Statistical Approaches

to Industrial Monitoring Problems -

Fault Detection and Isolation

M. Basseville, Q. Zhang, A. Benveniste
IRISA (CNRS and INRIA), Rennes, France

Three examples

Design of the algorithms : heuristics

Design of the algorithms : theory

Design of the algorithms : back to the examples

Introduction

Signal processing and on-board monitoring

Early detection of slight deviations

with respect to a characterization of the system

in usual working conditions

⇓

Condition-based maintenance, predictive maintenance

Three examples

Structural vibration monitoring

Physical *linear dynamic* model.

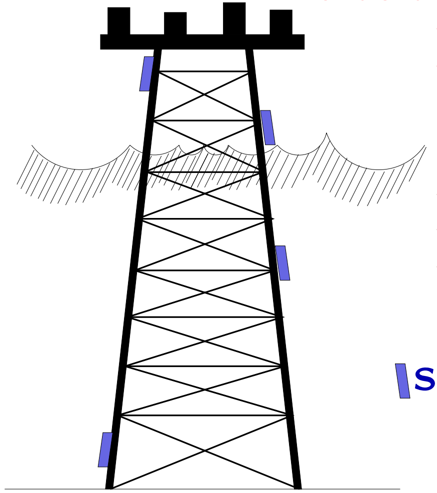
Combustion set of gas turbines

Semi-physical *non-linear static* model.

Catalytic converter of an automobile

Semi-physical *non-linear dynamic* model.

VIBRATIONS : OFFSHORE STRUCTURES

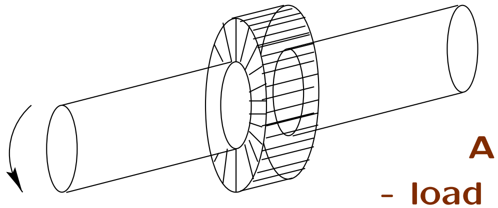


To be detected : changes in
- masses M, stiffness K
- modes

Natural excitation :
- swell
- turbulent, nonstationary
- non measured

▮ Sensors: accelerometers

VIBRATIONS : ROTATING MACHINES

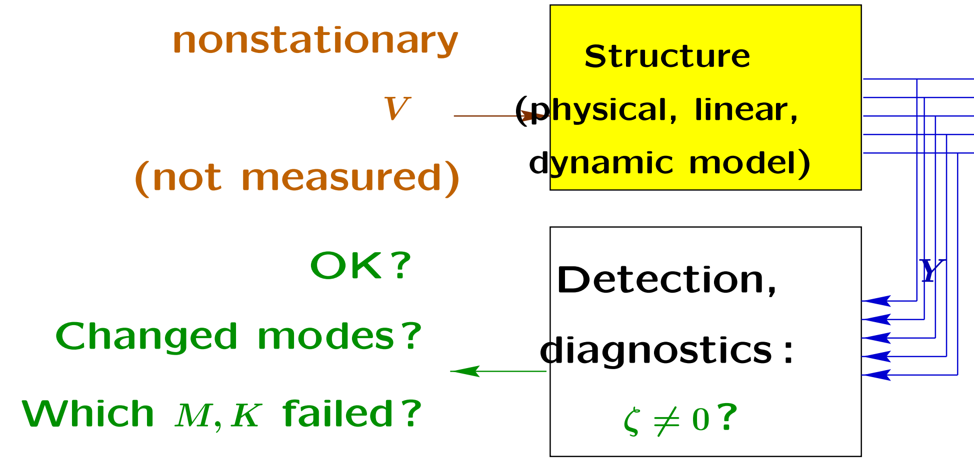


To be detected : changes in
- masses M, stiffness K
- modes

Ambiant excitation:
- load unbalancing, frictions, steam
- non stationary, non measured

Sensors: accelerometers (on the bearings!)

VIBRATIONS: MONITORING SCHEME



nonstationary

V

(not measured)

Structure
(physical, linear,
dynamic model)

Y

Detection,
diagnostics :
$\zeta \neq 0$?

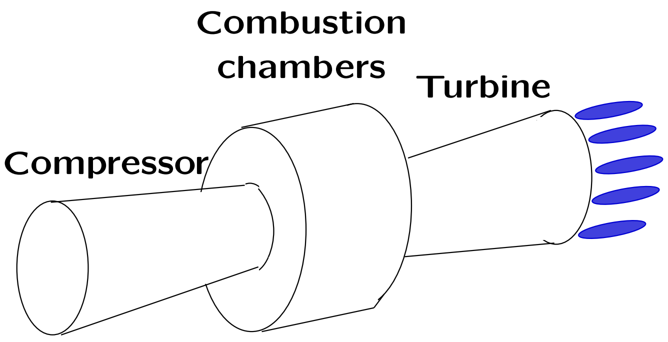
OK?

Changed modes?

