On robustness in statistical FDI methods

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Introduction

A priori information on faults and shape of hypotheses

Monitoring the mean of a Gaussian vector : a modified χ^2-test

Reducing FDI problems to the monitoring of a Gaussian vector

Example

Introduction

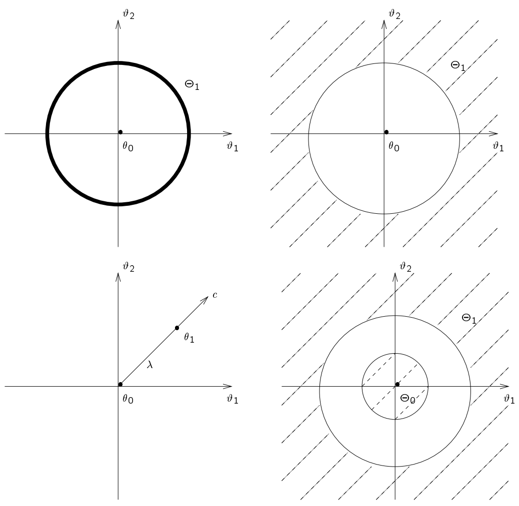
Robustness in model-based FDI methods

Residual generation : unknown input observers, disturbance decoupling

Residual evaluation : adaptive threshold, fuzzy decision logic

Alternative idea : a priori information, 'nonlinear' threshold

A priori information on faults and shape of hypotheses



Monitoring the mean of a Gaussian vector :

a modified χ^2-test

$$\Theta_0 \;: (\theta-\theta_0)^T\,\Sigma^{-1}\,(\theta-\theta_0) \le \rho_0, \quad \Theta_1 \;: (\theta-\theta_0)^T\,\Sigma^{-1}\,(\theta-\theta_0) \ge \rho_1$$

$$\zeta \sim \mathcal{N}(\theta,\Sigma), \quad l_\theta(\zeta) \triangleq (\zeta-\theta)^T\,\Sigma^{-1}\,(\zeta-\theta), \quad t \triangleq \min_{\theta_0\in\Theta_0} l_\theta - \min_{\theta_1\in\Theta_1} l_\theta$$

$$t = \begin{cases} -(\chi_0-\rho_1)^2 & \text{for } \chi_0 \le \rho_0 \\ -(\chi_0-\rho_1)^2 + (\chi_0-\rho_0)^2 & \text{for } \rho_0 \le \chi_0 \le \rho_1, \quad \chi_0 = \sqrt{l_{\theta_0}(\zeta)} \\ +(\chi_0-\rho_0)^2 & \text{for } \chi_0 \ge \rho_1 \end{cases}$$

Reducing FDI problems to

monitoring the mean of a Gaussian vector

Additive faults : repeated equations

Non-additive faults : local approach

Additive faults : repeated equations

$$\left\{\begin{array}{rcl} X_{k+1} & = & FX_k + GU_k + \Gamma\,\Upsilon_x + W_k \\ Y_k & = & HX_k + JU_k + \Xi\,\Upsilon_y + V_k \end{array}\right.$$

Reducing the problem to a static one

$$\mathcal{Y}_{k,\varrho} \triangleq \begin{pmatrix} Y_k \\ \vdots \\ Y_{k+\varrho-1} \end{pmatrix} \qquad (\varrho - \text{size window})$$

$$\underline{\mathcal{Y}}_{k,\varrho} \triangleq \mathcal{Y}_{k,\varrho} - \mathcal{M}_\ell(G,J)\ \mathcal{U}_{k,\varrho}, \qquad \underline{\mathcal{V}}_{k,\varrho} \triangleq \mathcal{M}_\ell(I_n,0)\ \mathcal{W}_{k,\varrho} + \mathcal{V}_{k,\varrho}$$

$$\underline{\mathcal{Y}}_{k,\varrho} = \mathcal{O}_\ell\ X_k + \underline{\mathcal{M}_\ell}(\Gamma)\ \Upsilon_x + (\mathbb{1}_\ell \otimes \Xi)\ \Upsilon_y + \underline{\mathcal{V}}_{k,\varrho}$$

X_k independent of $\underline{\mathcal{V}}_{k,\varrho}$, fault gain matrices f.c.r., cov $\underline{\mathcal{V}}_{k,\varrho}$ not block-diagonal.

Non-additive faults : local approach

Linear dynamic systems

$$\left\{\begin{array}{rcl} X_{k+1} & = & FX_k + \varepsilon_k \\ Y_k & = & HX_k \end{array}\right. \qquad \begin{array}{l} F\,\varphi_\lambda = \lambda\,\varphi_\lambda \\ \Phi_\lambda \triangleq H\,\varphi_\lambda \end{array} \qquad \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$$

Non-linear static systems

$$Y_k = h(\theta, X_k, U_k) + \varepsilon_k$$

Non-linear dynamic systems

$$\left\{\begin{array}{rcl} \dot{X} & = & f(\theta, X, U) \\ Y & = & g(\theta, X, U) \end{array}\right. \qquad \left\{\begin{array}{rcl} \delta X_k & = & f(\theta, X_k, U_k) \\ Y_k & = & g(\theta, X_k, U_k) + \varepsilon_k \end{array}\right.$$

Design of statistical FDI algorithms

Primary residuals : estimating functions $K(\theta, \mathcal{Y}_{k,\varrho})$

$$\mathbf{E}_{\theta_0}\, K(\theta, \mathcal{Y}_{k,\varrho}) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$$

$$M(\theta_0) \triangleq -\,\mathbf{E}_{\theta_0}\, \frac{\partial}{\partial\theta}\, K(\theta, \mathcal{Y}_{k,\varrho})\Big|_{\theta=\theta_0} \qquad \text{f.c.r.}$$

Improved residuals : local approach

(*Safe system*) $\mathbf{H}_0 :\ \theta = \theta_0$ and (*Faulty system*) $\mathbf{H}_1 :\ \theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$

$$\zeta_N(\theta_0) \triangleq \frac{1}{\sqrt{N}} \sum_{k=1}^{N} K(\theta_0, \mathcal{Y}_{k,\varrho}) \ \rightarrow\ \left\{\begin{array}{lll} \mathcal{N}(\ 0,\ \Sigma(\theta_0)) & \text{under} & \mathbf{P}_{\theta_0} \\ \mathcal{N}(\ M(\theta_0)\,\delta\theta,\ \Sigma(\theta_0)) & \text{under} & \mathbf{P}_{\theta_0 + \frac{\delta\theta}{\sqrt{N}}} \end{array}\right.$$

Example : combustion set of gas turbines

Semi-physical model

$$y_j = f(U) \sum_{i=1}^{10} \alpha_i \, g(\phi_j - h(U) - \beta) \qquad (1 \leq j \leq 18)$$

$$\theta \triangleq \begin{pmatrix} \alpha_1 & \ldots & \alpha_{10} & \beta \end{pmatrix}^T$$

$$Y_k(\theta) \triangleq \begin{pmatrix} y_1 & \ldots & y_{18} \end{pmatrix}^T (k)$$

LS-score

$$K(\theta, Y_k, U_k) \;=\; -\frac{1}{2} \left(\frac{\partial \epsilon_k(\theta)}{\partial \theta} \right)^T \epsilon_k(\theta)$$

$$\epsilon_k(\theta) \;=\; Y_k - Y_k(\theta)$$