On fault detectability and isolability

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1. Intrinsic or performance-based definitions

2. Additive faults in linear systems: some equivalences

3. Component faults in nonlinear systems: one example

Model-based FDI

System models and fault types - Residual generation

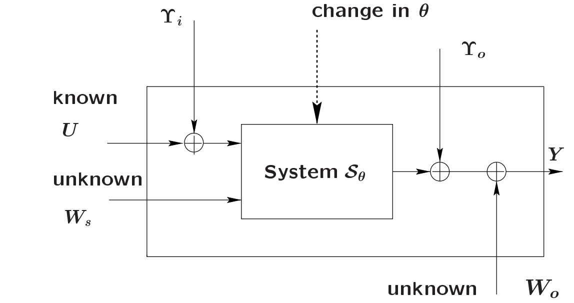
Detectability and isolability definitions

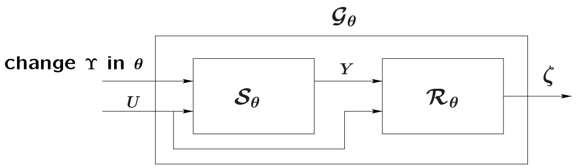
Fault incidence matrix

Fault information content, distance between systems

Fault transfer, fault extended state space

System models and fault types - Residual generation





$$\zeta = \mathcal{R}_\theta(U,Y), \qquad \zeta = \mathcal{G}_\theta\ (\Upsilon), \qquad \mathcal{G}_\theta \triangleq \mathcal{R}_\theta \circ \mathcal{S}_\theta$$

Detectability and isolability definitions

Intrinsic definitions:

Capture a 'signature' of the fault on the system output

Performance-based definitions:

Capture a 'signature' of the fault on the residual

Fault incidence matrix

1. **Additive faults on linear (static, dynamic) Gaussian systems**

$$Y \sim \mathcal{N}(L\ \Upsilon, Q)$$

$$\zeta \triangleq \mathcal{R}\ Y \sim \begin{cases} \mathcal{N}(0,\Sigma) & (\textit{fault-free}) \\ \mathcal{N}(M\ \Upsilon,\Sigma) & (\textit{faulty}) \end{cases} \qquad (\dagger)$$

$$M \triangleq \mathcal{R}\ L, \qquad \Sigma \triangleq \mathcal{R}\ Q\ \mathcal{R}^T \quad (\textit{not diagonal})$$

2. **(Small) component faults on nonlinear systems**

($\dagger$) **holds true** (asymptotically), **with:** $M \triangleq -\ \mathrm{E}_{\theta_0} \left.\dfrac{\partial \mathcal{G}_\theta}{\partial \theta}\right|_{\theta=\theta_0}$

How much fault information in the data?

Observed outputs

$$Y \sim \mathcal{N}(L\ \Upsilon, Q), \qquad \mathrm{F}_Y \triangleq L^T\ Q^{-1}\ L$$

$$\mathrm{F}_Y(\theta_0) \triangleq \lim_{N\to\infty} \frac{1}{N}\ \mathrm{E}_{\theta_0}\ \frac{\partial \ln p_{\theta_0}(\mathcal{Y}_N)}{\partial \theta}\ \frac{\partial \ln p_{\theta_0}(\mathcal{Y}_N)}{\partial \theta}^T$$

Residuals

$$\mathrm{F}_\zeta(\theta_0) \triangleq \lim_{N\to\infty} \frac{1}{N}\ \mathrm{E}_{\theta_0}\ \frac{\partial \ln r_{\theta_0}(\mathcal{Z}_N)}{\partial \theta}\ \frac{\partial \ln r_{\theta_0}(\mathcal{Z}_N)}{\partial \theta}^T$$

If ($\dagger$) **holds:** $\qquad \mathrm{F}_\zeta = M^T\ \Sigma^{-1}\ M$

Detectability definitions : Fault-to-noise ratios

$$\mathrm{J}_Y \triangleq \Upsilon^T\ \mathrm{F}_Y\ \Upsilon \quad > \ 0$$

$$\mathrm{J}_\zeta \triangleq \Upsilon^T\ \mathrm{F}_\zeta\ \Upsilon \quad > \ 0$$

Isolability definitions :

F_Y **close to a diagonal matrix**

F_ζ **close to a diagonal matrix**

How far apart the faulty and the fault-free systems?