Which M, K failed?

ζ shows if the model still fits the data

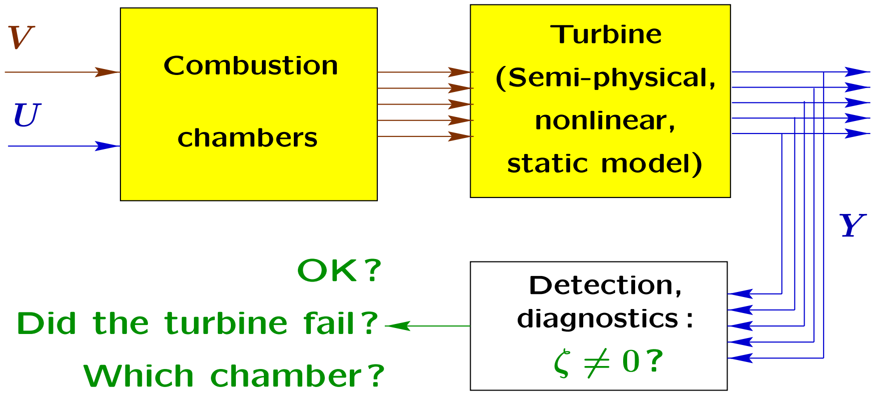
COMBUSTION : GAS TURBINES



Sensors :
thermocouples
at the exhaust

To be detected : changes in burners and turbine

COMBUSTION : MONITORING SCHEME



V

U

Combustion
chambers

Turbine
(Semi-physical,
nonlinear,
static model)

Y

Detection,
diagnostics :
$\zeta \neq 0$?

OK?

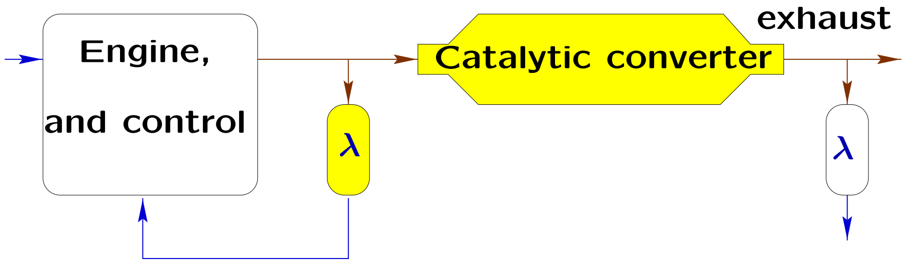
Did the turbine fail?

Which chamber?

