

Handling the **temperature effect in SHM**:
 combining a subspace-based **statistical test**
 and a **temperature-adjusted null space**

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Introduction

- Usefulness of global **vibration-based SHM** methods
- Limitations due to **temperature effects on the dynamics** of civil engineering structures
- Wanted: discriminate between changes in modal parameters due to damages and changes due to temperature effects
- A **statistical subspace-based damage detection** algorithm: **null space** of a matrix built on **reference modes/modeshapes** at a known temperature
- Proposed solution to temperature handling: measured temperatures, thermal effect modeling: **analytical null space updating**

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Parametric subspace-based damage detection

$$\begin{cases} X_{k+1} = F X_k + V_k & F \varphi_\lambda = \lambda \varphi_\lambda \\ Y_k = H X_k & \phi_\lambda \triangleq H \varphi_\lambda \end{cases}$$

$$R_i \triangleq E(Y_k Y_{k-i}^T), \quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

$$R_i = H F^i G \implies \mathcal{H} = \mathcal{O} \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (G \quad FG \quad F^2G \quad \dots)$$

$$G \triangleq E(X_k Y_k^T)$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \phi_\lambda)$$

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Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$ modes
mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \vdots \\ \Phi \Delta^p \end{pmatrix}$

θ_0 : reference parameter for safe structure

Left null space: $S^T S = I_s$, $S^T \mathcal{O}_{p+1}(\theta_0) = 0$

Y_k : N -size sample of new measurements

Residual for SHM:

$$\zeta_N(\theta_0) \triangleq \text{vec}(S^T(\theta_0) \hat{\mathcal{H}})$$

$\mathcal{J}(\theta_0)$: sensitivity of ζ w.r.t. modal changes; $\Sigma(\theta_0)$: covariance

$$\chi^2\text{-test: } \zeta_N^T \Sigma^{-1} \mathcal{J}(\mathcal{J}^T \Sigma^{-1} \mathcal{J})^{-1} \mathcal{J}^T \Sigma^{-1} \zeta_N \geq h$$

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N_0 remains spatially constant
(no external axial body forces, no axial surface tractions, gravity effects negligible).

Only one measure at a given point is necessary:
Thermally compensated strain gauges measure $\varepsilon(x) = N_0/EA$.

Solutions of the eigen problem:

$$2\gamma_n^- \gamma_n^+ (1 - \cos(\gamma_n^- L) \cosh(\gamma_n^+ L)) + (\gamma_n^{+2} - \gamma_n^{-2}) \sin(\gamma_n^- L) \sinh(\gamma_n^+ L) = 0$$

with :

$$\gamma_n^+ = \left(\sqrt{\frac{\rho A}{EI} \omega_n^2 + \left(\frac{N_0}{2EI} \right)^2} + \frac{N_0}{2EI} \right)^{1/2}$$

Solved numerically (no analytical solutions for the clamped case).

Analytic expression for mode shapes.

Pre-stress effects on mode shapes are negligible
(numerical results).

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Modeling the temperature effect

Clamped, planar, axially pre-stressed, Euler-Bernoulli beam

$$\text{Eigen problem } \begin{cases} EI \frac{d^4 w(x)}{dx^4} - N_0 \frac{d^2 w(x)}{dx^2} - \rho A \omega^2 w(x) = 0 \\ w(x)|_{x=0,L} = 0 ; \quad \frac{dw(x)}{dx} \Big|_{x=0,L} = 0 \end{cases}$$

with $\omega \triangleq \ln |\lambda|$, $w = w_\omega \triangleq \phi_\lambda$, $w(x)$: transversal displacement.

E, I, ρ, A : Young's modulus, cross-section inertia momentum, density and cross-sectional area.

N_0 : quasi-static axial preload in Newton:

$$N_0 = EA \varepsilon(x), \quad \varepsilon(x) \triangleq \varepsilon_0(x) - \alpha \delta T, \quad \delta T \triangleq T_0(x) - T_{\text{ref}}$$

ε_0 : mechanical strain, T_0 : current temperature, T_{ref} : reference (no stress) temperature, α : thermal expansion coefficient.

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Temperature-adjusted null space detection

- θ_0 : reference modal parameter for safe structure
- Y_k : N -size sample of new measurements;
T recorded
- Update the modal parameter θ_T :

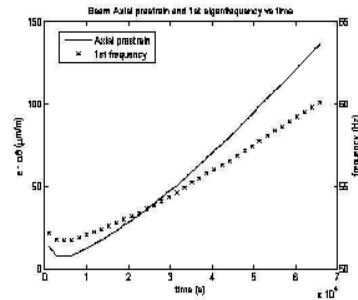
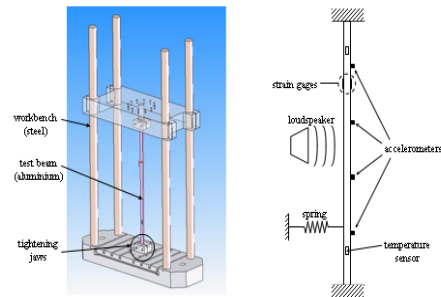
$$\delta T \longrightarrow \varepsilon(x) \longrightarrow (\omega_n)_n \longrightarrow (\lambda_n, \varphi_n)_n \longrightarrow \theta_T$$

