Statistical detection approach to flutter monitoring

Laurent Mével, Michèle Basseville, Albert Benveniste,

IRISA (CNRS & INRIA, Rennes, France)

basseville@irisa.fr - http://www.irisa.fr/sigma2/

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In-flight modal analysis and aircraft stability

- Ensuring aircraft stability:
  in-flight tests with increasing altitude and airspeed

- Limited choices for measured excitation inputs,
  natural excitation input (turbulence):
  not controlled, not measured, and nonstationary

- Flutter: monitoring critical damping coefficients:
  accuracy and real-time issues in identification

- Idea: detection algorithms (shorter response time)

Modelling - Eigenstructure problem

FE model:

$$\begin{cases} M\ \ddot{\mathcal{Z}}(s) + C\ \dot{\mathcal{Z}}(s) + K\ \mathcal{Z}(s) & = \nu(s) \\ Y(s) = L\ \mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\ \Psi_\mu = 0\ , \quad \psi_\mu = L\ \Psi_\mu$$

State space:

$$\begin{cases} X_{k+1} = F\ X_k + V_k \\ Y_k = H\ X_k \end{cases}$$

$$F\ \varphi_\lambda = \lambda\ \varphi_\lambda\ , \quad \phi_\lambda \triangleq H\ \varphi_\lambda$$

Parameter:

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}}\ , \quad \underbrace{\psi_\mu = \phi_\lambda}_{\text{mode shapes}}\ ; \ \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\ \Phi \end{pmatrix}$$

Output-only covariance-based subspace identification

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k \; Y_{k-i}^T\right)}_{\text{ok if stationary !}} \; , \quad \mathcal{H} = \text{Hank}(R_i) \; , \qquad R_i = H \; F^i \; G$$

$$G \triangleq \mathbf{E}\left(X_k \; Y_k^T\right), \quad \mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} , \; \mathcal{C} \triangleq \left(G \;\; FG \;\; F^2G \;\; \ldots \right)$$

$$\mathcal{H} = \mathcal{O} \; \mathcal{C} \quad , \; \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

Implementation: $\underbrace{\hat{R}_i \triangleq 1/n \sum_{k=1}^{n} Y_k \; Y_{k-i}^T}_{\text{ok when nonstationary !}} \; , \quad \hat{\mathcal{H}} = \text{Hank}(\hat{R}_i)$

$$\textbf{SVD}(\hat{\mathcal{H}}) + \textbf{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

Eigenstructure monitoring

θ_0 : reference parameter, known (or identified)

Y_k: n-size sample of new measurements

System parameter characterization:

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the same left kernel.

$$\exists U, \quad U^T U = I_s, \qquad U^T \mathcal{O}_{p+1}(\theta_0) = 0; \qquad \text{say } U(\theta_0)$$

$$\theta_0 \leftrightarrow (R_i^0)_i \quad \text{characterized by:} \quad U^T(\theta_0)\, \hat{\mathcal{H}}^0_{p+1,q} = 0$$

Subspace-based residual for eigenstructure monitoring

$$\zeta_n(\theta_0) \triangleq \sqrt{n}\ \mathrm{vec}(U^T(\theta_0)\, \hat{\mathcal{H}}_{p+1,q})$$

The residual is asymptly Gaussian (local approach)

Mean sensitivity (Jacobian) $\mathcal{J}(\theta_0)$ and covariance $\Sigma(\theta_0)$

$$\zeta_n(\theta_0) \;\rightarrow\; \begin{cases} \mathcal{N}(\; 0,\; \Sigma(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(\; \mathcal{J}(\theta_0)\,\delta\theta,\; \Sigma(\theta_0)) & \text{under} \quad \mathrm{P}_{\theta_0+\frac{\delta\theta}{\sqrt{n}}} \end{cases}$$