ζ shows if the model still fits the data

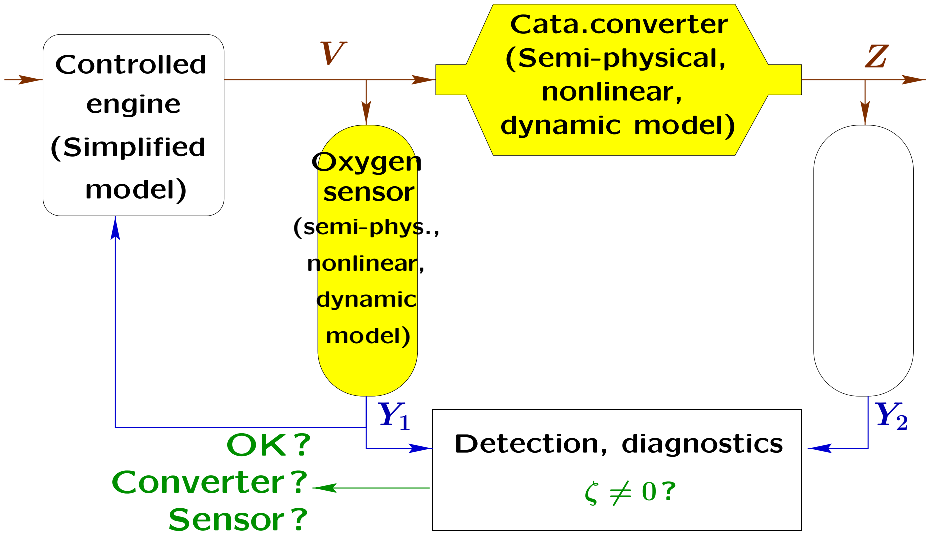
AUTOMOBILE : DEPOLLUTION SYSTEM

To be diagnosed (OBD2 norm) :
catalytic converter, front λ oxygen sensors



Sensors : λ oxygen gauges

DEPOLLUTION : MONITORING SCHEME



Design of the algorithms : heuristics

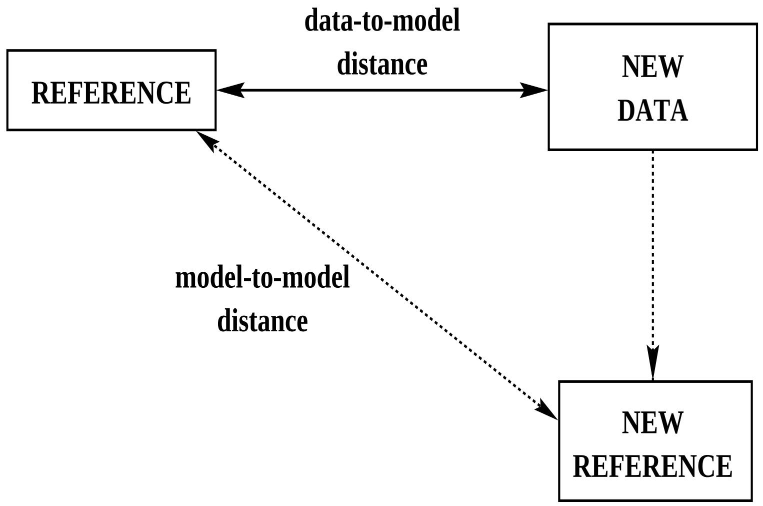
(Model validation)

Modeling and identification

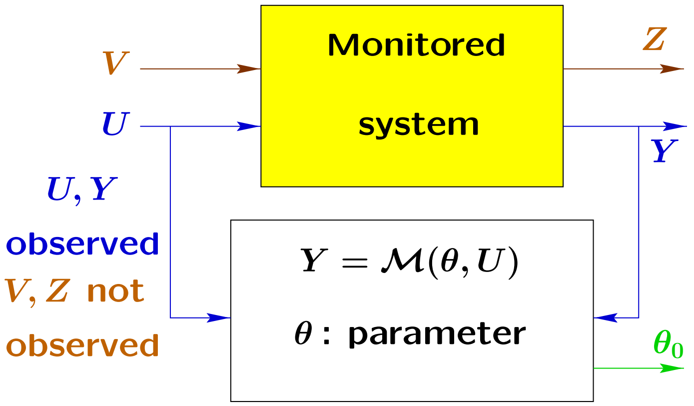
Monitoring

Reduction to a universal problem

On-Board Detection : don't re-identify!



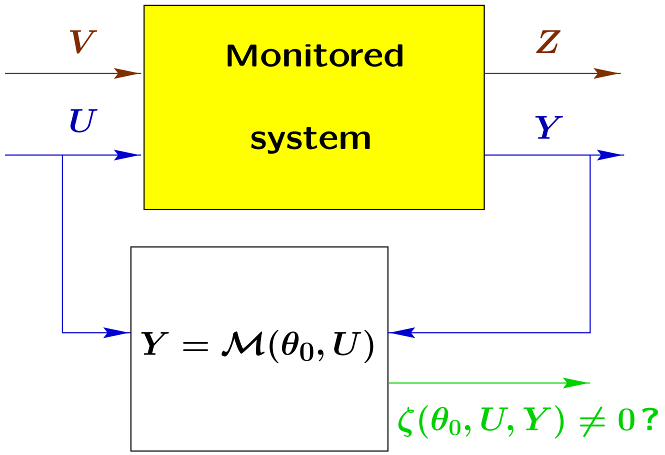
Modeling and identification



$$\theta_0 = \ \arg\min_\theta \ \Sigma_k \ (Y_k - \mathcal{M}(\theta, U_k))^2$$

or θ : $\ \Sigma_k \ \mathcal{C}(Y_k, \mathcal{M}(\theta_0, U_k)) = 0$

Monitoring



$$\zeta(\theta, U, Y) \triangleq \frac{\partial}{\partial \theta} \Sigma_k \ (Y_k - \mathcal{M}(\theta, U_k))^2$$

or: $\zeta(\theta, U, Y) \triangleq \ \Sigma_k \ \mathcal{C}(Y_k, \mathcal{M}(\theta, U_k))$

Reduction to a universal problem

$$\zeta_N \triangleq \zeta\left(\theta_0, \mathcal{Y}_1^N\right) \quad \neq \quad 0 \quad ?$$

Local approach

Test $\quad \mathcal{H}_0 \ : \ \theta = \theta_0$ against $\quad \mathcal{H}_1 \ : \ \theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$

$$\zeta_N \sim \mathcal{N}\left(0, \ \Sigma(\theta_0)\right) \qquad\qquad \zeta_N \sim \mathcal{N}\left(M(\theta_0) \ \delta\theta, \ \Sigma(\theta_0)\right)$$

χ^2 in ζ_N

Noises and uncertainty on θ_0 taken into account.

