Detection and Diagnosis of Small Changes in Nonlinear Systems - Application to the Monitoring of a Gas Turbine *

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Abstract

In this paper, we describe a general approach for detecting small changes, which can be applied to a wide class of *nonlinear* systems, and which can manage with a *reduced order* and *biased* signature of the monitored system. We show how to implement this general approach for solving a real problem, namely the monitoring of the combustion chambers of a gas turbine. Furthermore, we also describe a statistical method for solving the problem of diagnosing the detected changes in the combustion chambers.

Keywords: failure detection, identification, diagnosis, gas turbines

1 Introduction

These last years, in a great number of industrial applications, an increasing demand appeared for replacing systematic maintenance by *conditional* maintenance strategies which are based upon the continuous supervision of the evolution of the considered machine or structure or process. In this framework, it has been shown that a possible solution consists in the *early warning* of *small deviations* with respect to a signature of the machine or structure corresponding to its usual working conditions. In this paper, we present an approach for this problem. This approach is based upon the so-called "asymptotic local" approach for change detection previously introduced by some of the authors [4, 5]. Its original principle consists in characterizing a system via some identified (possibly nonlinear) model, and then to monitor its changes using some data-to-model distance also derived from identification techniques and from the statistical local approach.

In [3] and earlier related works, we have shown that, using the (accelerometers or constraint gauges) measurements which are recorded on a mechanical structure or rotating machine, it is possible to detect and diagnose small changes in the vibrating characteristics (eigenfrequencies and modal shapes), even though the measurements do contain a mixing of information related to the structure *and* to its possibly nonstationary environment. In other words, we do take decisions concerning states of the system which are not measured. Moreover it has been also shown in [3] that, if additional information concerning the physical model of the system is available, it is possible to design *measurement processing* methods which do infer the physical origin of the detected changes, even when the parametric space of the modal signature is of much lower dimension than the space of the mechanical characteristics (masses, stiffness, damping, volumic masses, Young modulus,...), which is the case when only a few sensors are available.

Similarly, in [8, 9], the idea is to monitor the combustion chambers of the gas turbine of an electrical power generator, using the temperature measurements which are taken at the output of the expansion turbine. Preliminary attempts to apply the above mentioned general approach to the monitoring of the combustion chambers led us to develop an extension of this method [9] in the following manner. Model reduction can be enforced; biased identification procedures can be used; and finally one can even get rid of identification for getting the signature, and use instead some much simpler Monte-Carlo estimation technique prior to change detection. In this paper, we show how to implement this extended asymptotic local approach for the monitoring of the combustion chambers.

Moreover we describe how to use the so-called min-max robust approach (known for statistical inference in the presence of nuisance parameters) for solving the problem of diagnosing the detected changes.

The paper is organized as follows. In section 2, we describe the extension of the local approach, following the ideas developed in [2]. In section 3, we describe the application of this general detection approach to the thermodynamical model of the combustion chambers of a gas turbine, and we show experimental results concerning on-line change detection. In section 4, we discuss the corresponding diagnosis problem. We first recall the min-max robust approach following again [2] and we propose a new variant of this method

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which seems to be more appropriate for the present real problem.

2 Detecting small changes in a biased nonlinear signature

In this section, we describe the extension of the local approach [4] to the case of a biased and/or undermodelled signature, following the presentation which is given in [2]. First we recall our approach to the detection of small changes in nonlinear systems, and then we describe the proposed extension.

2.1 The local approach in nonlinear systems

In [2], we explain that the so-called local approach is a relevant and less complex alternative to the general likelihood ratio approach for the detection of non additive changes. We also introduce to another use of the local approach for deriving non-likelihood based algorithms for the detection of small non additive changes in a wide class of nonlinear systems which we recall now.

We consider the observed processes $(X_k)_k$ which can be thought as being the nonlinear counterpart of ARMA models and are called semi-Markov processes :

$$\begin{aligned} P(\xi_n \in G \mid \xi_{n-1}, \xi_{n-2} \ldots) &= \int_G \pi_\Theta\,(\xi_{n-1}, dx) \\ X_n &= f(\xi_n) \end{aligned}$$

where Θ is the parameter which characterizes the true system; π_Θ is the transition probability of a Markov chain ξ_n controlled by the parameter Θ, f is a function, and E_Θ is the expectation under the law for which (ξ_n) is a stationary Markov chain with transition probability π_Θ. The problem is then to detect and diagnose changes on Θ.

