

In-operation structural health monitoring: a statistical approach

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Toolboxes: LMS CADA-X, and free Scilab

<http://www.irisa.fr/sigma2/constructif/modal.htm>

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Identification and merging

- **Output-only eigenstructure** identification,
- In the presence of **nonstationary excitation**,
- Handling **moving** sensor pools, with some reference sensors: **avoid merging identification results** from the different pools, **merge the data** instead, and process them globally, using a **standard** subspace algorithm.

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Problems: In-operation modal identification and damage detection and localization

- The **excitation** is typically:
 - natural, **not controlled**.
 - **not measured**:
 - * buildings, bridges, offshore structures,
 - * rotating machinery (e.g. steam flowing),
 - * cars, trains, aircrafts.
 - **nonstationary** (e.g., turbulent).
- How to **merge** multiple measurements setups e.g. in case of **moving** sensors?
- How to **detect** and **localize small** damages?

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Damage detection and localization

- **Output-only** damage **detection** and **localization**,
- In the presence of **nonstationary excitation**,
- **On-board** handling of **small** damages.

Wanted:

- **Early warning** and interpretation of damages,
- **Avoid re-identification** prior to detection,
- **Avoid inverse problem solving** prior to damage localization.

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Contents

- Modelling
- Output-only **covariance**-driven **subspace** identification
- **Robustness** to **nonstationary** excitation
- Damage **detection**
- Damage **diagnostics**
- **Examples**

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Output-only **covariance**-driven **subspace** identification

$$\begin{cases} X_{k+1} = F X_k + V_k \\ Y_k = H X_k \end{cases}$$

$$R_i \triangleq \mathbb{E}(Y_k Y_{k-i}^T), \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \dots & \vdots \end{pmatrix}$$

ok if stationary !

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (G \quad FG \quad F^2G \quad \dots)$$

$$G \triangleq \mathbb{E}(X_k Y_k^T)$$

$$R_i = H F^i G \implies \mathcal{H} = \mathcal{O} \mathcal{C}$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

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Modelling

FE model:
$$\begin{cases} M \ddot{Z}(s) + C \dot{Z}(s) + K Z(s) = \nu(s) \\ Y(s) = L Z(s) \end{cases}$$

$$(M \mu^2 + C \mu + K) \Psi_\mu = 0, \quad \psi_\mu = L \Psi_\mu$$

State space:
$$\begin{cases} X_{k+1} = F X_k + V_k \\ Y_k = H X_k \end{cases}$$

$$F \Phi_\lambda = \lambda \Phi_\lambda, \quad \varphi_\lambda \triangleq H \Phi_\lambda$$

$$\underbrace{e^{\delta \mu}}_{\text{modes}} = \lambda, \quad \underbrace{\psi_\mu}_{\text{mode shapes}} = \varphi_\lambda$$

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Implementation

$$\hat{R}_i \triangleq 1/N \sum_{k=1}^N Y_k Y_{k-i}^T, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \dots & \vdots \end{pmatrix}$$

ok when nonstationary !

$$\hat{\mathcal{H}} \approx \hat{\mathcal{O}} \hat{\mathcal{C}}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

$$\hat{\mathcal{H}} = U \Delta W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T; \quad \hat{\mathcal{O}} = U \Delta_1^{1/2}$$

$$\mathcal{O}_p^\dagger(H, F) = \mathcal{O}_p(H, F) F$$

$$\det(F - \lambda I) = 0, \quad F \Phi_\lambda = \lambda \Phi_\lambda, \quad \varphi_\lambda = H \Phi_\lambda$$

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Robustness to nonstationary excitation

Approximate factorization of covariances : $\hat{R}_i \approx H F^i \hat{G}$

Consistency : $T^{-1} \hat{F} T \rightarrow F, \hat{H} \rightarrow H; (\hat{\lambda}, \hat{\varphi}_\lambda) \rightarrow (\lambda, \varphi_\lambda)$

Theory and experience show that the combination of:

- the key **factorization** property of the covariances,
- the **averaging** operation in their computation,

allows to cancel out nonstationarities in the excitation.

