In-operation damage detection and localization

Steelquake and Z24 bridge benchmarks

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Toolboxes: LMS CADA-X, and Scilab

ftp://ftp.inria.fr/INRIA/Projects/Meta2/Scilab/contrib/MODAL/

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Modelling

FE model:

$$\begin{cases} M\ddot{\mathcal{Z}}(s) + C\dot{\mathcal{Z}}(s) + K\mathcal{Z}(s) = \nu(s) \\ Y(s) = L\mathcal{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0 \ , \quad \psi_\mu = L\Psi_\mu$$

State space:

$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \ \Phi_\lambda \ , \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

Parameter:

$$\underbrace{e^{\delta\mu} = \lambda}_{\text{modes}} \ , \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{mode shapes}} \ ; \ \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec}\,\Phi \end{pmatrix}$$

Damage detection

Local approach (small deviations)

θ_0 : reference parameter, known (or identified)

Y_k: N-size sample of new measurements

Build a residual ζ significantly non zero when damage

Test $\quad \mathcal{H}_0 : \quad \theta = \theta_0 \quad$ against $\quad \mathcal{H}_1 : \quad \theta = \theta_0 + \frac{\delta\theta}{\sqrt{N}}$

Residual $\leftrightarrow$ Estimating function

$$\zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} K(\theta_0, Y_k)$$

Characterized by: $\mathbf{E}_{\theta_0} K(\theta, Y_k) = 0 \quad \Longleftrightarrow \quad \theta = \theta_0$

Warning: Prediction error for sensor faults ONLY !

Mean sensitivity (Jacobian) and covariance

$$\mathcal{M}(\theta_0) \triangleq -\mathbf{E}_{\theta_0} \frac{\partial K(\theta_0, Y_k)}{\partial \theta}, \quad \Sigma(\theta_0) \triangleq \lim_{N \to \infty} \mathbf{E}_{\theta_0} \zeta_N(\theta_0) \zeta_N^T(\theta_0)$$

The residual is asymptotically Gaussian

$$\zeta_N(\theta_0) \;\longrightarrow\; \begin{cases} \mathcal{N}(\quad\quad\quad\quad 0,\ \Sigma(\theta_0)) & \textbf{under} \quad \mathrm{P}_{\theta_0} \\ \mathcal{N}(\ \mathcal{M}(\theta_0)\ \delta\theta,\ \Sigma(\theta_0)) & \textbf{under} \quad \mathrm{P}_{\theta_0+\frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

(On-board) χ^2-test

$$\zeta_N^T\ \Sigma^{-1}\, \mathcal{M}\ (\mathcal{M}^T\, \Sigma^{-1}\, \mathcal{M})^{-1}\, \mathcal{M}^T\, \Sigma^{-1}\ \zeta_N \ \ \geq h$$

Invariant / pre-multiplication of ζ with invertible gain

Back to eigenstructure monitoring

$$\left\{ \begin{array}{rcl} X_{k+1} & = & F X_k + V_k \\ Y_k & = & H X_k \end{array} \right. \qquad \begin{array}{l} F \, \varphi_\lambda = \lambda \, \varphi_\lambda \\ \Phi_\lambda \triangleq H \, \varphi_\lambda \end{array}$$

Canonical parametrization : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec} \, \Phi \end{pmatrix}$

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$

System parameter characterization:

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the same left kernel.

