

Handling the **temperature effect**

in vibration monitoring of civil structures:

a combined **subspace**-based
and **nuisance rejection** approach

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Introduction

- Usefulness of global **vibration-based SHM** methods
- Limitations due to **temperature effects on the dynamics** of civil engineering structures
- Wanted: discriminate between changes in modal parameters due to damages and changes due to temperature effects
- A **statistical subspace-based damage detection** algorithm:
null space of a matrix built on **reference modes/modeshapes** at a known temperature
- Proposed solution to temperature handling:
no temperature measurement,
thermal effect modeling,
statistical nuisance rejection

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Modeling

$$\text{FE model: } \begin{cases} M\ddot{Z}(s) + C\dot{Z}(s) + KZ(s) = \nu(s) \\ Y(s) = LZ(s) \end{cases}$$

$$(M\mu^2 + C\mu + K)\phi_\mu = 0, \quad \psi_\mu = L\phi_\mu$$

$$\text{State space: } \begin{cases} X_{k+1} = FX_k + V_k \\ Y_k = HX_k \end{cases}$$

$$F\Phi_\lambda = \lambda \Phi_\lambda, \quad \varphi_\lambda \triangleq H\Phi_\lambda$$

$$\underbrace{e^{\tau\mu} = \lambda}_{\text{modes}}, \quad \underbrace{\psi_\mu = \varphi_\lambda}_{\text{modeshapes}}$$

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Parametric subspace-based damage detection

$$\begin{cases} X_{k+1} = F X_k + V_k & F \Phi_\lambda = \lambda \Phi_\lambda \\ Y_k = H X_k & \varphi_\lambda \triangleq H \Phi_\lambda \end{cases}$$

$$R_i \triangleq E(Y_k Y_{k-i}^T), \quad \mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \dots & \vdots \end{pmatrix}$$

$$R_i = H F^i G \implies \mathcal{H} = \mathcal{O} \mathcal{C}$$

$$\mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (G \quad FG \quad F^2G \quad \dots)$$

$$G \triangleq E(X_k Y_k^T)$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \phi_\lambda)$$

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Focused monitoring

$$\text{Fisher information } F(\theta_0) \triangleq \mathcal{J}(\theta_0)^T \Sigma(\theta_0)^{-1} \mathcal{J}(\theta_0)$$

$$\theta = \begin{pmatrix} \theta_a \\ \theta_b \end{pmatrix}, \quad \mathcal{J} = (\mathcal{J}_a \quad \mathcal{J}_b), \quad F = \begin{pmatrix} F_{aa} & F_{ab} \\ F_{ba} & F_{bb} \end{pmatrix} = \begin{pmatrix} \mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_a & \mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_b \\ \mathcal{J}_b^T \Sigma^{-1} \mathcal{J}_a & \mathcal{J}_b^T \Sigma^{-1} \mathcal{J}_b \end{pmatrix}$$

$$F_a^* \triangleq F_{aa} - F_{ab} F_{bb}^{-1} F_{ba}$$

Sensitivity approach - Partial residual

$$\tilde{\zeta}_a \triangleq \mathcal{J}_a^T \Sigma^{-1} \zeta, \quad \tilde{\chi}_a^2 \triangleq \tilde{\zeta}_a^T F_{aa}^{-1} \tilde{\zeta}_a$$

Min-max approach - Robust residual

$$\zeta_a^* \triangleq \tilde{\zeta}_a - F_{ab} F_{bb}^{-1} \tilde{\zeta}_b, \quad \chi_a^{*2} = \zeta_a^{*T} F_a^{*-1} \zeta_a^*$$

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Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$ modes
mode shapes

$$\text{Observability in modal basis : } \mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \vdots \\ \Phi \Delta^p \end{pmatrix}$$

θ_0 : reference parameter for safe structure

Left null space: $S^T S = I_s$, $S^T \mathcal{O}_{p+1}(\theta_0) = 0$

Y_k : N -size sample of new measurements

Residual for SHM:

$$\zeta_N(\theta_0) \triangleq \sqrt{N} \text{vec}(S^T(\theta_0) \hat{\mathcal{H}})$$

