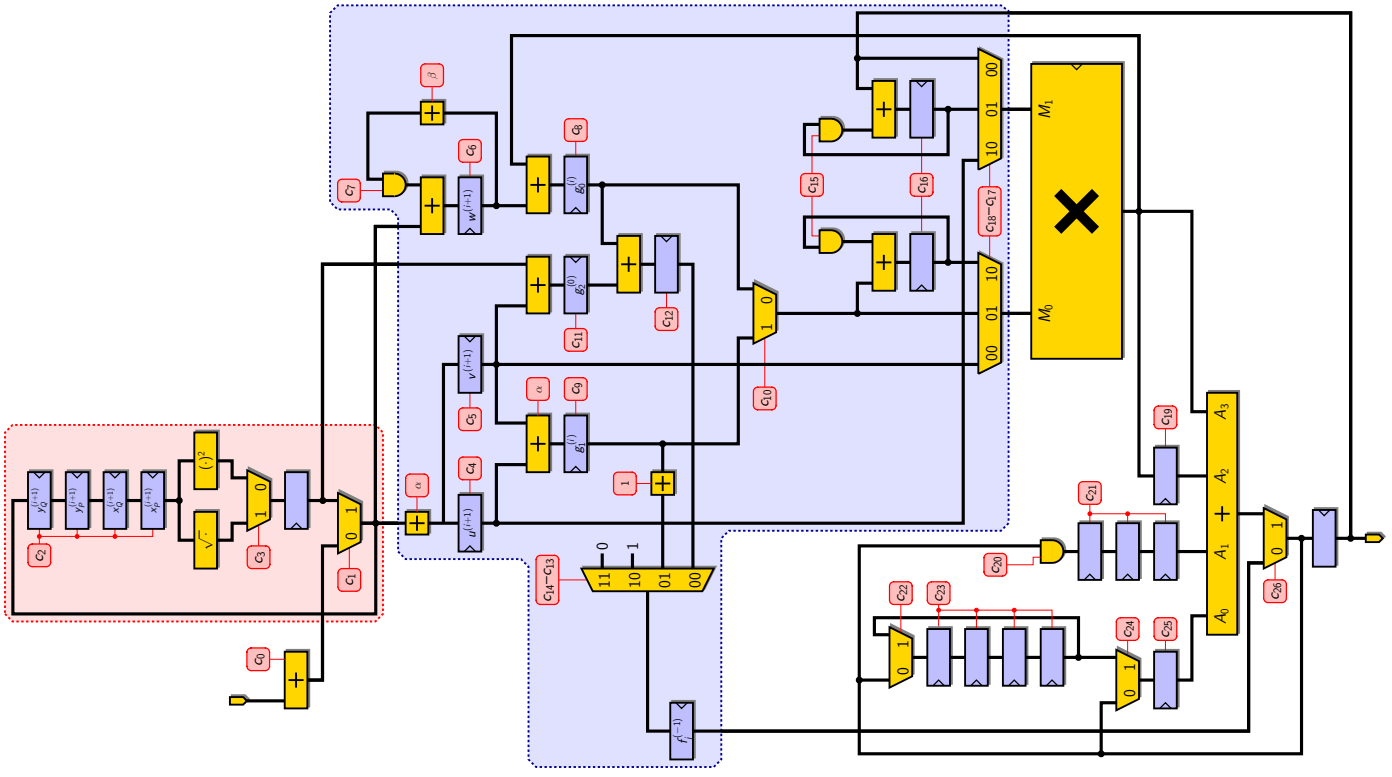

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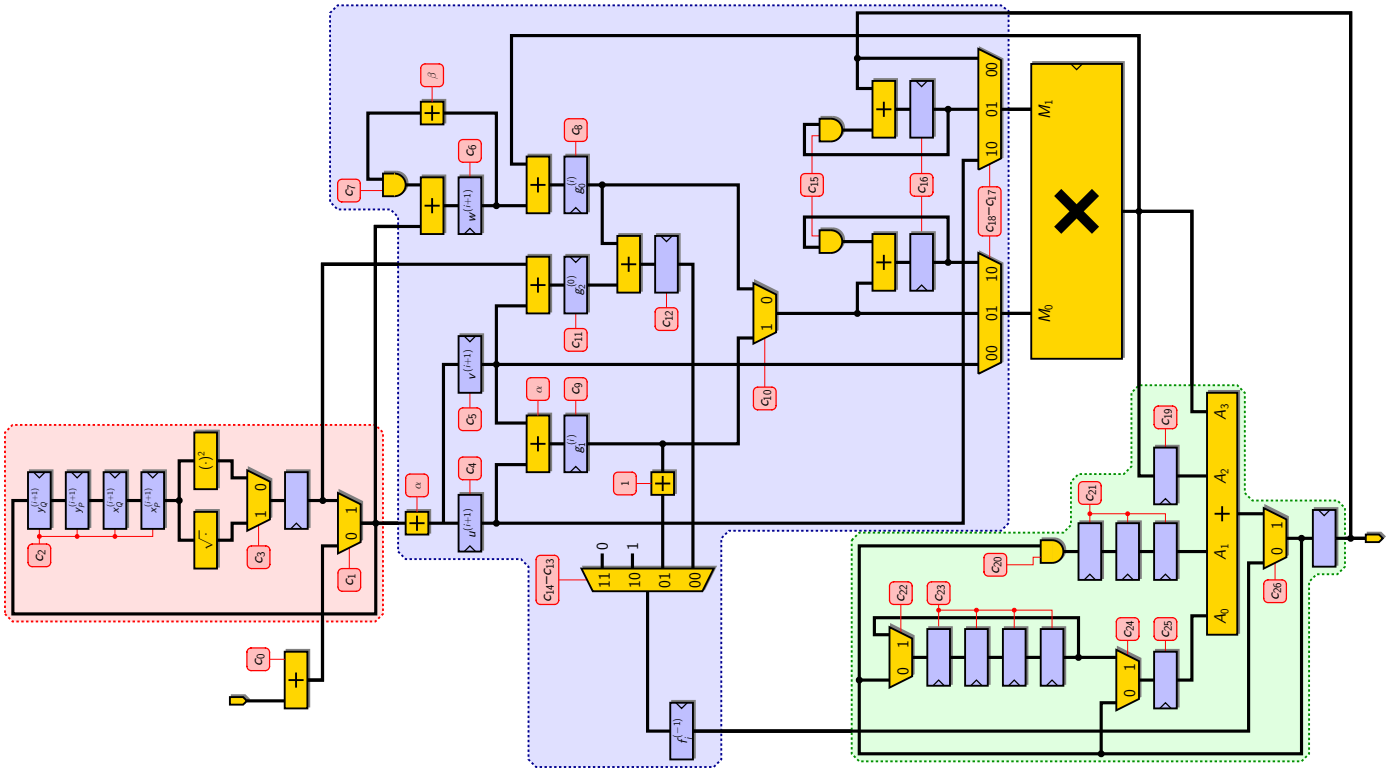

Outline of the talk

- Context
- Reduced Tate pairing
- Non-reduced Tate pairing
- Fully parallel Karatsuba multiplier
- Final Exponentiation
- Results & Conclusion
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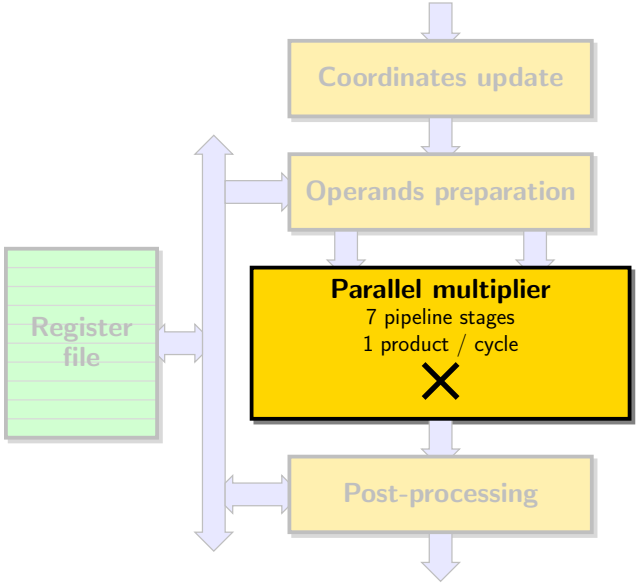

Finite field representation and Karatsuba's formula

- Polynomial basis:
  - $\mathbb{F}_{p^m} \cong \mathbb{F}_p[x]/(f(x))$
  - $f(x)$ irreducible polynomial of degree m in $\mathbb{F}_p[x]$
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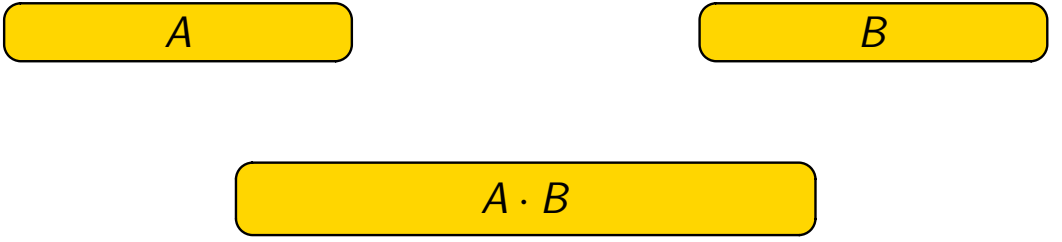

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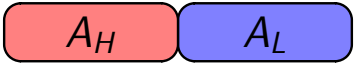

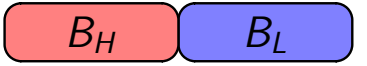

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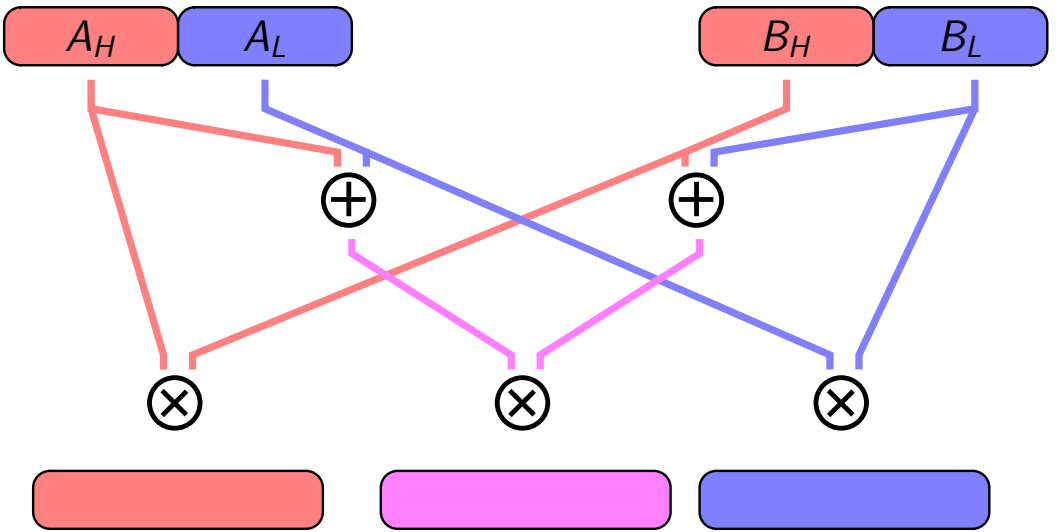

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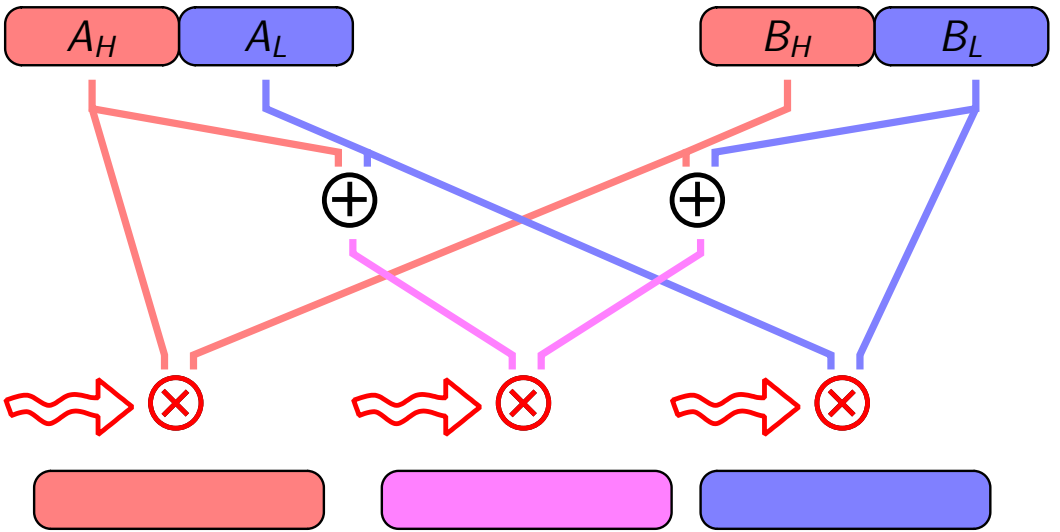

