Fiber-based Fracture Model for Simulating Soft Tissue Tearing

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Abstract. In this paper, we propose a novel approach for simulating soft tissue tearing, using a model that takes into account the existence of fibers within the tissue. These fibers influence the deformation by introducing anisotropy, and impact the direction of propagation for the fracture during tearing. We describe our approach for simulating, in real-time, the deformation and fracture of anisotropic membranes, and we illustrate our method with the simulation of capsulorhexis, one of the critical steps of cataract surgery.

Keywords. Medical Simulation, Soft tissue Tearing, Fracture, Capsulorhexis

Introduction

A cataract is an opacity in the natural lens of the eye which represents an important cause of visual impairment, sometimes leading to blindness if not treated. The best treatment for this pathology remains surgery. Cataract surgery has made important advances over the past twenty years, and every year, more than five million people in the United States and in Europe undergo cataract surgery. While those recent advances benefit the patient, they require an higher level of training for the surgeon. To address this need, new training simulation systems for cataract surgery have been recently developed. The main objectives of the simulators are to reproduce with great accuracy the three main steps of cataract surgery: capsulorhexis, phacoemulsification and implantation of an intraocular lens. In this paper, we will focus on capsulorhexis, the technique used to create a circular opening in the lens capsule, which relies essentially on the application of shear and stretch forces to propagate a fracture throughout the membrane.

1. Related Work

Previous work on cataract surgery simulation has mainly focused on capsulorhexis and phacoemulsification [1]. In [2] the authors have proposed a rigid model of the lens, and a spring-mass model for the capsule. In [3] and [4] the lens capsule is modeled by a mass-spring network based on a triangular mesh. Most of the existing models use discrete

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methods which require specific mesh structures (e.g. radial and concentric springs with different stiffness values) to approximate the anisotropic properties of the capsule. However, when simulating the capsulorhexis, complex re-meshing techniques are required to maintain a similar mesh structure. On the other hand, models derived from continuum mechanics (typically based on elasticity theory) have the potential to produce more realistic behaviors [5,6]. When combined with a finite element technique, such approaches can no longer rely on the mesh topology to specify preferred directions of anisotropy. The approach presented in this paper relies on a continuous model based on elasticity theory for which specific fiber directions can be defined. The underlying finite element model can handle geometrically non-linear anisotropic deformations at interactive rates. This model is combined with a novel technique for determining the fracture direction, which can be propagated along existing edges of the topology or across existing faces by using a remeshing algorithm.

2. Methods

2.1. Anisotropic Soft Tissue Model

As many biological soft tissues are composite fibrous materials, additional information needs to be introduced to enable realistic tearing: the principal direction of fibers within the soft tissue. Indeed, the orientation of these fibers highly influences the direction of propagation of the tear in the tissue. It also introduces anisotropy in the model. The proposed model for describing deformation and tearing of thin soft tissue (such as membranes, capsules, etc.) relies on a transversely isotropic FEM formulation using triangular elements, in which a specific fiber direction can be defined on each element. The fiber direction on each element, called θ, is defined with respect to a local (co-rotational) reference frame (x, y). This local frame of reference is in turn defined with respect to a global frame of reference (X,Y) via a rigid transformation. The frame defined by the fiber is named (F,T), where F is along the fiber, and T is the transverse (orthogonal) direction (see Figure 1). This leads to the following definition of the anisotropic material stiffness matrix of an element:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{1}{1-\nu_F\nu_T} \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (1)$$

with:

$$K_{11} = c^4 E_F + s^4 E_T + 2c^2 s^2(\nu_T E_F + 2G_F)$$
$$K_{22} = s^4 E_F + c^4 E_T + 2c^2 s^2(\nu_T E_F + 2G_F)$$
$$K_{33} = c^2 s^2(E_F + E_T - 2\nu_T E_F) + (c^2 - s^2)G_F)$$
$$K_{12} = c^2 s^2(E_F + E_T - 4G_F) + (c^4 + s^4)\nu_T E_F)$$
$$K_{13} = -cs[c^2 E_F - s^2 E_T - (c^2 - s^2)(\nu_T E_F + 2G_F)]$$
$$K_{23} = -cs[s^2 E_F - c^2 E_T + (c^2 - s^2)(\nu_T E_F + 2G_F)]$$