Design of the algorithms : theory

Local approach : likelihood

Local approach : other estimating/monitoring functions

Local approach : likelihood

Log-likelihood function

$$\ln\ p_\theta(\mathcal{Y}_1^N) = \sum_{k=1}^{N} \ln\ p_\theta(Y_k|\mathcal{Y}_1^{k-1})$$

Efficient score

$$\zeta_N(\theta_0) \triangleq \frac{1}{\sqrt{N}} \left.\frac{\partial \ln p_\theta(\mathcal{Y}_1^N)}{\partial\theta}\right|_{\theta=\theta_0} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \cdots$$

Fisher information matrix

$$\mathcal{I}(\theta) \triangleq \lim_{N\to\infty} \mathcal{I}_N(\theta), \quad \mathcal{I}_N(\theta) \triangleq \operatorname{cov} \zeta_N(\theta) = -\frac{1}{N}\, \mathrm{E}_\theta\, \frac{\partial^2 \ln p_\theta(\mathcal{Y}_1^N)}{\partial\theta^2}$$

Second order Taylor expansion of the log-likelihood ratio

$$\theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$$

$$S_N(\theta_0,\theta) \triangleq \ln \frac{p_\theta(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} \approx \delta\theta^T\, \zeta_N(\theta_0) - \frac{1}{2}\, \delta\theta^T\, \mathcal{I}(\theta_0)\, \delta\theta$$

$$\begin{aligned} \mathrm{E}_{\theta_0}\, S_N &\approx -\frac{1}{2}\, \delta\theta^T\, \mathcal{I}(\theta_0)\, \delta\theta \\ \mathrm{E}_{\theta}\, S_N &\approx +\frac{1}{2}\, \delta\theta^T\, \mathcal{I}(\theta_0)\, \delta\theta \quad \approx -\,\mathrm{E}_{\theta_0}\, S_N \end{aligned}$$

$$\operatorname{cov}_{\theta_0} S_N \approx \delta\theta^T\, \mathcal{I}(\theta_0)\, \delta\theta \quad \approx \operatorname{cov}_\theta S_N$$

First order Taylor expansion of the efficient score

$$\begin{aligned} \zeta_N(\theta) &\approx \zeta_N(\theta_0) + \frac{1}{N} \left.\frac{\partial^2 \ln p_\theta(\mathcal{Y}_1^N)}{\partial\theta^2}\right|_{\theta=\theta_0} \delta\theta \\ \mathrm{E}_{\theta_0}\, \zeta_N(\theta) &\approx -\,\mathcal{I}(\theta_0)\, \delta\theta \end{aligned}$$

Efficient score = ML estimating function

$$\mathrm{E}_{\theta_0}\, \zeta_N(\theta) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$$

Caution : Efficient score ≠ innovation !

Example : Gaussian scalar AR process

$$Y_k = \sum_{i=1}^{p} a_i\, Y_{k-i} + E_k, \qquad \theta^T = (\ a_1\ \ \ldots\ \ a_p\ )$$

$$\zeta_N(\theta) = \frac{1}{\sqrt{N}}\, \frac{1}{\sigma^2} \sum_{k=1}^{N} \mathcal{Y}_{k-1,p}^{-}\, \varepsilon_k(\theta)$$

$$\mathrm{I}(\theta) = \frac{1}{\sigma^2}\, \mathrm{T}_p$$

Efficient score ζ : vector-valued function

Innovation ε : scalar function

Hypotheses testing - CLT

Locally Asymptotic Normal (LAN) family (Le Cam, 1960)

$$\begin{aligned} S_N(\theta_0,\theta_N) &\triangleq \ln \frac{p_{\theta_N}(\mathcal{Y}_1^N)}{p_\theta(\mathcal{Y}_1^N)} \\ &\approx \delta\theta^T\,\zeta_N(\theta_0) - \tfrac{1}{2}\,\delta\theta^T\,\mathcal{I}_N(\theta_0)\,\delta\theta + \alpha_N(\theta_0,\mathcal{Y}_1^N,\delta\theta) \\ \zeta_N(\theta_0) &\to \mathcal{N}(0,\mathcal{I}(\theta_0)) \\ \alpha_N &\to 0 \text{ a.s. under } \mathcal{H}_0 \end{aligned}$$

Examples :
i.i.d. variables, stationary Gaussian processes, stationary Markov processes.

CLT in LAN families

(i.i.d. variables, stationary Gaussian processes, stationary Markov processes)

$$S_N(\theta_0,\theta) \;\to\; \begin{cases} \mathcal{N}(\;-\tfrac{1}{2}\,\delta\theta^T\,\mathcal{I}(\theta_0)\,\delta\theta,\;\; \delta\theta^T\,\mathcal{I}(\theta_0)\,\delta\theta) & \text{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(\;+\tfrac{1}{2}\,\delta\theta^T\,\mathcal{I}(\theta_0)\,\delta\theta,\;\; \delta\theta^T\,\mathcal{I}(\theta_0)\,\delta\theta) & \text{under} \quad \mathrm{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

$$\zeta_N(\theta_0) \;\to\; \begin{cases} \mathcal{N}(\quad 0,\;\; \mathcal{I}(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(\;\mathcal{I}(\theta_0)\,\delta\theta,\;\; \mathcal{I}(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

