

On-line monitoring

of **slow to fast evolving** aeronautic structures

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COSMAD : Free Scilab Toolbox for modal analysis and SHM

<http://www.irisa.fr/sisthem/cosmad/>

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Aircraft **flutter monitoring**

- Aero-elastic flutter: critical instability phenomenon
- Flight flutter testing procedure
- Objective: **on-line** in-flight exploitation of test data
- On-line flight **flutter monitoring** problem:
monitoring some specific **damping coefficient**
- Decide whether $\rho < \rho_c$, critical value

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Problems : **In-operation** on-line

structural identification and **monitoring**

- In-operation modal analysis:

The **excitation** is typically **not controlled** (natural),
not measured, and **nonstationary** (e.g., turbulent)

- Objectives :

- **On-line** in-flight exploitation of test data
- On-line flight **flutter monitoring**

- Monitoring as an **identification** problem
- Monitoring as a **detection** problem

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Contents

- Modelling
- Covariance-driven **subspace identification**
- **Automated modal analysis** for monitoring
- Subspace-based **damage detection** test
- **On-line test for flutter monitoring**
- **Application** to real datasets

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Modelling

FE model:
$$\begin{cases} M\ddot{\mathbf{Z}}(s) + C\dot{\mathbf{Z}}(s) + K\mathbf{Z}(s) = \nu(s) \\ Y(s) = L\mathbf{Z}(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\Psi_\mu = 0, \quad \psi_\mu = L\Psi_\mu$$

State space:
$$\begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \Phi_\lambda, \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\delta\mu}}_{\text{modes}} = \lambda, \quad \underbrace{\psi_\mu}_{\text{mode shapes}} = \varphi_\lambda$$

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Implementation

$$\hat{R}_i \triangleq \frac{1}{N} \sum_{k=1}^N Y_k Y_{k-i}^T, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \dots & \vdots \end{pmatrix}$$

ok when nonstationary!

$$\hat{\mathcal{H}} \approx \hat{\mathcal{O}} \hat{\mathcal{C}}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

Two steps:

- **Stabilization diagrams:** increasing SVD trunc. order
- **MAC plots:** mode-shapes in a selected freq. band

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Output-only covariance-driven subspace identification

$$\begin{cases} X_{k+1} = F X_k + V_k \\ Y_k = H X_k \end{cases}$$

$$\underbrace{R_i \triangleq E(Y_k Y_{k-i}^T)}_{\text{ok if stationary!}}, \quad \mathcal{H} = \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \dots & \vdots \end{pmatrix}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (G \quad FG \quad F^2G \quad \dots)$$

$$G \triangleq E(X_k Y_k^T)$$

$$R_i = H F^i G \implies \mathcal{H} = \mathcal{O} \mathcal{C} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

With input/output data: Handle **projections** $Y/u, Y/u^\perp$

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Implementation - Details

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

$$\hat{\mathcal{H}} = (U_1 \ U_2) \Delta W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T; \quad \hat{\mathcal{O}} = U_1 \Delta_1^{1/2}$$

$$\mathcal{O}_p^\uparrow(H, F) = \mathcal{O}_p(H, F) F$$

$$\det(F - \lambda I) = 0, \quad F \Phi_\lambda = \lambda \Phi_\lambda, \quad \varphi_\lambda = H \Phi_\lambda$$

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Automated modal analysis for monitoring

Automatic post-processing of stabilization diagrams,
GUI within **COSMAD** toolbox.

- Extracted frequencies and damping coefficients plotted over time
- Zoom on a selected frequency
- Interactive graphical output
- Time evolution of frequencies and damping coeff. and associated confidence intervals

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Structural monitoring : Eigenstructure monitoring

$$\begin{cases} X_{k+1} = F X_k + V_k & F \Phi_\lambda = \lambda \Phi_\lambda \\ Y_k = H X_k & \varphi_\lambda \triangleq H \Phi_\lambda \end{cases}$$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$ modes
mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \vdots \\ \Phi \Delta^p \end{pmatrix}$

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Damage detection

θ_0 : reference parameter, known (or identified)

Y_k : N -size sample of new measurements

Build a residual ζ significantly non zero when damage

Local approach (small deviations)

Test $\mathcal{H}_0 : \theta = \theta_0$ against $\mathcal{H}_1 : \theta = \theta_0 + \delta\theta/\sqrt{N}$

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Subspace model/data correlation (1)

Fresh data $\longrightarrow \hat{R}_i \longrightarrow \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$

Nominal model : $\mathcal{O}(\theta_0) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \Phi \Delta^2 \\ \vdots \end{pmatrix}$ Observability
in modal basis