We first consider the case where Θ can be reached by identification, for example with a recursive algorithm of the form :

$$\theta_n = \theta_{n-1} \,+\, \gamma_n\, H(\theta_{n-1}, X_n) \tag{1}$$

namely we assume that this algorithm does converge towards the true parameter Θ. At the corresponding equilibrium point $\theta = \Theta$, the mean vector :

$$h(\theta) = \lim_{n \to +\infty} E_\Theta H(\theta, X_n)$$

is zero.

As we explain in [4, 2], for detecting and analyzing changes in the true system represented by Θ, we suggest to monitor the deviations, with respect to zero, of the mean value of $H(\theta, X_n)$. The idea is to detect a change in the parameter Θ when this function of observations is significantly different from zero. It turns out that there exists a mathematical result which makes this statement having a precise sense. We recall it below.

It should be clear that we have *two* parameters :

- Θ, which represents the true system : Θ is unknown and we are interested in monitoring its changes;
- θ, which is an adjustable and accessible parameter and which will be used for monitoring changes in Θ.

2.2 Extension to the cases of undermodelled or biased signatures

In our previous derivation of the asymptotic local approach [4, 5], the following conditions have played a key role : the true system is in the model set; the monitoring procedure is derived from a bias-free identification algorithm; the same identification procedure is used, both to derive the monitoring procedure, and to estimate the nominal model when it is indeed fitted on some training data set.

Now, when investigating the application reported in section 3, we were faced to a more complex situation, and we had to show that these requirements can be drastically relaxed : the true system does not need to belong to the model set (in other words, we can monitor the true system with the aid of *simplified* models), biased identification algorithms can be used to design the tests, and we can even avoid using system identification procedures to characterize the nominal behavior.

We only summarize this extension here. The interested reader is referred to [9] for details.

We continue to assume that the observations are represented by the model :

$$\begin{aligned} P(\xi_n \in G \mid \xi_{n-1}, \xi_{n-2} \ldots) &= \int_G \pi_\Theta\,(\xi_{n-1}, dx) \\ X_n &= f(\xi_n) \end{aligned}$$

with the same hypotheses as before. We monitor the system with the aid of the random vector field :

$$H(\theta, X_n)$$

where θ is a parameter which is adjustable and known by the user. But the fact that θ represents the true system is now stated in the condition :

$$\mathcal{F}(\Theta) = \theta \tag{2}$$

where $\mathcal{F}$ is called the *bias function*. Consequently, if we define :

$$h(\theta, \Theta) = \lim_{n \to +\infty} E_\Theta H(\theta, X_n)$$

(so that $h(\theta, \Theta)$ is the mean of $H(\theta, X_n)$ when the true system is Θ), the vector field $H(\theta, X_n)$ should satisfy the condition :

$$h(\theta, \Theta) = 0 \Leftrightarrow\; \mathcal{F}(\Theta) = \theta \tag{3}$$

A monitoring procedure which uses this fact is made of the following steps :

1. On the process in safe state, identify the parameter θ which represents the true system (in the sens of (2)); let θ_0 be the parameter obtained in this manner, we call it the *nominal model*, or signature. We denote by Θ^* the true system (which is unknown and subject to changes);

2. Using a *new record* of observations, estimate the quantity $h(\theta_0, \Theta^*)$ in order to know if the true system has changed or not; this test consists in answering to the question :

$$h(\theta_0, \Theta^*) = 0 \ ? \tag{4}$$

   We give below the relevant function of both the signature and the new measurements which allows us to answer to this question.

Now a simple way for defining a function $H(\theta, X_n)$ which satisfies the condition (3) is to proceed as follows :

1. Choose a function $H(\theta, X_n)$ (a "bad" identification method — of the form (1) — is convenient, even if it results in a non negligible bias);

2. Choose a nominal model θ_0 by identification or by using prior knowledge; of course the closer it is to the true system, the better the test works; see the discussion in [9] concerning the consequence of this choice on the performances of the test;

3. Using measurements on the process in (safe) reference state, estimate the quantity :

$$\widehat{h}_0 = h(\theta_0, \Theta^*)$$

   (the corresponding formula is given below);

4. Then the bias adjusted vector field :

$$H(\theta, X_n) - \widehat{h}_0$$

   satisfies the condition (3), and thus can be used for detecting changes in the process, by monitoring its mean value. This is exactly the above mentioned function of the signature and of the new measurements which allows us to answer the question (4).

2.3 Main mathematical result

Now we give without proof a summary of the main result which gives the solution to two types of change detection problems. The interested reader is referred to [4, 5] for the technical details.

The two types of change detection problems which we consider are as follows. For the first one, we have a sample of measurements for which we wish to check if it corresponds to the signature; this problem is often called *model validation*. In the second one, we wish to test for the presence of the change inside the sample; this problem is called on-line change detection. These two problems can be stated as hypotheses testing problems for n-size samples in the following manner.

