In-operation structural health monitoring: a statistical approach

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Toolboxes: LMS CADA-X, and free Scilab
http://www.irisa.fr/sigma2/constructif/modal.htm

Problems: In-operation modal identification and damage detection and localization

• The excitation is typically:
  – natural, not controlled.

  – not measured:
    * buildings, bridges, offshore structures,
    * rotating machinery (e.g. steam flowing),
    * cars, trains, aircrafts.

  – nonstationary (e.g., turbulent).

• How to merge multiple measurements setups e.g. in case of moving sensors?

• How to detect and localize small damages?

Identification and merging

• Output-only eigenstructure identification,

• In the presence of nonstationary excitation,

• Handling moving sensor pools, with some reference sensors: avoid merging identification results from the different pools, merge the data instead, and process them globally, using a standard subspace algorithm.

Damage detection and localization

• Output-only damage detection and localization,

• In the presence of nonstationary excitation,

• On-board handling of small damages.

Wanted:

• Early warning and interpretation of damages,

• Avoid re-identification prior to detection,

• Avoid inverse problem solving prior to damage localization.
Contents

- Modelling
  - Output-only covariance-driven subspace identification
  - Robustness to nonstationary excitation
  - Damage detection
  - Damage diagnostics
  - Examples

Output-only covariance-driven subspace identification

\[
\begin{align*}
X_{k+1} &= F X_k + V_k \\
Y_k &= H X_k
\end{align*}
\]

\[R_i \triangleq \mathbb{E}(Y_k Y_k^T), \quad \mathcal{H} = \begin{pmatrix}
R_0 & R_1 & R_2 & \ldots \\
R_1 & R_2 & R_3 & \ldots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

\[\mathcal{O} \triangleq \begin{pmatrix}
H \\
H F G \\
H F^2 G \\
\vdots
\end{pmatrix}, \quad C \triangleq \mathbb{E}(X_k Y_k^T)
\]

\[R_i = H F^i G \implies \mathcal{H} = \mathcal{O} C
\]

\[\mathcal{H} \rightarrow \mathcal{O} \rightarrow (H, F) \rightarrow (\lambda, \varphi_{\lambda})
\]

Modelling

FE model:

\[
\begin{align*}
M \ddot{Z}(s) + C \dot{Z}(s) + K Z(s) &= \nu(s) \\
Y(s) &= L Z(s)
\end{align*}
\]

\[ (M \mu^2 + C \mu + K) \Psi_{\mu} = 0, \quad \psi_{\mu} = L \Psi_{\mu} \]

State space:

\[
\begin{align*}
X_{k+1} &= F X_k + V_k \\
Y_k &= H X_k
\end{align*}
\]

\[F \Phi_{\lambda} = \lambda \Phi_{\lambda}, \quad \varphi_{\lambda} \triangleq H \Phi_{\lambda}
\]

\[e^{\delta \mu} = \lambda, \quad \psi_{\mu} = \varphi_{\lambda}
\]

Implementation

\[
\hat{R}_i \triangleq 1/N \sum_{k=1}^{N} Y_k Y_{k-i}^T, \quad \hat{\mathcal{H}} = \begin{pmatrix}
\hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \ldots \\
\hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \ldots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

\[\hat{\mathcal{H}} \approx \mathcal{O} \hat{C}
\]

\[\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \rightarrow \mathcal{O} \rightarrow (\hat{H}, \hat{F}) \rightarrow (\hat{\lambda}, \hat{\varphi}_{\lambda})
\]

\[\mathcal{H} = U \Delta W = U \begin{pmatrix}
\Delta_1 & 0 \\
0 & \Delta_2
\end{pmatrix} W^T; \quad \mathcal{O} = U \Delta_1^{1/2}
\]

\[\mathcal{O}_p(H, F) = \mathcal{O}_p(H, F) F
\]

\[\det(F - \lambda I) = 0, \quad F \Phi_{\lambda} = \lambda \Phi_{\lambda}, \quad \varphi_{\lambda} = H \Phi_{\lambda}
\]
Robustness to nonstationary excitation

Approximate factorization of covariances: \( \hat{R}_i \approx H F^i \hat{G} \)

Consistency: \( T^{-1} \hat{F} T \rightarrow F; \hat{H} \rightarrow H; (\hat{\lambda}, \varphi_\lambda) \rightarrow (\lambda, \varphi_\lambda) \)

Theory and experience show that the combination of:

- the key factorization property of the covariances,
- the averaging operation in their computation,

allows to cancel out nonstationarities in the excitation.