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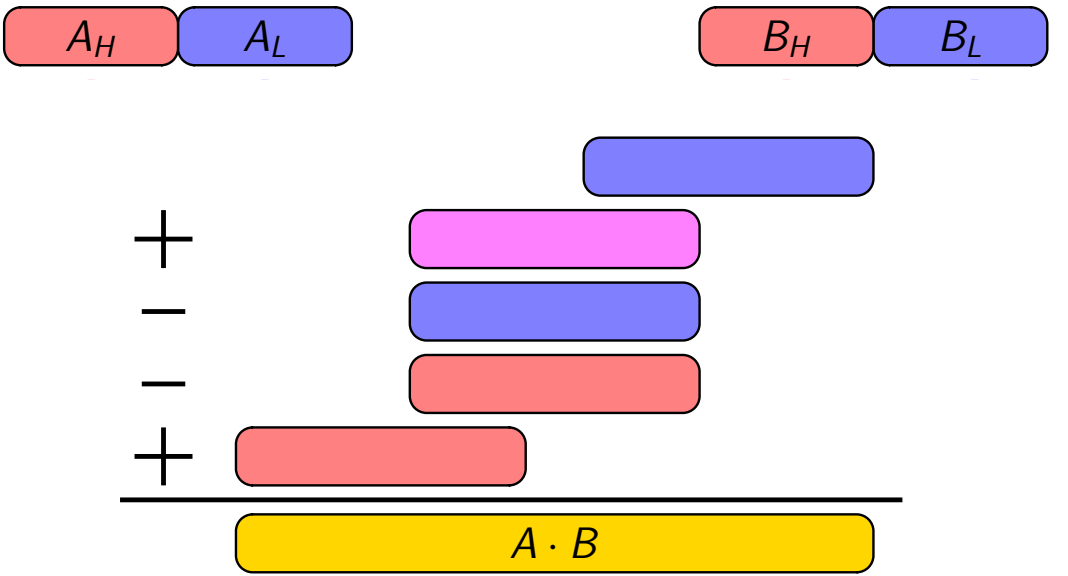
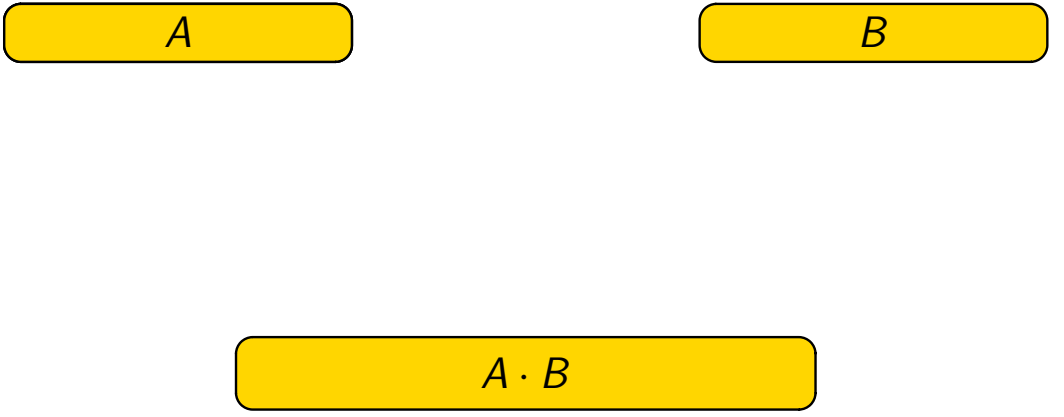


Odd–even split for Karatsuba multiplication



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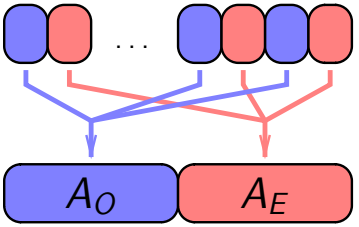

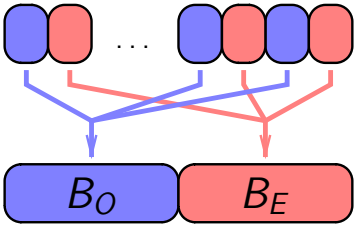

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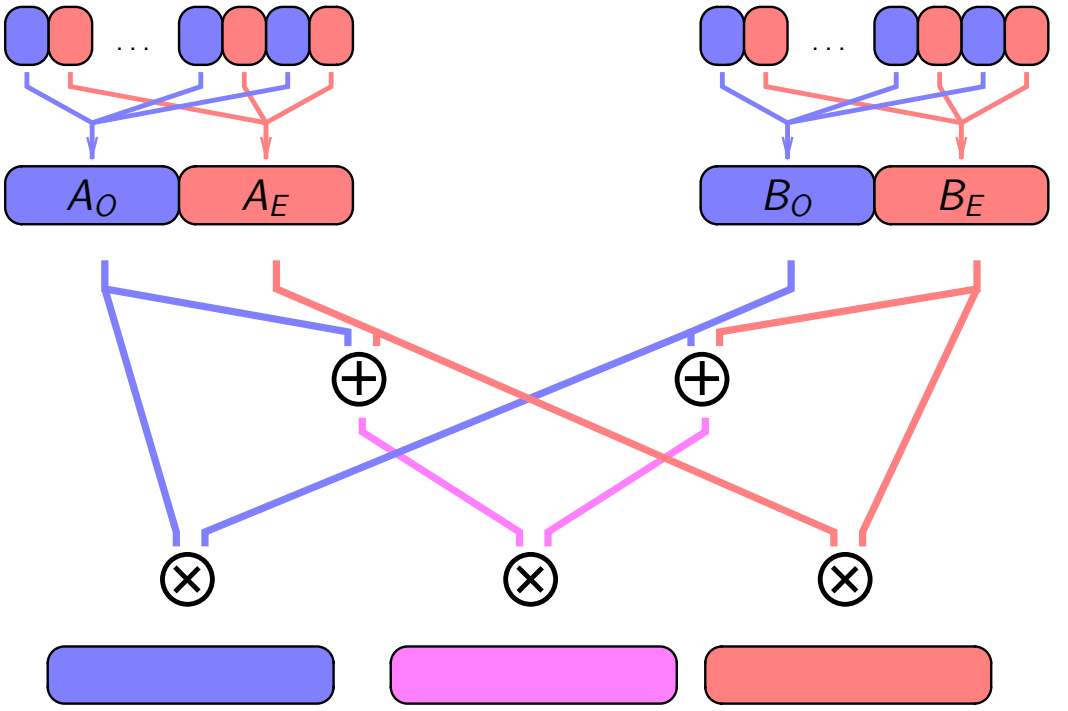

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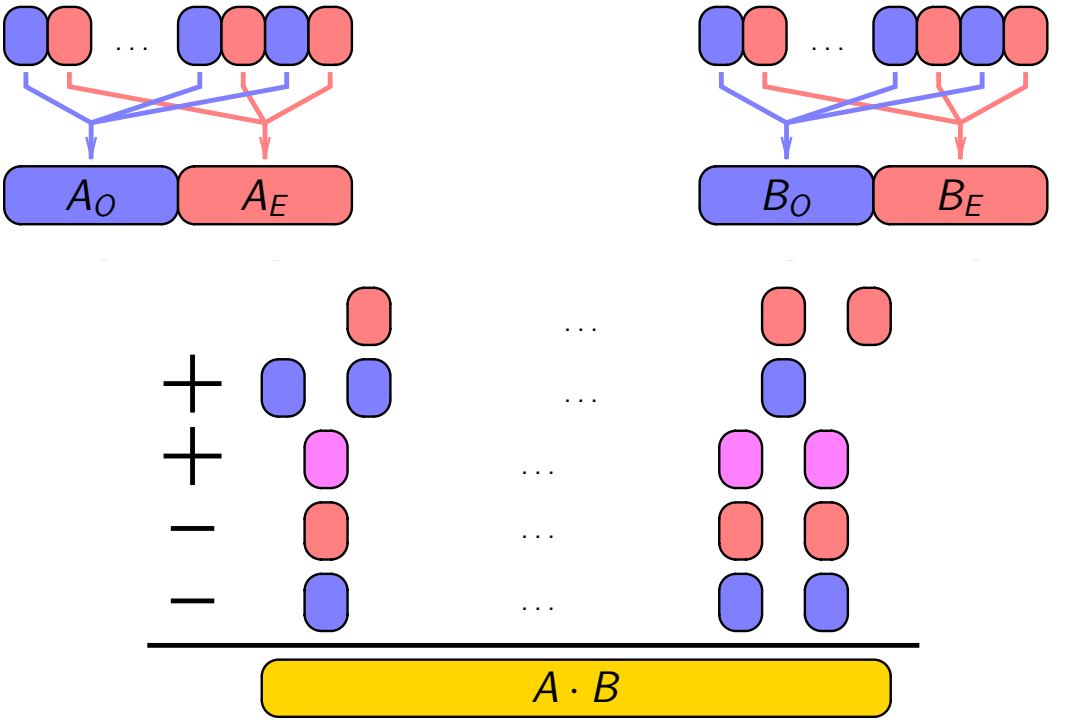

Some other subquadratic multiplication algorithms

- Karatsuba-like algorithms: detailed algorithm
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- Select best method for each stage of recursion

Multiplier architecture

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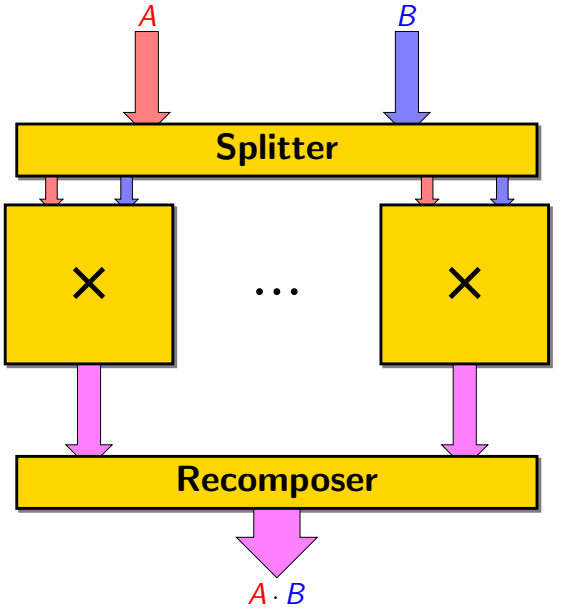

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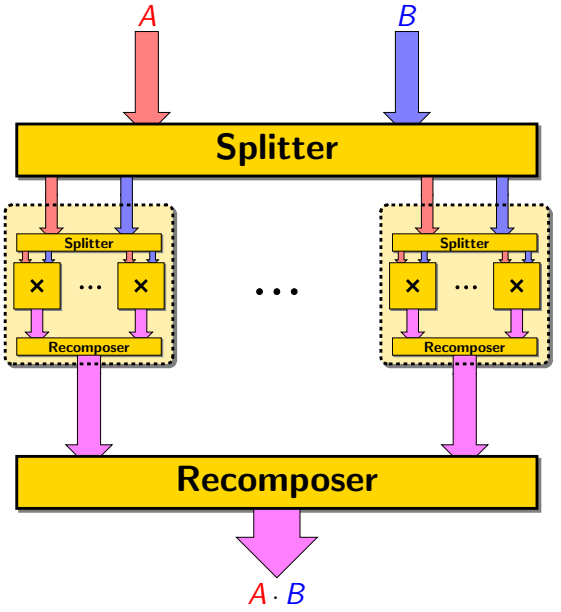

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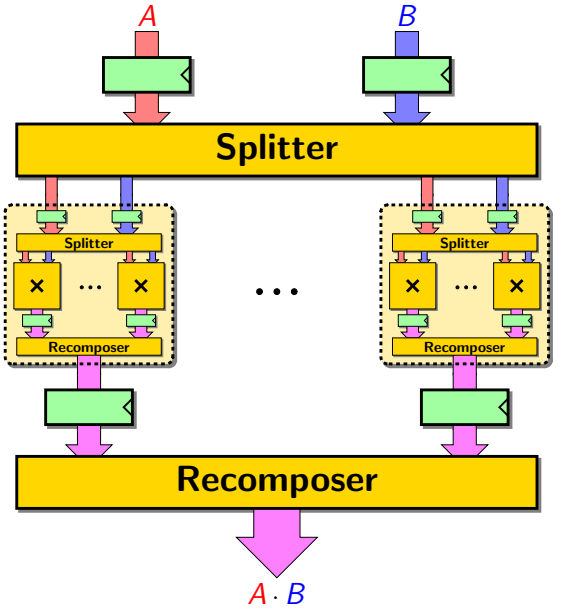