For *model validation*, we consider the hypotheses :

$$\begin{aligned} \mathcal{H}_0 : \mathcal{F}(\Theta^*) &= \theta_0 \\ \mathcal{H}_1 : \mathcal{F}(\Theta^*) &= \theta_0 + \frac{\Upsilon}{\sqrt{n}} \end{aligned} \tag{5}$$

For *on-line detection* , we use the hypotheses :

$$\begin{aligned} \mathcal{H}_0 &: \mathcal{F}(\Theta^*) = \theta_0 \\ \mathcal{H}_1' &: \text{there exists } \tau \in [0,1] \text{ such that} \\ & \quad \mathcal{F}(\Theta^*) = \theta_0 \ \text{ for } k < \tau n \\ & \quad \mathcal{F}(\Theta^*) = \theta_0 + \frac{\Upsilon}{\sqrt{n}} \text{ for } \tau n \le k \le n \end{aligned} \tag{6}$$

In both cases, $\Upsilon \neq 0$ is an unknown change vector.

For $m \le n$, let us introduce the following quantity :

$$\Delta_{n,m}(\theta_0, \Upsilon) = \frac{1}{\sqrt{n}} \sum_{k=1}^{m} \left(H(\theta_0,\ X_k) - \widehat{h}_0 \right)$$

where :

$$\widehat{h}_0 = \mathbf{E}_{\Theta_*} H\left(\theta_0, X_k\right) \tag{7}$$

is estimated with the parameter Θ^* corresponding to the hypothesis $\mathcal{H}_0$. We define $\Delta_{n,t}$ as the linear piecewise interpolation of $\Delta_{n,m}$ at the points $t = \frac{m}{n}$. Under the hypothesis $\mathcal{H}_1'$ with $\tau \in [0,1]$, the behavior of $\Delta_{n,t}$ when n goes to infinity is governed by the following theorem [4, 5] :

Theorem 1 *Under $\mathcal{H}_1'$, the process $(\Delta_{n,t})$ converges weakly towards the diffusion process (Δ_t) which is solution of the differential equation :*

$$d\Delta_t = -1_{\{t \ge \tau\}} \cdot h'_{\theta_0} \cdot \Upsilon \cdot dt + R^{\frac{1}{2}}(\theta_0)\, dW_t$$

where (W_t) is the standard Brownian motion and :

$$\begin{aligned} R(\theta_0) &= \sum_{i=-\infty}^{+\infty} cov_{\Theta^*}\left[H(\theta_0; X_i), H(\theta_0; X_0)\right] \\ h'_{\theta_0} &= \left. \frac{\partial h(\theta, \Theta^*)}{\partial \theta} \right|_{\theta=\theta_0} \end{aligned} \tag{8}$$

The behaviors of $\Delta_{n,t}$ under the other hypotheses can be obtained as particular cases of this general theorem : $\mathcal{H}_0$ by letting $\Upsilon = 0$, and $\mathcal{H}_1$ with $\tau = 0$.

In other words, this theorem replaces the complex initial problem of detecting a non additive change in a nonlinear process into the much simpler problem of detecting a change in the mean of an independent Gaussian sequence with known and constant covariance matrix.

The connection between this local non-likelihood approach and the classical local likelihood approach is discussed in [2, 1].

2.4 Computing the tests

2.4.1 Estimating the unknown quantities

The two quantities $R(\theta_0)$ and h'_{θ_0} of the previous theorem are estimated *on a reference sample* (X_k^0) corresponding to a safe state of the process, with the aid of the following formulas :

$$\begin{aligned} R(\theta_0) &\approx \sum_{i=-\infty}^{+\infty} \frac{1}{n} \sum_{k=1}^{n} \left(H(\theta_0, X_{k+i}^0) - \widehat{h}_0\right) \\ &\quad \left(H(\theta_0, X_k^0) - \widehat{h}_0\right)^T \quad (9) \\ \text{where } \widehat{h}_0 &\approx \frac{1}{n} \sum_{k=1}^{n} H(\theta_0, X_k^0) \quad (10) \\ h'_{\theta_0} &\approx \frac{1}{n} \sum_{k=1}^{n} \left.\frac{\partial H\left(\theta, X_k^0\right)}{\partial \theta}\right|_{\theta=\theta_0} \quad (11) \end{aligned}$$

The statistics which is used in all our tests is :

$$\Delta_n \triangleq \frac{1}{\sqrt{n}} \sum_{k=1}^{n} \left(H(\theta_0, X_k) - \widehat{h}_0\right) \quad (12)$$

where (X_k) is the *new sample* of observations.

