What can be Computed in a Distributed System?

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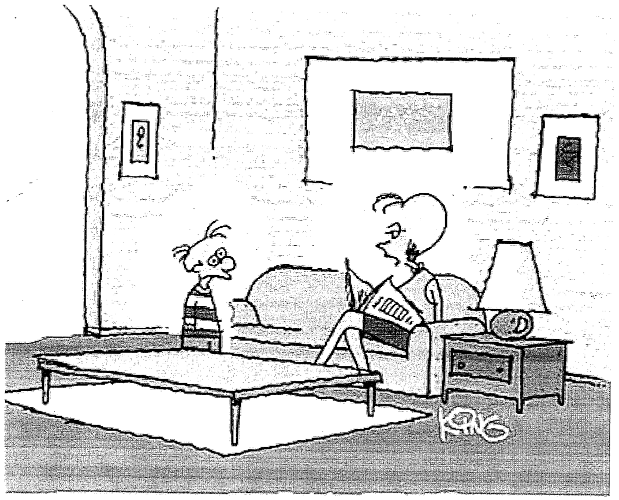
Never forget ....



"No, you weren't downloaded. Your were born."

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On distributed computing

God made the bits,
all else is the work of man

God made the integers, all else is the work of man (L. Kronecker)

Distributed computing

- A birth certificate: *Time, clocks and the ordering of events in a distributed system*, Leslie Lamport, CACM 1978
- DC arises when one has to solve a problem in terms of entities (processes, agents, sensors, peers, actors, nodes, processors, ...) such that each entity has only a partial knowledge of the many parameters involved in the problem that has to be solved

What is distributed computing about?

- Real-time: masters On-time computing
- Parallelism: provides Efficiency
- Distributed computing:

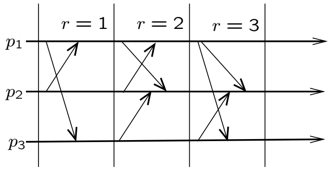
  masters Uncertainty

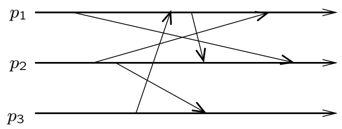
Distributed system

- n sequential deterministic processes: p_1, ..., p_n
- Communication:
  - ⋆ message-passing (send/receive) or
  - ⋆ read/write atomic registers
- Deterministic: the behavior of a process is entirely determined by its initial state, its algorithm, and
  - ⋆ the sequence of values read from atomic registers or
  - ⋆ the sequence of messages it receives

Synchronous vs asynchronous system

- Asynchronous:
  process speed and message transfer delays: arbitrary
- Synchronous
  - ⋆ Round-based computation
  - ⋆ A round is made up of three phases: send, receive, local computation
  - ⋆ A message sent during a round is received during the very same round





Process failure model

- Crash: unexpected halt
- t-resilient model
  Model parameter t= max # processes that may crash
- Wait-free model: $t = n - 1$

- Herlihy M.P., Wait-free synchronization. *ACM Transactions on Programming Languages and Systems*, 13(1):124-149, 1991

Textbooks:

- Attiya H. and Welch J.L., *Distributed computing: fundamentals, simulations and advanced topics*, Wiley-Interscience, 414 pages, 2004

- Lynch N.A., *Distributed algorithms*. Morgan Kaufmann, 872 pages, 1996.

- Raynal M., *Concurrent programming: algorithms, principles, and foundations*. Springer, 530 pages, 2013

Notion of an environment in DC

- Environment: set of failures and (a)synchrony patterns in which the system may evolve
- The system does not master its environment, it only suffers it
- This is a fundamental difference with sequential (or parallel) computing

Computability and complexity in DC

- Computability and complexity are the two lenses that allows us to understand and master computing
- In DC we have the following:

	Synchronous	Asynchronous
Failure-free	complexity	complexity
Crash-prone	complexity	computability

A (the?) fundamental issue

Are asynchronous crash-prone distributed systems universal?

On computability and universal constructions (1)

- In sequential computing, computability is understood through the Church-Turing's thesis (anything that can be computed, can computed by a Turing machine)
- The notion of Computability is intimately related to the notion of universality
- A fundamental issue of DC:

  Is it possible to design a universal construction (algorithm) on top of an asynchronous distributed system prone to crash failure?

