Abstract—In this paper, the Peak-to-Average-Power-Ratio (PAPR) for multicarrier systems with different pulse shapes is studied. First, the influence of the pulse shaping characteristics on the PAPR is investigated. Then, from the gained insights, an innovative approach is proposed where the waveform is adapted to the environment. To this end, a Wavelet Packet Modulation is introduced and its PAPR is compared with the one of Orthogonal Frequency Domain Multiplexing modulation. Simulation results show that multicarrier systems using the wavelet approach significantly reduces the PAPR for a low number of channels and hence the cost of the transceiver.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is currently the modulation used for high data rate transmission in frequency-selective fading channels [1]. A major drawback of OFDM modulation at the transmitter side is the high Peak-to-Average Power Ratio (PAPR) of the transmitted signal. PAPR reduction techniques [2]-[5] have been proposed to reduce the PAPR problem in OFDM transmitter. Some techniques use coding, in which the data sequence is embedded in a larger sequence and only a subset of all the possible sequences is used to exclude patterns with high PAPR [2]. These coding techniques reduce PAPR, but they also reduce the transmission rate. Recently, multiple signal representation techniques have been proposed. These include the partial transmit sequence technique [3], selected mapping technique [4] and interleaving technique [5]. These techniques require side informations to be transmitted from the transmitter to the receiver to recover the original data block from the received signal.

In this paper, we propose a novel PAPR solution based on the pulse shaping characteristics of multicarrier modulation. We show that the PAPR decreases with an appropriate choice of the pulse shaping.

In order to design a multicarrier modulation with a low PAPR, a given solution is to use wavelet theory. Its application to filter bank and its extension to wavelet packet decomposition allow the construction of orthogonal bases used to modulate the data as a multicarrier system. A multicarrier modulation based on wavelet packet transform is called WPM (Wavelet Packet Modulation).

Wavelet theory applied to multicarrier modulation has been studied in previous works [6]-[9]: it has been shown that the WPM is efficient for wired transmission [6]. In a wireless environment [7][8], performances of WPM depend on the equalization techniques and a study [9] shows that the use of complex wavelets could reduce time and frequency dispersive channel interferences.

Indeed, major improvements of this paper are the study of the link between PAPR and pulse shaping for multicarrier modulation transmitter, and the evaluation of PAPR for the WPM transmitter.

In the following, a PAPR analysis is performed for multicarrier modulation in order to define a selection criterion of the pulse shaping (Sec. II). In Sec. III, wavelet packet modulation is introduced. In Sec. IV, the choice of the proposed criterion and a useful asymptotic function is discussed in function of the number of channels. Simulation results are given in Sec. V to show the performance of the system in different situations. Finally, conclusions from simulations are drawn in Sec. VI.

II. PAPR ANALYSIS IN MULTICARRIER MODULATION

Let $M$ be the number of channels in the multicarrier scheme. The multicarrier modulated signal $s(t)$ results from a linear combination of the pulse shaping functions $\{\psi_{m,n}(t), n \in \mathbb{Z}, m = 0, \ldots, M-1\}$ weighted with the transmitted symbols $x_m[n]$:

$$s(t) = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{M-1} x_m[n] \psi_{m,n}(t). \quad (1)$$

OFDM modulation [1] uses a rectangular pulse shaping of duration $T_s$ and orthogonality is obtained with a carrier spacing $1/T_s$. By noting $\Pi^T_0(t) = \begin{cases} 1 & \text{if } 0 \leq t < T_s \\ 0 & \text{else} \end{cases}$ the rectangular function, $\psi_{m,n}(t)$ is then expressed by:

$$\psi_{m,n}(t) = e^{-j2\pi \frac{m}{T_s}} \Pi^T_0(t-nT_s). \quad (2)$$

In the linear case (when no power amplifier is used) and for large values of $M$, the OFDM spectrum goes to an ideal bandlimited rectangular spectrum. This means that the OFDM signal within each block appears as Gaussian with very high variations from one sample to the next. Thus, the power spectral density of the modulated signal will be broadened by the nonlinear distortions of a High Power Amplifier (HPA).