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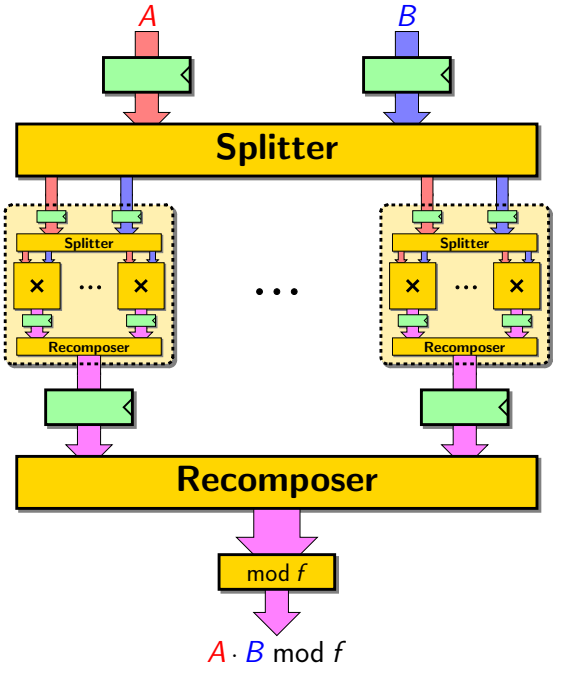
- Final reduction modulo f
  - small operator if f has low Hamming weight



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- Why the odd–even trick does not always work
  - depends also on characteristic and FPGA
  - have to estimate the area in number of LUT not in number of additions

An example of multiplier over $\mathbb{F}_{3^{239}}$

- Polynomial basis

$$\mathbb{F}_{3^{239}} \cong \mathbb{F}_3[X]/(X^{239} - X^5 + 1)$$

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Polynomial size	Used algorithm
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- Post-place-and-route estimation for Xilinx Virtex-II Pro
  - 49984 slices
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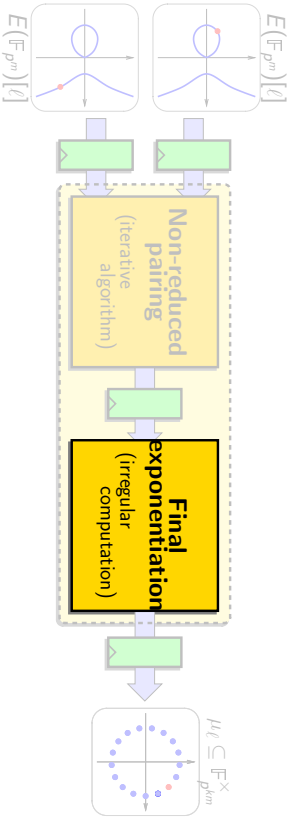
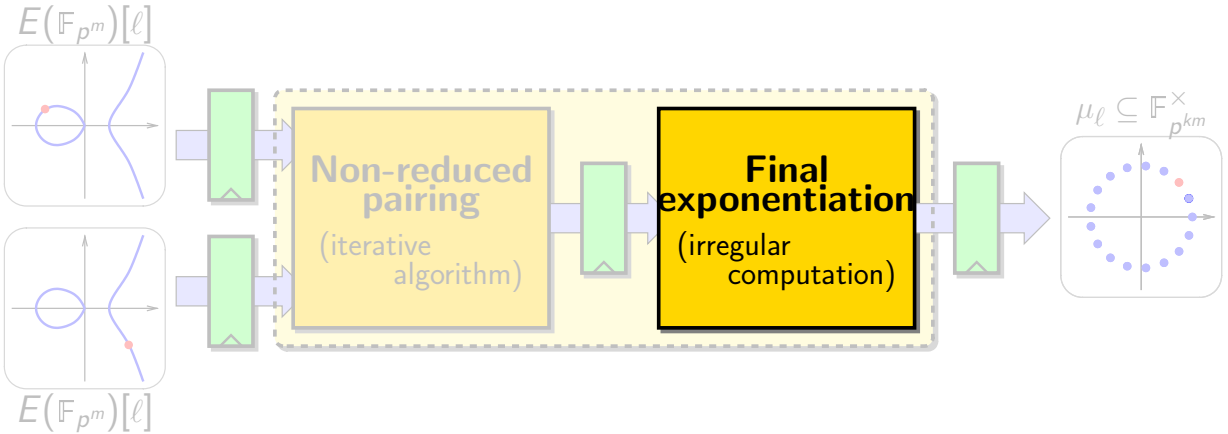


Final exponentiation



- Design rationale:
  - as small as possible
  - at least as fast as the computation of the non-reduced pairing

Coprocessor for the final exponentiation (char. 3)

- Highly sequential computation
- Very heterogeneous

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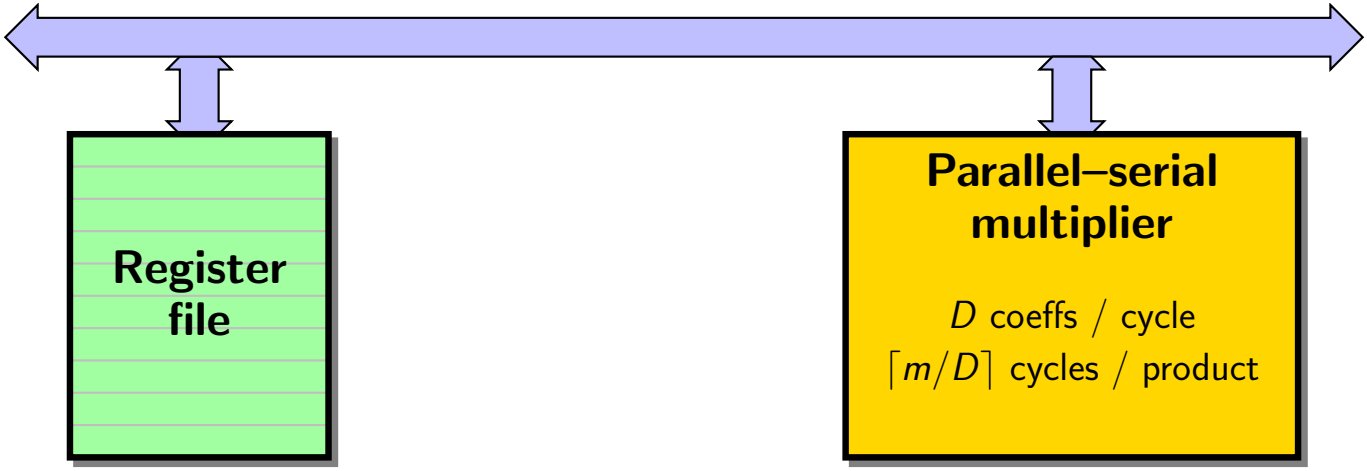

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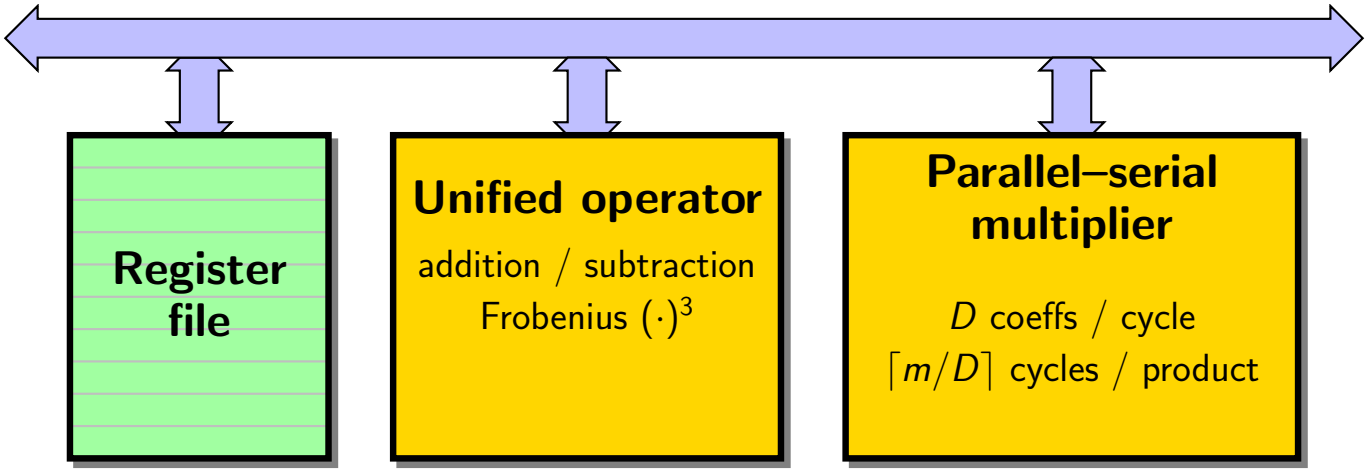

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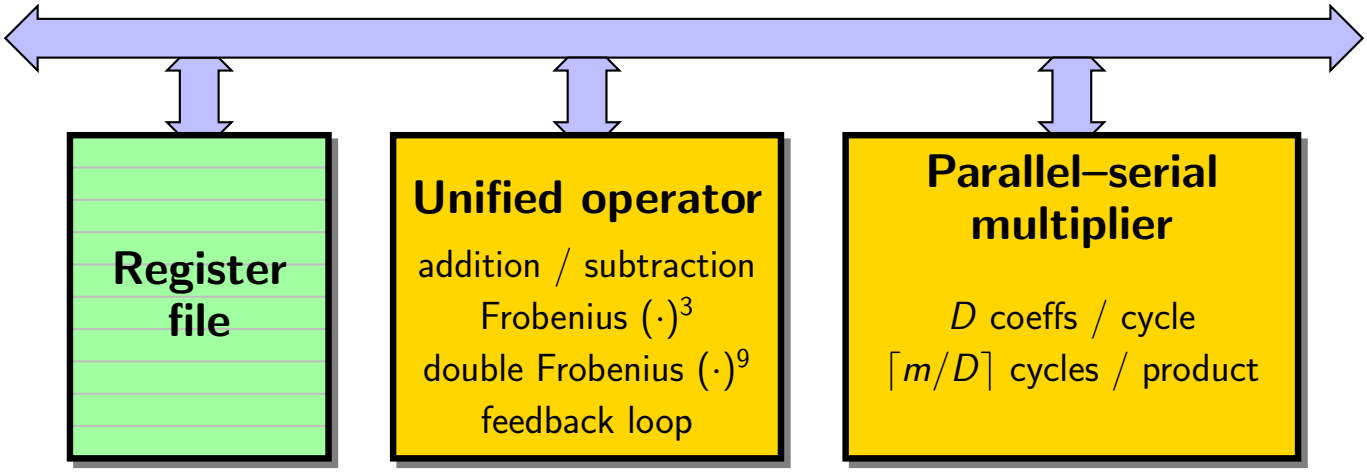

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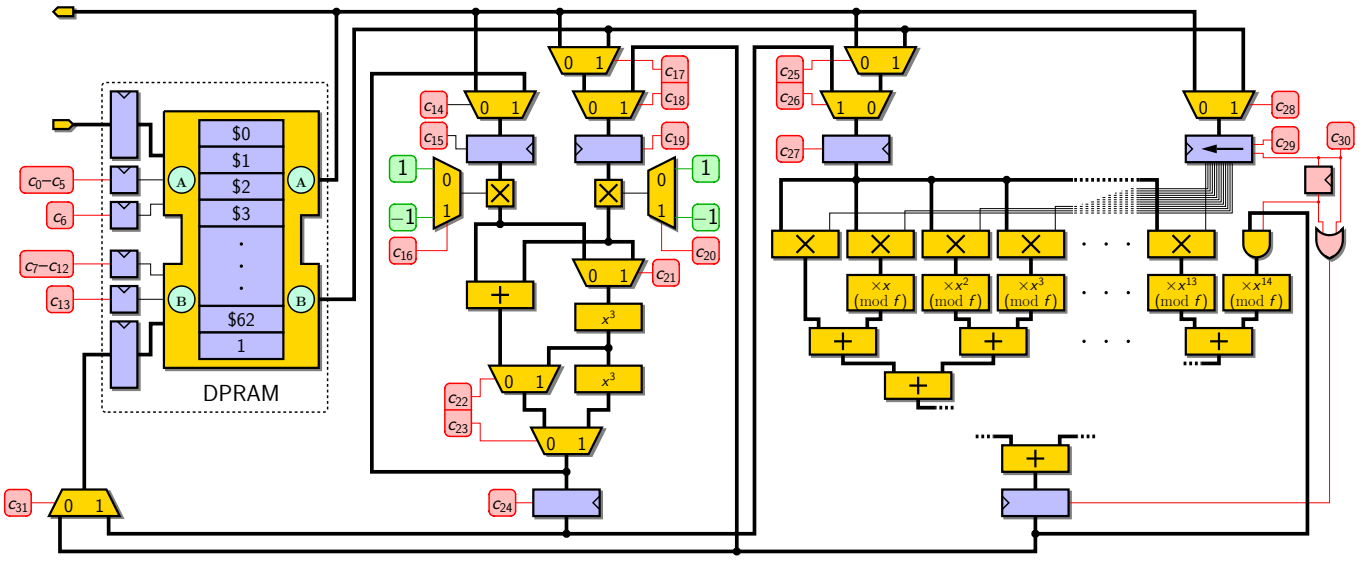