ζ_N **is asymptotically a sufficient statistics.**

Asymptotically optimum tests for composite hypotheses

Asymptotically equivalent problems :

$$(1) \;:\; \mathcal{P} = \{\mathrm{P}_\theta\}_{\theta\in\Theta\subset\mathrm{R}^\ell} \;\; \textbf{LAN}$$

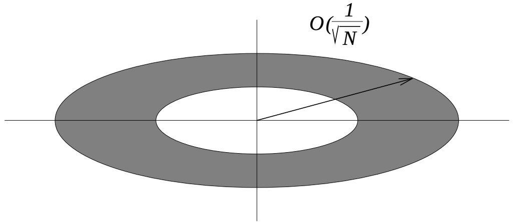
$$\mathcal{Y}_1^N, \quad N \to \infty$$

$$\mathcal{H}_0 = \{\theta \in \Theta_0\}, \quad \mathcal{H}_1 = \{\theta \in \Theta_1\}, \qquad \Theta_i = \theta_0 + \frac{\Gamma_i}{\sqrt{N}}$$

$$(2) \;:\; \zeta \;\sim\; \mathcal{N}(\Upsilon,\, \mathcal{I}^{-1}(\theta_0))$$

$$\mathcal{H}_0 = \{\Upsilon \in \Gamma_0\}, \quad \mathcal{H}_1 = \{\Upsilon \in \Gamma_1\}$$

Asymptotically equivalent and UMP tests
(Kushnir-Pinski, 1971; Nikiforov, 1982)



$$\begin{aligned} \frac{\sup_{\theta\in\Theta_1}\; p_\theta(\mathcal{Y}_1^N)}{\sup_{\theta\in\Theta_0}\; p_\theta(\mathcal{Y}_1^N)} &\geq \lambda \\ \frac{p_{\hat\theta}(\mathcal{Y}_1^N)}{p_{\theta_0}(\mathcal{Y}_1^N)} &\geq \lambda \\ N\;(\hat\theta-\theta_0)^T\,\mathcal{I}(\theta_0)\;(\hat\theta-\theta_0) &\geq \lambda \\ \zeta_N^T(\theta_0)\;\mathcal{I}^{-1}(\theta_0)\;\zeta_N(\theta_0) &\geq \lambda \end{aligned}$$

Local approach : other estimating functions

Quasi-score

$$\zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} H(\theta_0, Y_k)$$

Estimating function

$$\mathrm{E}_{\theta_0}\, H(\theta, Y_k) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$$

Mean deviation

$$M(\theta_0) \triangleq -\, \mathrm{E}_{\theta_0}\, \frac{\partial}{\partial\theta}\, H(\theta, Y_k)\Big|_{\theta=\theta_0}$$

Computation of the mean deviation

$$\begin{aligned} M(\theta_0) &\triangleq -\, \mathrm{E}_{\theta_0}\, \frac{\partial}{\partial\theta}\, H(\theta, Y_k)\Big|_{\theta=\theta_0} \\ &= -\, \frac{\partial}{\partial\theta}\, \mathrm{E}_{\theta_0}\, H(\theta, Y_k)\Big|_{\theta=\theta_0} \\ &= +\, \frac{\partial}{\partial\theta}\, \mathrm{E}_{\theta}\, H(\theta_0, Y_k)\Big|_{\theta=\theta_0} \\ &= \mathrm{cov}_{\theta_0}\left(\zeta_N(\theta_0),\ \zeta_N^{ML}(\theta_0)\right) \quad \textbf{(i.i.d. case)} \end{aligned}$$

First order Taylor expansion of a quasi-score

$$\theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$$

$$\begin{tikzcd}
\zeta_N(\theta) \approx & \underbrace{\zeta_N(\theta_0)} \arrow[d, "\text{CLT}"', "\text{under } \theta_0"] & + \sqrt{N}\, \underbrace{\frac{1}{N} \left(\sum_{k=1}^{N} \frac{\partial}{\partial\theta}\, H(\theta, Y_k)\Big|_{\theta=\theta_0} \right)} \arrow[d, "\text{LLN}"', "\text{under } \theta_0"] & \frac{\delta\theta}{\sqrt{N}} \\
& \mathcal{N}(0, \Sigma(\theta_0)) & \underbrace{\mathrm{E}_{\theta_0}\, \frac{\partial}{\partial\theta}\, H(\theta, Y_k)\Big|_{\theta=\theta_0}}_{=\, -\, M(\theta_0)} &
\end{tikzcd}$$