where $c = cos\theta$, $s = sin\theta$, $G_F = E_F/(1 + \nu_F)$ and $\nu_T/E_T = \nu_F/E_F$. Since we are dealing with the simulation of fracture and tearing, topological changes will occur. Therefore no matrix assembly is performed, instead we solve the system for the whole mesh through a dedicated conjugate gradient solver [7].

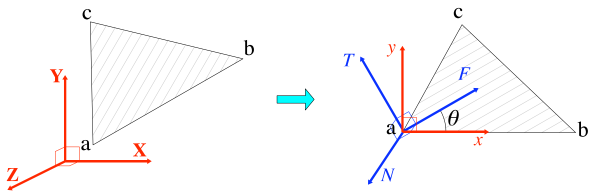

Figure 1. Global reference frame (X,Y,Z), local frame (x,y), and frame (F,T,N) based on fiber orientation.

2.2. *Tearing Criterion*

Although a significant amount of work exists in the field of finite element modeling for the simulation of fractures (see [8,9] for instance), little has been done in the area of interactive medical simulation. Principal difficulties come from the real-time simulation requirement, and the need to account for the presence of fibers (previous work essentially deal with isotropic materials). In general, tearing or fracture occurs when a certain stress threshold is reached. For isotropic materials, this threshold is the same in every direction, and a fracture criterion can be determined using an eigenvalue decomposition of the stress tensor in each element [10]. If the largest eigenvalue is above the given threshold, the element is then fractured along a direction perpendicular to the eigenvector associated to the principal stress direction. This is however not applicable in the case of anisotropic materials, as the presence of fibers leads to preferred fracture directions. Also, the stress threshold is generally not identical along the fiber and transverse directions. Fracture occurs when:

$$\sigma_F > \bar{\sigma}_F \ \text{ or } \ \sigma_T > \bar{\sigma}_T \tag{2}$$

2.2.1. *Definition*

We propose the following measure of the fracture condition. Basically, tearing occurs if:

$$c(\mathbf{d}, \sigma, \mathbf{f}, \mathbf{p}) > 1 \tag{3}$$

where $\mathbf{d}$ is the potential direction of propagation of the fracture, $\sigma = (\sigma_x \sigma_y \tau_{xy})$ is the stress tensor, $\mathbf{f}$ is the fiber direction, and $\mathbf{p}$ is the previous fracture direction (useful to avoid backtracking suddenly when we are continuing from an existing fracture, as otherwise the stress and fiber directions are undirected). This criterion is typically evaluated at the tip of a pre-existing fracture or at the center of each potentially fracturing element. In this paper we propose a first formulation of the fracture criteria, based on the stress threshold in each direction $\bar{\sigma}$. This criterion includes a parameter limiting the angle between the previous and next directions of propagation to be at most θ_P:

$$c(\mathbf{D}, \sigma, \mathbf{f}, \mathbf{p}) = \quad \frac{\sigma_{D^\perp}}{\bar{\sigma}_{\mathbf{d}^\perp}} \quad H\left((\mathbf{d}\cdot\mathbf{p}) - \cos\theta_P\right) \ \text{with}\ \mathbf{d}^\perp = \mathbf{d}\times\mathbf{n} \tag{4}$$

where H is the Heaviside step function: $H(a-b) = \{0 \text{ if } a < b, \quad 1 \text{ if } a \geq b\}$.
The stress along a direction $\mathbf{u}$ can be computed by the coordinate transformation formula, using the angle θ_u between $\mathbf{u}$ and the x axis of the local frame of reference:

$$\sigma_u = \cos^2\theta_u\ \sigma_x + \sin^2\theta_u\ \sigma_y + 2\sin\theta_u\cos\theta_u\ \tau_{xy} \tag{5}$$