$\mathcal{J}(\theta_0)$: sensitivity of ζ w.r.t. modal changes; $\Sigma(\theta_0)$: covariance

$$\chi^2\text{-test: } \zeta_N^T \Sigma^{-1} \mathcal{J}(\mathcal{J}^T \Sigma^{-1} \mathcal{J})^{-1} \mathcal{J}^T \Sigma^{-1} \zeta_N \geq h$$

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The temperature effect - Example 1

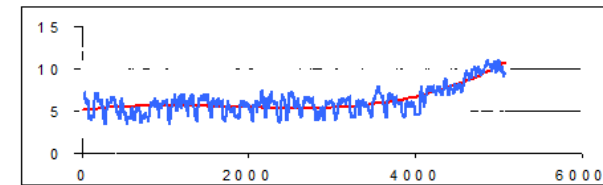
Z24 bridge

- A benchmark of the BRITE/EURAM project SIMCES and of the European COST action F3
- Response to traffic excitation under the bridge measured over one year in 139 points
- Two damage scenarios (DS1 and DS2): pier settlements of 20mm and 80mm.

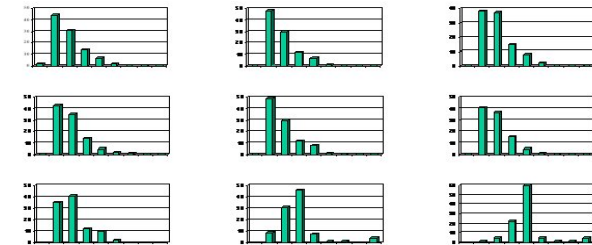
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Identified first four natural frequencies / Test values
(Results with four sensors)

	Mode	1	2	3	4	χ^2
Undamaged	Freq.(Hz)	3.88	5.01	9.80	10.30	$8.80 \cdot 10e2$
Damaged (1)	Freq.(Hz)	3.87	5.06	9.79	10.32	$8.00 \cdot 10e5$
Damaged (2)	Freq.(Hz)	3.76	4.93	9.74	10.25	$3.96 \cdot 10e6$



Evolution of the test values over nine months (log-scale).



Distribution of the test values for each of the nine months.

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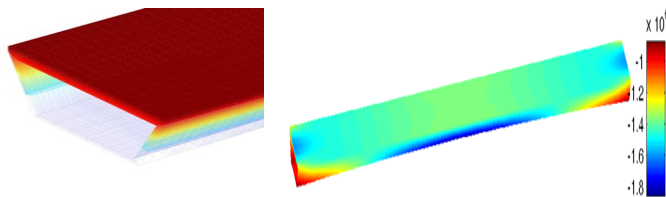
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The temperature effect - Example 2

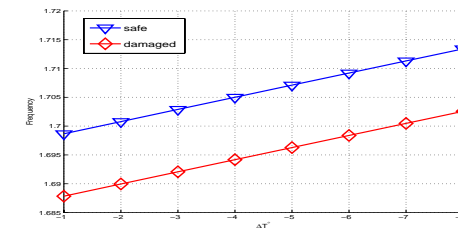
Simulated bridge deck

Finite elements toolbox OpenFEM (with Matlab or Scilab).
60 m span, 9600 volume elements, 13668 nodes.

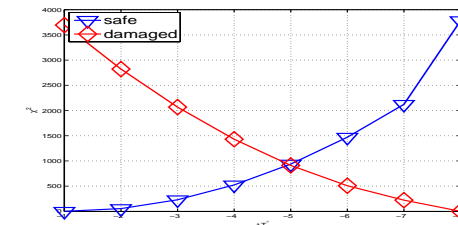
Temperature variations: either a uniform temperature elevation or a linear variation with z .



Linear thermal field (Left) and induced axial stress (Right).
The warmer deck expands while the cooler bottom contracts.



Decreasing temperature effect on the first frequency.



Partial χ^2 -test values.