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Damage detection

θ_0 : reference parameter, known (or identified)

Y_k : N -size sample of new measurements

Build a **residual** ζ **significantly non zero** when damage

Local approach (**small** deviations)

Test $\mathcal{H}_0 : \theta = \theta_0$ against $\mathcal{H}_1 : \theta = \theta_0 + \delta\theta/\sqrt{N}$

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Structural monitoring : Eigenstructure monitoring

$$\begin{cases} X_{k+1} = F X_k + V_k & F \Phi_\lambda = \lambda \Phi_\lambda \\ Y_k = H X_k & \varphi_\lambda \triangleq H \Phi_\lambda \end{cases}$$

Canonical parameter: $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$ **modes**
mode shapes

Observability in modal basis: $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \vdots \\ \Phi \Delta^p \end{pmatrix}$

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Subspace **model/data correlation** (1)

Fresh data $\longrightarrow \hat{R}_i \longrightarrow \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$

Nominal model : $\mathcal{O}(\theta_0) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \Phi \Delta^2 \\ \vdots \end{pmatrix}$ **Observability**
in modal basis

! $\mathcal{H} = \mathcal{O} \mathcal{C}$! **$\ker \hat{\mathcal{H}}^T \stackrel{?}{=} \ker \mathcal{O}^T(\theta_0)$**

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Subspace **model/data correlation** (2)

$$\exists S, \quad S^T S = I_s, \quad S^T \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } S(\theta_0)$$

Check if: $S^T(\theta_0) \hat{\mathcal{H}} \approx 0$

Residual for **structural damage** monitoring

$$\zeta_N(\theta_0) \triangleq \text{vec}(S^T(\theta_0) \hat{\mathcal{H}})$$

? How to assess the significance of: $S^T(\theta_0) \hat{\mathcal{H}} \approx 0$?

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Subspace **model/data correlation** (3)

The **residual** is asymptotically **Gaussian**

$$\zeta_N(\theta_0) \rightarrow \begin{cases} \mathcal{N}(\mathbf{0}, \Sigma(\theta_0)) & \text{under } P_{\theta_0} \\ \mathcal{N}(\mathcal{M}(\theta_0) \delta\theta, \Sigma(\theta_0)) & \text{under } P_{\theta_0 + \delta\theta/\sqrt{N}} \end{cases}$$

$\mathcal{M}(\theta_0)$: mean **sensitivity** (Jacobian) of residual ζ w.r.t. modal changes

(On-board) **χ^2 -test** in the residual

$$\zeta_N^T \Sigma^{-1} \mathcal{M} (\mathcal{M}^T \Sigma^{-1} \mathcal{M})^{-1} \mathcal{M}^T \Sigma^{-1} \zeta_N \geq h$$

(On-board) **modal χ^2 -test**

$$\zeta_N^T \Sigma^{-1} \mathcal{M}_i (\mathcal{M}_i^T \Sigma^{-1} \mathcal{M}_i)^{-1} \mathcal{M}_i^T \Sigma^{-1} \zeta_N \geq h$$

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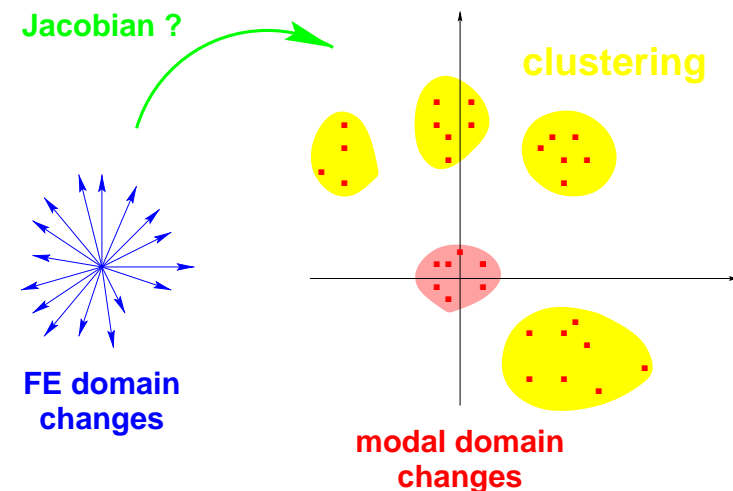
Model/data correlation - **Generalization**