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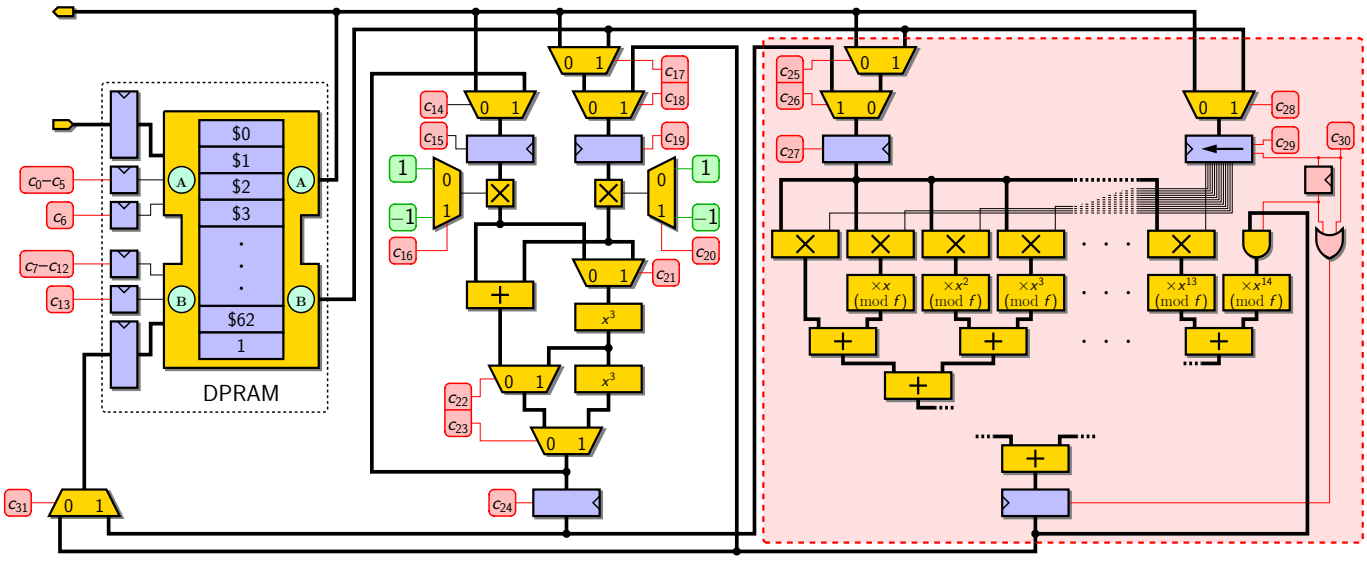

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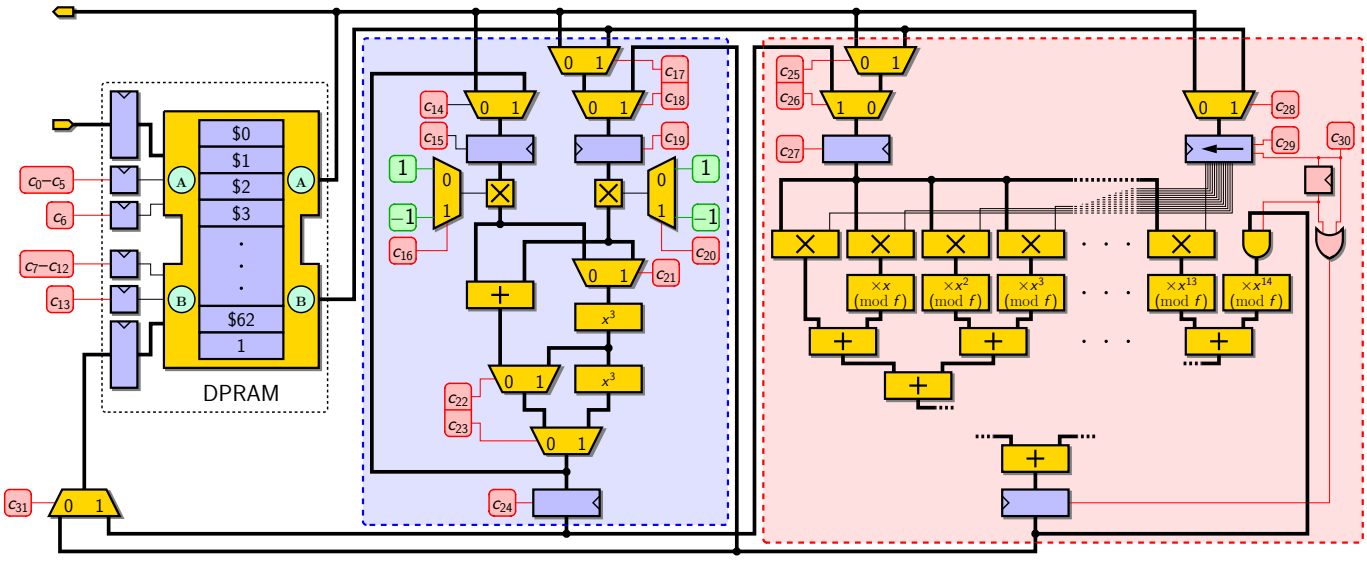

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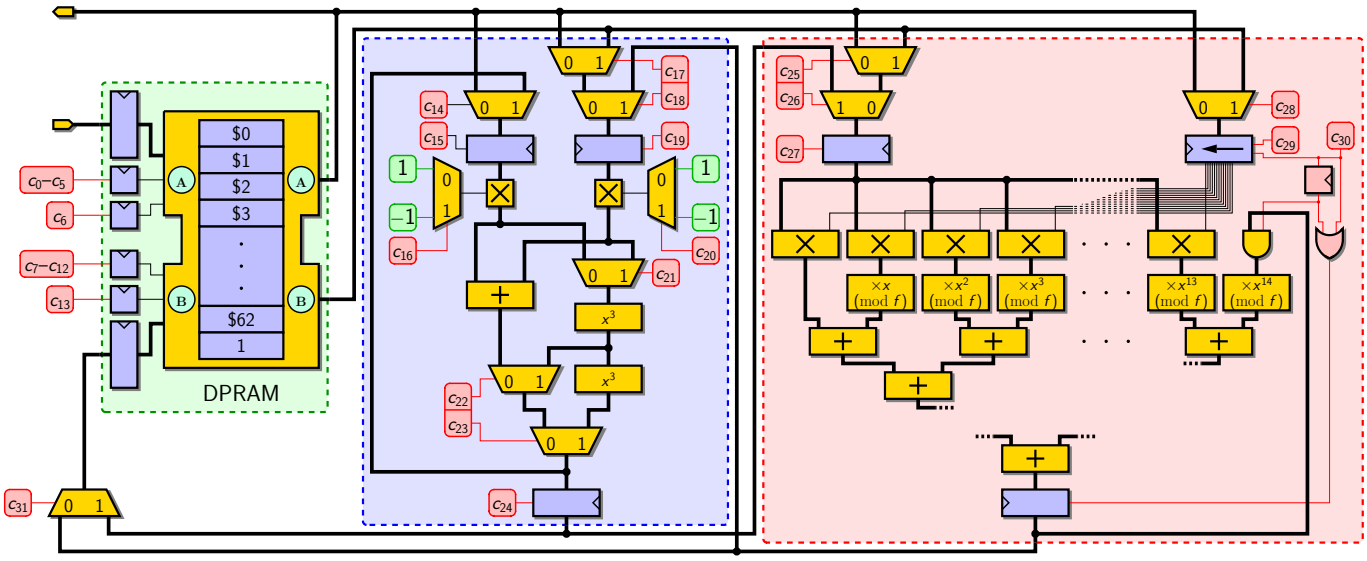

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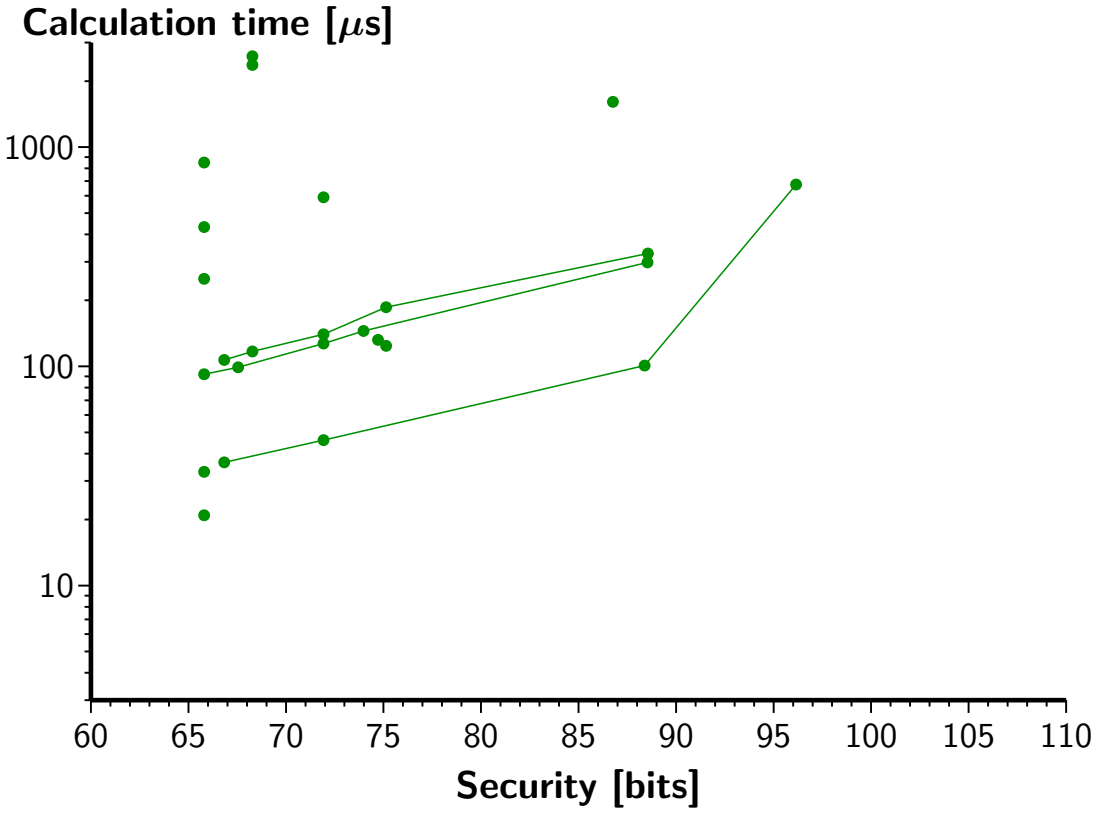
Experimental setup

- Full Tate pairing computation:
  - non-reduced pairing and
  - final exponentiation
- Prototyped on Xilinx Virtex-II Pro and Virtex-4 LX FPGAs
- Post-place-and-route timing and area estimations

Calculation time



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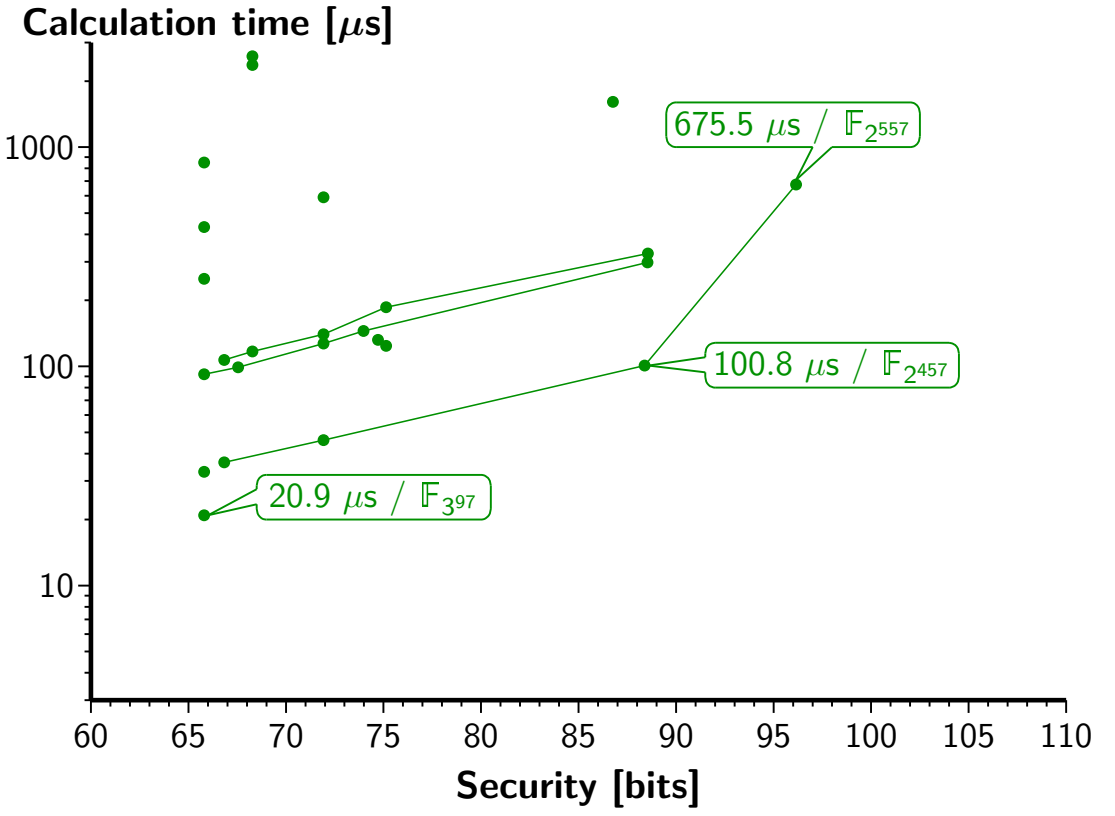