2.4.2 Model validation

We consider the hypotheses (5). In this case, it results from the above theorem that :

$$\begin{aligned} \text{under } \mathcal{H}_0 : \Delta_n &\sim \mathcal{N}\left(0 \; ; \; R\left(\theta_0\right)\right) \quad (13) \\ \text{under } \mathcal{H}_1 : \Delta_n &\sim \mathcal{N}\left(-h'_{\theta_0} \cdot \Upsilon \; ; \; R\left(\theta_0\right)\right) \quad (14) \end{aligned}$$

The maximization of the log-likelihood ratio leads to the test :

$$\chi_n^2 = \Delta_n^T \, R^{-1}(\theta_0) \, \Delta_n \quad (15)$$

where Δ_n is defined in (12). $\mathcal{H}_1$ is decided when the quantity χ_n^2 is greater than a threshold.

2.4.3 On-line detection

We now consider the hypotheses (6), and we denote by r the discrete change time. In this case, it results from the previous theorem that we can work as if the increments of the cumulative sum Δ were independent with distribution :

$$\begin{aligned} \mathcal{H}_0 : H(\theta_0, X_k) - \widehat{h}_0 &\sim \mathcal{N}\left(0 \; ; \; R(\theta_0)\right) \\ \mathcal{H}_1 : H(\theta_0, X_k) - \widehat{h}_0 &\sim \mathcal{N}\left(-h'_{\theta_0} \cdot \Upsilon \; ; \; R(\theta_0)\right) \end{aligned}$$

Note that we could have used this formulation for the model validation problem, but it would have led to exactly the same test as in (15).

In the present on-line situation, the corresponding test is based upon :

$$t_a = \min\{n \geq 1, g_n \geq \lambda\}$$

$$g_n = \max_{1 \leq j \leq n} (\chi_j^n)^2$$

where :

$$(\chi_j^n)^2 = (\Delta_j^n)^T \, R^{-1}(\theta_0) \, \Delta_j^n \quad (16)$$

and :

$$\Delta_j^n = \frac{1}{\sqrt{n-j+1}} \sum_{k=j}^{n} \left(H(\theta_0, X_k) - \widehat{h}_0\right)$$

In order to reduce the cost of the search for the maximum in j of $(\chi_j^n)^2$, we constrain this search inside a window $\mathcal{M}$ with fixed length $L = n_1 - n_0 + 1$:

$$\begin{aligned} \mathcal{M} &= \{(n - n_1), \ldots, (n - n_0)\} \quad (17) \\ g_n &= \max_{j \in \mathcal{M}} (\chi_j^n)^2 \quad (18) \\ \hat{r} &= \arg\max_{j \in \mathcal{M}} (\chi_j^{t_a})^2 \quad (19) \end{aligned}$$

In [9] can be found simulation results concerning the monitoring of AR(p) processes with the aid of AR(q) models with $q \ll p$, showing the robustness of the proposed detection algorithm with respect to under-modelling. We now describe the real application which we have investigated.

3 Monitoring the combustion chambers of a gas turbine

We present in this section the application of our local approach to the monitoring of a gas turbine, and more precisely of the sub-system composed of the combustion chambers and the expansion turbine. We refer the reader to [6] for a presentation of the computer architecture designed for this industrial application. Here we give a relevant thermodynamical model for this combustion set, which turns out to be nonlinear with noisy inputs and outputs. An unbiased parametric signature is thus impossible to get [7]. But we show here that this bias does not prevent a precise monitoring of small changes in the combustion set.

3.1 Simplified mathematical model

We are interested in monitoring the 10 combustion chambers and the expansion turbine, using the measurements given by the 18 thermocouples. It is shown in [8] that the temperatures measured by the 18 thermocouples y_j can be expressed in the following way:

$$\begin{aligned} y_j &= \frac{T_s}{T_e} \left(T_e + \sum_{i=0}^{9} a_i g(\phi_j^t - \phi_i^c - d) \right) \quad (20) \\ &+ \; b \cos(\phi_j^t - \phi^s) + w_j, \quad j = 0, \ldots, 17 \\ d &= D(T_e, T_s, N) + d_0 + d_6 \sin(\phi_j^t - \phi^s) \end{aligned}$$