! $\mathcal{H} = \mathcal{O} \mathcal{C}$! **$\ker \hat{\mathcal{H}}^T \stackrel{?}{=} \ker \mathcal{O}^T(\theta_0)$**

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Subspace **model/data correlation** (2)

$$\exists U, \quad U^T U = I_s, \quad U^T \mathcal{O}_{p+1}(\theta_0) = 0; \quad \text{say } U(\theta_0)$$

$$\text{Check if: } U^T(\theta_0) \hat{\mathcal{H}} \approx 0$$

Residual for **structural damage** monitoring

$$\zeta_N(\theta_0) \triangleq \sqrt{N} \text{vec}(U^T(\theta_0) \hat{\mathcal{H}})$$

? How to assess the significance of: $U^T(\theta_0) \hat{\mathcal{H}} \approx 0$?

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Monitoring a **damping** coefficient

- Write the **subspace-based residual** ζ as a **cumulative sum**
- Test $\rho \geq \rho_c$ against $\rho < \rho_c$
Non local! Use a different asymptotics for ζ
- Introduce a **minimum change magnitude** (actual change magnitude unknown)
- Run two **CUSUM tests** in parallel (actual change direction unknown)

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Subspace **model/data correlation** (3)

The **residual** is asymptotically **Gaussian**

$$\zeta_N(\theta_0) \rightarrow \begin{cases} \mathcal{N}(\quad 0, \Sigma(\theta_0)) & \text{under } P_{\theta_0} \\ \mathcal{N}(\mathcal{J}(\theta_0) \delta\theta, \Sigma(\theta_0)) & \text{under } P_{\theta_0 + \delta\theta/\sqrt{N}} \end{cases}$$

$\mathcal{J}(\theta_0)$: mean **sensitivity** (Jacobian) of residual ζ w.r.t. modal changes

(On-board) **χ^2 -test** in the residual

$$\zeta_N^T \Sigma^{-1} \mathcal{J} (\mathcal{J}^T \Sigma^{-1} \mathcal{J})^{-1} \mathcal{J}^T \Sigma^{-1} \zeta_N \geq h$$

(On-board) **modal χ^2 -test**

$$\zeta_N^T \Sigma^{-1} \mathcal{J}_i (\mathcal{J}_i^T \Sigma^{-1} \mathcal{J}_i)^{-1} \mathcal{J}_i^T \Sigma^{-1} \zeta_N \geq h$$

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Monitoring a **damping** coefficient - Details

$$\bar{\zeta}_n(\rho_0) \triangleq \mathcal{J}(\rho_0)^T \Sigma(\theta_0)^{-1} \zeta_n(\theta_0) = \frac{n-p}{\sum_{k=q}^{n-p}} Z_k(\rho_0) / \sqrt{n}$$

$$Z_k(\rho_0) \triangleq \mathcal{J}(\rho_0)^T \Sigma(\theta_0)^{-1} \text{vec}(U(\theta_0)^T \mathcal{Y}_{k,p+1}^+ \mathcal{Y}_{k,q}^{-T})$$

$$\nu \triangleq \mathbb{E}(Z_k), \quad \bar{H}_0 : \nu \geq + \nu_m/2 \quad \text{and} \quad \bar{H}_1 : \nu < - \nu_m/2$$

Decreasing mean

$$S_n^{(-)} \triangleq \bar{\Sigma}^{-1/2} \sum_{k=q}^{n-p} (Z_k + \nu_m)$$

$$T_n^{(-)} \triangleq \max_{q \leq k \leq n-p} S_k^{(-)}$$

$$g_n^- \triangleq T_n^{(-)} - S_n^{(-)}$$

$$g_n^- \geq \gamma^-$$

Increasing mean

$$S_n^{(+)} \triangleq \bar{\Sigma}^{-1/2} \sum_{k=q}^{n-p} (Z_k - \nu_m)$$

$$T_n^{(+)} \triangleq \min_{q \leq k \leq n-p} S_k^{(+)}$$

$$g_n^+ \triangleq S_n^{(+)} - T_n^{(+)}$$

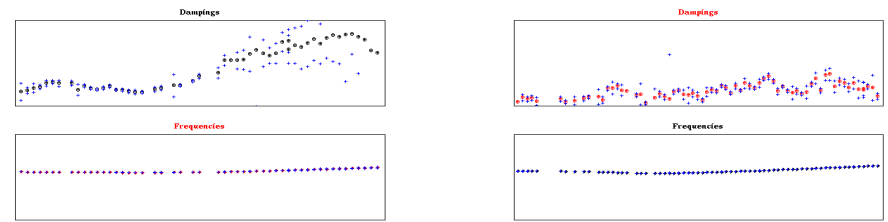
$$g_n^+ \geq \gamma^+$$

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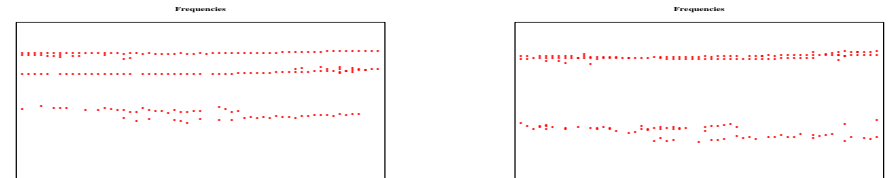
Application to real datasets