The PAPR is one way to measure such variation of the transmitted signal. The PAPR is defined as [10]:

$$PAPR = \max \left\{ \frac{|s(t)|^2}{E_s} \right\}, \quad (3)$$

with $E_s = E \left\{ |s(t)|^2 \right\}$ where $E \{.\}$ denotes the statistical average operator.
In order to concentrate the study on the effect of $\psi_{m,n}(t)$ on the PAPR, we only choose multicarrier modulation with $E_s = 1$ and we limit the study to 4-QAM symbols with constant envelope. We also consider that the same pulse shaping $\psi(t)$ is used on every subcarrier, i.e. $\{\psi_{m,n}(t) = \psi(t-nT_s)e^{-j2\pi mm^2n2\pi nT_s}, m=0,\ldots,M-1\}$. In this case, we can write the theorem below:

**Theorem 1:** For a multicarrier modulation with $M$ channels that uses a modulated pulse shaping $\psi(t)$, the PAPR of the transmitted signal is upper bound by

$$PAPR \leq M \max_{0 \leq t \leq T_s} |\psi(t)|^2,$$

(4)

and $PAPR \leq M$ when $\psi(t)$ is a rectangular pulse shaping used by OFDM modulation.

For the use in multicarrier modulation, we make use of the pulse shaping characteristics and we postulate for low PAPR transmission:

$$\max_{0 \leq t \leq T_s} |\psi(t)|^2 < 1.$$  

(5)

**Proof:** Assuming a 4-QAM multicarrier system with $E_s = 1$, Eq. 3 becomes:

$$PAPR \leq PAPR_{\text{MAX}} = \frac{1}{M} \max_{0 \leq t \leq T_s} \left( \sum_{m=0}^{M-1} |\psi_{m,n}(t)|^2 \right)^{\frac{1}{2}},$$

(6)

If the same pulse shaping is used on every subcarrier, the maximum PAPR becomes:

$$PAPR_{\text{MAX}} = \frac{1}{M} \max_{0 \leq t \leq T_s} \left( \sum_{m=0}^{M-1} |\psi(t-nT_s)e^{-j2\pi mm^2n2\pi nT_s}|^2 \right)^{\frac{1}{2}},$$

(7)

$$= \frac{1}{M} \max_{0 \leq t \leq T_s} \left( \sum_{m=0}^{M-1} |\psi(t)|^2 \right)^{\frac{1}{2}},$$

(8)

$$= \frac{1}{M} \max_{0 \leq t \leq T_s} (M|\psi(t)|)^{\frac{1}{2}},$$

(9)

$$= M \max_{0 \leq t \leq T_s} |\psi(t)|^2.$$  

(10)

Eq. (10) is the upper bound defined in Eq. 4.

According to theorem 1, the proposed solution is to use pulse shaping that reduces $\max|\psi(t)|^2$ without decreasing $E_s$. In this paper, we propose to use the WPM to reduce the PAPR. Indeed, this modulation allows a large choice of pulse shaping $\psi(t)$ by choosing different wavelets.

**III. WPM SYSTEM DESCRIPTION**

The idea of wavelet packet modulation [6] is to use wavelet packets as the pulse shaping functions. 

Wavelet packets: The concept of wavelet transform has been extended by establishing the theory for libraries of orthonormal bases which were obtained by filling out the binary tree to some uniform depth as shown in Fig. 1. This decomposition allows a uniform analysis of the spectrum. The obtained functions are wavelet packets, which are recursively defined by:

$$p^{2m}(t) = \sqrt{2} \sum_{n} h_n p^m(2t-n),$$

(11)

$$p^{2m+1}(t) = \sqrt{2} \sum_{n} g_n p^m(2t-n).$$

(12)

$h_n$ and $g_n$ are a quadrature mirror filter (QMF) pair. $h_n$ is a lowpass filter while $g_n$ is a highpass filter. They are connected by the relation $g_n = (-1)^n h_{1-n}$.

![Uniform wavelet packet decomposition.](image)

**Definition 1 [11]:** A wavelet packet base of $L^2(\mathbb{R})$ is given by all orthonormal bases chosen among the functions:

$$\{p_{l,n}^m(t) = 2^\frac{m}{2} p^m(2^l t - n), (l,n) \in (\mathbb{Z}, \mathbb{Z}), m \in \mathbb{N}\}.$$  

(13)

Therefore, any function $f(t)$ of $L^2(\mathbb{R})$ can be decomposed on the base $\{p_{l,n}^m(t), (l,n) \in (\mathbb{Z}, \mathbb{Z})\}$:

$$s(t) = \sum_{m,n} a_{l,n}^m p_{l,n}^m(t).$$

(14)

All these coefficients $a_{l,n}^m$, constitute the DWPT (Discrete Wavelet Packet Transform) of $s(t)$ and the inverse transform is called IDWPT (Inverse Discrete Wavelet Packet Transform).

