

# Online detection of aircraft instability precursors

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## Introduction - (1)

- **Flutter**: critical aircraft **instability** phenomenon  
unfavorable interaction of aerodynamic, elastic and inertial forces; may cause major failures
- **Flight flutter testing**, very expensive and time consuming :  
Design the flutter free flight envelope
- Flutter clearance techniques:  
In-flight **identification**: output-only, or using input excitations  
Data processing: time-frequency, wavelet, envelope function  
Flutter **prediction** based on model-based approaches:  
flutterometer ( $\mu$ -robustness), physical model updating
- Some **challenges**:  
Real time **on-board monitoring**,  
robustness to noise and uncertainties
- Our approach:  
**Statistical detection** for monitoring instability indicators

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## Introduction

**Subspace**-based residual for **modal monitoring**

**CUSUM** test for monitoring a **scalar** instability index

Using and **tuning** the CUSUM test

Experimental **results**

Conclusion

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## Introduction - (2)

- Aim of in-flight **online flutter monitoring**:  
**Early** detection of a deviation in the aircraft modal parameters  
**before** it develops into flutter.
- **Change-point** detection: natural approach
- For a scalar **instability criterion**  $\psi$  and a **critical value**  $\psi_c$ ,  
online **hypotheses** testing:  
$$H_0 : \psi > \psi_c \text{ and } H_1 : \psi \leq \psi_c$$
- **CUSUM** test as an approximation to the optimal test
- Performances of the CUSUM test  
for different statements of  $H_0$

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## Subspace-based residual for modal monitoring

$$\begin{cases} \mathbf{X}_{k+1} = \mathbf{F} \mathbf{X}_k + \mathbf{V}_k & \mathbf{F} \phi_\lambda = \lambda \phi_\lambda \\ \mathbf{Y}_k = \mathbf{H} \mathbf{X}_k & \varphi_\lambda \triangleq \mathbf{H} \phi_\lambda \end{cases}$$

$$\mathbf{R}_i \triangleq \mathbb{E}(\mathbf{Y}_k \mathbf{Y}_{k-i}^T), \quad \mathcal{H} \triangleq \begin{pmatrix} \mathbf{R}_0 & \mathbf{R}_1 & \mathbf{R}_2 & \dots \\ \mathbf{R}_1 & \mathbf{R}_2 & \mathbf{R}_3 & \dots \\ \mathbf{R}_2 & \mathbf{R}_3 & \mathbf{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$\mathbf{R}_i = \mathbf{H} \mathbf{F}^i \mathbf{G} \implies \mathcal{H} = \mathcal{O} \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} \mathbf{H} \\ \mathbf{H}\mathbf{F} \\ \mathbf{H}\mathbf{F}^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (\mathbf{G} \quad \mathbf{F}\mathbf{G} \quad \mathbf{F}^2\mathbf{G} \quad \dots)$$

$$\mathbf{G} \triangleq \mathbb{E}(\mathbf{X}_k \mathbf{Y}_k^T)$$

Output-only covariance-driven **subspace identification**

$$\text{SVD of } \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (\mathbf{H}, \mathbf{F}) \longrightarrow (\lambda, \varphi_\lambda)$$

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## Local approach to testing

$$\bar{\mathbf{H}}_0: \theta = \theta_\star \quad \text{and} \quad \bar{\mathbf{H}}_1: \theta = \theta_\star + \Upsilon/\sqrt{n}$$

Mean **sensitivity** and **covariance** matrices:

$$\mathcal{J}_n(\theta_\star, \theta) \triangleq 1/\sqrt{n} \partial/\partial\tilde{\theta} \mathbb{E}_\theta \zeta_n(\tilde{\theta})|_{\tilde{\theta}=\theta_\star}, \quad \Sigma_n(\theta_\star, \theta) \triangleq \mathbb{E}_\theta (\zeta_n(\theta_\star) \zeta_n(\theta_\star)^T)$$

If  $\Sigma_n(\theta_\star, \theta)$  is positive definite, and for all  $\Upsilon$ , under both hypoth:

$$\Sigma_n(\theta_\star, \theta)^{-1/2} (\zeta_n(\theta_\star) - \mathcal{J}_n(\theta_\star, \theta) \Upsilon) \xrightarrow{n \rightarrow \infty} \mathcal{N}(0, I)$$