As illustrated in Eq. 2, anisotropic materials are typically characterized by two stress thresholds, $\bar{\sigma}_F$ in the direction of the fibers, and $\bar{\sigma}_T$ in the transverse direction. It is not clear what this threshold is for other directions. In our approach, we interpolate between $\bar{\sigma}_F$ and $\bar{\sigma}_T$ based on the angle between $\mathbf{u}$ and the fiber direction. To favor directions that are close to the fiber direction, it is useful to specify a peek function with a controllable steepness. This is achieved through a user-defined power α in the interpolation factor:

$$\bar{\sigma}_u = \bar{\sigma}_T + (\bar{\sigma}_L - \bar{\sigma}_T)\left(1 - \frac{2}{\pi}\cos^{-1}\left(|\mathbf{u}\cdot\mathbf{f}|\right)\right)^{\alpha} \tag{6}$$

With this formulation, the criterion along the fiber and transverse directions match the original conditions given by the anisotropic model.

2.2.2. Evaluation

Locally refining the mesh and lowering the timestep would allow a precise evaluation of the stress at the time of fracture [10]. A single direction will reach the threshold, thus a single candidate will be selected for fracturing. However, if the simulation needs to meet real-time constraints, this solution might prove prohibitive. Therefore, we propose the following approximations. First, in the case of linear materials, the computed stress inside each element does not precisely represent the real material stress, unless very fine meshes are used. To approximate the stress σ at the tip of the fracture, we propose to compute an weighted average of the stress inside each element in the neighborhood of the point of interest. Second, if large timesteps are used, several points and directions can reach the fracture threshold at the end of the timestep. To avoid substepping, which would be incompatible with real-time simulations, we propose the following heuristic: we choose as the fracture direction the direction $\mathbf{d_{fracture}}$ where the criterion $c(\mathbf{d}, \sigma, \mathbf{f}, \mathbf{p})$ is maximum, i.e. $c(\mathbf{d_{fracture}}, \sigma, \mathbf{f}, \mathbf{p}) = \max c(\mathbf{d}, \sigma, \mathbf{f}, \mathbf{p})$. For the special case of an isotropic fracture criterion ($\bar{\sigma}_F = \bar{\sigma}_T$ and $\theta_\sigma = 90^o$), the solution can be computed by eigenvalue decomposition of the stress tensor. Thus, our algorithm is equivalent to the *maximum principal stress criterion* for the special case of isotropic materials. For general anisotropic surfaces, an exhaustive search on all the directions within the triangles connected to the fracture tip can be used, as the evaluation of our criterion can be implemented very efficiently.

2.3. Tearing Propagation

Once the fracture direction has been determined, topological changes need to be applied to the finite element mesh to describe fracture propagation. At this point, there are two possible strategies. In the case where the initial mesh is sufficiently fine, or if the edges are oriented along the fiber or transverse direction, it is possible to restrict the potential fracture direction to an existing edge. This solution is simple to implement, and the simulation cost as well as the quality of the initial mesh is preserved (as no new element is created). However the resulting fracture will be highly dependent on the original mesh. For a precise, mesh-independent, tearing propagation, it is necessary to allow for arbitrary fracture directions. Once the fracture direction $\mathbf{d_{fracture}}$ has been determined, triangles need to be split to create new edges along the fracture direction (see Fig. 2). Particular care must be taken to avoid creating degenerated triangles, as explained in [10], which would otherwise negatively impact the computation of the finite element model.

