Subspace algorithms for data from varying sensor locations

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Context :

In-operation modal identification of large structures

- Use moving sensors to mimic a (much) larger set of sensors.
- The excitation is typically:
  - natural, not controlled.
  - not measured:
    - buildings, bridges, offshore structures,
    - rotating machinery,
    - cars, trains, aircrafts.
  - nonstationary (e.g., turbulent).

The problem

utput-only eigenstructure identification,

ased on moving sensor pools, with some reference sensors,

the presence of nonstationary excitation.

ced:

void merging identification results from the different pools,

lerge the data instead, and process them globally,

se a standard subspace algorithm.

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1. Modelling

FE model: $\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) = \nu(s) \\ Y(s) = L\mathcal{Z}(s) \end{cases}$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \ , \quad \psi_\mu = L\Psi_\mu$$

ate space: $\begin{cases} X_{k+1} = F X_k + V_k \\ Y_k = H X_k \end{cases}$

$$F\Phi_\lambda = \lambda \ \Phi_\lambda \ , \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \ , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}}$$

2. Output-only subspace identification

$$\underbrace{R_i \triangleq \mathbf{E}\left(Y_k \, Y_{k-i}^T\right)}_{\text{ok if stationary !}} \ , \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H \ F^i \ G \ , \quad G \triangleq \mathbf{E}\left(X_k \ Y_k^T\right)$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix} \ , \ \mathcal{C} \triangleq \left(G \ \ FG \ \ F^2G \ \ \dots \right)$$

$$\mathcal{H} = \mathcal{O} \ \mathcal{C} \ , \ \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

Implementation

$$\underbrace{\hat{R}_i \triangleq \frac{1}{N} \sum_{k=1}^{N} Y_k \ Y_{k-i}^T}_{\text{ok when nonstationary !}} \ , \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$\hat{R}_i \approx H \ F^i \ \hat{G}$, hence $\hat{\mathcal{H}} \approx \mathcal{O} \ \hat{\mathcal{C}}$

VD$(\hat{\mathcal{H}})$ + truncation $\longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$

$\hat{\mathcal{O}}$ robust against nonstationary excitation !

3. Polyreference subspace identification (1)

$$\underbrace{\begin{bmatrix} Y_k^{(0,1)} \\ Y_k^{(1)} \end{bmatrix}}_{\text{Record 1}} \ \underbrace{\begin{bmatrix} Y_k^{(0,2)} \\ Y_k^{(2)} \end{bmatrix}}_{\text{Record 2}} \ \dots \ \underbrace{\begin{bmatrix} Y_k^{(0,J)} \\ Y_k^{(J)} \end{bmatrix}}_{\text{Record J}}$$

$$\begin{cases} X_{k+1}^{(j)} = F \ X_k^{(j)} + V_k^{(j)} & \\ Y_k^{(0,j)} = H_0 \ X_k^{(j)} & \text{(the reference)} \\ Y_k^{(j)} = H_j \ X_k^{(j)} & \text{(sensor pool n}^o j\text{)} \end{cases}$$

$$R_i^{0,j} \triangleq \mathbf{E} \ Y_k^{(0,j)} \ Y_{k-i}^{(0,j)^T} \ , \quad R_i^j \triangleq \mathbf{E} \ Y_k^{(j)} \ Y_{k-i}^{(0,j)^T}$$

onary excitation

$$\mathrm{cov} V_k^{(j)} = Q \ , \quad G \triangleq \mathrm{E} \ X_k^{(j)} \ Y_k^{(0,j)^T}$$

$$R_i^{0,j} = H_0 \ F^i \ G \ \triangleq \ R_i^0 \ , \quad R_i^j = H_j \ F^i \ G$$

$$R_i^\pi \triangleq \begin{bmatrix} R_i^0 \\ R_i^1 \\ \vdots \\ R_i^J \end{bmatrix} = H \ F^i \ G \ , \quad H \triangleq \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_J \end{bmatrix}$$

tationary excitation

$$\mathrm{cov} V_k^{(j)} = Q_j \ , \quad G_j \triangleq \mathrm{E} \ X_k^{(j)} \ Y_k^{(0,j)^T}$$

$$R_i^{0,j} = H_0 \ F^i \ G_j \ , \quad R_i^j = H_j \ F^i \ G_j$$

4. Nonstationary excitation: data renormalization

Reference sensors: using the redundancy

$$R_i^{0,j} = H_0 \ F^i \ G_j \ , \quad G \triangleq \left(G_1 \ \ G_2 \ \ \ldots \ \ G_J \right)$$