**Normalized** residual:

$$\bar{\zeta}_n(\theta_\star) \triangleq \mathcal{K}_n(\theta_\star, \theta) \zeta_n(\theta_\star)$$

$$\mathcal{K}_n(\theta_\star, \theta) \triangleq \Sigma_n^{-1/2} \mathcal{J}_n^T \Sigma_n^{-1}, \quad \bar{\Sigma}_n(\theta_\star, \theta) \triangleq \mathcal{J}_n^T \Sigma_n^{-1} \mathcal{J}_n$$

$$(\bar{\zeta}_n(\theta_\star) - \bar{\Sigma}_n(\theta_\star, \theta)^{1/2} \Upsilon) \xrightarrow{n \rightarrow \infty} \mathcal{N}(0, I)$$

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**Canonical parameter** :  $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$  **modes**  
**mode shapes**

$$\text{Observability in modal basis} : \mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix}$$

Given:

- a **reference parameter**  $\theta_\star$ , by SVD of  $\bar{\mathcal{H}}_{p+1,q}^\star$  (reference data)

$$U(\theta_\star)^T \bar{\mathcal{H}}_{p+1,q}^\star = 0 \quad \text{parameter estimating function}$$

$$U(\theta_\star)^T \mathcal{O}_{p+1}(\theta_\star) = 0, \quad U(\theta_\star)^T U(\theta_\star) = I$$

- a  $n$ -size sample of **new data**;  $\bar{\mathcal{H}}_{p+1,q}$

For **testing**  $\theta = \theta_\star$ , **statistics** (residual) :

$$\zeta_n(\theta_\star) \triangleq \sqrt{n} \text{vec} (U(\theta_\star)^T \bar{\mathcal{H}}_{p+1,q})$$

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## Data-driven computation for **online** detection

$$\bar{\zeta}_n(\theta_\star) = \sum_{k=q}^{n-p} \mathbf{Z}_k(\theta_\star) / \sqrt{n}$$

$$\mathbf{Z}_k(\theta_\star) \triangleq \mathcal{K}_n(\theta_\star, \theta) \text{vec} (U(\theta_\star)^T \mathcal{Y}_{k,p+1}^+ \mathcal{Y}_{k,q}^{-T})$$

$\sum_{k=q}^{n-p} \mathbf{Z}_k(\theta_\star) / \sqrt{n}$  asymptotically Gaussian distributed,  
with mean zero under  $\bar{\mathbf{H}}_0$  and  $\bar{\Sigma}(\theta_\star, \theta)^{1/2} \Upsilon$  under  $\bar{\mathbf{H}}_1$ .

## Another approximation

For  $n$  large enough, and  $k = 1, \dots, n$ ,

$\mathbf{Z}_k(\theta_\star) \approx$  **Gaussian i.i.d.**, mean 0 before change and  $\neq 0$  after.

## Monitoring any function $\psi(\theta)$

Replace  $\mathcal{J}_n(\theta_\star, \theta)$  with  $\mathcal{J}_n(\theta_\star, \theta) \mathcal{J}_{\theta_\star}^\psi$ , where  $\mathcal{J}_{\theta_\star}^\psi = \partial\theta/\partial\psi|_{\theta=\theta_\star}$ .

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## CUSUM test for monitoring a scalar index

The crossing of a critical  $\psi_c$  by  $\psi$  is reflected into a change with the same sign in the mean  $\nu$  of the i.i.d. Gaussian  $Z_k(\theta_*)$ .

The CUSUM test may be used for testing between:

$$H_0 : \nu > 0 \quad \text{and} \quad H_1 : \nu \leq 0$$

Procedure for unknown sign and magnitude of change in  $\psi$

i) Set a min. change magnitude  $\nu_m > 0$ , and test between:

$$H_0 : \nu > \nu_m/2 \quad \text{and} \quad H_1 : \nu \leq -\nu_m/2$$

$$S_n(\theta_*) \triangleq \sum_{k=q}^{n-p} (Z_k(\theta_*) + \nu_m), \quad T_n(\theta_*) \triangleq \max_{k=q, \dots, n-p} S_k(\theta_*)$$

$$g_n(\theta_*) \triangleq T_n(\theta_*) - S_n(\theta_*) \underset{H_0}{\overset{H_1}{>}} \varrho \quad \text{threshold}$$

- ii) Run 2 tests in parallel, for decreasing and increasing  $\psi$ ;
- iii) Make a decision from the first test which fires;
- iv) Reset all sums and extrema to 0, switch to the other test.