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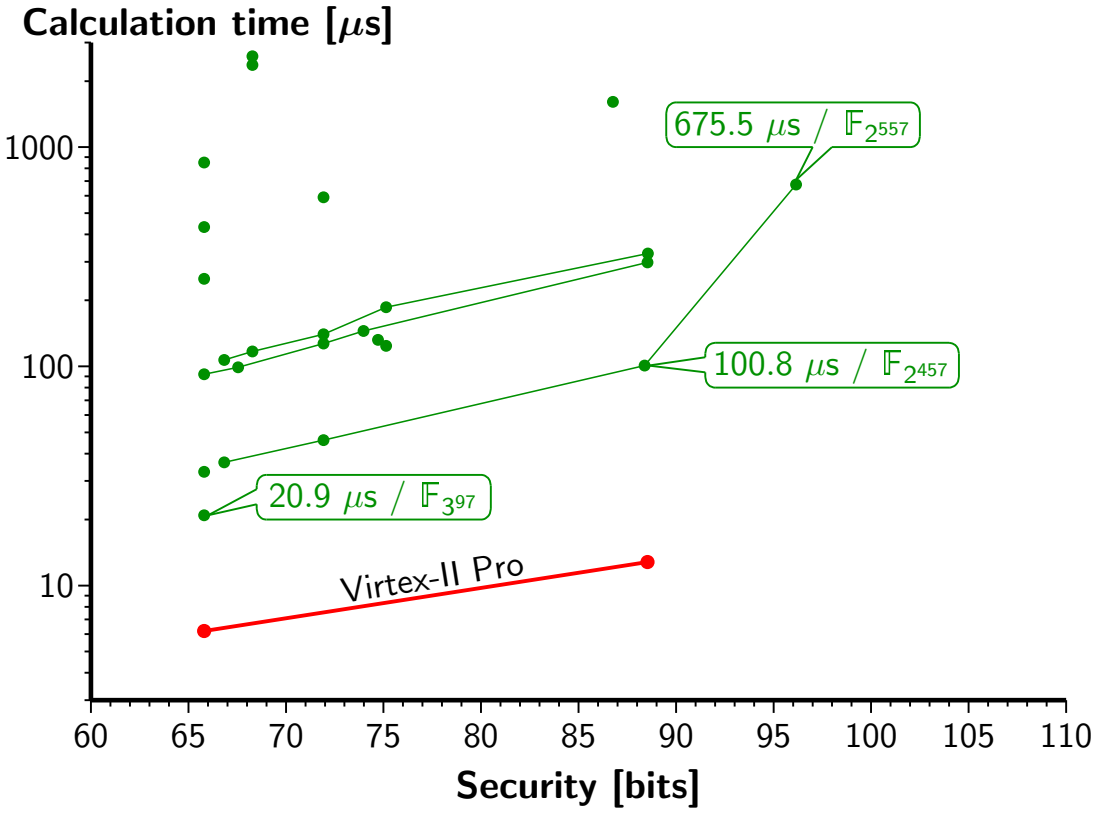

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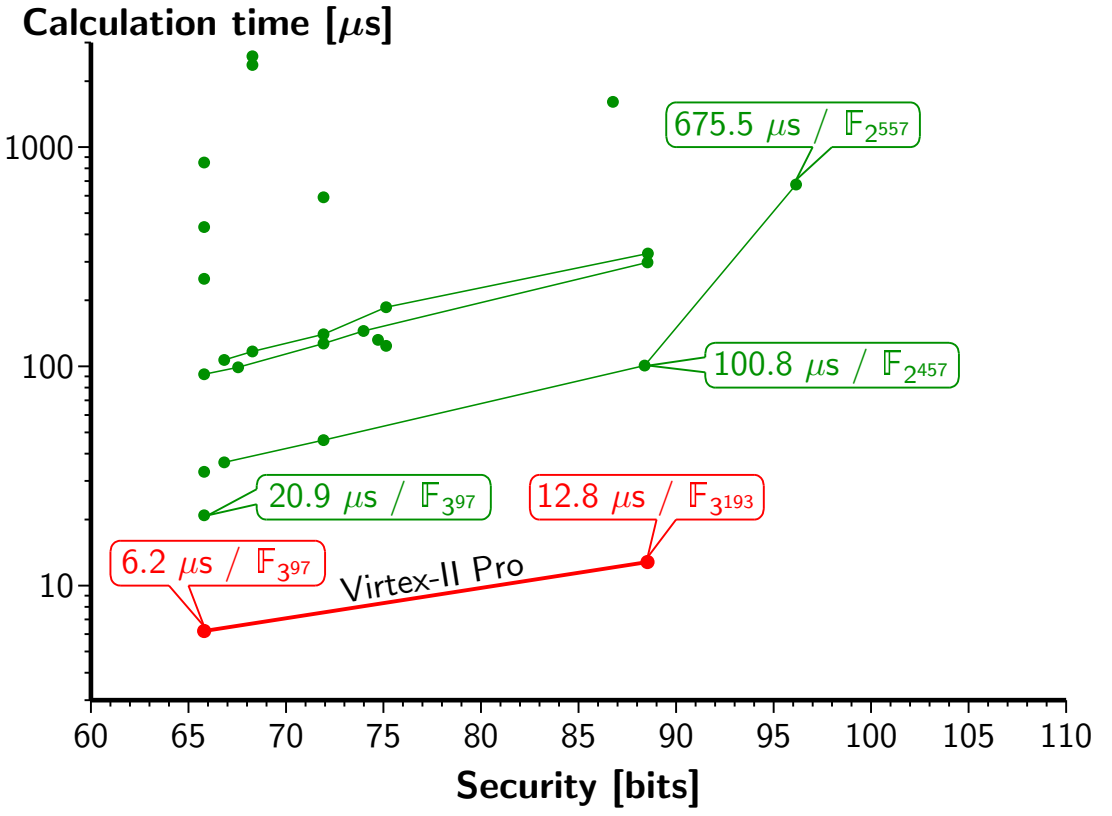

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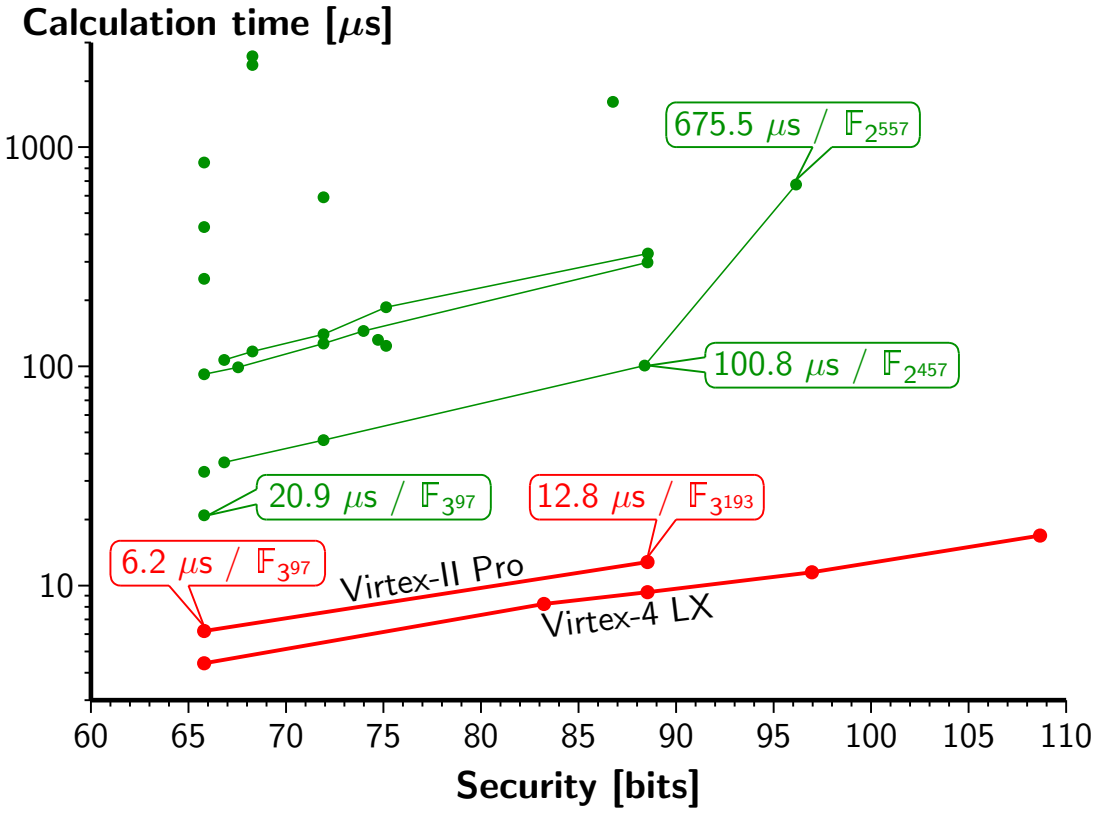
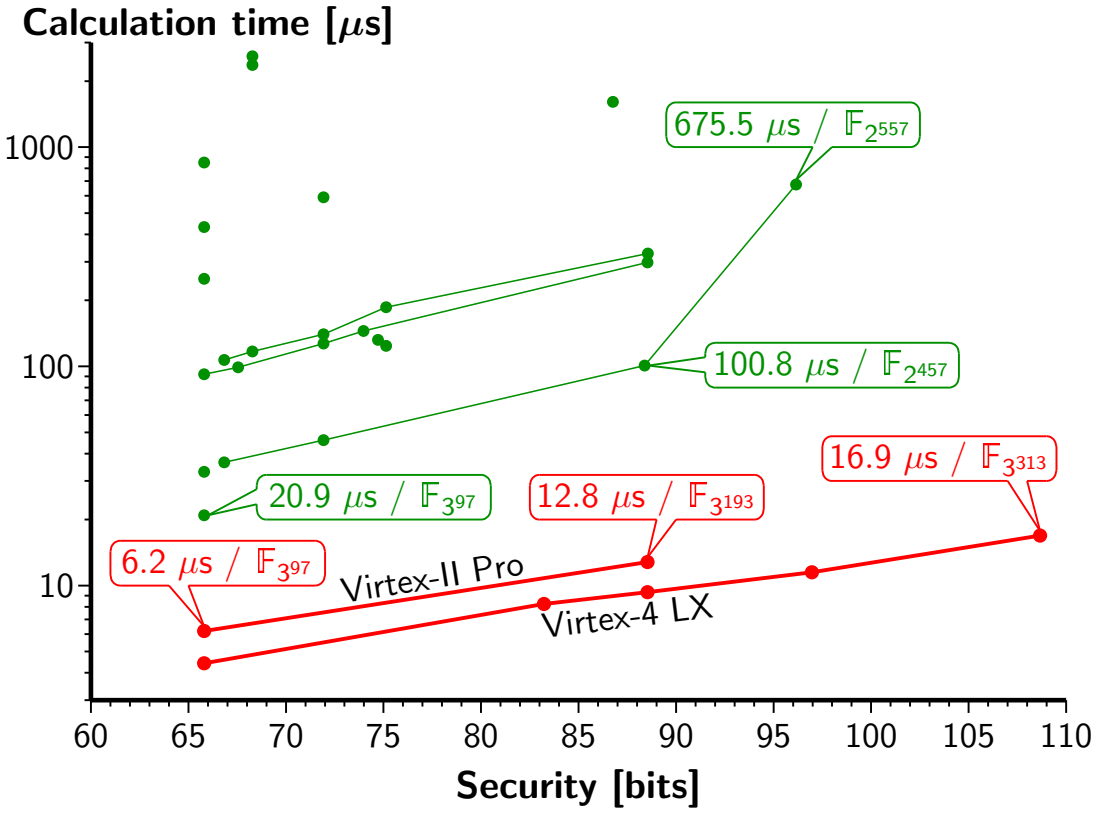
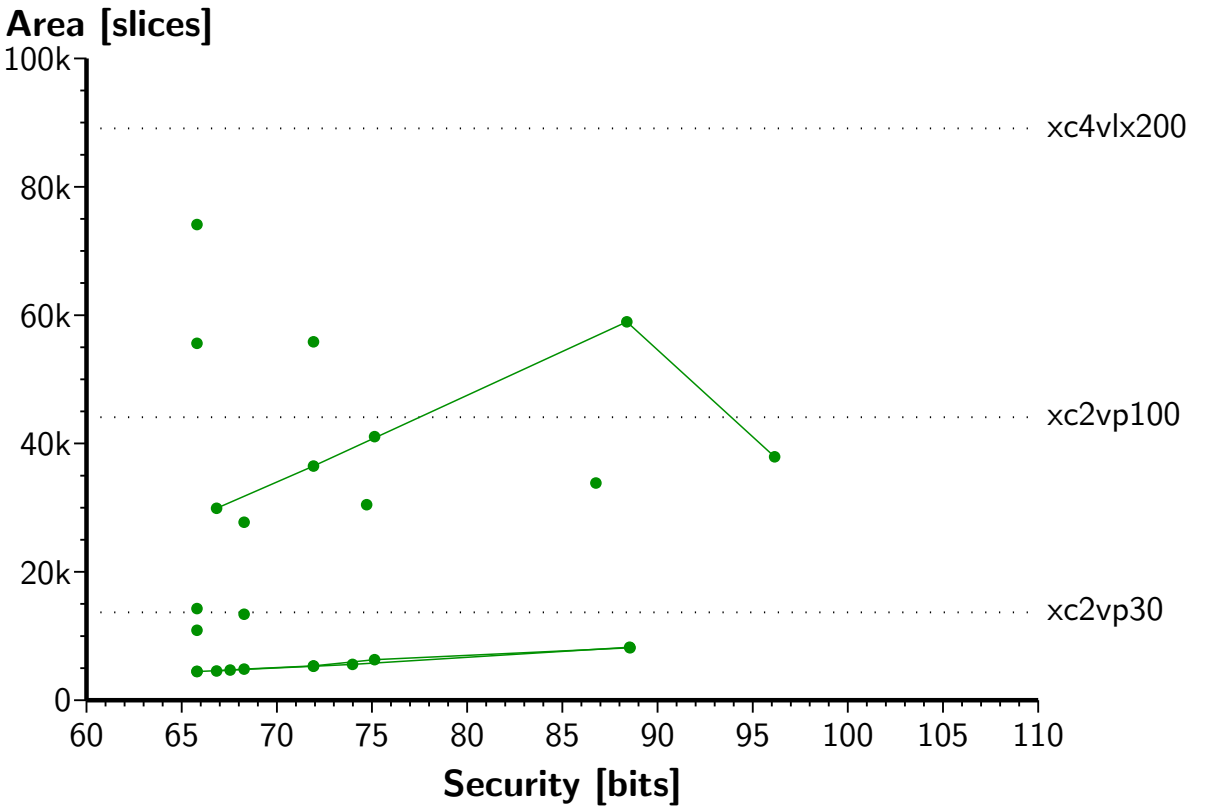


