An adaptive statistical approach to flutter detection

Rafik Zouari, Laurent Mevel, Michèl Basseville
IRISA (INRIA & CNRS), Rennes, France
Eurêka project no 3341 FliTE2

michele.basseville@irisa.fr -- http://www.irisa.fr/sisthem/

Content

Introduction

Subspace-based residual for modal monitoring

CUSUM test for monitoring a scalar instability index

Using and tuning the CUSUM test

A moving reference version

Experimental results

Conclusion

Introduction - (1)

- Flutter: critical aircraft instability phenomenon
  unfavorable interaction of aerodynamic, elastic and inertial forces; may cause major failures
- Flight flutter testing, very expensive and time consuming:
  Design the flutter free flight envelope
- Flutter clearance techniques:
  In-flight identification: output-only, or using input excitations
  Data processing: time-frequency, wavelet, envelope function
- Flutter prediction based on model-based approaches:
  flutterometer ($\mu$-robustness), physical model updating
- Some challenges:
  Real time on-board monitoring, robustness to noise and uncertainties
- Our approach:
  Statistical detection for monitoring instability indicators

Introduction - (2)

- Aim of in-flight online flutter monitoring:
  Early detection of a deviation in the aircraft modal parameters before it develops into flutter.
- Change-point detection: natural approach
- For a scalar instability criterion $\psi$ and a critical value $\psi_c$, online hypotheses testing:
  $H_0 : \psi > \psi_c$ and $H_1 : \psi \leq \psi_c$
- CUSUM test as an approximation to the optimal test
- A moving reference version


Subspace-based residual for modal monitoring

\[
\begin{align*}
X_{k+1} &= F X_k + V_k \\
Y_k &= H X_k
\end{align*}
\]

\[F \phi_\Lambda = \lambda \phi_\Lambda\]

\[\varphi_\Lambda \triangleq H \phi_\Lambda\]

\[R_i \triangleq \mathbb{E} \{ Y_k Y_{k-i}^T \} , \quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \ldots \\
R_1 & R_2 & R_3 & \ldots \\
R_2 & R_3 & R_4 & \ldots \\
\vdots & \vdots & \vdots & \ddots \end{pmatrix}\]

\[R_i = H F^i G \implies \mathcal{H} = \mathcal{O} \mathcal{C}\]

\[
\begin{align*}
\mathcal{O} &\triangleq \begin{pmatrix} H \\
HF \\
HF^2 \\
\vdots \end{pmatrix} , \quad \mathcal{C} \triangleq \begin{pmatrix} G & FG & F^2 G & \ldots \end{pmatrix} \\
&\quad \mathbb{E} \{ X_k Y_k^T \}
\end{align*}
\]

Output-only covariance-driven subspace identification

SVD of \( \mathcal{H} \rightarrow \mathcal{O} \rightarrow (H,F) \rightarrow (\lambda, \varphi_\Lambda) \)

**Local approach to testing**

\( H_0 : \theta = \theta_* \) and \( H_1 : \theta = \theta_* + Y/\sqrt{n} \)

Mean sensitivity and covariance matrices:

\[
\mathcal{J}_n(\theta_*, \theta) \triangleq 1/\sqrt{n} \partial \theta \partial \hat{\theta} \mathbb{E} \zeta_n(\hat{\theta})|_{\hat{\theta} = \theta} , \quad \Sigma_n(\theta_*, \theta) \triangleq \mathbb{E}_\theta \{ \zeta_n(\theta_*) \zeta_n(\theta_*)^T \}
\]

If \( \Sigma_n(\theta_*, \theta) \) is positive definite, and for all \( Y \), under both hypoth:

\[
\Sigma_n(\theta_*, \theta)^{-1/2} \{ \zeta_n(\theta_*) - \mathcal{J}_n(\theta_*, \theta) Y \} \xrightarrow{n \to \infty} \mathcal{N} (0, I)
\]

Normalized residual:

\[
\zeta_n(\theta_*) \triangleq \mathcal{K}_n(\theta_*, \theta) \zeta_n(\theta_*) \\
\mathcal{K}_n(\theta_*, \theta) \triangleq \Sigma_n^{-1/2} \zeta_n^{-1} \zeta_n^{-1} , \quad \Sigma_n(\theta_*, \theta) \triangleq \mathcal{J}_n^T \Sigma_n^{-1} \mathcal{J}_n \\
\mathcal{J}_n \triangleq \mathbb{E} \{ (\mathcal{O}_n(\theta_*) \mathcal{C}_n(\theta_*) - \mathcal{O}_n(\theta_*) \mathcal{C}_n(\theta_*)^T) \}
\]

\[
\left( \zeta_n(\theta_*) - \Sigma_n(\theta_*, \theta)^{1/2} Y \right) \xrightarrow{n \to \infty} \mathcal{N} (0, I)
\]

