

Variations on **CUSUM tests** for **flutter monitoring**

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Introduction - (1)

- **Flutter**: critical aircraft **instability** phenomenon
unfavorable interaction of aerodynamic, elastic and inertial forces; may cause major failures
- **Flight flutter testing**, very expensive and time consuming :
Design the flutter free flight envelope
- Flutter clearance techniques:
In-flight **identification**: output-only, or using input excitations
Data processing: time-frequency, wavelet, envelope function
Flutter **prediction** based on model-based approaches:
flutterometer (μ -robustness), physical model updating
- Some **challenges**:
Real time **on-board monitoring**,
Handling **transients** between steady flight test points
- Our approach:
Statistical detection for monitoring instability indicators

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Introduction

Subspace-based residual for **modal monitoring**

CUSUM test for monitoring a **scalar** instability index

Variations on the CUSUM test

Experimental **results**

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Introduction - (2)

- Aim of in-flight **online flutter monitoring**:
Early detection of a deviation in the aircraft modal parameters
before it develops into flutter.
- **Change-point** detection: natural approach
- For a scalar **instability criterion** ψ and a **critical value** ψ_c ,
online **hypotheses** testing:
$$H_0 : \psi > \psi_c \text{ and } H_1 : \psi \leq \psi_c$$
- **CUSUM** test as an approximation to the optimal test
- **Variations** on the CUSUM test

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Subspace-based residual for modal monitoring

$$\begin{cases} X_{k+1} = F X_k + V_k & F \phi_\lambda = \lambda \phi_\lambda \\ Y_k = H X_k & \varphi_\lambda \triangleq H \phi_\lambda \end{cases}$$

$$R_i \triangleq E(Y_k Y_{k-i}^T), \quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H F^i G \implies \mathcal{H} = \mathcal{O} \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (G \quad FG \quad F^2G \quad \dots)$$

$$G \triangleq E(X_k Y_k^T)$$

Output-only covariance-driven **subspace identification**

$$\text{SVD of } \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

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Local approach to testing

$$\bar{H}_0: \theta = \theta_\star \quad \text{and} \quad \bar{H}_1: \theta = \theta_\star + \Upsilon/\sqrt{n}$$

Mean **sensitivity** and **covariance** matrices:

$$\mathcal{J}_n(\theta_\star, \theta) \triangleq 1/\sqrt{n} \partial/\partial\theta E_\theta \zeta_n(\tilde{\theta})|_{\tilde{\theta}=\theta_\star}, \quad \Sigma_n(\theta_\star, \theta) \triangleq E_\theta (\zeta_n(\theta_\star) \zeta_n(\theta_\star)^T)$$

If $\Sigma_n(\theta_\star, \theta)$ is positive definite, and for all Υ , under both hypoth:

$$\Sigma_n(\theta_\star, \theta)^{-1/2} (\zeta_n(\theta_\star) - \mathcal{J}_n(\theta_\star, \theta) \Upsilon) \xrightarrow{n \rightarrow \infty} \mathcal{N}(0, I)$$

Normalized residual:

$$\bar{\zeta}_n(\theta_\star) \triangleq \mathcal{K}_n(\theta_\star, \theta) \zeta_n(\theta_\star)$$

$$\mathcal{K}_n(\theta_\star, \theta) \triangleq \bar{\Sigma}_n^{-1/2} \mathcal{J}_n^T \Sigma_n^{-1}, \quad \bar{\Sigma}_n(\theta_\star, \theta) \triangleq \mathcal{J}_n^T \Sigma_n^{-1} \mathcal{J}_n$$

$$(\bar{\zeta}_n(\theta_\star) - \bar{\Sigma}_n(\theta_\star, \theta)^{1/2} \Upsilon) \xrightarrow{n \rightarrow \infty} \mathcal{N}(0, I)$$

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Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$ modes
mode shapes

$$\text{Observability in modal basis} : \mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \vdots \\ \Phi \Delta^p \end{pmatrix}$$

Given:

- a **reference parameter** θ_\star , by SVD of $\widehat{\mathcal{H}}_{p+1,q}^\star$ (reference data)

$$U(\theta_\star)^T \widehat{\mathcal{H}}_{p+1,q}^\star = 0 \quad \text{parameter estimating function}$$

$$U(\theta_\star)^T \mathcal{O}_{p+1}(\theta_\star) = 0, \quad U(\theta_\star)^T U(\theta_\star) = I$$