where the notations are explained as follows. We use a zero mean function $g(\cdot)$ to express the shape of the temperature profile generated by each combustion chamber and a_i is the scaling factor of the i-th chamber. ϕ_i^c is the angular coordinate of the i-th combustion chamber, and ϕ_j^t is the angular coordinate of the j-th thermocouple. T_e is the temperature at the entrance of the turbine. T_s denotes the mean temperature at the exhaust of the expansion turbine, so T_s/T_e is the drop in temperature during the expansion. Besides this drop in temperature, the gas rotates around the axis of the turbine during its expansion. We call this rotation angle *phase shift* and denote it by d. Using prior knowledge, we can compute the phase shift in an approximate way, and this computation is expressed by a known function $D(T_e, T_s, N)$ where N is the rotation speed of the turbine. In view of the approximate feature of $D(T_e, T_s, N)$, we use an additional parameter d_0 to compensate for its error and a corrective term for the dissymetry $d_6 \sin(\phi_j^t - \phi^s)$. The term $b\cos(\phi_j^t - \phi^s)$ takes into account the geometric dissymmetry of the expansion turbine where ϕ^s depends on the direction of the exhauster and b gives a measurement of this dissymmetry. Finally w_j's are noises introduced due to modeling and measurement errors.

To summarize, the parameter vector of the model is written as:

$$\theta = (a_0, \ldots, a_9, b, d_0, d_6)^T$$

and the available observations are:

$$X_k = (T_e(k), T_s(k), N(k), y_0(k), \ldots, y_{17}(k))^T \quad (21)$$

where T_s is a simple average of the 18 thermocouples. Since (20) is a static system and we assume the noise to be white, the sample X is an i.i.d. sequence. We monitor changes in the process using records of X which we denote by $\{X_k\}$.

An identification procedure is proposed in [8] to identify θ using a record $\{X_k\}$. It consists in attempting to minimize the cost function:

$$C(\theta) = \mathbf{E}\{c(\theta, X_k)\} \quad (22)$$

where :

$$c(\theta, X_k) = \frac{1}{2}\sum_{j=0}^{17}(F_j(\theta, k) - y_j(k))^2 \quad (23)$$

and $F_j(\theta, k)$ is the value of the measurement predicted by the model (20). Note that a stochastic gradient algorithm of Robbins-Monro type would be of the form:

$$\theta_k = \theta_{k-1} + \gamma_k H(\theta_{k-1}, X_k) \quad (24)$$

with :

$$H(\theta, X_k) = -\frac{\partial c(\theta, X_k)}{\partial \theta} \quad (25)$$

Unfortunately, we were unable to handle this recursive algorithm in order to make it stable and convergent. The reasons for this are the nonlinearity of the model, and the presence of noise at both the output and *the input* of the considered system. So the basic idea of the identification procedure proposed in [8] was to separate d_0 from the other parameters during the identification and this results in an off-line identification procedure. In this way, we manage to give a stable estimation of θ, but this estimation is known to be biased because of the noisy input [7]. The bias is essentially caused by the noises at the input of the identified process, that is, the measurement noises on T_e and N. Although a more complicated identification procedure may possibly remove this bias, we preferred to keep our relatively simple identification procedure to identify the nominal model of the system, because the purpose of this identification is to monitor changes in the process, and as we previously discussed, the local approach for change detection can be implemented using biased nominal models.

3.2 Monitoring the combustion set - Refinement

We apply the above non-likelihood method with the aid of the vector field defined by (25) and (23), where F is the output predicted by the model (20). Note that, because the measurements (X_n) correspond to an independent sequence, this leads to a simplification of the formula (9), because the only non zero term corresponds to $i = 0$. For simplicity, we now discuss only model validation problems.

Preliminary experiments with the aid of real data have shown that, even under $\mathcal{H}_0$, *the values of* χ_n^2 *(15) are large*, which can be explained by the nonstationarity induced by the changes in the functioning mode of the turbine itself.

In order to increase the robustness of the detection algorithm with respect to this practically important nonstationarity, we modify the algorithm in such a way to allow the measurements to deviate a little bit from the nominal model. Actually the detection of the changes in the mean of Δ_n - remember (13)-(14) - can then be achieved through two possible ways.

The first way consists in choosing a minimum magnitude of jump and to try to detect changes greater than this minimum magnitude with the aid of the classical Page-Hinkley test [2, 1]. This algorithm is known for its simplicity and its robustness. But it turned out that, on real data, we could not find a relevant choice of the minimum magnitude of change to be detected, which could lead to the requested robustness with respect to thes change in the functioning mode. A second method had thus been applied.