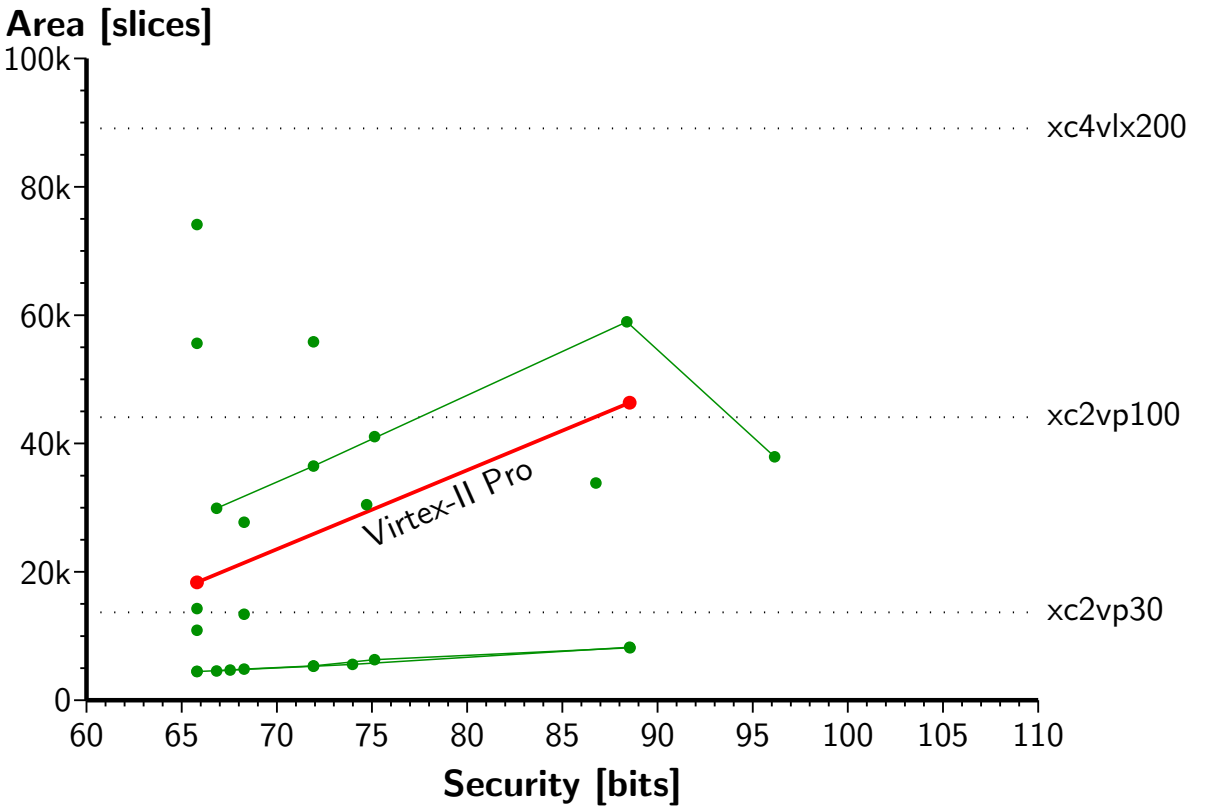
Calculation time



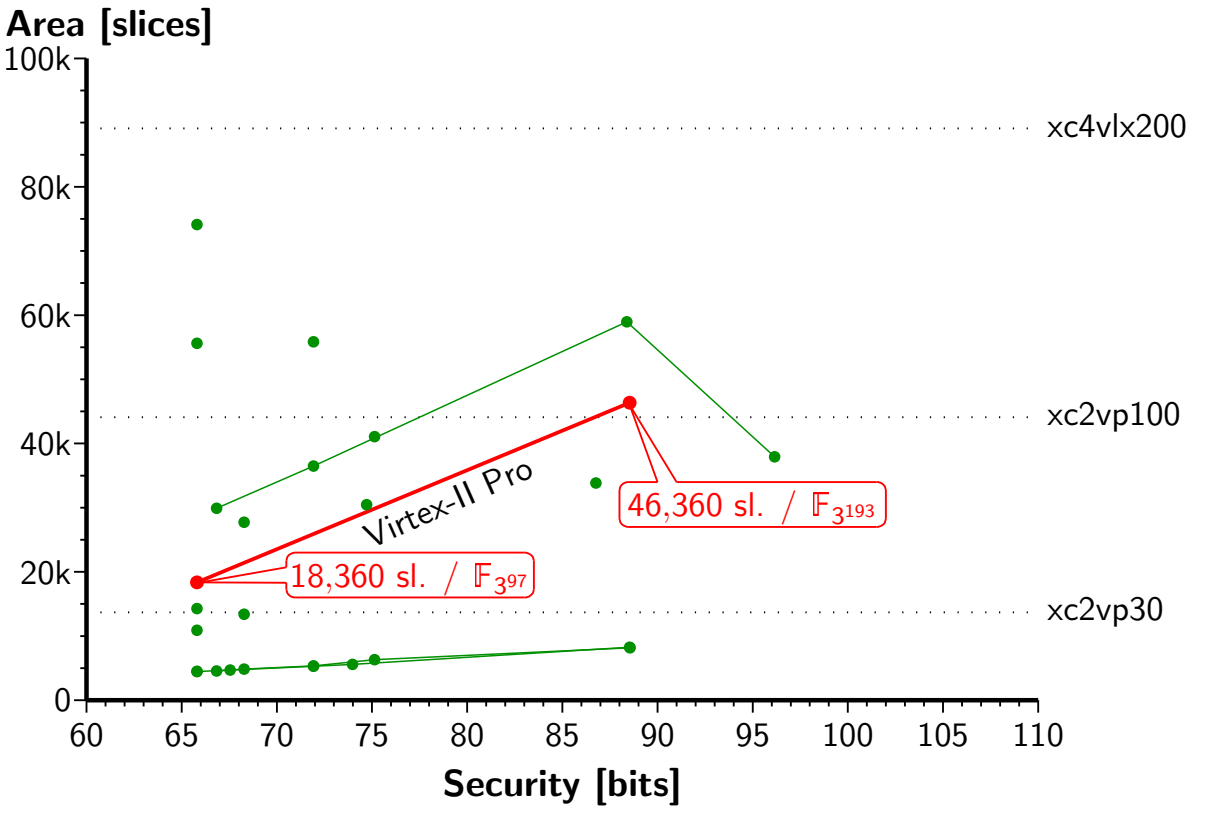
Area


Area [slices]
100k
80k
60k
40k
20k
0
xc4vlx200
xc2vp100
xc2vp30
60
65
70
75
80
85
90
95
100
105
110
Security [bits]

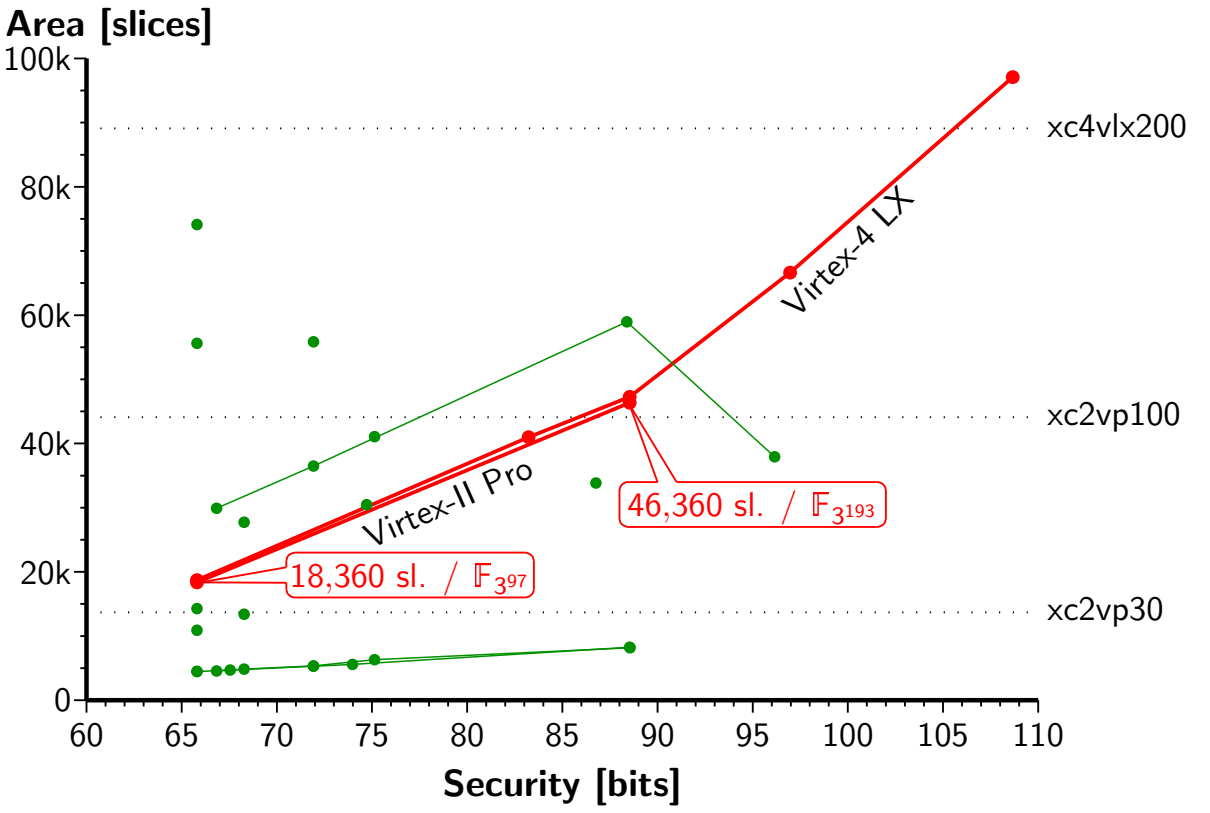
Area


Area [slices]
100k
80k
60k
40k
20k
0
xc4vlx200
xc2vp100
xc2vp30
Virtex-II Pro
60
65
70
75
80
85
90
95
100
105
110
Security [bits]

