Handling parametric and non-parametric additive faults in LTV Systems

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Problem and approaches

FDI for LTV systems
- Relevant approach to FDI of NL systems (linearization along the actual or nominal trajectory)
- LTV systems more general than widely used LPV systems

Three main approaches
- Detection filter, game theoretic approach to filter design, unknown input decoupled filter, UIO, finite horizon fault detection filter
  Keviczky, Edelmayer, Chung-Speyer, Chen-Patton, Hou-Muller, Zhong-Ding, ...
- Adaptive observers, set-valued observers, time domain solutions to different $\mathcal{H}_- / \mathcal{H}_\infty$ problems
  Zhang-Xu, Rosa-Shamma-Athans, Li-Zhou, ...
- Parity-based fault estimation Zhong-Ding
Different fault types

- **Parametric** fault: (rare) changes in a parameter vector
- **Non-parametric** fault: arbitrary unknown function of time

Most FDI methods for LTV systems address the non-parametric fault case, or the parametric one.

Contribution

- A statistical approach exists for constant parametric faults
- Extension to both TV parametric and non-parametric faults

Two solutions:

- Assuming (piecewise) constant parametric fault and rejecting the non-parametric fault
- Adapting to the TV parametric fault
Model and assumptions

**MIMO LTV system ($\mathbb{H}_0$)**

\[
\begin{align*}
X_{k+1} &= F_k X_k + G_k U_k + W_k \\
Y_k &= H_k X_k + J_k U_k + V_k
\end{align*}
\]

- $F_k, G_k, H_k, J_k$: bounded TV matrices
- $W_k, V_k$: independent white Gaussian noises, TV cov. $Q_k, R_k$
- $(H_k, F_k)$ observable & $(F_k, Q_k^{1/2})$ controllable, both uniformly

**Additive faults ($\mathbb{H}_1$)**

\[
\begin{align*}
X_{k+1} &= F_k X_k + G_k U_k + W_k + \Psi_k \theta_k + E_k f_k \\
Y_k &= H_k X_k + J_k U_k + V_k
\end{align*}
\]

- known fault profile matrix $\Psi_k$, unknown fault vector $\theta_k$
- known fault incidence matrix $E_k$, unknown fault profile vector $f_k$
Different fault cases

- $E_k f_k$ and $\psi_k \theta_k$ typically represent actuator faults
- $f_k$: no a priori information; $\theta_k$: constant or slowly varying
- Parametric faults in both state and output equations (sensor faults) can be handled
- This modeling framework encompasses multiple faults
- Non-additive faults are not handled

Particular cases

- Actuator bias: $U_k \rightarrow U_k + \theta$; then $\psi_k = G_k$
- Actuator gain loss: $U_k \rightarrow (I - \text{diag}(\theta))U_k$; then $\psi_k = -G_k \text{diag}(U_k)$
- $\psi_k = \delta_{r,k+1} I$: investigated by Willsky-Jones, Gustafsson with $F_k$ assumed exponentially stable
Fault effect on the innovation of a linear filter

State prediction error and innovation - Fault free case

\[ \tilde{X}_k \triangleq X_k - \hat{X}_{k|k-1} \]
\[ \varepsilon_k \triangleq Y_k - J_k U_k - H_k \hat{X}_{k|k-1} \]

\[ \tilde{X}_{k+1}^0 = F_k(I - K_k H_k)\tilde{X}_k^0 - F_k K_k V_k + W_k \]
\[ \varepsilon_k^0 = H_k \tilde{X}_k^0 + V_k \]

State prediction error and innovation - Faulty case

\[ \tilde{X}_{k+1} = F_k(I - K_k H_k)\tilde{X}_k - F_k K_k V_k + W_k + \psi_k \theta_k + E_k f_k \]
\[ \varepsilon_k = H_k \tilde{X}_k + V_k \]
Introducing a matrix gain

\[
\eta_k \triangleq \tilde{X}_k - \Gamma_k \theta_k
\]

\[
\Gamma_{k+1} \triangleq F_k (I - \mathcal{K}_k H_k) \Gamma_k + \Psi_k , \quad \Gamma_0 \triangleq 0
\]

\[
\eta_{k+1} = F_k (I - \mathcal{K}_k H_k) \eta_k - F_k \mathcal{K}_k V_k + W_k
\]

\[- \Gamma_{k+1} (\theta_{k+1} - \theta_k) + E_k f_k \]

