

An **adaptive** statistical approach to **flutter detection**

Rafik Zouari, Laurent Mevel, Michèle Basseville

IRISA (INRIA & CNRS), Rennes, France

Eurêka project no 3341 **FLiTE2**

michele.basseville@irisa.fr -- <http://www.irisa.fr/sisthem/>

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Introduction - (1)

- **Flutter**: critical aircraft **instability** phenomenon
unfavorable interaction of aerodynamic, elastic and inertial forces; may cause major failures
- **Flight flutter testing**, very expensive and time consuming :
Design the flutter free flight envelope
- Flutter clearance techniques:
In-flight **identification**: output-only, or using input excitations
Data processing: time-frequency, wavelet, envelope function
Flutter **prediction** based on model-based approaches:
flutterometer (μ -robustness), physical model updating
- Some **challenges**:
Real time **on-board monitoring**,
robustness to noise and uncertainties
- Our approach:
Statistical detection for monitoring instability indicators

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Introduction

Subspace-based residual for **modal monitoring**

CUSUM test for monitoring a **scalar** instability index

Using and **tuning** the CUSUM test

A **moving reference** version

Experimental **results**

Conclusion

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Introduction - (2)

- Aim of in-flight **online flutter monitoring**:
Early detection of a deviation in the aircraft modal parameters
before it develops into flutter.
- **Change-point** detection: natural approach
- For a scalar **instability criterion** ψ and a **critical value** ψ_c ,
online **hypotheses** testing:
$$H_0 : \psi > \psi_c \text{ and } H_1 : \psi \leq \psi_c$$
- **CUSUM** test as an approximation to the optimal test
- A **moving reference** version

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Subspace-based residual for modal monitoring

$$\begin{cases} X_{k+1} = F X_k + V_k & F \phi_\lambda = \lambda \phi_\lambda \\ Y_k = H X_k & \varphi_\lambda \triangleq H \phi_\lambda \end{cases}$$

$$R_i \triangleq E(Y_k Y_{k-i}^T), \quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \dots & \vdots \end{pmatrix}$$

$$R_i = H F^i G \implies \mathcal{H} = \mathcal{O} \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (G \quad FG \quad F^2G \quad \dots)$$

$$G \triangleq E(X_k Y_k^T)$$

Output-only covariance-driven **subspace identification**

$$\text{SVD of } \mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

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Local approach to testing

$$\bar{H}_0: \theta = \theta_\star \quad \text{and} \quad \bar{H}_1: \theta = \theta_\star + \Upsilon/\sqrt{n}$$

Mean **sensitivity** and **covariance** matrices:

$$\mathcal{J}_n(\theta_\star, \theta) \triangleq 1/\sqrt{n} \partial/\partial \tilde{\theta} E_\theta \zeta_n(\tilde{\theta})|_{\tilde{\theta}=\theta_\star}, \quad \Sigma_n(\theta_\star, \theta) \triangleq E_\theta (\zeta_n(\theta_\star) \zeta_n(\theta_\star)^T)$$

If $\Sigma_n(\theta_\star, \theta)$ is positive definite, and for all Υ , under both hypoth:

$$\Sigma_n(\theta_\star, \theta)^{-1/2} (\zeta_n(\theta_\star) - \mathcal{J}_n(\theta_\star, \theta) \Upsilon) \xrightarrow{n \rightarrow \infty} \mathcal{N}(0, I)$$

Normalized residual:

$$\bar{\zeta}_n(\theta_\star) \triangleq \mathcal{K}_n(\theta_\star, \theta) \zeta_n(\theta_\star)$$

$$\mathcal{K}_n(\theta_\star, \theta) \triangleq \Sigma_n^{-1/2} \mathcal{J}_n^T \Sigma_n^{-1}, \quad \bar{\Sigma}_n(\theta_\star, \theta) \triangleq \mathcal{J}_n^T \Sigma_n^{-1} \mathcal{J}_n$$

$$(\bar{\zeta}_n(\theta_\star) - \bar{\Sigma}_n(\theta_\star, \theta)^{1/2} \Upsilon) \xrightarrow{n \rightarrow \infty} \mathcal{N}(0, I)$$

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Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$ modes
mode shapes

$$\text{Observability in modal basis} : \mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \vdots \\ \Phi \Delta^p \end{pmatrix}$$

Given:

- a **reference parameter** θ_\star , by SVD of $\hat{\mathcal{H}}_{p+1,q}^\star$ (reference data)

$$U(\theta_\star)^T \hat{\mathcal{H}}_{p+1,q}^\star = 0 \quad \text{parameter estimating function}$$

$$U(\theta_\star)^T \mathcal{O}_{p+1}(\theta_\star) = 0, \quad U(\theta_\star)^T U(\theta_\star) = I$$

- a n -size sample of **new data**; $\hat{\mathcal{H}}_{p+1,q}$

For **testing** $\theta = \theta_\star$, **statistics** (residual) :

$$\zeta_n(\theta_\star) \triangleq \sqrt{n} \text{vec} (U(\theta_\star)^T \hat{\mathcal{H}}_{p+1,q})$$

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Data-driven computation for **online** detection

$$\bar{\zeta}_n(\theta_\star) \approx \frac{n-p}{\sum_{k=q}^{n-p}} Z_k(\theta_\star) / \sqrt{n}$$

$$Z_k(\theta_\star) \triangleq \mathcal{K}_k(\theta_\star, \theta) \text{vec} (U(\theta_\star)^T \mathcal{Y}_{k,p+1}^+ \mathcal{Y}_{k,q}^{-T})$$

Another approximation

For n large enough, and $k = 1, \dots, n$,

$Z_k(\theta_\star) \approx$ **Gaussian i.i.d.**, mean 0 before change and $\neq 0$ after.

Monitoring any function $\psi(\theta)$

Replace $\mathcal{J}_n(\theta_\star, \theta)$ with $\mathcal{J}_n(\theta_\star, \theta) \mathcal{J}_{\theta_\star}^\star \psi$, where $\mathcal{J}_{\theta_\star}^\star \psi = \partial \psi / \partial \theta |_{\theta=\theta_\star}$.

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CUSUM test for monitoring a scalar index

The crossing of a **critical** ψ_c by ψ is reflected into a change with the **same sign** in the mean ν of the i.i.d. Gaussian $Z_k(\theta_*)$.

The CUSUM test may be used for testing between:

$$H_0 : \nu > 0 \quad \text{and} \quad H_1 : \nu \leq 0$$

Procedure for **unknown** sign and magnitude of **change in** ψ

i) Set a **min. change magnitude** $\nu_m > 0$, and test between:

$$H_0 : \nu > \nu_m/2 \quad \text{and} \quad H_1 : \nu \leq -\nu_m/2$$

$$S_n(\theta_*) \triangleq \sum_{k=q}^{n-p} (Z_k(\theta_*) + \nu_m), \quad T_n(\theta_*) \triangleq \max_{k=q, \dots, n-p} S_k(\theta_*)$$

$$g_n(\theta_*) \triangleq T_n(\theta_*) - S_n(\theta_*) \underset{H_0}{\underset{H_1}{>}} \varrho \quad \text{threshold}$$

- ii) Run **2 tests** in parallel, for **decreasing** and **increasing** ψ ;
- iii) Make a decision from the first test which fires;
- iv) Reset all sums and extrema to 0, switch to the other test.

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Using and tuning the CUSUM test

For detecting aircraft instability precursors, **select**:

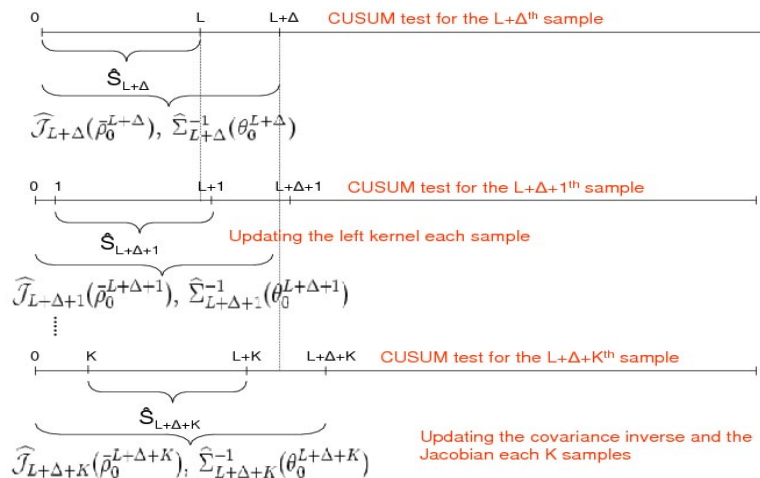
- a) An instability criterion ψ and a critical value ψ_c ;
- b) A left kernel matrix $U(\cdot)$;
- c) **Estimates** of $\mathcal{J}_n(\theta_*, \theta)$ and $\Sigma_n(\theta_*, \theta)$;
- d) A min. change magnitude ν_m and a threshold ϱ .