**Data-driven computation for online detection**

\[
\zeta_n(\theta_*) \approx \frac{1}{\sqrt{n}} \sum_{k=q}^{n-p} Z_k(\theta_*)/\sqrt{n}
\]

\[
Z_k(\theta_*) \triangleq \mathcal{K}_n(\theta_*, \theta) \mathbb{E} \{ \mathcal{C}_n(\theta_*) \mathcal{C}_n(\theta_*)^T \}
\]

Another approximation

For \( n \) large enough, and \( k = 1, \ldots, n \), \( Z_k(\theta_*) \approx \) Gaussian i.i.d., mean 0 before change and \( \neq 0 \) after.

**Monitoring any function \( \psi(\theta) \)**

Replace \( \mathcal{J}_n(\theta_*, \theta) \) with \( \mathcal{J}_n(\theta_*, \theta) \mathcal{J}_n^\psi \), where \( \mathcal{J}_n^\psi = \partial \theta/\partial \psi|_{\theta = \theta_*} \).
CUSUM test for monitoring a scalar index

The crossing of a critical $\psi_c$ by $\psi$ is reflected into a change with the same sign in the mean $\nu$ of the i.i.d. Gaussian $Z_k(\theta_s)$.

The CUSUM test may be used for testing between:

$$H_0 : \nu > 0 \quad \text{and} \quad H_1 : \nu \leq 0$$

Procedure for unknown sign and magnitude of change in $\psi$

i) Set a min. change magnitude $\nu_m > 0$, and test between:

$$H_0 : \nu > \nu_m/2 \quad \text{and} \quad H_1 : \nu \leq -\nu_m/2$$

$$S_n(\theta_s) \triangleq \sum_{k=q}^{n-p} (Z_k(\theta_s) + \nu_m), \quad T_n(\theta_s) \triangleq \max_{k=q,...,n-p} S_k(\theta_s)$$

$$g_n(\theta_s) \triangleq T_n(\theta_s) - S_n(\theta_s) \quad \text{H}_1 \text{ threshold}$$

ii) Run 2 tests in parallel, for decreasing and increasing $\psi$;

iii) Make a decision from the first test which fires;

iv) Reset all sums and extrema to 0, switch to the other test.

Using and tuning the CUSUM test

For detecting aircraft instability precursors, select:

a) An instability criterion $\psi$ and a critical value $\psi_c$;

b) A left kernel matrix $U(.)$;

c) Estimates of $J_n(\theta_s, \theta)$ and $\Sigma_n(\theta_s, \theta)$;

d) A min. change magnitude $\nu_m$ and a threshold $\theta$.

Two solutions for b)-c):

1. $\theta_s \triangleq \theta_0$ identified on reference data for the stable system; $U(\theta_s)$ computed, $J_n, \Sigma_n$ estimated once for all with those data.

2. $U(.) \triangleq \bar{U}_n$ estimated on test data; $J_n, \Sigma_n$ estimated recursively with those test data.

Example - Aeroelastic Hancock wing model

Rigid wing with constant chord; 2 d.o.f. in bending and torsion.

Matrix $F$, and eigenvalues $\lambda$: functions of airspeed $V$.

Flutter airspeed: $V_f = 88.5 m/s$.

Stability indicator: Damping coefficient

20700-size 2D-samples simulated (300 for each $V=20:1:88 m/s$).
Example - Numerical results

CUSUM test run with $\nu_m = 0.1$, $\varrho = 100$, and the damping as $\psi$.

Solution 1. with $\theta_0 = \theta_0$ at $V = 20m/s$ and fixed $\mathcal{J}, \Sigma$.

Solution 2. with online $\hat{\theta}_n, \hat{\mathcal{J}}_n, \hat{\Sigma}_n$.

Solution 1. $\theta_0$ far from instability, too early alarm at $V=69m/s$. The test detects that torsional damping decreases under the predefined threshold.

Solution 2. Alarm at $V = 82m/s$ much closer to flutter. The test detects that the torsional damping decreases abruptly.

Both algorithms do what they are intended to do. Only Solution 2 is a flutter detection algorithm.

Conclusion

Online detection of instability precursors

Model-free subspace statistics, local approach, CUSUM

Analytical model for flutter prediction

Recursive computation of covariance matrix

Relevance on a small simulated structure

Limitations: cost of online kernel and covariance computation

Major issues: dimension of $\theta$, large number of correlated criteria