On computability and universal constructions (2)

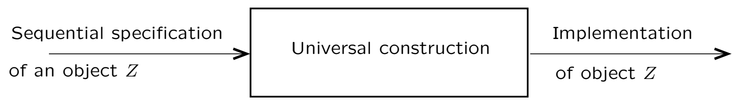
- Due to the environment (asynchrony and failures), distributed computability has a different flavor than sequential computability
- Moreover, this is independent of the fact that communication is by message-passing or read/write registers
- A famous quote of Leslie Lamport on distributed computing:

  A distributed system is one in which the failure of a computer you didn't even know existed can render your own computer unusable
- It follows that the limits of distributed computability reflect the difficulty of making decisions in the face of uncertainty, and has little to do with the computational power of each participant

-Herlihy M.P., Rajsbaum S., and Raynal M., Power and limits of distributed computing shared memory models. *Theoretical Computer Science*, 509:3-24, 2013

A universality notion for distributed computing

- Let Z be any concurrent object, which can be defined by a sequential specification on total operations
- From a practical point of view: Z is a service that we want to make reliable in the presence of failures
- The universality notion in which we are interested concerns the possibility to implement any such object Z despite asynchrony and any number of process crashes



- Herlihy M.P., Wait-free synchronization. *ACM Transactions on Programming Languages and Systems*, 13(1):124-149, 1991

Lamport L., Time, clocks, and the ordering of events in a distributed system. *Communications of the ACM*, 21(7):558–565, 1978

On the progress conditions of object Z

- The wait-freedom progress condition states that: An invocation of an operation on Z can fail to terminate only if the invoking process crashes

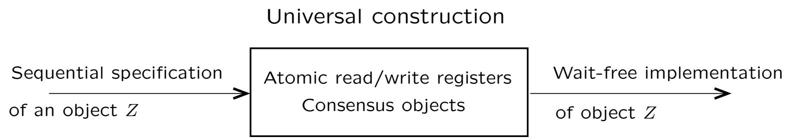
  Corresponding implementations are said "wait-free"
- A wait-free implementation prevents the use of locks!
- Non-blocking and obstruction-freedom are progress conditions weaker than wait-freedom

- Herlihy M.P., Luchangco V., and Moir M., Obstruction-free synchronization: double-ended queues as an example. *Proc. 23th Int'l IEEE Conference on Distributed Computing Systems (ICDCS'03)*, IEEE Press, pp. 522-529, 2003

- Herlihy M.P. and Wing J.M, Linearizability: a correctness condition for concurrent objects. *ACM Transactions on Programming Languages and Systems*, 12(3):463-492, 1990

Consensus-based universal constructions

- The consensus object is universal in the sense it allows the design of wait-free implementations of any object Z defined by a sequential specification



- Herlihy M.P., Wait-free synchronization. *ACM Transactions on Programming Languages and Systems*, 13(1):124-149, 1991

Consensus object

- A consensus object is a one-shot concurrent object that provides processes with a single operation denoted propose(v) where v is an input parameter (called "proposed value")
- A consensus object is defined by the following properties
  - ⋆ Validity. If a process decides a value, this value has been proposed by a process
  - ⋆ Agreement. No two processes decide different values
  - ⋆ Termination. An invocation of propose() by a process that does not crash terminates
- Consensus objects allow the processes to agree on the same sequence of operations applied to Z, despite any concurrency, asynchrony, and failure pattern

A fundamental result of distributed computability

- There is no deterministic algorithm that wait-free implements a consensus object
  - ⋆ Whatever the number of processes $n \geq 2$
  - ⋆ Whatever the communication medium (read/write registers or message-passing)
  - ⋆ Even if a single process may crash
  - ⋆ Even if processes have to agree on a single bit!