Wavelet Packet Modulation: From eq. (14), we can see that any function $s(t)$ of $L^2(\mathbb{R})$ can be expressed as the sum of weighted wavelet packets. In communication systems, this means that a signal can be seen as the sum of modulated wavelet packets, which gives the idea of wavelet packet modulation [6]: the transmitter transforms the symbols from the wavelet domain to the time domain with an IDWPT and the receiver transforms the received signal from the time domain to the wavelet domain with a DWPT.

By choosing $M = 2^L$, $\psi_{m,n}(t) = p_{l,n}^m(t)$ and $y_m[n] = a_{l,n}^m$, the tree shown in Fig. 1 represents an example of WPM demodulation for a $M=8$ subcarriers system. The orthogonality of wavelet packets gives a perfect reconstruction system for an ideal transmission without interferences.

**Complexity:** In [8], we compute the complexity $C_{WPM}$ of WPM. Based on [12], we show that the number of MAC
(Multiplication and ACcumulation) needed to compute one WPM symbol is
\[ C_{WPM} = LM \log_2 M, \quad (15) \]
where \( L \) denotes the number of coefficients of filters \( h \) and \( g \). We note that the complexity of WPM increases with the order of the wavelet but it varies in the same way as OFDM modulation according to the number of carriers \( M \).

**PAPR illustration:** WPM allows flexibility in the pulse shaping design and the choice depends on the transmission environment. In this study, we use Daubechies wavelets which give significant results on PAPR reduction. They are parameterized by their order. The higher the wavelet’s order is, the higher the time dispersion but the computation complexity will be higher (filters length \( L \) increase with the order). Coefficients \( h_n \) have been published by I. Daubechies in [13].

The idea that WPM gives lower PAPR than OFDM modulation is illustrated by Fig. 2. It shows time representations of an OFDM signal and a WPM signal using \( 6^{th} \) Daubechies wavelet. We can see that the maximum peak of WPM signal is lower than the one of OFDM signal.

\[ \text{Fig. 2. Time vision of wavelet's peak reduction.} \]

To limit the PAPR, one method is the clipping [14]. A clipping level is set on Fig. 2 at \(|\text{Amplitude}| = 1.5\). We can see that the number of peaks above this reference is higher for OFDM modulation than for WPM. For the same clipping level, WPM should be less sensitive to nonlinear HPA than OFDM modulation.

**IV. ASYMPTOTIC DISTRIBUTION OF THE PAPR**

In II, PAPR is evaluated based on the maximum peak power as shown in eq. (4). However, for the hardware implementation, the evaluation of the power distribution is more important than only the maximum power evaluation. In this part, we present a study of asymptotic PAPR distribution that stands for both OFDM and WPM modulation.

In both OFDM and WPM systems, the signal \( s(t) \) going into the channel is a sum of random symbols modulating orthogonal basis functions. From the central limit theorem, it is claimed [15] that for a large value of channels \( M \), the multicarrier signal can be modelled as a zero-mean Gaussian distributed random variable with variance \( \sigma_s^2 \). Introducing the variable \( y = |s|^2 \), we obtain the central chi-square distribution [16]:
\[ p_y(y) = \frac{1}{\sqrt{2\pi y\sigma_s^2}} e^{-\frac{y}{2\sigma_s^2}} \quad (16) \]

The complementary cumulative distribution function (CCDF) of the PAPR is defined as:
\[ \text{CCDF}(PAPR_0) = \text{Prob}[\text{PAPR} > PAPR_0]. \quad (17) \]
Using the result in (16), so, the CCDF of the PAPR can be calculated as:
\[ \text{CCDF}(PAPR_0) = 1 - (1 - e^{-PAPR_0})^N. \quad (18) \]

The distributions obtained by the conventional analysis, however, does not fit those of the PAPR of the OFDM signals obtained by computer simulations, even for very large \( M \). In [15], Van Nee and Prasad gave an empirical approximation:
\[ \text{CCDF}(PAPR_0) = 1 - (1 - e^{-PAPR_0})^{\alpha N}, \quad (19) \]
where \( \alpha \) is a parameter determined by computer simulation to be 2.8. It should be noted that this approximation lacks theoretical justification.