- **Automated modal analysis**
AIRBUS France in-flight aircraft dataset
EADS Launch Vehicles ground test dataset
 - **Window size** effect
 - **Sensors selection** influence
 - **Data filtering** effect
- **Detection test** for flutter monitoring
Ariane booster launcher during a launch scenario on the ground (not during a real flight test)
 - Running the test with 2 critical values

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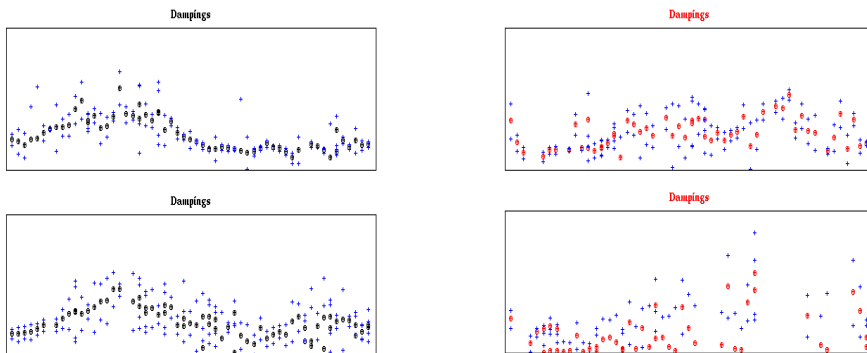


Window size: 4000 (left) and 2000 (right).

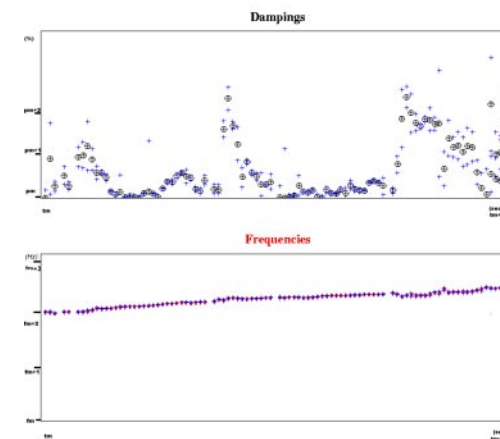


Time evolution of some extracted modes
 First sensor selection (left), 2nd sensor selection (right).

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Damping evolution over time for Exp.1 - First (left) and second (right) mode. With (Top) and without (Bottom) filtering.



Automatic identification. Each symbol: processing 5 sec. data.

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Drawbacks, advantages and conclusion

- Automated identification

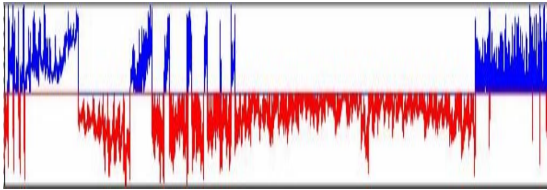
- Higher **computing resources**
- **Bias / variance** trade-off
- **Good results**, especially for frequencies
- **Mature** approach

- Detection

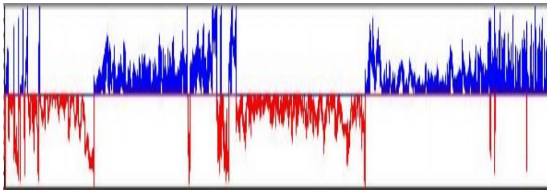
- **Calibration** (estimation of Σ)
- **Actual** case $\rho_c < \rho_0$ not solved yet
- **Sample point-wise** processing
- **Robustness** to non-stationarity (unsteady aerodynamics)

- Complementary approaches

- Further investigations on flutter detection within **FLiTE2**



Test for $\rho_c = \rho_0 = \rho_c^{(1)}$.



Test for $\rho_c = \rho_0 = \rho_c^{(2)} < \rho_c^{(1)}$.

Bottom: $-g_n^-$ reflects $\rho < \rho_c$. **Top:** g_n^+ reflects $\rho > \rho_c$.