This method [1] consists in distinguishing between two new hypotheses :

$$\mathcal{H}_0 : \left(h'_{\theta_0} \cdot \widetilde{\theta}\right)^T R^{-1}(\theta_0) \left(h'_{\theta_0} \cdot \widetilde{\theta}\right) \le r_0^2$$

$$\mathcal{H}_1 : \left(h'_{\theta_0} \cdot \widetilde{\theta}\right)^T R^{-1}(\theta_0) \left(h'_{\theta_0} \cdot \widetilde{\theta}\right) \ge r_1^2$$

where r_0^2 and r_1^2 are two thresholds to be chosen. The introduction of r_0^2 allows the robustness with respect to nonstationarities. The maximization of the log-likelihood ratio between these two hypotheses leads to [1] :

$$\frac{1}{n} S_n = \begin{cases} -(\tilde{\chi}_n - r_1)^2 & \text{for } \tilde{\chi}_n \leq r_0 \\ -(\tilde{\chi}_n - r_1)^2 + (\tilde{\chi}_n - r_0)^2 & \\ \quad \text{for } r_0 \leq \tilde{\chi}_n \leq r_1 & \\ +(\tilde{\chi}_n - r_0)^2 & \text{for } \tilde{\chi}_n \geq r_1 \end{cases} \tag{26}$$

where :

$$\tilde{\chi}_n = \frac{1}{\sqrt{n}} \chi_n$$

and where χ_n is given in (15). We decide $\mathcal{H}_1$ when S_n is greater than a threshold.

Similarly an on-line decision procedure can be built in the same way [1], using the decision rules given in the subsection 2.4.3 and replacing $(\chi_j^n)^2$ by S_j^n. The behavior of this on-line decision function on some real data is shown in the figures 1 and 2, corresponding respecticely to no change, and to a decrease of 20 K in the first thermocouple.

4 Diagnosing the detected changes

After an alarm, it is desirable to decide which part of the machine has failed. In [4, 3], we describe a *sensitivity* technique for solving this diagnosis problem, in the general case where the physical model in terms of which the diagnosis is desired is of much greater dimension than the parameter θ used as a signature. Here the situation is different in the sense that the parameter θ already contains physical information, and thus the diagnosis in terms of the components of θ themselves is of interest. As the number of sensors - 18 - is greater than the dimension of θ - 13 -, it is relevant [1] to use the *minmax* robust approach for solving this diagnosis problem. The idea is, for each component (or subset of components) of θ of interest, to build a test which monitors changes in this component, while being insensitive to the other components which are considered as nuisance parameters. In [1] it is shown that this approach is equivalent to the GLR approach for treating the nuisance parameters.

4.1 Minmax approach

Let Y be a Gaussian random variable $\mathcal{N}(\mu, \Sigma)$, for which we test between the hypotheses $\mathcal{H}_0 \ : \ \mu = 0$ and $\mathcal{H}_1 \ : \ \mu = M\eta = [M_\theta, M_\psi] \begin{bmatrix} \Theta \\ \Psi \end{bmatrix}$, where M is a *full column rank matrix* and Σ is nonsingular. The minmax approach for testing a change in Θ while ignoring changes in Ψ consists in building the test which is optimal for the worst (also called least favorable) nuisance parameter Ψ which minimizes the power of the test. Maximizing the log-likelihood ratio related to η leads to :

$$t_0 = Y^T \Sigma^{-1} M (M^T \Sigma^{-1} M)^{-1} M^T \Sigma^{-1} Y$$

which is a χ^2 random variable with dimension of Y degrees of freedom, centered under $\mathcal{H}_0$ and with non-centrality parameter :

$$\gamma = \eta^T M^T \, \Sigma^{-1} \, M \eta$$

under $\mathcal{H}_1$. As the power of this test is, for any value of the threshold, an increasing function of the non-centrality parameter, the minmax approach in fact consists in minimizing γ with respect to the nuisance parameter Ψ. Using the notations :

$$M^T \Sigma^{-1} M = \begin{bmatrix} \mathcal{J}_{\theta\,\theta} & \mathcal{J}_{\theta\,\psi} \\ \mathcal{J}_{\psi\,\theta} & \mathcal{J}_{\psi\,\psi} \end{bmatrix},$$

the minimum of γ in Ψ, which is obtained at :

$$\Psi^* = -\, \mathcal{J}_{\psi\,\psi}^{-1} \, \mathcal{J}_{\psi\,\theta} \, \Theta$$

is :

$$\begin{aligned} \gamma^* = \min_{\Psi} \gamma \quad &= \quad \Theta^T \left\{ [I, -\, \mathcal{J}_{\theta\,\psi} \, \mathcal{J}_{\psi\,\psi}^{-1}] M^T \Sigma^{-1} \right\} \Sigma \\ & \quad \left\{ \Sigma^{-1} M [I, -\, \mathcal{J}_{\theta\,\psi} \, \mathcal{J}_{\psi\,\psi}^{-1}]^T \right\} \Theta \end{aligned}$$