(GLR) χ^2-test for modal monitoring

$$\zeta_n^T \; \Sigma^{-1} \, \mathcal{J} \, (\mathcal{J}^T \, \Sigma^{-1} \, \mathcal{J})^{-1} \, \mathcal{J}^T \, \Sigma^{-1} \; \zeta_n \quad \geq h$$

(GLR) Directional χ^2-test for modal diagnosis

$$\zeta_n^T \; \Sigma^{-1} \, \mathcal{J}_i \, (\mathcal{J}_i^T \, \Sigma^{-1} \, \mathcal{J}_i)^{-1} \, \mathcal{J}_i^T \, \Sigma^{-1} \; \zeta_n \quad \geq h$$

Flutter monitoring with residual ζ - Quadratic tests

- χ^2 focussed on damping ρ : BUT $\rho \neq \rho_0$ irrelevant;

- Hypothesis of interest: $\rho < \rho_c$

1. $\rho_c = \rho_0$: Use GLR test for (local) $\delta\rho \geq 0$ against $\delta\rho < 0$:

$$l(\theta_0) = -\,\mathrm{sign}(\overline{\zeta}) \cdot \overline{\chi}^2, \qquad \overline{\chi}^2 \triangleq \overline{\zeta}^T \overline{\Sigma}^{-1} \overline{\zeta}$$

2. $\rho_c < \rho_0$: non local hypotheses $\rho \geq \rho_c$ against $\rho < \rho_c$:

$\tilde{\theta}_0 \triangleq \theta_0$ except that $\rho_0 \leftarrow \rho_c$; use $l(\tilde{\theta}_0)$: BUT biased

Flutter monitoring with residual ζ - Linear tests

- Another approximation for ζ:

$$\zeta(\tilde{\theta}_0) = \sum_{k=1}^{n} Z_k(\tilde{\theta}_0)/\sqrt{n}, \quad Z_k(\tilde{\theta}_0) \triangleq U(\tilde{\theta}_0)^T \ \mathcal{Y}^+_{k,p+1} \ \mathcal{Y}^-_{k,q}$$

$$\sum_{k=1}^{n} Z_k/\sqrt{n} \ \longrightarrow \ \mathcal{N}(0,\cdot) \quad \text{under } \rho = \rho_c$$

$$\Rightarrow \ Z_k \ \longrightarrow \ \mathcal{N}(0,\cdot),$$ and the Z_k's are iid

- Idea: for overcoming a bias, handle deviations !

Flutter monitoring - Linear tests (Contd.)

Use CUSUM test for $\rho = \rho_c + \epsilon$ against $\rho = \rho_c - \epsilon$:

$$S_n(\tilde{\theta}_0) \triangleq \sum_{k=1}^{n} Z_k(\tilde{\theta}_0), \quad M_n(\tilde{\theta}_0) \triangleq \max_{1 \leq k \leq n} S_k(\tilde{\theta}_0)$$

$$g_n(\tilde{\theta}_0) \triangleq M_n(\tilde{\theta}_0) - S_n(\tilde{\theta}_0) \geq 0$$

under $\rho = \rho_c + \epsilon$: $\quad \overline{g}_n(\tilde{\theta}_0) \approx 0$