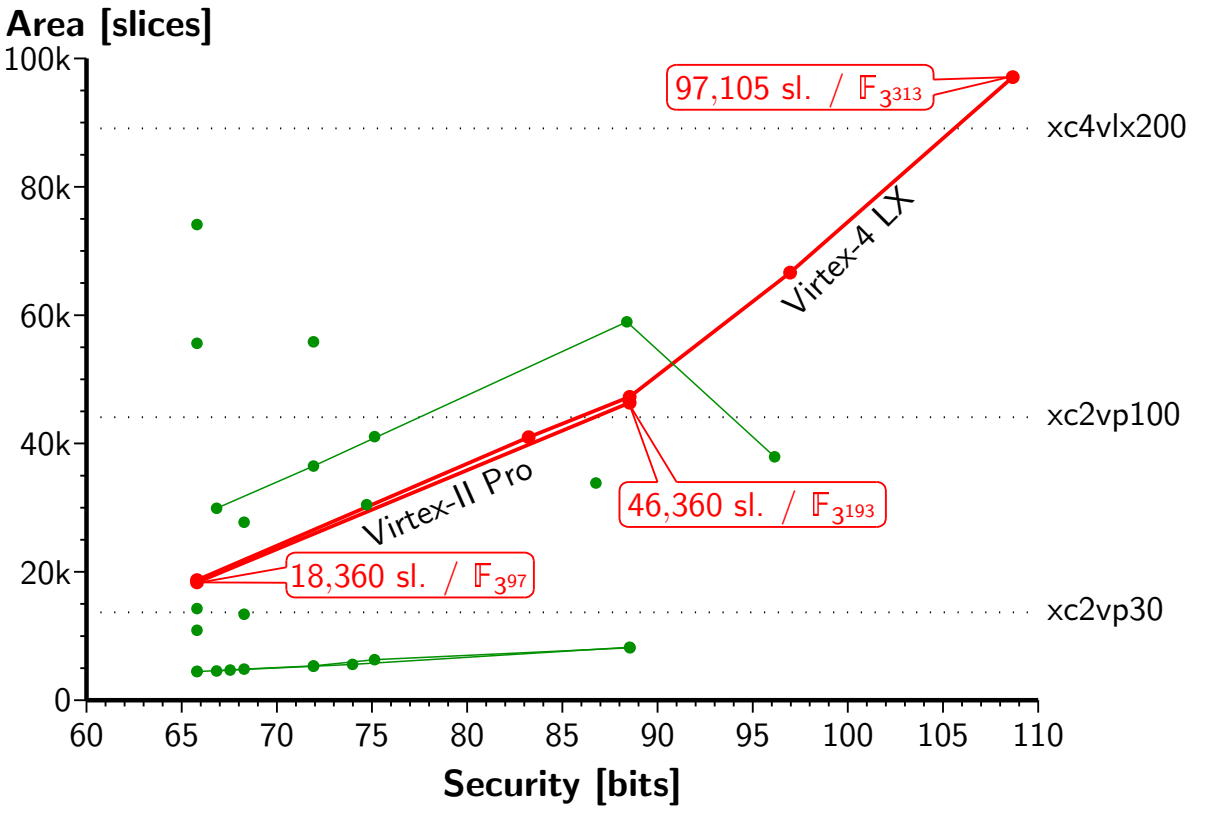
Area


Area [slices]
100k
80k
60k
40k
20k
0
xc4vlx200
xc2vp100
xc2vp30
Virtex-II Pro
46,360 sl. / $\mathbb{F}_{3^{193}}$
18,360 sl. / $\mathbb{F}_{3^{97}}$
60
65
70
75
80
85
90
95
100
105
110
Security [bits]

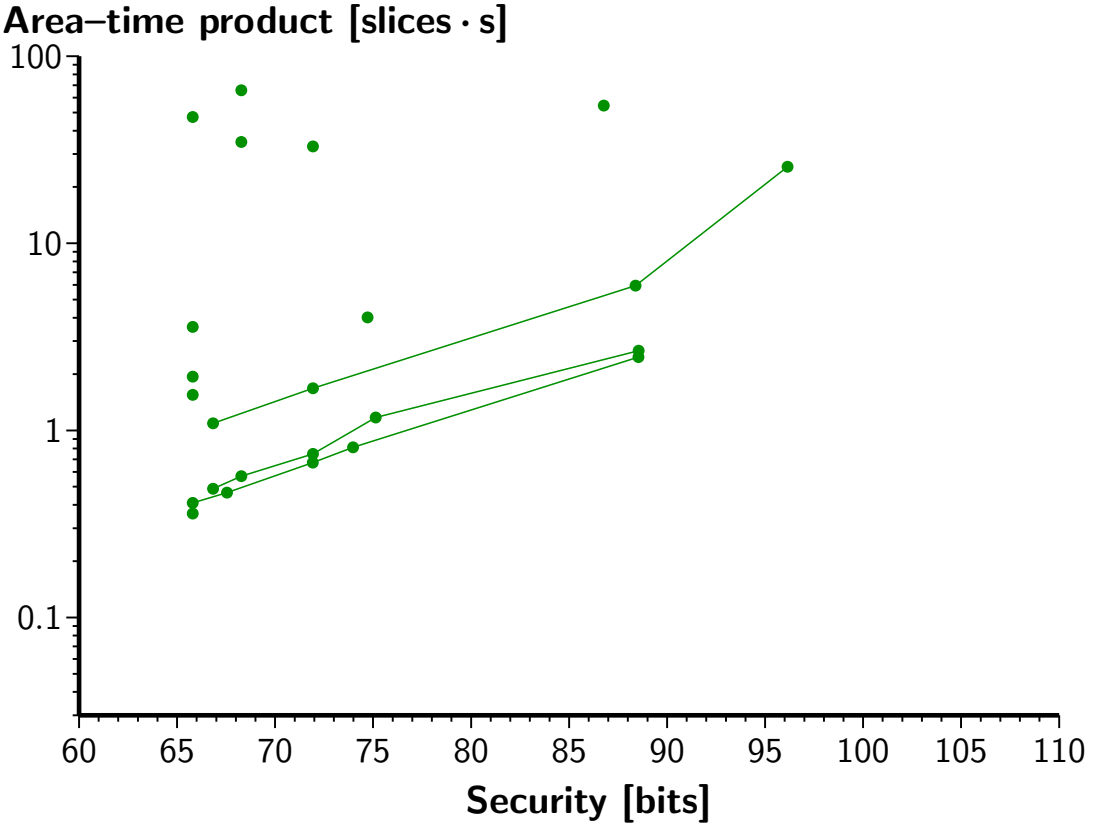
Area


Area [slices]
100k
80k
60k
40k
20k
0
60
65
70
75
80
85
90
95
100
105
110
Security [bits]
xc4vlx200
xc2vp100
xc2vp30
Virtex-4 LX
Virtex-II Pro
46,360 sl. / $\mathbb{F}_{3^{193}}$
18,360 sl. / $\mathbb{F}_{3^{97}}$

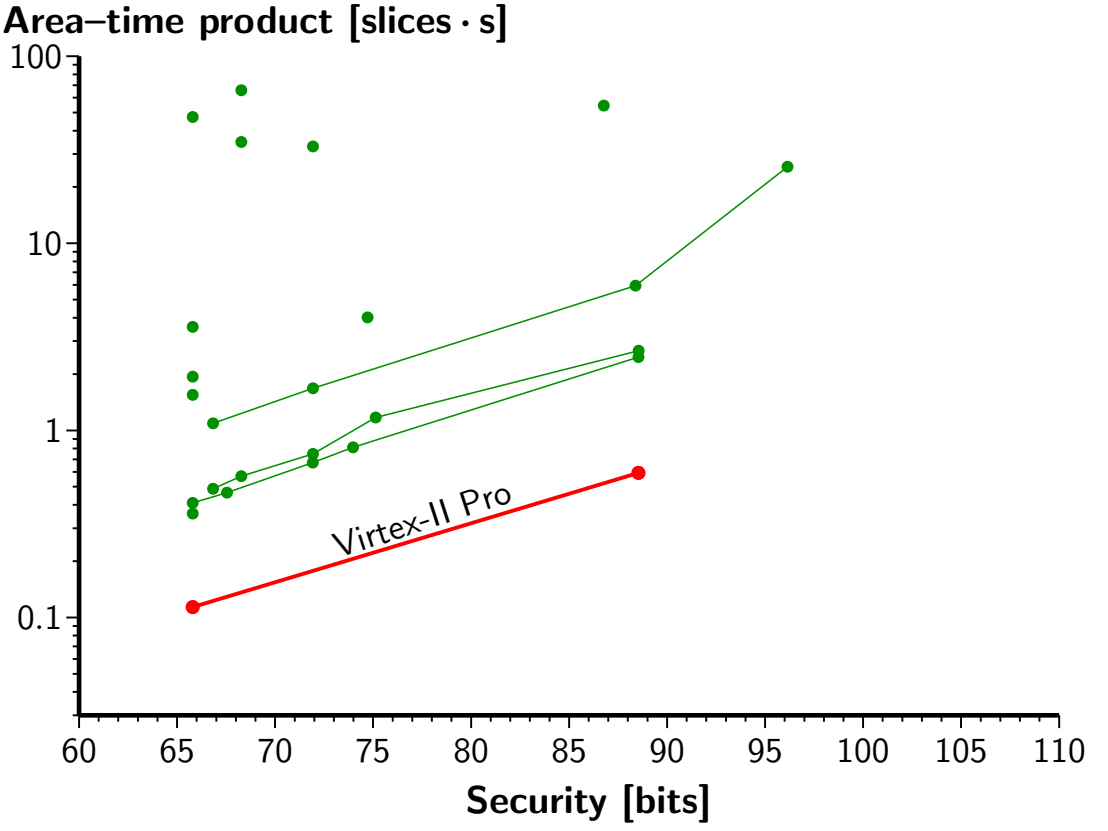
Area


Area [slices]
100k
80k
60k
40k
20k
0
60
65
70
75
80
85
90
95
100
105
110
Security [bits]
97,105 sl. / $\mathbb{F}_{3^{313}}$
46,360 sl. / $\mathbb{F}_{3^{193}}$
18,360 sl. / $\mathbb{F}_{3^{97}}$
Virtex-4 LX
Virtex-II Pro
xc4vlx200
xc2vp100
xc2vp30