Distinguishing two cases for the parametric fault vector

- Constant \( \theta \)
- TV \( \theta_k \)
Fault effect

First solution: rejecting the non-parametric fault

Second solution: adapting to the parametric fault

Constant parametric fault vector

\[ \theta_k \triangleq \theta \]

Let \[ \zeta_{k+1} \triangleq F_k (I - \mathcal{K}_k H_k) \zeta_k + E_k f_k , \quad \zeta_0 = 0 \]

cf. \[ \Gamma_{k+1} \triangleq F_k (I - \mathcal{K}_k H_k) \Gamma_k + \Psi_k , \quad \Gamma_0 \triangleq 0 \]

\[ \eta_{k+1} = F_k (I - \mathcal{K}_k H_k) \eta_k - F_k \mathcal{K}_k V_k + W_k + E_k f_k \]

\[ \eta_k = \tilde{X}_k^0 + \zeta_k \]

Additive fault effect

\[ \varepsilon_k = \varepsilon_k^0 + H_k \Gamma_k \theta + H_k \zeta_k \]
Guaranteed properties of the recursive $\Gamma_k$ and $\zeta_k$

- $\Gamma_k$ depends on the fault gain $\Psi_k$, not on the fault vector $\theta$.
- The matrix gain $\Gamma_k$ computed from the bounded $\Psi_k$ is bounded even when the system is not stable.
- Similarly, if $f_k$ is bounded, $\zeta_k$ is bounded.
- The persistent excitation condition:
  $$\sum_k \Gamma_k^T H_k^T \Sigma_k^{-1} H_k \Gamma_k$$
  is strictly positive definite
  is satisfied even when the number of sensors is smaller than the number of faults.

Difference with the Willsky-Jones algorithm

- Computations based on recursive formulas involving $F_k$ (thus required to be stable)
**TV parametric fault vector**

\[
\theta_{k+1} = \theta_k + e_k, \quad |e_k| \leq \delta
\]

\[
\delta_{k+1} \triangleq F_k (I - \mathcal{K}_k H_k) \delta_k - \Gamma_{k+1} e_k, \quad \delta_0 \triangleq 0
\]

\[
\eta_k = \tilde{\chi}_k^0 + \delta_k + \zeta_k
\]

**Additive fault effect**

\[
\varepsilon_k = \varepsilon_k^0 + H_k \Gamma_k \theta_k + H_k \delta_k + H_k \zeta_k
\]

\(\Gamma_k\) is bounded

\(F_k (I - \mathcal{K}_k H_k)\) defines an exponentially stable LTV system
Kitanidis filter (UI-KF) for rejecting $E_k f_k$

\[
\begin{aligned}
\left\{
\begin{array}{l}
X_{k+1} = F_k X_k + G_k U_k + W_k + E_k f_k \\
Y_k = H_k X_k + J_k U_k + V_k
\end{array}
\right.
\]

\[
\begin{align*}
\hat{X}_{k+1} &= F_k \hat{X}_k + G_k U_k + F_k L_k (Y_k - J_k U_k - H_k \hat{X}_k) \\
L_k &= K_k + (I - K_k H_k) E_{k-1} (E^T_{k-1} H_k^T \Sigma_{k}^{-1} H_k E_{k-1})^{-1} E^T_{k-1} H_k^T \Sigma_{k}^{-1} \\
K_k &= P_k H_k^T \Sigma_{k}^{-1} \\
P_{k+1} &= F_k (I - L_k H_k) P_k (I - L_k H_k)^T F_k^T + F_k L_k R_k L_k^T F_k^T + Q_k \\
\Sigma_k &= H_k P_k H_k^T + R_k
\end{align*}
\]

$\mathcal{K}_k \triangleq L_k$
Monitoring a constant parametric fault

**Fault effect on the Kitanidis filter innovation**

\[ \varepsilon_k = \varepsilon_k^0 + H_k \Delta_k \theta \]

\[ \Delta_{k+1} = F_k (I - L_k H_k) \Delta_k + \psi_k, \quad \Delta_0 = 0 \]

**The Kitanidis filter innovation is white**

Proof in the notes.