Back to structural subspace identification

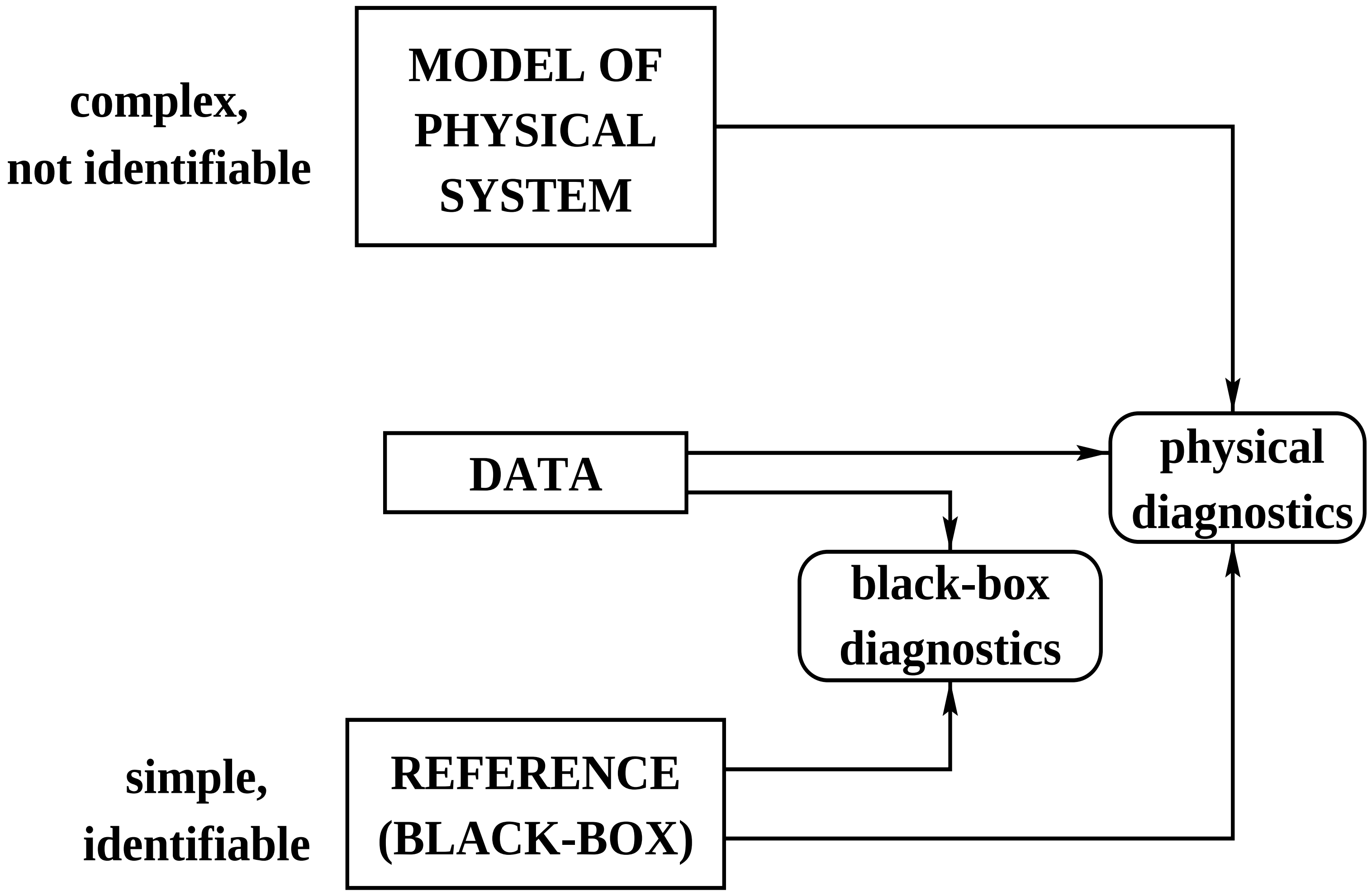
$$\exists S, \quad S^T S = I_s, \qquad S^T \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } S(\theta_0)$$

$$\theta_0 \leftrightarrow (R_i^0)_i \text{ characterized by: } S^T(\theta_0)\, \hat{\mathcal{H}}^0_{p+1,q} = 0$$

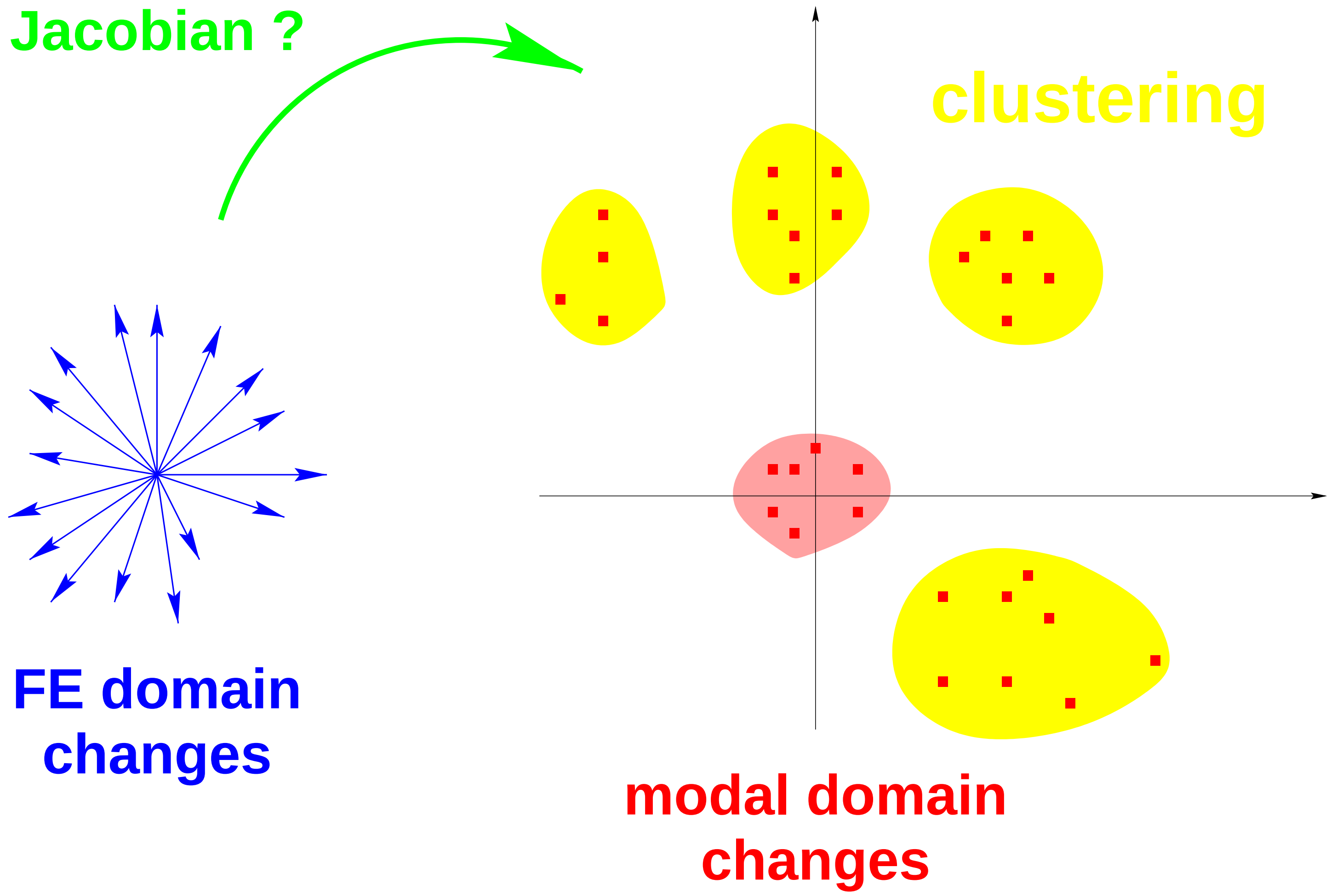
Residual for structural damage monitoring