Any **estimating function** can play the role of a **residual**

Warning:

The **prediction error** is OK for sensor faults,
NOT for structural damages !

On-board damage diagnostics: **projecting** changes



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$$\zeta \sim \mathcal{N}(\mathcal{M} \delta\theta, \Sigma), \quad \delta\theta = \mathcal{I} \mathcal{J}_{(M_0^*, K_0^*)} \begin{pmatrix} \delta M \\ \delta K \end{pmatrix}$$

(M_0^*, K_0^*) : design model

Jacobian : $(\delta M, \delta K) \xrightarrow{\mathcal{J}_{(M_0^*, K_0^*)}} (\delta\mu, \delta\psi_\mu)$

Reduction: \mathcal{I} matching computed/identified modes

Problem : $\dim \begin{pmatrix} M \\ K \end{pmatrix} \gg \dim \theta$

Hint: **Cluster** the vectors $(\delta\mu, \delta\psi_\mu)$ using the χ^2 -metric

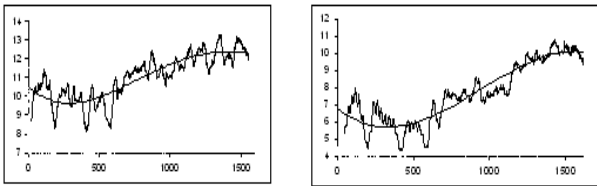
- Sports car
- **Z24 bridge**
- Reticular structure
- Slat track
- **Aircraft flutter** monitoring

Z24 bridge

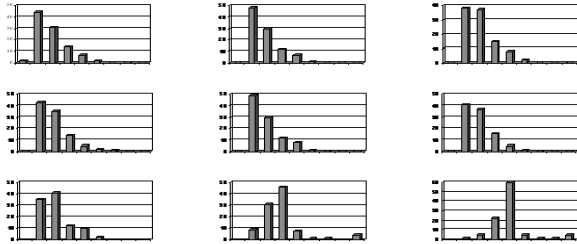
- A benchmark of the BRITE/EURAM project SIMCES and of the European COST action F3
- Response to traffic excitation under the bridge measured over one year in 139 points
- Two damage scenarios (DS1 and DS2): pier settlements of 20mm and 80mm.

Identified first four natural frequencies / Test values (Results with four sensors)

	Mode	1	2	3	4	χ^2
Undamaged	Freq.(Hz)	3.88	5.01	9.80	10.30	8.80 · 10e2
Damaged (1)	Freq.(Hz)	3.87	5.06	9.79	10.32	8.00 · 10e5
Damaged (2)	Freq.(Hz)	3.76	4.93	9.74	10.25	3.96 · 10e6



Evolution of the test values over three months on a log-scale amplitude. Two different sets of sensors (Left and Right).



Distribution of the test values for each of the nine months.

Aircraft flutter monitoring

- Aero-elastic flutter: critical instability phenomenon
- Flight flutter testing procedure
- Objective : on-line in-flight exploitation of test data
- On-line flight flutter monitoring problem: monitoring some specific **damping** coefficient
- Using a different computation of the **residual** ζ , introducing a minimum magnitude of change, and using the **CUSUM test**

Test for $\rho_c = \rho_c^{(1)}$.

Test for $\rho_c = \rho_c^{(2)} < \rho_c^{(1)}$.

Bottom: $-g_n^-$ reflects $\rho < \rho_c$. **Top:** g_n^+ reflects $\rho > \rho_c$.