Damage detection

\( \theta_0 \): reference parameter, known (or identified)

\( Y_k \): \( N \)-size sample of new measurements

Build a residual \( \zeta \) significantly non zero when damage

Structural monitoring: Eigenstructure monitoring

\[
\begin{align*}
X_{k+1} &= F X_k + V_k \\
Y_k &= H X_k
\end{align*}
\]

\[
F \Phi_\lambda = \lambda \Phi_\lambda \\
\varphi_\lambda \triangleq H \Phi_\lambda
\]

Canonical parameter: \( \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec} \Phi \end{pmatrix} \) modes mode shapes

Observability in modal basis: \( O_{p+1}(\theta) \)

Subspace model/data correlation (1)

Fresh data \( \rightarrow \hat{R}_i \rightarrow \mathcal{H} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \ldots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \ldots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \)

Nominal model: \( \mathcal{O}(\theta_0) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \Phi \Delta^2 \\ \vdots \\ \end{pmatrix} \) Observability in modal basis

\( \mathcal{H} = \mathcal{O} \mathcal{C} \) ! ker \( \mathcal{H}^T \triangleq \ker \mathcal{O}^T(\theta_0) \)

Local approach (small deviations)

Test \( \mathcal{H}_0: \theta = \theta_0 \) against \( \mathcal{H}_1: \theta = \theta_0 + \delta \theta / \sqrt{N} \)
Subspace model/data correlation (2)

\[ S, \quad S^T S = I_s, \quad S^T \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } S(\theta_0) \]

Check if: \[ S^T(\theta_0) \mathcal{H} \approx 0 \]

Residual for structural damage monitoring

\[ \zeta_N(\theta_0) \triangleq \text{vec}(S^T(\theta_0) \mathcal{H}) \]

? How to assess the significance of: \[ S^T(\theta_0) \mathcal{H} \approx 0 \]?

Subspace model/data correlation (3)

The residual is asymptotically Gaussian

\[
\begin{align*}
\zeta_N(\theta_0) &\rightarrow \begin{cases} 
\mathcal{N}(0, \Sigma(\theta_0)) \quad \text{under } P_{\theta_0} \\
\mathcal{N}(\mathcal{M}(\theta_0) \delta \theta, \Sigma(\theta_0)) \quad \text{under } P_{\theta_0+\delta \theta/\sqrt{N}}
\end{cases}
\end{align*}
\]

\[ \mathcal{M}(\theta_0) : \text{mean sensitivity (Jacobian) of residual } \zeta \text{ w.r.t. modal changes} \]

(On-board) \( \chi^2 \)-test in the residual

\[ \zeta_N^T \Sigma^{-1} \mathcal{M} (\mathcal{M}^T \Sigma^{-1} \mathcal{M})^{-1} \mathcal{M}^T \Sigma^{-1} \zeta_N \geq h \]

(On-board) modal \( \chi^2 \)-test

\[ \zeta_N^T \Sigma^{-1} \mathcal{M}_i (\mathcal{M}_i^T \Sigma^{-1} \mathcal{M}_i)^{-1} \mathcal{M}_i^T \Sigma^{-1} \zeta_N \geq h \]

On-board damage diagnostics: projecting changes

Model/data correlation - Generalization

Any estimating function can play the role of a residual

Warning:
The prediction error is OK for sensor faults,
NOT for structural damages!
Damage diagnostics: (local) sensitivity approach

\[ \zeta \sim \mathcal{N}(\mathcal{M} \delta \theta, \Sigma), \quad \delta \theta = \mathcal{I} \mathcal{J}(\mathcal{M}_0^+, \mathcal{K}_0^+) \left( \delta \mathcal{M} \right) \]

[\(M_0^+, K_0^+\) : design model]

Jacobian: \((\delta \mathcal{M}, \delta \mathcal{K}) \mathcal{J}(\mathcal{M}_0^+, \mathcal{K}_0^+) \rightarrow (\delta \mu, \delta \psi_\mu)\)

Reduction: \(\mathcal{I}\) matching computed/identified modes

Problem: \(\dim \left( \begin{bmatrix} M \\ K \end{bmatrix} \right) \gg \dim \theta\)

Hint: Cluster the vectors \((\delta \mu, \delta \psi_\mu)\) using the \(\chi^2\)-metric

Examples

- Sports car
- Z24 bridge
- Reticular structure
- Slat track
- Aircraft flutter monitoring

Z24 bridge

- A benchmark of the BRITE/EURAM project SIMCES and of the European COST action F3
- Response to traffic excitation under the bridge measured over one year in 139 points
- Two damage scenarios (DS1 and DS2): pier settlements of 20mm and 80mm.

Identified first four natural frequencies / Test values (Results with four sensors)

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>(\chi^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undamaged</td>
<td>3.88</td>
<td>5.01</td>
<td>9.80</td>
<td>10.30</td>
<td>8.80 \cdot 10e2</td>
</tr>
<tr>
<td>Damaged (1)</td>
<td>3.87</td>
<td>5.06</td>
<td>9.79</td>
<td>10.32</td>
<td>8.00 \cdot 10e5</td>
</tr>
<tr>
<td>Damaged (2)</td>
<td>3.76</td>
<td>4.93</td>
<td>9.74</td>
<td>10.25</td>
<td>3.96 \cdot 10e6</td>
</tr>
</tbody>
</table>
Aircraft flutter monitoring

- Aero-elastic flutter: critical instability phenomenon
- Flight flutter testing procedure
- Objective: on-line in-flight exploitation of test data
- On-line flight flutter monitoring problem: monitoring some specific damping coefficient
- Using a different computation of the residual $\zeta$, introducing a minimum magnitude of change, and using the CUSUM test

Test for $\rho_c = \rho_c^{(1)}$

Test for $\rho_c = \rho_c^{(2)} < \rho_c^{(1)}$

Bottom: $-g_n^-$ reflects $\rho < \rho_c$. Top: $g_n^+$ reflects $\rho > \rho_c$. 