- Update the null space $S(\theta_T)$
- Compute the residual $\zeta_N(\theta_T) \triangleq \text{vec}(S^T(\theta_T) \hat{\mathcal{H}})$
- Compute the χ^2 -test

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Example - Beam within a climatic chamber

- A laboratory test-case provided by LCPC Climatic chamber in Nantes
- Vertical clamped beam subject to **decreasing temperatures**
- **Small local damage**: horizontal clamped spring attached to the beam, with tunable stiffness and height



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Implementation issues

- Frequencies : computed \neq identified (the clamped boundary condition, obtained with tightening jaws, is not perfect)

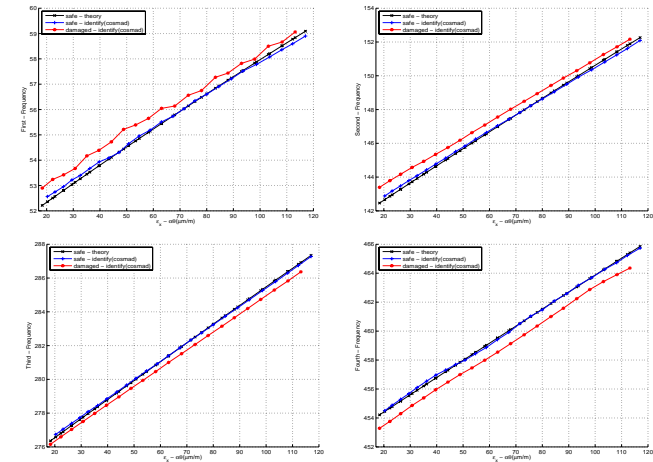
De-biased temperature-adjusted modal parameter:

$$(\forall \varepsilon) \quad \bar{\theta}_T(\varepsilon) \triangleq \theta_T(\varepsilon) + \bar{\theta}_0(\varepsilon_0) - \theta_T(\varepsilon_0)$$

- Compute the key matrices: residual sensitivity \mathcal{J} and covariance Σ for every realization of each scenario

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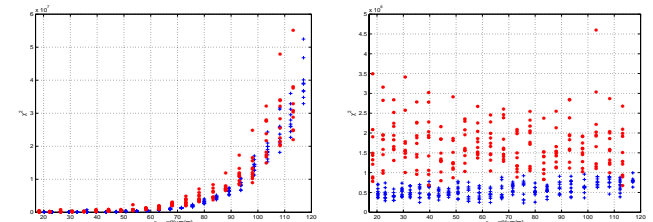
Decreasing temperature effect on the first 4 frequencies



First 4 frequencies vs. **thermal constraint**.
Computed (black) and identified (**safe**, **damaged**)

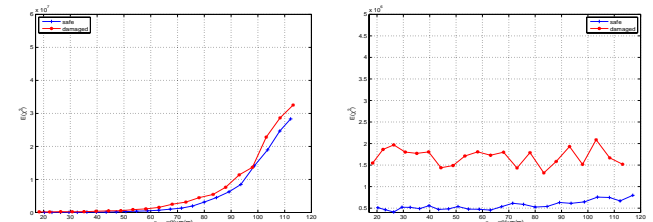
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Handling the temperature effect



Original χ^2 -test

New χ^2 -test



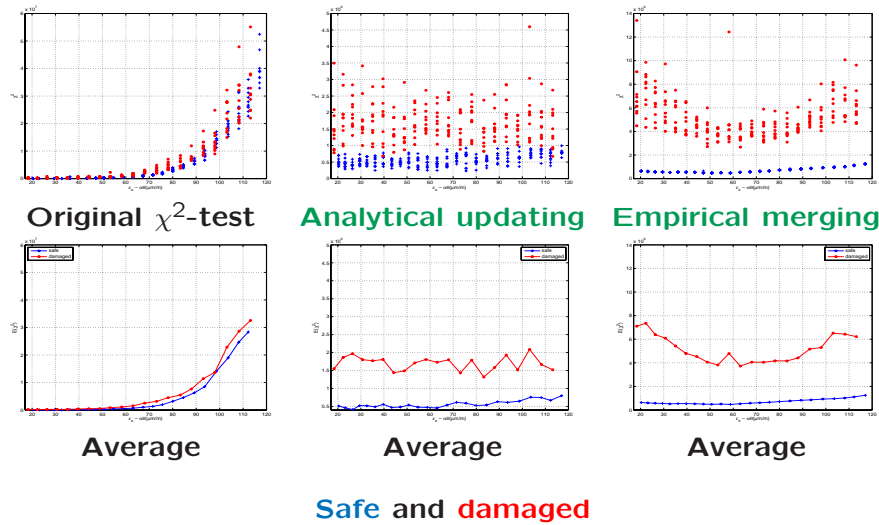
Original - Average

New - Average

Safe and **damaged**

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Comparison with a non parametric approach:
empirical null space merging data at # temperatures



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Temperature effect in vibration-based SHM

Statistical parametric model-based approach

Statistical subspace-based damage detection algorithm

Temperature-adjusted null space

Example: clamped beam within climatic chamber

Comparison with a non parametric approach
(empirical null space, merging data at # temperatures)

Ongoing: statistical nuisance rejection

Future: in-operation examples,
extension to 3D temperature fields,
thermal model parameterization

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