Then the transformed variable :

$$Y^* = [I, -\, \mathcal{J}_{\theta\,\psi} \, \mathcal{J}_{\psi\,\psi}^{-1}] M^T \Sigma^{-1} Y$$

is Gaussian with mean $\mu^* = \mathcal{J}^* \Theta$ and covariance $\mathcal{J}^* = \mathcal{J}_{\theta\,\theta} - \mathcal{J}_{\theta\,\psi} \mathcal{J}_{\psi\,\psi}^{-1} \mathcal{J}_{\psi\,\theta}$. The optimal test which monitors changes in Θ while being insensitive to changes in Ψ is then :

$$t_0^* = Y^{*T} \mathcal{J}^{*^{-1}} Y^*$$

When $M = I$, we recover the result in [2].

When looking for the components which actually changed, in the real situation where these components are not *a priori* known, it is useful to re-formulate the previous result in terms of a permutation of the components of θ, in order to put first the components which we wish to monitor. We assume for the moment that the number of changed components is known. Let σ be a permutation matrix, we define $\eta_\sigma = \sigma \, \eta$ by $\eta_\sigma(i) = \eta(\sigma(i))$ for all i. We can write that Y has the law $\mathcal{N}(M_\sigma \, \eta_\sigma, \Sigma)$ where $M_\sigma = M \, \sigma^T$. If we partition $\eta_\sigma = \begin{pmatrix} \Theta_\sigma \\ \Psi_\sigma \end{pmatrix}$, then by definition of M_σ, we have :

$$\begin{aligned} M_\sigma^T \Sigma^{-1} M_\sigma \quad &= \quad \begin{bmatrix} \mathcal{J}_{\theta_\sigma\,\theta_\sigma} & \mathcal{J}_{\theta_\sigma\,\psi_\sigma} \\ \mathcal{J}_{\psi_\sigma\,\theta_\sigma} & \mathcal{J}_{\psi_\sigma\,\psi_\sigma} \end{bmatrix} \\ &= \quad \sigma \begin{bmatrix} \mathcal{J}_{\theta\,\theta} & \mathcal{J}_{\theta\,\psi} \\ \mathcal{J}_{\psi\,\theta} & \mathcal{J}_{\psi\,\psi} \end{bmatrix} \sigma^T \end{aligned}$$

The minmax test is then :

$$\begin{aligned} t_\sigma^* \quad &= \quad Y_\sigma^{*T} \mathcal{J}_\sigma^{*^{-1}} Y_\sigma^{*T} \\ Y_\sigma^{*T} \quad &= \quad [I, -\, \mathcal{J}_{\theta_\sigma\,\psi_\sigma} \, \mathcal{J}_{\psi_\sigma\,\psi_\sigma}^{-1}] \, \sigma \, M^T \, \Sigma^{-1} \, Y \\ \mathcal{J}_\sigma^* \quad &= \quad \mathcal{J}_{\theta_\sigma\,\theta_\sigma} - \mathcal{J}_{\theta_\sigma\,\psi_\sigma} \mathcal{J}_{\psi_\sigma\,\psi_\sigma}^{-1} \mathcal{J}_{\psi_\sigma\,\theta_\sigma} \end{aligned}$$

4.2 Using a variant and estimating the number of changed components

When applying this approach to the above mentioned gas turbine example, namely with :

$$\begin{aligned} \Sigma &= R(\theta_0) \\ M &= h'_{\theta_0} \\ Y &= \Delta_n \end{aligned}$$

it appeared that the following variant of the minmax approach is of interest.

This variant consists in using a converse approach, namely considering the parameters of interest as nuisance parameters, monitoring the other ones, and selecting the components corresponding to the minimum test. More precisely, let i be the number of changed components, which we assume to be known for the moment, and let G_i be the subset of permutations which exchange the last i components, such that if $j < k$ then $\sigma(j) < \sigma(k)$. To each subset with i elements corresponds a single permutation of G_i which puts this subset at the end. Let :

$$\hat{\sigma}_i = \arg\min_{\sigma \in G_i} t_\sigma$$

where we assume that the argument of the minimum is unique. The components of the mean vector where the changes occured are then given by the i last values of y, where $y = \hat{\sigma}_i x$, $x = [1, 2, ..., \ell]^T$ and ℓ is the dimension of the parameter vector θ.