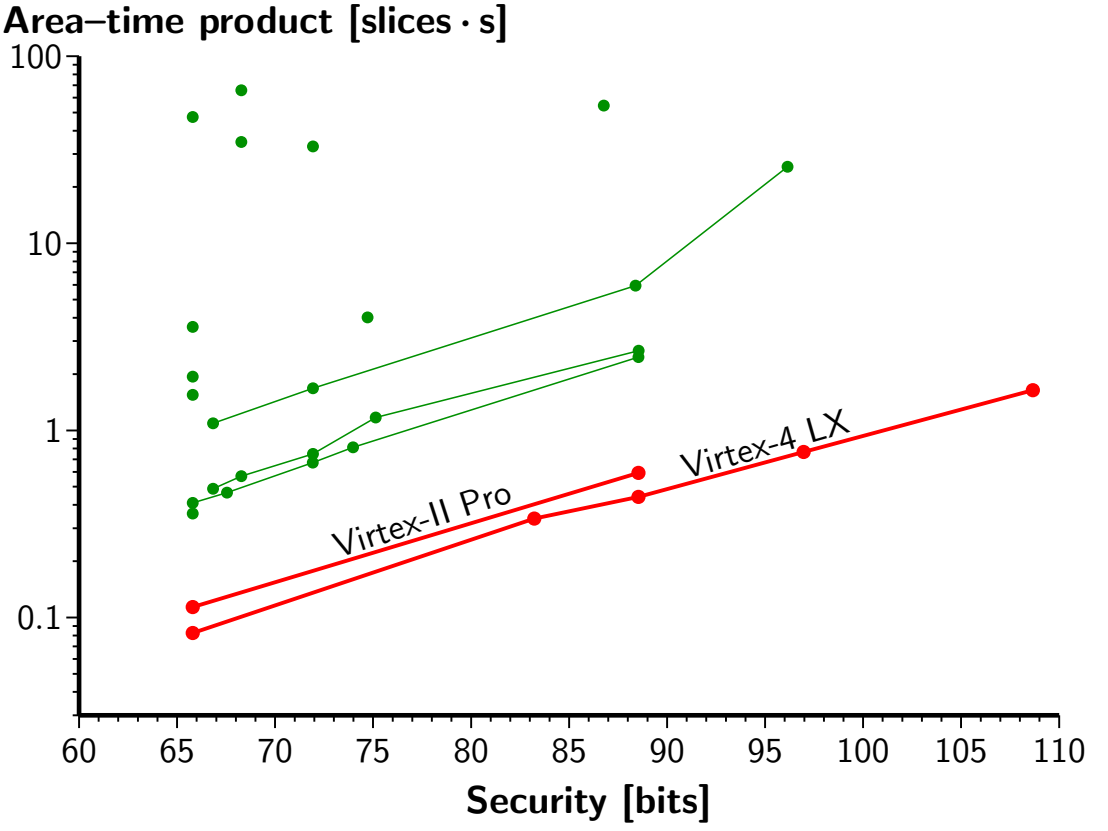
Area–Time product



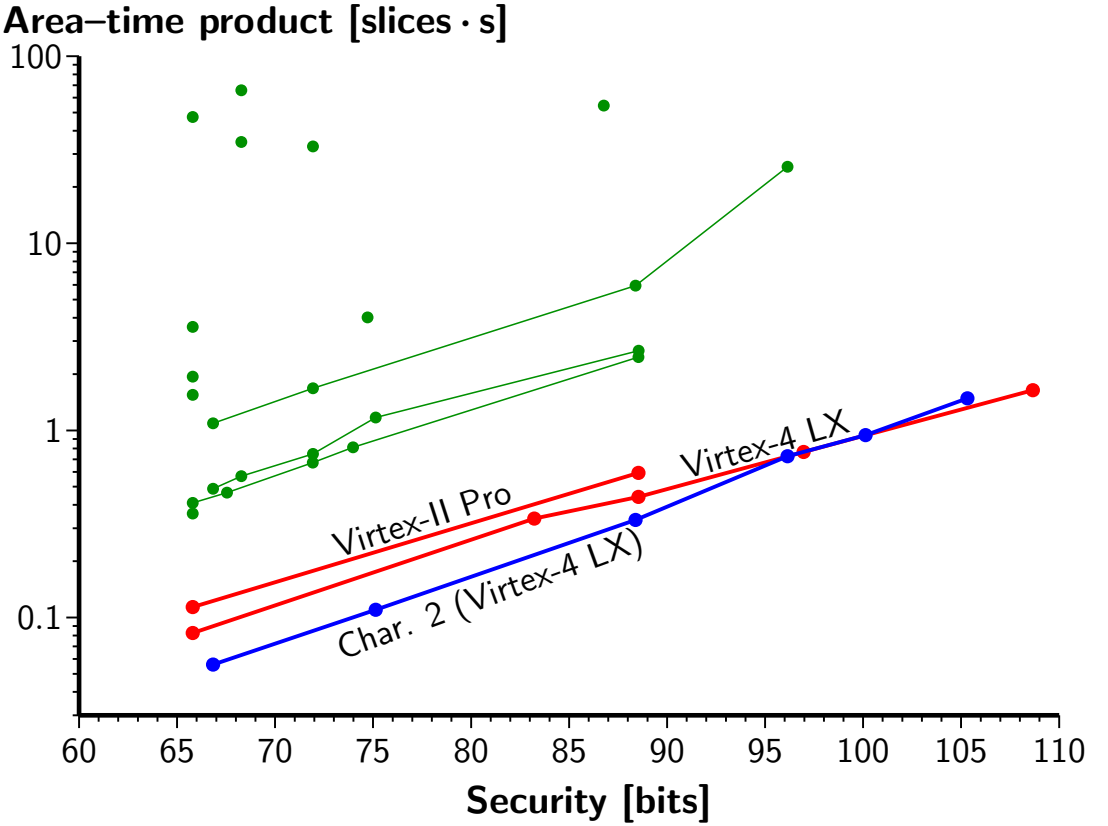
Area–Time product



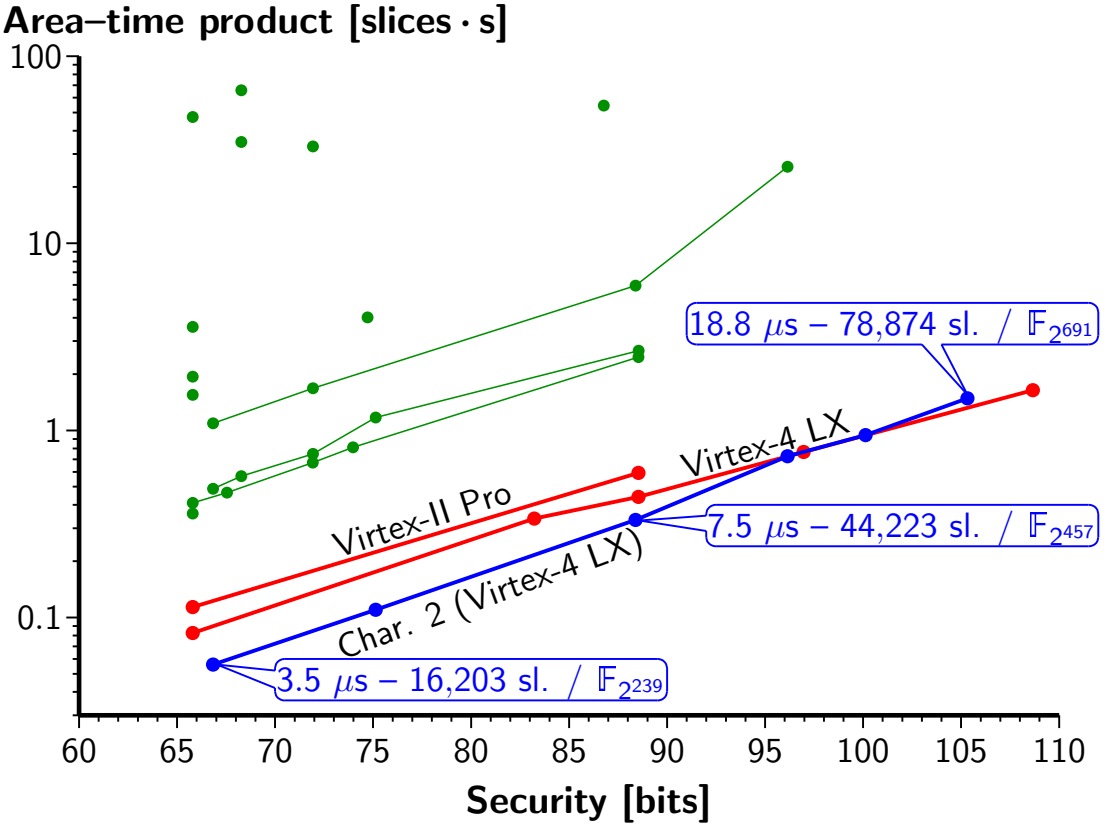
Area–Time product



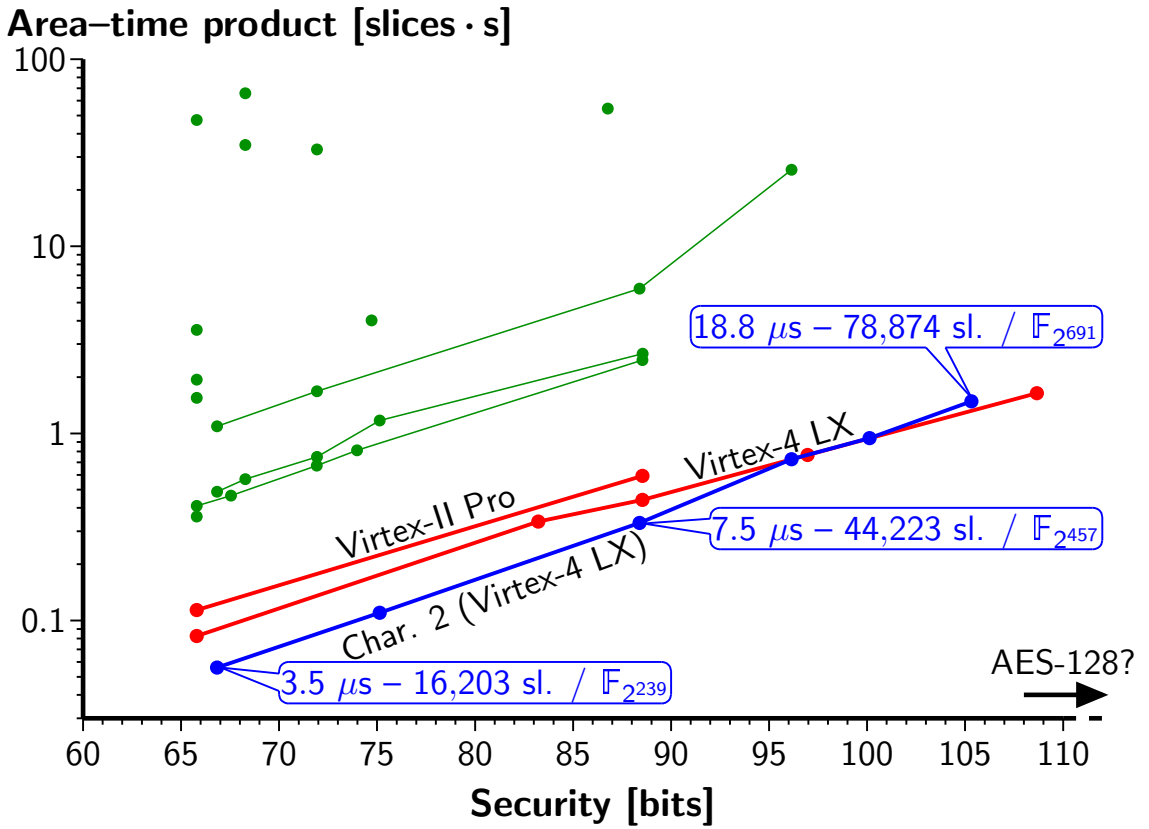
Area–Time product



Area–Time product



Area–Time product



Conclusion

- A new architecture for pairing computation
  - two specialized coprocessors
  - bet on parallelizing multiplier
  - based on Karatsuba multiplication scheme
  - importance of architecture–algorithm co-design
  - careful bubble-free scheduling of Miller's loop

Conclusion

- A new architecture for pairing computation
  - two specialized coprocessors
  - bet on parallelizing multiplier
  - based on Karatsuba multiplication scheme
  - importance of architecture–algorithm co-design
  - careful bubble-free scheduling of Miller's loop
- High-performance accelerator
  - the fastest coprocessor (17 μs for 109 bits of security)
  - the best area–time trade-off
  - scales to higher security levels

Future work

- Fully parallel multipliers
  - tune finely Karatsuba algorithm and multiplier architecture
  - try other algorithms: Toom–Cook, Montgomery's formulae
  - try less parallel multipliers: slower but smaller

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Future work

- Fully parallel multipliers
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  - try other algorithms: Toom–Cook, Montgomery's formulae
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- Final-exponentiation coprocessor
  - full-featured finite-field processor
  - compute the full pairing with it (promising first experimental results)
- Toward AES-128 security level
  - explore supersingular pairing over $\mathbb{F}_{2^{mm'}}$ and $\mathbb{F}_{3^{mm'}}$ (work in progress)
  - genus-2 supersingular curves in characteristic 2 (work in progress)
  - Barreto–Naehrig curves

Thank you for your attention

Questions?

Outline of the talk

- Context
- Reduced Tate pairing
- Non-reduced Tate pairing
- Fully parallel Karatsuba multiplier
- Final Exponentiation
- Results & Conclusion
- Appendix

Detailed Karatsuba algorithms

Algorithm	Splitting	Subproducts	Recomposition
2-way split	$A \to A_L + X^{\lceil m/2 \rceil} A_H$ $B \to B_L + X^{\lceil m/2 \rceil} B_H$ $A_M \leftarrow A_H + A_L$ $B_M \leftarrow B_H + B_L$	$p_H \leftarrow A_H * B_H$ $p_M \leftarrow A_M * B_M$ $p_L \leftarrow A_L * B_L$	$S \leftarrow p_H X^{2\lceil m/2 \rceil}$ $+ (p_M - p_H - p_L) X^{\lceil m/2 \rceil}$ $+ p_L$
3-way split	$A \to A_0 + X^{\lceil m/3 \rceil} A_1 + X^{2\lceil m/3 \rceil} A_2$ $B \to B_0 + X^{\lceil m/3 \rceil} B_1 + X^{2\lceil m/3 \rceil} B_2$ $A_{S_0} \leftarrow A_1 + A_2$ $A_{S_1} \leftarrow A_0 + A_2$ $A_{S_2} \leftarrow A_1 + A_0$ $B_{S_0} \leftarrow B_1 + B_2$ $B_{S_1} \leftarrow B_0 + B_2$ $B_{S_2} \leftarrow B_1 + B_0$	$p_0 \leftarrow A_0 * B_0$ $p_1 \leftarrow A_1 * B_1$ $p_2 \leftarrow A_2 * B_2$ $p'_0 \leftarrow A_{S_0} * B_{S_0}$ $p'_1 \leftarrow A_{S_1} * B_{S_1}$ $p'_2 \leftarrow A_{S_2} * B_{S_2}$	$S \leftarrow p_2 X^{4\lceil m/3 \rceil}$ $+ (p'_0 - p_1 - p_2) X^{3\lceil m/3 \rceil}$ $+ (p'_1 - p_0 + p_1 - p_2) X^{2\lceil m/3 \rceil}$ $+ (p'_2 - p_1 - p_0) X^{\lceil m/3 \rceil}$ $+ p_0$

Detailed odd–even split Karatsuba algorithm

Algorithm	Splitting	Subproducts	Recomposition
2-way split	$A \to A_E(X^2) + XA_O(X^2)$ $B \to B_E(X^2) + XB_O(X^2)$ $A_M \leftarrow A_O + A_E$ $B_M \leftarrow B_O + B_E$	$p_O \leftarrow A_O * B_O$ $p_M \leftarrow A_M * B_M$ $p_E \leftarrow A_E * B_E$	$S \leftarrow (p_E + Xp_O)(X^2)$ $+ X(p_M - p_E - p_O)(X^2)$
3-way split	$A \to A_0(X^3) + XA_1(X^3) + X^2A_2(X^3)$ $B \to B_0(X^3) + XB_1(X^3) + X^2B_2(X^3)$ $A_{S_0} \leftarrow A_1 + A_2$ $A_{S_1} \leftarrow A_0 + A_2$ $A_{S_2} \leftarrow A_1 + A_0$ $B_{S_0} \leftarrow B_1 + B_2$ $B_{S_1} \leftarrow B_0 + B_2$ $B_{S_2} \leftarrow B_1 + B_0$	$p_0 \leftarrow A_0 * B_0$ $p_1 \leftarrow A_1 * B_1$ $p_2 \leftarrow A_2 * B_2$ $p'_0 \leftarrow A_{S_0} * B_{S_0}$ $p'_1 \leftarrow A_{S_1} * B_{S_1}$ $p'_2 \leftarrow A_{S_2} * B_{S_2}$	$S \leftarrow (p_0 + X(p'_0 - p_1 - p_2))(X^3)$ $+ X(p'_2 - p_0 - p_1 + Xp_2)(X^3)$ $+ X^2(p_1 + p'_1 - p_2 - p_0)(X^3)$

Multiplication algorithms