Estimation efficiency and covariance of estimating fct

$$\sqrt{N}\, \left(\hat{\theta}_N - \theta_0\right) \approx \underbrace{-\left(\mathrm{E}_{\theta_0}\, \frac{\partial}{\partial\theta}\, H(\theta, Y_k)\Big|_{\theta=\theta_0}\right)^{-1}}_{M(\theta_0)^{-1}} \zeta_N(\theta_0)$$

Hence

$$\mathrm{cov}\left(\hat{\theta}_N - \theta_0\right) = \left(M^T(\theta_0)\ \Sigma^{-1}(\theta_0)\ M(\theta_0)\right)^{-1}$$

Quasi score - CLT
(McLeish, 1977; Heyde, 1989; Delyon and Juditsky, 1997)

$$\zeta_N(\theta_0) \; \rightarrow \; \begin{cases} \mathcal{N}(\quad 0, \; \Sigma(\theta_0)) & \text{under} \quad \mathbf{P}_{\theta_0} \\ \mathcal{N}(\; M(\theta_0)\,\delta\theta, \; \Sigma(\theta_0)) & \text{under} \quad \mathbf{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

Tests for composite hypotheses
(Basawa, 1985; Benveniste et al., 1987)

$$\zeta_N^T \; \underbrace{\Sigma^{-1} \, M \, \left(M^T \, \Sigma^{-1} \, M\right)^{-1} \, M^T \, \Sigma^{-1}}_{\mathcal{I}^{-1}} \; \zeta_N \;\; \geq \lambda$$

χ^2-test invariant w.r.t. pre-multiplication of H
(and thus ζ) by an invertible matrix gain.

Design of the algorithms : back to the examples

Vibrations

Gas turbines

Catalytic converter

Structural vibration monitoring

$$\begin{cases} M\ddot{\mathcal{Z}}_t + C\dot{\mathcal{Z}}_t + K\mathcal{Z}_t & = & E_t \\ Y_t & = & L\mathcal{Z}_t \end{cases} \qquad \begin{array}{l} (M\mu^2 + C\mu + K)\Psi_\mu = 0 \\ \psi_\mu = L\Psi_\mu \end{array}$$

$$\begin{cases} X_{k+1} & = & FX_k + W_k \\ Y_k & = & HX_k \end{cases} \qquad \begin{array}{l} F\,\varphi_\lambda = \lambda\,\varphi_\lambda \\ e^{\tau\mu} = \lambda, \quad \psi_\mu = H\varphi_\lambda \end{array}$$

$$Y_k = \sum_{i=1}^{p} A_i \, Y_{k-i} + \sum_{j=0}^{q} B_j(k) \, V_{k-j} \qquad H \; F^p = \sum_{i=1}^{p} A_i \; H \; F^{p-i}$$

Monitor AR part, with nonstationary MA part.
Modal changes not visible on spectra (1%).
Likelihood : no hope !

Eigenstructure monitoring

$$\begin{cases} X_{k+1} & = & FX_k + W_k \\ Y_k & = & HX_k \end{cases} \qquad \begin{array}{l} F\,\varphi_\lambda = \lambda\,\varphi_\lambda \\ \Phi_\lambda \triangleq H\,\varphi_\lambda \end{array}$$

Canonical system parameter

$$\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}, \qquad \mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$$

System parameter characterization

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the same left kernel space

1. **Input-output (IV-based) monitoring functions:**
(Rougée, 1985)

$$\begin{pmatrix} A^T & -I_r \end{pmatrix} \widehat{\mathcal{H}}^0_{p+1,q} = 0$$

$$\left\{ \begin{array}{l} A^T \triangleq \begin{pmatrix} A_p & \dots & A_1 \end{pmatrix}, \quad \widehat{\mathcal{H}}_{p+1,q} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \dots & \hat{R}_{q-1} \\ \hat{R}_1 & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \hat{R}_p & \dots & \dots & \hat{R}_{p+q-1} \end{pmatrix} \\ \\ \qquad\qquad \begin{pmatrix} A^T(\theta_0) & -I_r \end{pmatrix} \mathcal{O}_{p+1}(\theta_0) = 0 \end{array} \right.$$

$$\zeta_N(\theta_0) = \sqrt{N} \;\; \mathrm{vec} \left(\begin{pmatrix} A^T(\theta_0) & -I_r \end{pmatrix} \widehat{\mathcal{H}}_{p+1,q} \right)$$

$$\widehat{\mathcal{H}}_{p+1,q} = \frac{1}{N-p-q+1} \sum_{k=q}^{N-p} \mathcal{Y}^+_{k,p+1} \, \mathcal{Y}^{-\,T}_{k,q}$$