Now, for estimating the number of changed components, we use a combinatorial argument. Let s_i be the ratio between the two smallest values of the χ^2 tests over G_i :

$$s_i = \frac{\min_{\sigma \in G_i - \{\hat{\sigma}_i\}} t_\sigma}{t_{\hat{\sigma}_i}} \tag{27}$$

An estimate of the number of changed components is then :

$$\hat{i} = \arg\max_{i \in [1, \ldots, \ell-1]} s_i \tag{28}$$

Now the diagnosis method consisting in searching for the number of components and then for the subset of changed components, is costly, because it leads to compute 2^ℓ matrices with size ℓ^2. This complexity can be reduced with the aid of the following incremental algorithm :

1. Use the previous algorithm for 1 suspected component;
2. Use the same method for 2 components, but exploring only the pairs which contain the first selected one;
3. Iterate, adding each time one component;
4. stop when :

$$\max_{\sigma \in G_i - \{\hat{\sigma}_i\}} t_\sigma - t_{\hat{\sigma}_i}$$

is less than a threshold.

4.3 Application to the monitoring of a gas turbine

In order to show the relevance of this incremental method for diagnosing changes in the gas turbine, we modify the coefficient a_3 by 5 %, a_8 by 5 %, b by 3 % and d_0 by 50 %. We obtain the values given in the table 1 for a single sample of size 200. The first coefficient which is diagnosed is a_8. Then we build the 11 statistics which take as nuisance parameters all the pairs containing a_8 and another coefficient. The next minimum is reached in b. Then we take as nuisance all the triples containing a_8 and b. The third diagnosed coefficient is d_0. Next we consider all the quadruples containing b,d_0,a_8 and we find a_3. The next diagnosed coefficient is a_4, but the discrepancy between the two smallest values is too small. The last line of the table shows test values computed with 300 additional observations; the discrepancy is still too small, and a_4 is not selected.

The behavior of the incremental method on real data corresponding to a decrease of 20 K in the first thermocouple is summarized in the table 2. The first selected coefficient is a_9, the next one is d_0, then the discrepancy between the two smallest values is too small. Using the known correspondance between the thermocouples and the combustion chambers, it can be easily checked that this diagnosis is correct.

5 Conclusion

Techniques for early warning of slight changes in systems and plants are useful for condition-based maintenance. In this paper we present an approach for this problem. This approach is based on the so-called "asymptotic local" approach for change detection previously introduced by some of the authors. We have presented in this paper a significant generalization of the original method as presented in [4, 5] : the *models* used for monitoring can be different from the *systems* to which monitoring is applied; the central tool in our approach, which is the *basic statistics* $H(\theta, X_k)$, may originate from some recursive identification procedure, but this is not mandatory; the *nominal model* θ_0 can be either identified or arbitrarily chosen.

Moreover we have investigated the *diagnosis* or isolation problem using the statistical minmax robust approach for decoupling. We have reported preliminary results from a real application in the area of condition based maintenance of gas turbines.

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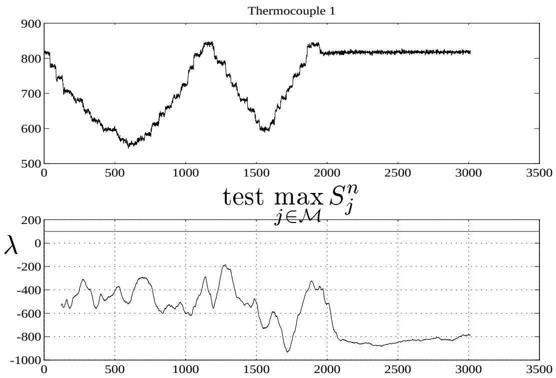

Figure 1: Test under $\mathcal{H}_0$ - Note the robustness with respect to the nonstationarity due to the changes in the functioning mode : the test statistics remains negative and does not cross the positive threshold.

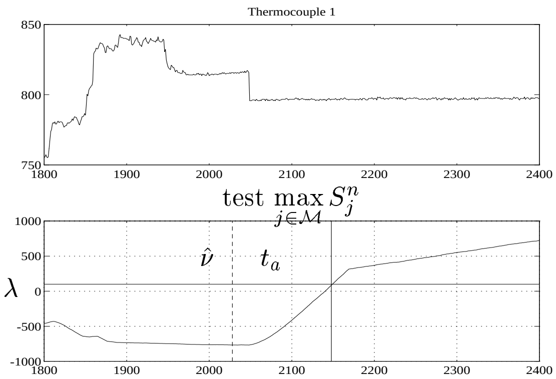

Figure 2: Detection of an abnormal decrease of 20 K in the first thermocouple.