- a n -size sample of **new data**; $\widehat{\mathcal{H}}_{p+1,q}$

For **testing** $\theta = \theta_\star$, **statistics** (residual) :

$$\zeta_n(\theta_\star) \triangleq \sqrt{n} \text{vec} (U(\theta_\star)^T \widehat{\mathcal{H}}_{p+1,q})$$

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Data-driven computation for online detection

$$\bar{\zeta}_n(\theta_\star) = \frac{1}{\sqrt{n}} \sum_{k=q}^{n-p} Z_k(\theta_\star)$$

$$Z_k(\theta_\star) \triangleq \mathcal{K}_k(\theta_\star, \theta) \text{vec} (U(\theta_\star)^T \mathcal{Y}_{k,p+1}^+ \mathcal{Y}_{k,q}^{-T})$$

Another approximation

For n large enough, and $k = 1, \dots, n$,

$Z_k(\theta_\star) \approx$ **Gaussian i.i.d.**, mean 0 before change and $\neq 0$ after.

Monitoring any function $\psi(\theta)$

Replace $\mathcal{J}_n(\theta_\star, \theta)$ with $\mathcal{J}_n(\theta_\star, \theta) \mathcal{J}_{\theta\psi}^\star$, where $\mathcal{J}_{\theta\psi}^\star = \partial\theta/\partial\psi|_{\theta=\theta_\star}$.

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CUSUM test for monitoring a scalar index

The crossing of a **critical** ψ_c by ψ is reflected into a change with the **same sign** in the mean ν of the i.i.d. Gaussian $Z_k(\theta_*)$.

The CUSUM test may be used for testing between:

$$H_0 : \nu > 0 \quad \text{and} \quad H_1 : \nu \leq 0$$

Procedure for **unknown** sign and magnitude of **change in** ψ :

i) Set a **min. change magnitude** $\nu_m > 0$, and test between:

$$H_0 : \nu > \nu_m/2 \quad \text{and} \quad H_1 : \nu \leq -\nu_m/2$$

$$S_n(\theta_*) \triangleq \sum_{k=q}^{n-p} (Z_k(\theta_*) + \nu_m), \quad T_n(\theta_*) \triangleq \max_{k=q, \dots, n-p} S_k(\theta_*)$$

$$g_n(\theta_*) \triangleq T_n(\theta_*) - S_n(\theta_*) \underset{H_0}{\overset{H_1}{>}} \varrho \quad \text{threshold}$$

ii) Run **2 tests** in parallel, for **decreasing** and **increasing** ψ ;

iii) Make a decision from the first test which fires;

iv) Reset all sums and extrema to 0, switch to the other test.

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Variations on the CUSUM test - (2)

Three solutions for b)-c):

1. $\theta_* \triangleq \theta_0$ **identified** on **reference data** for the stable system;

$U(\theta_*)$ **computed**,

$\mathcal{J}_n(\theta_0), \Sigma_n^{-1}(\theta_0)$ **estimated** recursively with the **test data**.

2. $\theta_* \triangleq \theta_c$, critical parameter closer to instability, **computed**

at each flight point using θ_0 and an **aeroelastic model**;

$U(\theta_*)$ **computed**,

$\mathcal{J}_n(\theta_c), \Sigma_n^{-1}(\theta_c)$ **estimated** recursively with the **test data**.

3. $U(\cdot) \triangleq \bar{U}_n$ **estimated** on **test data**,

$\mathcal{J}_n(\theta_0), \Sigma_n^{-1}(\theta_0)$ **estimated** recursively with the **test data**.

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Variations on the CUSUM test - (1)

For detecting aircraft instability precursors, **select**:

a) An instability criterion ψ and a critical value ψ_c ;

b) A left kernel matrix $U(\cdot)$;

c) A reference θ_* for estimating $\mathcal{J}_n(\theta_*)$ and $\Sigma_n^{-1}(\theta_*)$;

d) A min. change magnitude ν_m and a threshold ϱ .

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Solution 2. - Details

i) **Compute** the critical eigenvalues λ_c at flight point t using identified modal signatures $(\theta_1, \dots, \theta_t)$

and **extrapolation** of the characteristic polynomial associated with the quasi-steady aeroelastic model $M\ddot{q} + (D + VB)\dot{q} + (K + V^2C)q = 0$;

ii) **Build** the critical modal signature θ_c from λ_c and the mode-shapes φ_λ identified at flight point t ;

iii) Compute the $Z_k(\theta_c)$'s and $S_n(\theta_c)$;

Compute $\hat{\mathcal{J}}_n(\theta_c)$ and $\hat{\Sigma}_n^{-1}(\theta_c)$ with the test data;

iv) Run the **CUSUM** test between flight points t and $t + 1$;

v) **Repeat** these steps for flight point $t + 1$:

modal identification of θ_{t+1} to update the prediction of θ_c , computation of $\hat{\mathcal{J}}_n(\theta_c)$ and $\hat{\Sigma}_n^{-1}(\theta_c)$, CUSUM test between $t + 1$ and $t + 2$.