---

2. **Subspace-based monitoring functions:**
(Abdelghani et al., 1996)

$$\textbf{SVD of} \quad W_1 \, \widehat{\mathcal{H}}^0_{p+1,q} \, W_2^T$$

$$\left\{ \begin{array}{l} \qquad S^T(\theta) \, W_1 \, \widehat{\mathcal{H}}^0_{p+1,q} \, W_2^T = 0 \\ \\ S^T \, S = I_s, \qquad S^T \, W_1 \, \mathcal{O}_{p+1}(\theta) = 0 \end{array} \right.$$

$$\zeta_N(\theta_0) = \sqrt{N} \;\; \mathrm{vec} \left(S^T(\theta_0) \, W_1 \, \widehat{\mathcal{H}}_{p+1,q} \, W_2^T \right)$$

$$\widehat{\mathcal{H}}_{p+1,q} = \frac{1}{N-p-q+1} \sum_{k=q}^{N-p} \mathcal{Y}^+_{k,p+1} \, \mathcal{Y}^{-\,T}_{k,q}$$

Experimental results

- **Simulated and real data** (offshore platform, alternator):
- **Detect $< 1\%$ frequency changes**
- **Detect changes mainly in modal shapes**
- **In the LMS modal analysis software environment: current experiments on several laboratory experimental setups and international benchmarks**

Combustion set of gas turbines

$$y_j = f(U) \sum_{i=1}^{10} \alpha_i \, g(\phi_j - h(U) - \beta) \qquad (1 \leq j \leq 18)$$

$$\theta \triangleq \begin{pmatrix} \alpha_1 & \dots & \alpha_{10} & \beta \end{pmatrix}^T$$

$$Y_k(\theta) \triangleq \begin{pmatrix} y_1 & \dots & y_{18} \end{pmatrix}^T (k)$$

Noisy outputs Y and inputs U: biased reference θ_0.

Identification bias is not an obstacle to monitoring!
(Zhang, 1992)

LS-score

$$H(\theta, Y_k, U_k) \;=\; -\left(\frac{\partial \epsilon_k(\theta)}{\partial \theta}\right)^T \epsilon_k(\theta)$$

$$\epsilon_k(\theta) \;=\; Y_k - Y_k(\theta)$$

Bias-adjusted estimating function

$$\hat{h}_0 \;=\; \frac{1}{N} \sum_{k=1}^{N} H(\theta_0, Y_k^0, U_k^0)$$

$$\zeta_N(\theta_0) \;=\; \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \left(H(\theta_0, Y_k, U_k) - \hat{h}_0\right)$$

Robustness w.r.t. operating conditions :

shape of the hypotheses! (Mathis, 1994)

Experimental results

- **Simulated and real data**
  (a turbine operating during several months)
- **Two algorithms tuned and implemented differently :**
  one on-line: for large sudden drops in temperatures,
  the other off-line: for smaller and slower drops
- **Robustness improved using confidence ellipsoids**
- **Correct diagnostics**

Catalytic converter and oxygen sensor of an automobile

$$\begin{cases} \dot{X} &= f(\theta, X, U) \\ Y &= g(\theta, X, U) \end{cases}$$

$$\begin{cases} \delta X_k &= f(\theta, X_k, U_k) \\ Y_k &= g(\theta, X_k, U_k) \;+\; \varepsilon_k^{(Y)} \end{cases}$$

Unknown input U;   highly nonlinear functions f, g.

1. **Full state observer and LS-prediction-score :**

$$H(\theta, \mathcal{Y}_1^k, \mathcal{U}_1^k) = \left(\frac{\partial \hat{Y}_{k|k-1}(\theta)}{\partial \theta}\right)^T \left(Y_k - \hat{Y}_{k|k-1}(\theta)\right)$$

$$\begin{cases} \hat{X}_{k|k-1} & = \hat{f}(\theta, \hat{X}_{k-1|k-2}, Y_{k-1}, U_{k-1}) \\ \hat{Y}_{k|k-1}(\theta) & = g(\theta, \hat{X}_{k|k-1}, U_{k-1}) \end{cases}$$

$\dfrac{\partial \hat{Y}_{k|k-1}(\theta)}{\partial \theta}$ **solution of a differential system.**

2. **Input-output description and LS-score :** (Zhang, 1996)

$$F(\theta, Y_k, U_k, \delta) = \varepsilon_k^{(F)}$$

$$H(\theta, Y_k, U_k, \delta) = - \left(\frac{\partial F(\theta, Y_k, U_k, \delta)}{\partial \theta}\right)^T F(\theta, Y_k, U_k, \delta)$$

$$F(\theta, Y_k, U_k, \delta) = P(Y_k, U_k, \delta)\ \theta - Q(Y_k, U_k, \delta)$$

$$\begin{aligned} 2\ H(\theta, Y_k, U_k, \delta) = & - P^T(Y_k, U_k, \delta)\ P(Y_k, U_k, \delta)\ \theta \\ & + P^T(Y_k, U_k, \delta)\ Q(Y_k, U_k, \delta) \end{aligned}$$

Experimental results

- **Combined LS-score and observer :**
  Simulated and real data
  - catalytic converter and front oxygen sensor;
  - some subsystems of a nuclear power plant;
  - some subsystems of a thermal plant.
- **Combined LS-score and input-output equations :**
  - some subsystems of a nuclear power plant.