$$\zeta_N(\theta_0) \triangleq \mathrm{vec}(\ S^T(\theta_0)\ \hat{\mathcal{H}}_{p+1,q}\ )$$

On-board damage diagnostics without solving an inverse problem



Damage diagnostics: projecting changes



Damage diagnostics: (local) sensitivity approach

$$\zeta \sim \mathcal{N}(\mathcal{M}\ \delta\theta,\ \ \Sigma), \qquad \delta\theta = \mathcal{I}\ \mathcal{J}_{(M_0^\star, K_0^\star)} \begin{pmatrix} \delta M \\ \delta K \end{pmatrix}$$

$(M_0^\star, K_0^\star)$: **design** model

Jacobian : $(\delta M, \delta K) \xrightarrow{\mathcal{J}_{(M_0^\star, K_0^\star)}} (\delta\mu, \delta\psi_\mu)$

Reduction: $\mathcal{I}$ matching computed/identified modes

Problem : $\dim \begin{pmatrix} M \\ K \end{pmatrix} \gg \dim \theta$

Computing Jacobian

1. $(\delta M, \delta K) \xrightarrow[\text{mode selection}]{\mathcal{IJ}_{(M_0^\star, K_0^\star)}} (\delta\mu, \delta\psi_\mu)$

2. Apply $\mathcal{IJ}$ to unit vectors $(\delta M, \delta K)$

3. Truncate small vectors $(\delta\mu, \delta\psi_\mu)$

4. Cluster the remaining vectors $(\delta\mu, \delta\psi_\mu)$, using the χ^2-metric.

? Better reduction techniques, instead of 1., 2., 3.

Computing sensitivities

$$M\ddot{\mathcal{Z}}(t)+C\dot{\mathcal{Z}}(t)+K\mathcal{Z}(t) \;=\; \nu(t)\;;\quad \phi^T(\mu^2 M+\mu C+K)=0$$

$$\delta\mu\;(2\mu M+C)\;\Phi+\;(\mu^2 M+\mu C+K)\;\delta\Phi+\mu^2\;\delta M+\mu\;\delta C+\delta K=0$$

$$\delta\mu=-\;\frac{\phi^T\;(\mu^2\;\delta M+\mu\;\delta C+\delta K)\;\phi}{\phi^T\;(2\mu M+C)\;\phi}$$

$$(\mu^2 M+\mu C+K)\;\delta\phi=-\delta\mu\;(2\mu M+C)\;\phi-(\mu^2\;\delta M+\mu\;\delta C+\delta K)\;\phi$$

$$\phi^T\delta\phi=0$$

$$\delta\Phi_c=\mathcal{J}_{\Phi_c\psi}\;\delta\psi\;;\quad \delta\Phi_i=\delta\Phi_c\;(\Phi_i\neq\Phi_c)$$

Numerical results - Steelquake

Sensors 2, 8, 10, 13.

8 modes identified on $Q10$.

Theoretically, under no damage assumption: $\chi^2 = 80$.

Experimentally, for undamaged scenarios $Q09 - Q12$: $\chi^2 = 8 \cdot 10^3$.

For damaged scenarios $Q39 - Q42$: $\chi^2 = 8 \cdot 10^8$.

Numerical results - Z24 bridge

	Mode	1	2	3	4	χ^2
Safe	Freq.(Hz)	3.88	5.01	9.8	10.3	25
DS1	Freq.(Hz)	3.87	5.06	9.79	10.32	70000
DS2	Freq.(Hz)	3.76	4.93	9.74	10.25	70000

Damage localization - Z24 bridge