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## Using and tuning the CUSUM test

For detecting aircraft instability precursors, select:

- a) An instability criterion  $\psi$  and a critical value  $\psi_c$ ;
- b) A reference state for the system, for identifying (or computing)  $\theta_*$  and computing  $U(\theta_*)$ ;
- c) Estimates of  $\mathcal{J}_n(\theta_*, \theta)$  and  $\Sigma_n(\theta_*, \theta)$ ;
- d) A min. change magnitude  $\nu_m$  and a threshold  $\varrho$ .

Two solutions for b)-c):

- 1.  $\theta_* \triangleq \theta_0$  identified on reference data for the stable system;  $\mathcal{J}_n, \Sigma_n$  estimated once for all with those reference data.
- 2.  $\theta_* \triangleq \theta_c$ , critical parameter closer to instability, computed using  $\theta_0$  and an aeroelastic model;  $\mathcal{J}_n, \Sigma_n$  estimated recursively with the test data.

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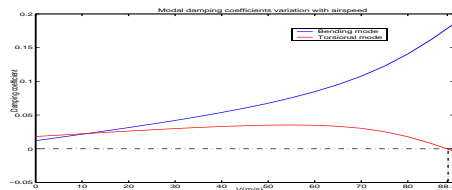
### Example - Aeroelastic Hancock wing model

Rigid wing with constant chord; 2 d.o.f. in bending and torsion.

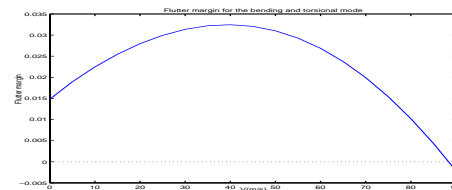
Matrix  $F$ , and eigenvalues  $\lambda$ : functions of airspeed  $V$ .  
Flutter airspeed:  $V_f = 88.5m/s$ .

Flutter margin: stability indicator (from Routh's criterion)

$$\psi_{jk} = \frac{|1 - \lambda_j \lambda_k|^2 |1 - \lambda_j \bar{\lambda}_k|^2 (1 - |\lambda_j|^2) (1 - |\lambda_k|^2)}{1 - |\lambda_j|^2 |\lambda_k|^2}$$



Damping coefficient



Flutter margin

10000-size 2D-samples for  $V = 20, 30, 40, 50, 60, 70, 80, 85, 88m/s$ .

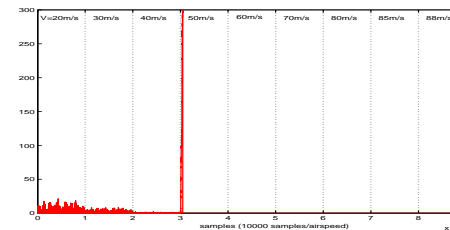
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### Example - Numerical results

CUSUM test run with  $\nu_m = 0.1$ , and the flutter margin as  $\psi$ .

Solution 1. with  $\theta_* = \theta_0$  at  $V = 40m/s$  and fixed  $\mathcal{J}, \Sigma$ .

Solution 2. with  $\theta_* = \theta_c$  at  $V = 85m/s$  and online  $\hat{\mathcal{J}}_n, \hat{\Sigma}_n$ .



Solution 1. ( $\theta_* = \theta_0$  too far from the instability):

alarm at  $V = 50m/s$  since  $\psi(\theta_0) = 0.0332 > \psi(\theta)|_{V=50}$

Solution 2. ( $\theta_* = \theta_c$  close to instability):

alarm at  $V = 88m/s$  since  $\psi(\theta_c) = \psi_c = 0.0037 \approx \psi(\theta)|_{V=89} = 0$

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## Conclusion

online **detection of instability** precursors

Model-free subspace statistics, local approach, CUSUM

Analytical model for flutter prediction

Recursive computation of covariance matrix

**Relevance** on a small simulated structure

**Limitations:** **cost** of online covariance computation

Availability of flutter prediction **model** in real cases

**Major issues:** **dimension of  $\theta$** , large number of **correlated** criteria