Distance between systems: $\qquad \Upsilon = \theta - \theta_0$

$$\bar{\mathrm{K}}_Y(\theta_0,\theta) \triangleq \lim_{N\to\infty} \mathrm{E}_{\theta_0} \left(\ln \frac{p_{\theta_0}(\mathcal{Y}_N)}{p_\theta(\mathcal{Y}_N)}\right), \quad \mathrm{K}_Y(\theta_0,\theta) \triangleq \bar{\mathrm{K}}_Y(\theta_0,\theta) + \bar{\mathrm{K}}_Y(\theta,\theta_0)$$

Add. faults, Gaussian syst.: $\qquad \mathrm{K}_Y = \Upsilon^T\ \mathrm{F}_Y\ \Upsilon = \mathrm{J}_Y$

$$\mathrm{K}_\zeta = \Upsilon^T\ \mathrm{F}_\zeta\ \Upsilon = \mathrm{J}_\zeta$$

Small component faults: $\qquad \mathrm{K}_Y \approx \Upsilon^T\ \mathrm{F}_Y\ \Upsilon = \mathrm{J}_Y$

$$\mathrm{K}_\zeta \approx \Upsilon^T\ \mathrm{F}_\zeta\ \Upsilon = \mathrm{J}_\zeta$$

Detectability definitions : $\qquad \mathrm{K}_Y > 0, \qquad \mathrm{K}_\zeta > 0$

Caution: **For any parity check:** $\qquad \mathrm{F}_\zeta \neq \mathrm{F}_Y$

Is the fault-to-data transfer function invertible?

$$\begin{cases} X_{k+1} = F X_k + G U_k + W_k + \Gamma \Upsilon_x \\ Y_k = H X_k + J U_k + V_k + \Xi \Upsilon_y \end{cases}$$

$$\mathcal{T}_{\Upsilon \to Y}(z) \triangleq \left(H(zI_n - F)^{-1}\Gamma \ \vdots \ \Xi \right) \quad \text{left invertible}$$

$$\mathcal{T}_{\Upsilon \to \zeta}(z) \triangleq \mathcal{T}_{Y \to \zeta}(z)\, \mathcal{T}_{\Upsilon \to Y}(z) \quad \text{left invertible}$$

If ζ from factorization of $\mathcal{T}_{U \to Y}$:

$$\mathcal{T}_{\Upsilon \to \zeta}(z) = \left(H(zI_n - \overline{F})^{-1}\Gamma \ \vdots \ (I_r - H(zI_n - \overline{F})^{-1} F K)\ \Xi \right)$$

where K s.t. $\bar{F} \triangleq F - FKH$ stable.

Fault-extended state space observable (detectable)?

$$\begin{cases} \mathcal{X}_{k+1} = F_\Upsilon\ \mathcal{X}_k + \begin{pmatrix} G\ U_k \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} W_k \\ 0 \\ 0 \end{pmatrix} \\ Y_k = H_\Upsilon\ \mathcal{X}_k + J U_k + V_k \end{cases}$$

$$\mathcal{X} = \begin{pmatrix} X \\ \Upsilon_x \\ \Upsilon_y \end{pmatrix}, \quad F_\Upsilon = \begin{pmatrix} F & \Gamma & 0 \\ 0 & I_{\tilde{n}} & 0 \\ 0 & 0 & I_{\tilde{r}} \end{pmatrix}, \qquad H_\Upsilon = \begin{pmatrix} H & 0 & \Xi \end{pmatrix}$$

Faults strongly (weakly) detectable if (H_Υ, F_Υ) observable (detectable).
Required condition:

$$\text{rank}\ H(I_n - \bar{F})^{-1}\Gamma = \text{dim. } \Upsilon_x$$

$$\text{rank}\ (I_r - H(I_n - \bar{F})^{-1} F K)\ \Xi = \text{dim. } \Upsilon_y$$

where K s.t. 1 is not eigenvalue of $\bar{F} = F - FKH$.

Additive faults in linear systems: some equivalences

The conditions ensuring:

- the positivity of the Kullback distance between the faulty and fault-free systems,
- the left invertibility of the fault-to-output transfer function,
- the observability (detectability) of the fault-extended state-space system,

are equivalent.

Component faults in nonlinear systems: one example

$$\begin{cases} \dot{x}_1 = -\ \theta_1\ \theta_2\ x_1\ x_2 + \theta_2\ u \\ \dot{x}_2 = +\ \alpha\ \theta_1\ x_1^2\ x_2 - \beta\ (x_2 - T_e) \\ y = x_1 \end{cases}$$

Faults
(1) $\theta_1 + 20\%$
(2) $\theta_2 - 9\%$

Two residuals ζ: state elimination or estimation.

For each residual: $\mathrm{J}_\zeta = \Upsilon^T\ \mathrm{F}_\zeta\ \Upsilon, \quad \mathrm{F}_\zeta = M^T\ \Sigma^{-1}\ M$

Evaluation of J_ζ:

Elimination: fault (1) less detectable than fault (2)

Estimation: fault (1) more detectable than fault (2)

F_ζ close to a diagonal matrix:

Estimation: fault isolation easier