$$R_i^0 \triangleq \left(R_i^{0,1} \ \ R_i^{0,2} \ \ \ldots \ \ R_i^{0,J} \right) \ = \ H_0 \ F^i \ G$$

$$\mathcal{H}_0 \triangleq \begin{pmatrix} R_0^0 & R_1^0 & R_2^0 & \ldots \\ R_1^0 & R_2^0 & R_3^0 & \ldots \\ R_2^0 & R_3^0 & R_4^0 & \ldots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix} = \mathcal{O}(H_0, F) \, \mathcal{C}(F, G) \ \longrightarrow \ \mathcal{C}(F, G_j)$$

ng sensors: normalizing the data

$$= H_j \ F^i \ G_j \ , \quad \mathcal{H}_j \triangleq \begin{pmatrix} R_0^j & R_1^j & R_2^j & \ldots \\ R_1^j & R_2^j & R_3^j & \ldots \\ R_2^j & R_3^j & R_4^j & \ldots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix} = \mathcal{O}(H_j, F) \, \mathcal{C}(F, G_j)$$

$$\triangleq \mathcal{H}_j \ \left(\mathcal{C}^T(F, G_j) \ \left(\mathcal{C}(F, G_j) \ \mathcal{C}^T(F, G_j) \right)^{-1} \ \mathcal{C}(F, G_1) \right)$$

$$\bar{\mathcal{H}}_j \ \leftrightarrow \ \bar{R}_i^j \triangleq H_j \ F^i \ G_1 \quad (1 \leq j \leq J)$$

$$\bar{R}_i \triangleq \begin{pmatrix} \bar{R}_i^0 \\ \bar{R}_i^1 \\ \vdots \\ \bar{R}_i^J \end{pmatrix} , \quad \bar{R}_i^0 \triangleq R_i^{0,1} = H_0 \ F^i \ G_1$$

$$\bar{\mathcal{H}} \triangleq \begin{pmatrix} \bar{R}_0 & \bar{R}_1 & \bar{R}_2 & \ldots \\ \bar{R}_1 & \bar{R}_2 & \bar{R}_3 & \ldots \\ \bar{R}_2 & \bar{R}_3 & \bar{R}_4 & \ldots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix} = \mathcal{O}(H, F) \ \mathcal{C}(F, G_1)$$

5. Polyreference subspace identification (2)

- Row-stack the $R_i^{(0,j)}$ into R_i^0
- Build $\hat{\mathcal{H}}_0$ on R_i^0
- SVD $(\hat{\mathcal{H}}_0)$ + truncation $\longrightarrow \ \mathcal{C}(F, G) \triangleq \mathcal{C}$
- Partition $\mathcal{C} = (\ \mathcal{C}_1 \ \ \mathcal{C}_2 \ \ \ldots \ \ \mathcal{C}_J \ )$
- Compute $(\mathcal{C}_j^T \ (\mathcal{C}_j \, \mathcal{C}_j^T \,)^{-1} \ \mathcal{C}_1)$
- Renormalize $\hat{\bar{\mathcal{H}}}_j = \hat{\mathcal{H}}_j \ (\mathcal{C}_j^T \ (\mathcal{C}_j \, \mathcal{C}_j^T \,)^{-1} \ \mathcal{C}_1)$
- Deduce $\bar{R}_i^j$
- Column-stack the $\bar{R}_i^j$ into $\bar{R}_i$
- Build $\bar{\mathcal{H}}$ on $\bar{R}_i$
- We are back to the standard problem.

. Nonstationary excitation within the records

$$\begin{cases} X_{k+1}^{(j)} = F\ X_k^{(j)} + V_k & \mathrm{cov} V_k = Q_k \\ Y_k^{(0,j)} = H_0\ X_k^{(j)} & (\text{the reference}) \\ Y_k^{(j)} = H_j\ X_k^{(j)} & (\text{sensor pool n}^o j) \end{cases}$$

$$A_j(K_j) = \frac{1}{K_j} \Sigma_{k\in\mathcal{I}_j} \left(\|Y_k^{(0,j)}\|^2 + \|Y_k^j\|^2 \right)$$

$$R_i^j(K_j) = \frac{1}{A_j(K_j)} \Sigma_{k\in\mathcal{I}_j}\ Y_k^j\ {Y_{k-i}^{(0,j)}}^T$$

$$R_i^{0,j}(K_j) = H_0\ F^i\ G_j(K_j) + o(K_j)$$

$$R_i^j(K_j) = H_j\ F^i\ G_j(K_j) + o(K_j)$$