Safe and damaged

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The temperature effect - Modeling

- Thermal field \rightarrow materials expansion \rightarrow thermal stress field.
- Temperature modification = external load.
- Static equilibrium under thermal loading \leftrightarrow pre-stress.
- Stiffness K , and thus modal frequencies, affected.
Mass M and damping C assumed not affected.
- Assuming thermal loads inducing small perturbations:
stiffness = linear function of the thermal field $T(x)$.
- If $T(x)$ = linear combination of constant thermal fields,
temperature effect on K :

$$K = K_0 + K_T \triangleq K_0 + \sum_i \alpha_i K_{T,i}$$

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The temperature effect - Rejection

- θ_0 : reference modal parameter for safe structure
Compute the null space $S(\theta_0)$,
the **sensitivities** $\mathcal{J}(\theta_0)$ and \mathcal{J}_T ,
the covariance $\Sigma(\theta_0)$ and Fisher matrix $F \triangleq F(\theta_0, T)$
- Y_k : N -size sample of new measurements
- Compute the residual $\zeta_N(\theta_0) \triangleq \sqrt{N} \text{vec}(S^T(\theta_0) \hat{\mathcal{H}})$
- Compute the **robust residual** $\zeta_{\theta_0}^* \triangleq \tilde{\zeta}_{\theta_0} - F_{\theta_0, T} F_{T, T}^{-1} \tilde{\zeta}_T$
- Compute the **χ^2 -test** : $\zeta_{\theta_0}^{*T} F_{\theta_0}^{*-1} \zeta_{\theta_0}^*$

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Computing the residual sensitivity w.r.t. T

Small deviations

$$E(\zeta_N) = \mathcal{J}(\theta_0) \delta\theta + \mathcal{J}_T \delta T, \quad \mathcal{J}_T \triangleq \mathcal{J}(\theta_0) \mathcal{J}_{\theta T}$$

$\mathcal{J}_{\theta T}$ involves computing $\delta\mu$ and $\delta\phi$ for $\delta K = K_{T,i}$, $\delta M = \delta C = 0$,
based on the differentiation of:

$$(M\mu^2 + C\mu + K)\phi_\mu = 0$$

that is:

$$\delta\mu = - \frac{\phi^T (\mu^2 \delta M + \mu \delta C + \delta K) \phi}{\phi^T (2\mu M + C) \phi}$$

and

$$(\mu^2 M + \mu C + K) \delta\phi = - \delta\mu (2\mu M + C) \phi - (\mu^2 \delta M + \mu \delta C + \delta K) \phi$$

with $\phi^T \delta\phi = 0$.

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Implementation issues

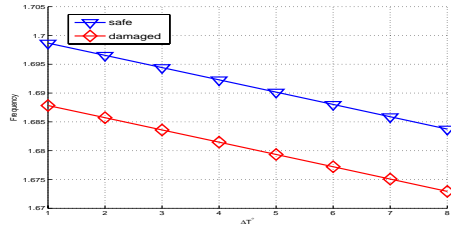
- Compute the key matrices:
sensitivities $\mathcal{J}(\theta_0)$ and \mathcal{J}_T and covariance Σ
on a long data sample for the safe structure.

In case of nonstationary excitation, computing Σ
on current data might be preferable.
- Σ computed with QR (Zhang, 2003).

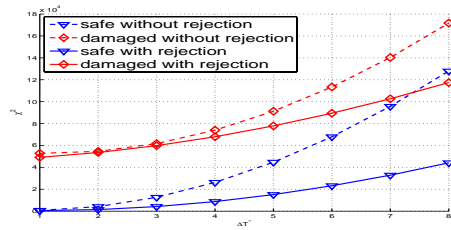
Null space S computed with QR (Nasser, 2006).

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Example - Back to the simulated bridge deck



Increasing temperature effect on the first frequency.



Global (dotted) and minmax (solid) χ^2 -test values.
Minmax operating range: $8 C \times 2$

Safe and damaged

Conclusion

Temperature effect in vibration-based SHM

Statistical parametric model-based approach

Subspace-based damage detection algorithm

Local rejection of the temperature seen as nuisance

Example: simulated bridge deck

Ongoing: empirical null space merging data at # temperatures,
analytical temperature-adjusted null space

Future: in-operation examples,
extension to 3D temperature fields,
thermal model parameterization