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Solution 3. - Details

i) Initialization

For an initial airspeed:

Estimate a reference θ_0 and compute the constant terms in $\hat{\mathcal{J}}_n$;

Select data sample size L , lag τ , block size K , ν_m , and ϱ ;

Compute $\hat{\Sigma}_{L+\tau}^{-1}$ and $\hat{\mathcal{J}}_{L+\tau}$ with the first $L + \tau$ samples;

Compute $\hat{U}_{L+\tau}$ with (Y_1, \dots, Y_L) ;

Compute the Z_k 's and $S_{L+\tau}$.

ii) Recursive loop

Running the **CUSUM** test: for each $n \geq L + \tau$:

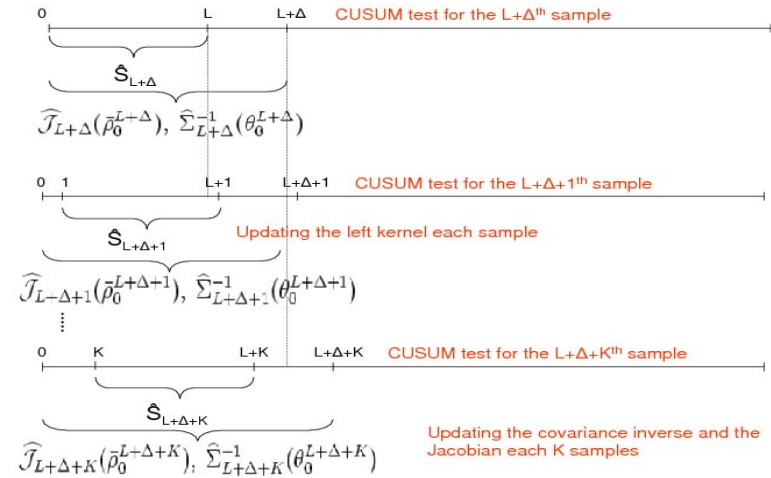
Compute recursively \hat{U}_n with $(Y_{n-\tau-L+1}, \dots, Y_{n-\tau})$;

Use \hat{U}_n with $\hat{\Sigma}_n^{-1}$ and $\hat{\mathcal{J}}_n$ to compute S_n and g_n until $g_n \geq \varrho$.

Update recursively $\hat{\Sigma}_n^{-1}$ and $\hat{\mathcal{J}}_n$ every K samples.

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Solution 3. - Details (Contd.)



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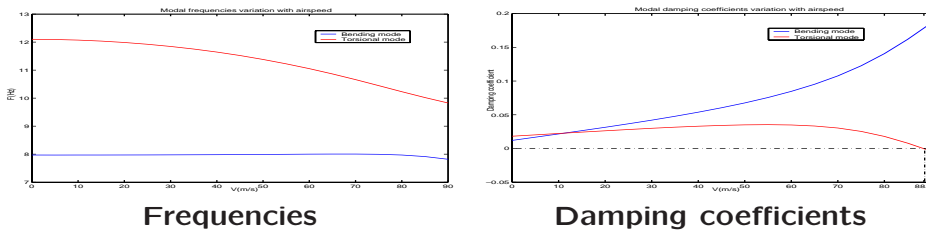
Example - Aeroelastic Hancock wing model

Rigid wing with constant chord; **2 d.o.f.** in bending and torsion.

Matrix F , and eigenvalues λ : functions of airspeed V .

Flutter airspeed: $V_f = 88.5m/s$.

Stability indicator ψ : Damping coefficient



Bending & torsion modes

20700-size 2D-samples simulated (300 for each $V=20:1:88m/s$).

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Example - Numerical results

CUSUM test run with $\nu_m = 0.1$, $\varrho = 100$, and the **damping** as ψ .

Solution 1. with $\theta_* = \theta_0$ at $V = 20m/s$, online recursive $\hat{\mathcal{J}}_n, \hat{\Sigma}_n$.