Statistical approaches to isolation

$$\zeta \sim \mathcal{N}(M\ \eta,\ \ \bar{\Sigma}\ \bar{\Sigma}^T), \quad \eta = \begin{pmatrix} \eta_a \\ \eta_b \end{pmatrix}, \quad M = \begin{pmatrix} M_a & M_b \end{pmatrix}, \quad p_{\eta_a, \eta_b}(\zeta)$$

Decide between $\eta_a = 0$ and $\eta_a \neq 0$; η_b unknown

$$\mathcal{I} = M^T\ \Sigma^{-1}\ M \triangleq \begin{pmatrix} \mathcal{I}_{aa} & \mathcal{I}_{ab} \\ \mathcal{I}_{ba} & \mathcal{I}_{bb} \end{pmatrix}$$

$\mathcal{I}_a^{*-1}$: **upper-left term of** $\mathcal{I}^{-1}$; $\mathcal{I}_a^* = \mathcal{I}_{aa} - \mathcal{I}_{ab}\, \mathcal{I}_{bb}^{-1}\, \mathcal{I}_{ba}$

Statistical projection (sensitivity)

$$2\ \ln \frac{\max_{\eta_a}\ p_{\eta_a,0}(\zeta)}{p_{0,0}(\zeta)} \;=\; \zeta_a^T\,\mathcal{I}_{aa}^{-1}\,\zeta_a\ , \qquad \zeta_a$$: partial score

Statistical rejection (minmax)

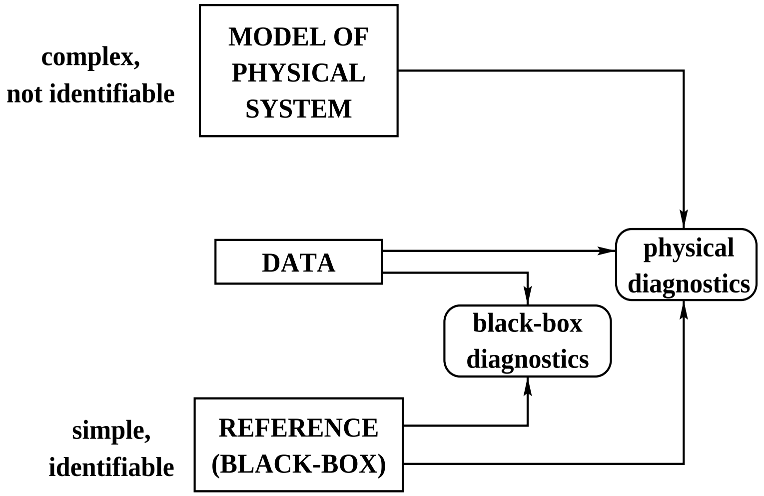
$$2\ \ln \frac{\max_{\eta_a,\eta_b}\ p_{\eta_a,\eta_b}(\zeta)}{\max_{\eta_b}\ p_{0,\eta_b}(\zeta)} \;=\; \zeta_a^{*T}\,\mathcal{I}_a^{*\,-1}\,\zeta_a^*\ , \qquad \zeta_a^*$$: effective score

$$\zeta_a^* \;=\; \zeta_a - \mathcal{I}_{ab}\,\mathcal{I}_{bb}^{-1}\,\zeta_b$$

regression of partial score on nuisance score (Neyman,1954).

Optimality (Spjotvoll, 1971; Mathis, 1994).

On-Board Diagnostics : don't solve the inverse problem!



complex,
not identifiable

MODEL OF PHYSICAL SYSTEM

DATA

physical diagnostics

black-box diagnostics

simple,
identifiable

REFERENCE (BLACK-BOX)

'Black-box' diagnostics : sensitivity

$$\delta\theta_i = \mathcal{J}_i\ \delta\phi_i, \quad \dim\phi = \dim\theta$$

'Physical' diagnostics : sensitivity with model reduction

$$\delta\theta_i = \mathcal{J}_i\ \delta\Phi_i, \quad \dim\Phi \gg \dim\theta$$

Aggregate the $\delta\theta_i$ with the metric of the χ^2-test.

Optimal sensor location

Maximize the power of the detection algorithms :

$$\mathrm{Trace}\ (M^T\ \Sigma^{-1}\ M)$$

Physical model necessary,
'compensate' for the number of d.o.f. of χ^2 tests.

Two possible uses :

- For a given sensor pool : which faults are detectable?
- For a given set of faults : how many sensors, and where?

Fault detectability

Conclusion

Actual and potential benefits of statistical local approach

Question: quasi-scores for (complex) state-space models?