Two solutions for b)-c):

- 1. $\theta_* \triangleq \theta_0$ **identified** on **reference data** for the stable system; $U(\theta_*)$ computed, \mathcal{J}_n, Σ_n **estimated** once for all with those data.
- 2. $U(\cdot) \triangleq \hat{U}_n$ **estimated** on **test data**; \mathcal{J}_n, Σ_n **estimated recursively** with those test data.

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The moving reference version



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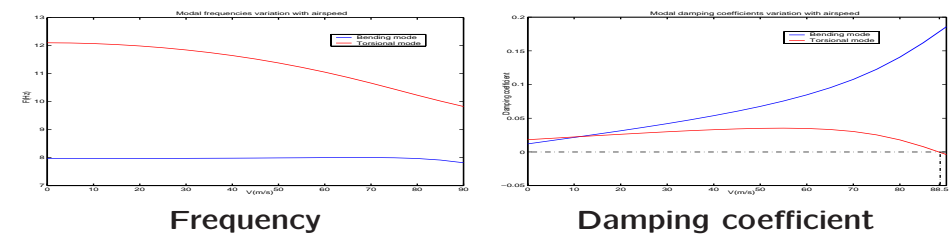
Example - Aeroelastic Hancock wing model

Rigid wing with constant chord; **2 d.o.f.** in bending and torsion.

Matrix F , and eigenvalues λ : functions of airspeed V .

Flutter airspeed: $V_f = 88.5m/s$.

Stability indicator: Damping coefficient



20700-size 2D-samples simulated (300 for each $V=20:1:88m/s$).

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Example - Numerical results

CUSUM test run with $\nu_m = 0.1$, $\rho = 100$, and the damping as ψ .

Solution 1. with $\theta_\star = \theta_0$ at $V = 20m/s$ and fixed \mathcal{J}, Σ .

Solution 2. with online $\hat{U}_n, \hat{\mathcal{J}}_n, \hat{\Sigma}_n$.

Solution 1. θ_\star far from instability, too early alarm at $V=69m/s$.

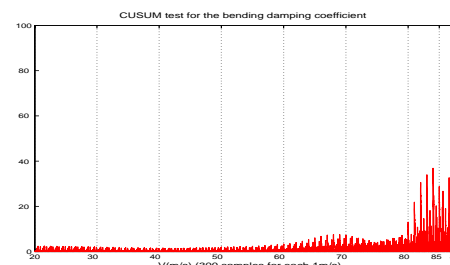
The test detects that torsional damping decreases under the predefined threshold.

Solution 2. Alarm at $V = 82m/s$ much closer to flutter.

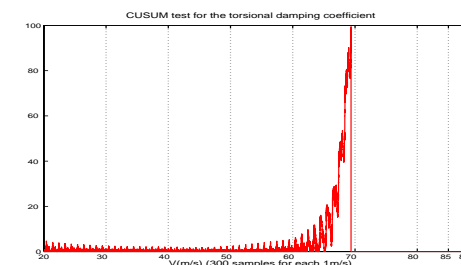
The test detects that the torsional damping decreases abruptly.

Both algorithms do what they are intended to do.

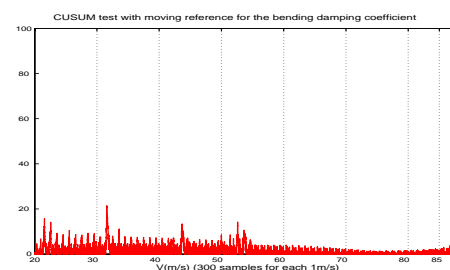
Only Solution 2 is a flutter detection algorithm.



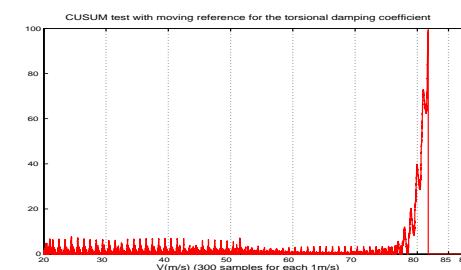
Solution 1: Bending mode



Solution 1: Torsion mode



Solution 2: Bending mode



Solution 2: Torsion mode

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Conclusion

Online **detection of instability** precursors

Model-free subspace statistics, local approach, CUSUM

Analytical model for flutter prediction

Recursive computation of covariance matrix

Relevance on a small simulated structure

Limitations: **cost** of online kernel and covariance computation

Major issues: **dimension of θ** , large number of **correlated** criteria

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