Solution 2. with $\theta_* = \theta_c$ at $V = 85m/s$, online recursive $\hat{\mathcal{J}}_n, \hat{\Sigma}_n$.

Solution 3. with online recursive $\hat{U}_n, \hat{\mathcal{J}}_n, \hat{\Sigma}_n$.

Alarm onset times depend on threshold; \hat{V}_f is more important.

Solution 1. θ_* far from instability, alarm at $V=67m/s$, $\hat{V}_f=65m/s$.

The test detects that torsional damping decreases under ψ_c .

Solution 2. θ_* close to instability, alarm at $V=88m/s$, $\hat{V}_f=85m/s$.

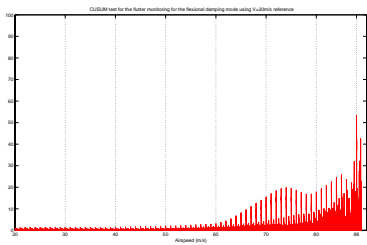
The test detects that flutter is happening between two steady points, and confirms the flutter prediction.

Solution 3. Alarm at $V=88m/s$, $\hat{V}_f=78m/s$ much closer to flutter.

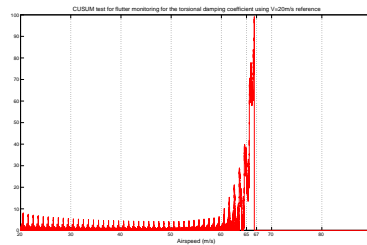
Good behavior for light damping decrease before alarm.

Detection (before flutter) of torsional damping drop.

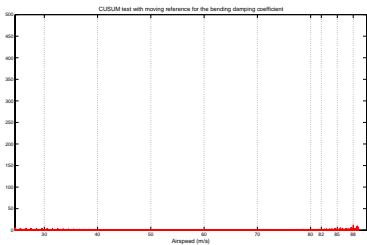
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Solution 1: Bending mode

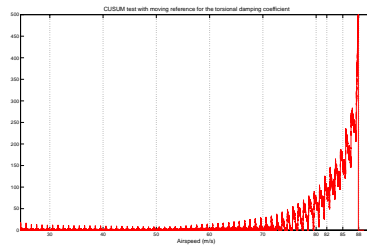


Solution 1: Torsion mode



Solution 3: Bending mode

No alarm



Solution 3: Torsion mode

Alarm of Sol.3 closer to flutter

Conclusion

Online **detection** for **flutter** monitoring

Model-free subspace statistics, local approach, **CUSUM**

Analytical model for flutter prediction

Recursive computation of Jacobian and covariance matrices

Three variants of CUSUM

Algo 1: detection of $\psi \leq \psi_0$

Algo 2: flutter detection

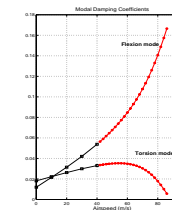
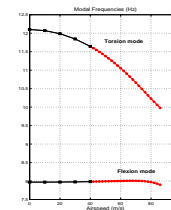
Algo 3: detection of abrupt drop in ψ

Relevance on a small simulated structure

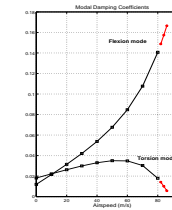
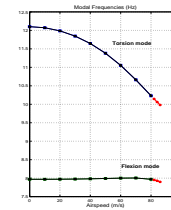
Limitations: **cost** of online covariance computation

Availability of flutter prediction **model** in real cases

Major issues: **dimension of θ** , large number of **correlated** criteria

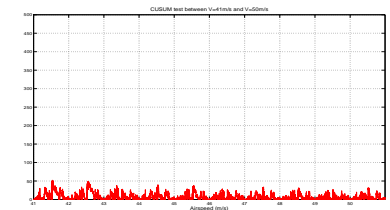


θ 's estimated up to $V = 40m/s$,
 θ_c predicted

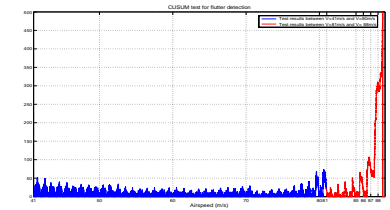


θ 's estimated up to $V = 80m/s$
 θ_c predicted

Drop of the torsion mode damping



Sol. 2 between flight points,
 $V = 40m/s$ and $50m/s$



Sol. 2 between flight points
 $V = 40m/s$ and $80m/s$ and
 $V = 81m/s$ and $90m/s$