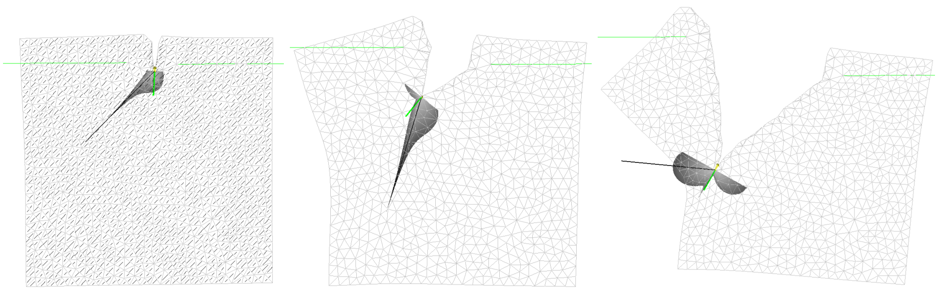

Figure 2. From left to right: tearing propagation throughout the tissue. Fibers are oriented with a 45 degree angle, and forces are applied outward, on the left and right sides of the square mesh. Our fracture criteria is represented as the shaded area. One can see on the left image how the anisotropy affects the deformation of the tissue, and on the rightmost images how the fracture direction depends on the previous fracture direction, as well as fiber direction. The square mesh is composed of 1,500 triangles. Anisotropic deformation, fracture and remeshing are performed in real-time.

3. Results

Figure 2 illustrates different aspects of our approach, and shows the relevance of using an anisotropic model to support realistic soft tissue deformation and tearing. We have also applied this method to the simulation of capsulorhexis, one of the critical steps of cataract surgery. The membrane of the lens capsule is modeled as an anisotropic material and a co-rotational finite element formulation (to allow for large displacements). Concentric fiber orientations are defined on the mesh to describe the actual structure of the lens capsule. An implicit integration scheme is used to enforce robustness of the deformation process. With this approach, realistic deformations can be computed in real-time, and tearing of the membrane can be simulated (see Fig. 3). The results are very encouraging, showing realistic deformations that take into account the anisotropic nature of the tissue, and fracture propagation that depends on the fiber direction.

4. Conclusion

In this paper we have show the important role played by fibers in both the deformation and fracture processes of soft tissues. Fibers are inherent to the structure of most soft tissues, yet are rarely accounted for. Our approach shows that, in the case of surface meshes, real-time computation of the deformation, fracture and remeshing is possible. Future work will include the extension of this method to volumetric meshes.

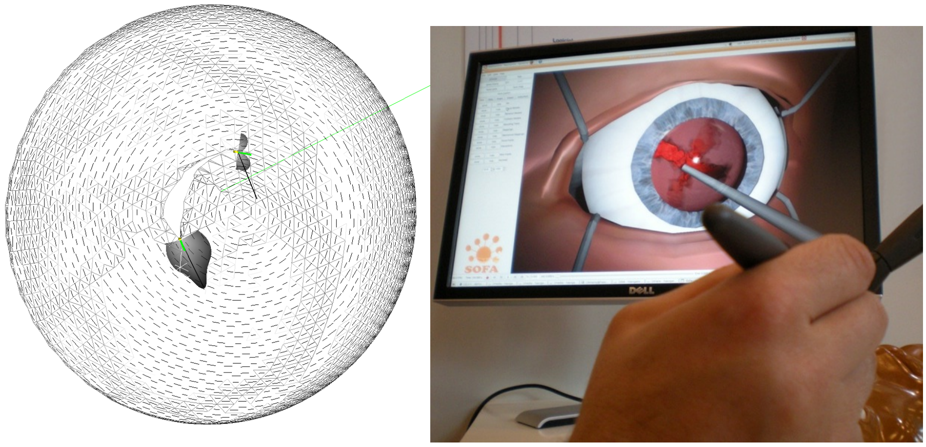

Figure 3. Left: simulation of capsulorhexis, a technique used to create a circular opening in the lens capsule. This technique relies essentially on the application of shear and stretch forces to propagate a fracture throughout the membrane. Right: the capsulorhexis simulation is only a part of a more complete simulation of cataract surgery, comprising two other important steps: phacoemulsification and implantation of an intraocular lens.

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