under $\rho = \rho_c - \epsilon$: $\quad \overline{g}_n(\tilde{\theta}_0) > 0$

if ρ decreases, $\overline{g}_n(\tilde{\theta}_0)$ increases

if ρ increases, $\overline{g}_n(\tilde{\theta}_0)$ decreases

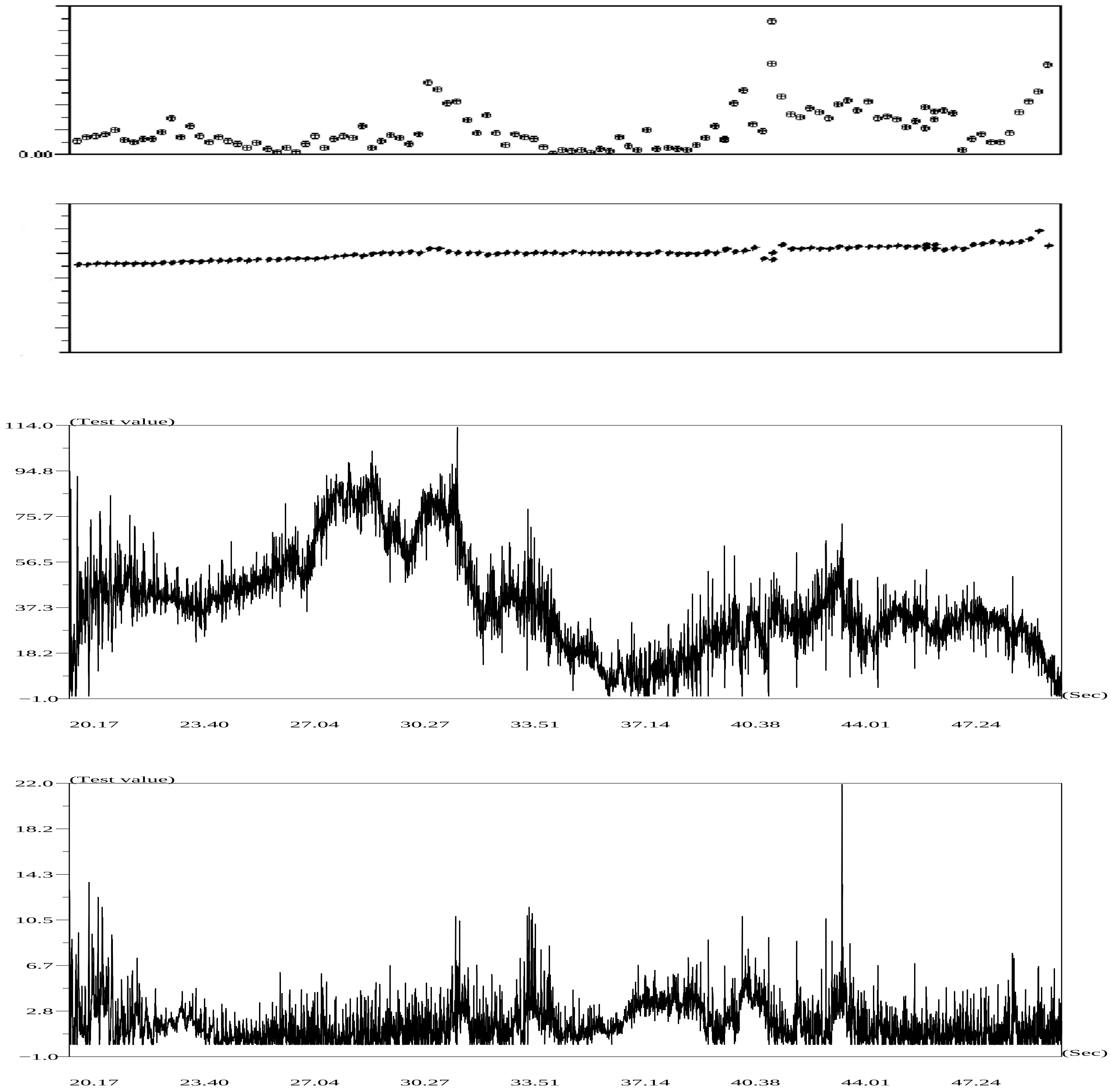
Numerical results - Ariane

- Identification: 2000-size blocks;
  Flutter monitoring: sample by sample

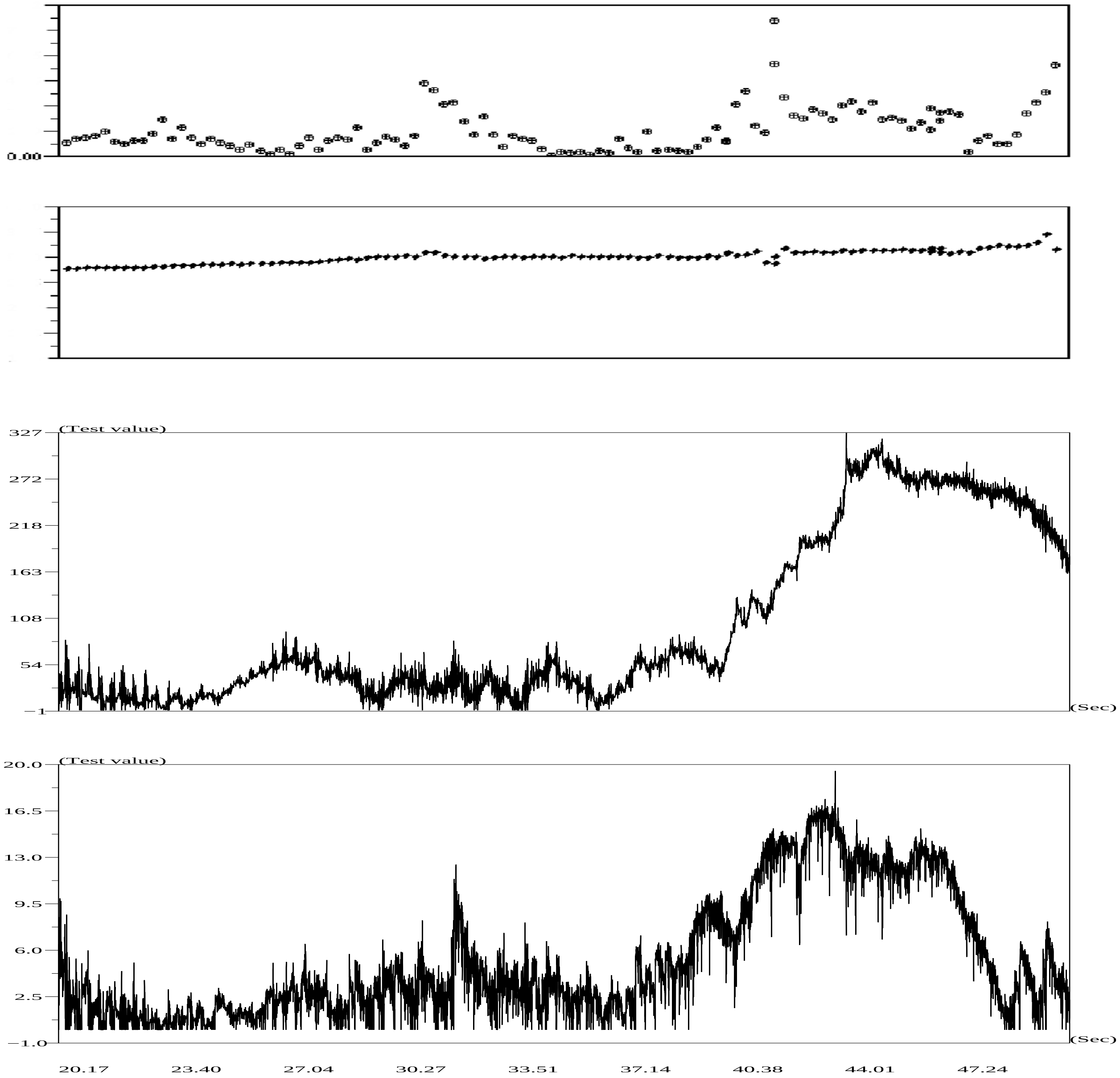
- Flutter monitoring: 2 reference data sets:
  one at the beginning $\longrightarrow \rho_c = 1.5\%$,
  one at the middle $\longrightarrow \rho_c = 0.5\%$

- Test without and with filtering

- Of interest: increase/decrease of the test g_n



Damping & frequency, CUSUM w/o filtering: $\rho_c = 1.5\%$ and 0.5%



Damping & frequency, CUSUM with filtering: $\rho_c = 1.5\%$ and 0.5%