**Use the GLR algorithm**
MLE of $\theta$ under $\mathbb{H}_1$ - Known fault profile matrix

$\mathbb{H}_0 : \varepsilon_k \sim \mathcal{N}(0, \Sigma_k)$, $\mathbb{H}_1 : \varepsilon_k \sim \mathcal{N}(H_k \Gamma_k \theta, \Sigma_k)$

$\hat{\theta}_k = \arg\min_{\tilde{\theta}} \sum_{j=1}^{k} (\varepsilon_j - H_j \Gamma_j \tilde{\theta})^T \Sigma_j^{-1} (\varepsilon_j - H_j \Gamma_j \tilde{\theta}) = C_k^{-1} d_k$

$C_k = C_{k-1} + \Gamma_k^T H_k^T \Sigma_k^{-1} H_k \Gamma_k$

$d_k = d_{k-1} + \Gamma_k^T H_k^T \Sigma_k^{-1} \varepsilon_k$

GLR test

$l_k \triangleq 2 \ln \frac{p(\varepsilon_1, \ldots, \varepsilon_k \mid \theta = \hat{\theta}_k)}{p(\varepsilon_1, \ldots, \varepsilon_k \mid \theta = 0)} = d_k^T C_k^{-1} d_k$
Monitoring the non-parametric fault

While the GLR does not detect anything

Run a Kalman filter based on the fault-free model

\[
\begin{align*}
\hat{X}_{k+1} &= F_k \hat{X}_k + G_k U_k + F_k K_k (Y_k - J_k U_k - H_k \hat{X}_k) \\
K_k &= P_k H_k^T \Sigma_k^{-1} \\
P_{k+1} &= F_k (I - K_k H_k) P_k F_k^T + Q_k \\
\Sigma_k &= H_k P_k H_k^T + R_k
\end{align*}
\]

Monitor its energy

OK when \( \text{dim}(f_k) \geq \text{dim}(Y_k) \), i.e. testing a Gaussian white noise against an arbitrary signal.
More sophisticated tests might be considered in the case where \( \text{dim}(f_k) < \text{dim}(Y_k) \).
Tracking a slowly time-varying $\theta_k$

\[\begin{align*}
\epsilon_k &= \epsilon_k^0 + H_k \Gamma_k \theta_k + H_k \delta_k + H_k \zeta_k \\
\end{align*}\]

**RLS**

\[\begin{align*}
\hat{\theta}_k &= \hat{\theta}_{k-1} + L_k \left( \epsilon_k - H_k \Gamma_k \hat{\theta}_{k-1} \right), \quad \hat{\theta}_0 \stackrel{\Delta}{=} 0 \\
S_k &= \left( \lambda \Sigma_k + H_k \Gamma_k P_{k-1} \Gamma_k^T H_k^T \right)^{-1} \\
L_k &= P_{k-1} \Gamma_k^T H_k^T S_k \\
P_k &= \lambda^{-1} \left( P_{k-1} - P_{k-1} \Gamma_k^T H_k^T S_k H_k \Gamma_k P_{k-1} \right), \quad P_0 \stackrel{\Delta}{=} I \\
\epsilon_k &\stackrel{\Delta}{=} \epsilon_k - H_k \Gamma_k \hat{\theta}_k; \quad \text{monitor its energy to detect } E_k f_k
\end{align*}\]
FDI for LTV systems with TV additive faults

- **Constant parametric faults**
  - Combining a recursive and stable filter that cancels out the fault dynamics and a GLR test
  - Handling additive parametric faults with weaker assumptions than usual on the system stability and the number of required sensors

- Handling both TV parametric and non-parametric faults
  - Two solutions
    - Assuming constant parametric fault and rejecting the non-parametric fault
    - Adapting to the TV parametric fault