Now, we evaluate the values of \( M \) where the pulse shaping criteria defined in II can be used. Fig. 3 shows the PAPR versus different amounts of channels \( M \) for OFDM modulation and WPM using the \( 6^{th} \) Daubechies wavelet. The theoretical distribution function is also plotted. It is shown that for large values of \( M \), OFDM and WPM have quite the same PAPR and the theoretical cumulative distribution function is accurate. However, for low values of \( M \), WPM outperforms OFDM modulation.

\[ \text{Fig. 3. PAPR as a function of the number of channels } M. \]

We conclude that Theorem 1 is right for low values of \( M \) whereas theoretical function defined by (19) is right for high
values. In the following, to evaluate the influence of WPM on the PAPR, we choose \( M = 128 \) subcarriers. In this case, Fig. 4 shows a 0.6 dB difference between OFDM and WPM using the 6\(^{th}\) Daubechies wavelet.

V. SIMULATION RESULTS

We use computer simulations to evaluate the performance of the proposed PAPR reduction technique. As a performance measure for the proposed technique, we use the CCDF of the PAPR and Bit Error Rate (BER) with a nonlinear power amplifier. Performances of the proposed system are compared to OFDM modulation for a multicarrier systems with 4-QAM symbols modulated on \( M=128 \) subcarriers. Fig. 4 shows the CCDF of the proposed technique for different Daubechies wavelets. It is shown that the OFDM signal has a PAPR which exceeds 11.3 dB for less than 0.1\% of the OFDM data blocks. The 0.1\% PAPR of the proposed technique is 11.1 dB when 3\(^{rd}\) Daubechies wavelet is used and 10.7 dB when 6\(^{th}\) Daubechies wavelet is used, resulting 0.2 dB and 0.6 dB reduction in 0.1\% PAPR respectively.

A 5.1 dB reduction is reached with the 20\(^{th}\) Daubechies wavelet; this result is very interesting but, according to (15), the complexity of this system is higher than the complexity of OFDM modulation.

Now, we evaluate the influence on the BER. To approximate the effect of nonlinear HPA, we adopt Rapp’s model for amplitude conversion [15]. The relation between amplitude of the normalized input signal \( s(t) \) and amplitude of the normalized output signal \( g(s) \) of the nonlinear power amplifier is given by:

\[
g(s) = \frac{s}{\left(1 + s^{2p}\right)^{1/(2p)}},
\]  

where \( p \) is a parameter that represents the nonlinear characteristic of the HPA. The power amplifier approaches linear amplifier as \( p \) gets larger. We choose \( p = 3 \), which is a good approximation of a general power amplifier [15]. The phase conversion of the power amplifier is neglected in this paper. Fig. 5 shows BER curve for OFDM modulation and WPM using the 3\(^{rd}\) and the 20\(^{th}\) Daubechies wavelets. They are compared to the BER curve of a 4-QAM modulation without distortion. This figure shows that WPM is less sensitive to the nonlinear HPA than OFDM modulation. Using the 3\(^{rd}\) Daubechies wavelet increases the SNR of 1 dB at \( BER = 10^{-3} \) compared to OFDM modulation. WPM using the 20\(^{th}\) Daubechies wavelet is very close to the ideal case with no distortion.

PAPR reduction capability and the BER of the proposed technique depend on the choice of the wavelet. Specifically, the amount of PAPR reduction increases as the wavelet’s order gets larger. We can achieve low BER with the 20\(^{th}\) Daubechies wavelet.

VI. CONCLUSIONS

In this paper, a novel proposition based on the characteristics of the pulse shaping has been introduced to reduce PAPR in multicarrier schemes. Applied to wavelets, this proposal leads to the multicarrier modulation based on wavelet pulse shaping. The proposed modulation is compared to OFDM modulation and achieves a significant reduction in PAPR for a low number of channels. Furthermore, it is possible to trade-off the amount of PAPR reduction and the complexity by using different versions of wavelet used in WPM. An important advantage of proposed technique is that wavelets allow flexibility in the system’s design and their choice depends on the transmission environment. We achieve an efficient reconfigurable multicarrier modulation.

REFERENCES