- Fischer M.J., Lynch N.A., and Paterson M.S., Impossibility of distributed consensus with one faulty process. *Journal of the ACM*, 32(2):374-382, 1985

- Loui M. and Abu-Amara H., Memory requirements for agreement among unreliable asynchronous processes. *Advances in Comp. Research*, 4:163-183, JAI Press, 1987

Underlying intuition with binary consensus (1)

- Let a global state be 0-valent if only 0 can be decided from this state
- Let a global state be 1-valent if only 1 can be decided from this state
- Univalent state: 0-valent or 1-valent
- Bivalent state: any of 0 or 1 can still be decided "the dice are not yet cast"
- Decision step: carries the construction from a bivalent state to a univalent state

- Fischer M.J., Lynch N.A., and Paterson M.S., Impossibility of distributed consensus with one faulty process. *Journal of the ACM*, 32(2):374-382, 1985

Underlying intuition with binary consensus (2)

- The impossibility theorem (FLP) is by contradiction
- It assumes there is an algorithm and shows that
  - ⋆ There is at least one initial bivalent state
  - ⋆ Among all possible executions there is at least one that makes the algorithm to always progress from a bivalent state to another bivalent state
- This shows that it is NOT always possible to break the non-determinism created by the environment

Sequential vs distributed computability

- A network of asynchronous Turing machines where even only one may crash, connected by a message-passing facility, or a read/write shared memory, is computationally less powerful than a single reliable Turing machine
- The nature of distributed computability issues is different from the nature of Turing's computability issues, namely, it is not related to the computational power of the individual participants

Enriching read/write systems with stronger objects

- Consensus number of an object X = largest n for which consensus can be be wait-free implemented in a read/write system of n processes enriched with objects X
  If there is no largest n, the consensus number is $+\infty$
- Herlihy's hierarchy:
  - ⋆ Consensus number 1: read/write atomic registers, ...
  - ⋆ Consensus number 2: test&set, swap, fetch&add, stack, queue, ...
  - ⋆ Consensus number $+\infty$: compare&swap, LL/SC, mem-to-mem swap, ...
- Any object with consensus number n is universal in a system of $\leq n$ processes

- Herlihy M.P., Wait-free synchronization. *ACM Transactions on Programming Languages and Systems*, 13(1):124-149, 1991

From read/write to message-passing systems

- Whatever the environment, it is possible to simulate message-passing on top of read/write
- It is impossible to simulate read/write on top of message-passing when $t \geq n/2$ (ABD impossibility)
- Intuition: indistinguishability argument
- A variant: CAP theorem
  - ⋆ CAP = Consistency, Availability, Partition-tolerance
  - ⋆ States that, when designing distributed services, it is impossible to design an algorithm that simultaneously ensures the three previous properties
  - ⋆ Impossibility variant of FLP + ABD

- Attiya H., Bar-Noy A. and Dolev D., Sharing memory robustly in message passing systems. *Journal of the ACM*, 42(1):121-132, 1995

- Brewer E.A., Pushing the CAP: strategies for consistency and availability. *IEEE Computer*, 45(2):23-29, 2012

Playing with progress conditions and consensus objects

- Obstruction-freedom: an invocation of an operation on an object is guaranteed to terminate when it executes alone for a "long enough period" (whatever the points at which the other invocations stopped)
- (y, x)-*liveness* if the object can be accessed by a subset of $y \leq n$ processes only, and wait-freedom is guaranteed for $x \leq y$ processes while obstruction-freedom is guaranteed for the remaining $y - x$ processes
- Impossibility to build an $(n, 1)$-live consensus object from read/write atomic registers and $(n-1, n-1)$-live consensus objects
- Another hierarchy: any (n, x)-live consensus object with $x < n$ has consensus number $x + 1$

- Imbs D., Raynal M., and Taubenfeld G., On asymmetric progress conditions. *Proc. 29rd ACM Symposium on Principles of Distributed Computing (PODC'10)*, ACM Press, pp. 55-64, 2010

How to circumvent consensus impossibility

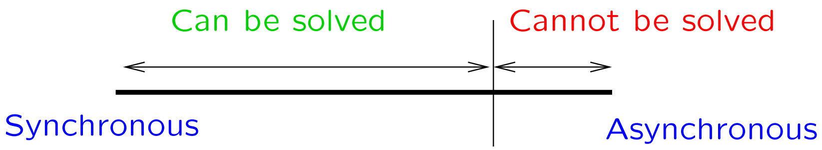
Remark:
No notion of objects with consensus number in MP systems

Three approaches

- Add an oracle
  (which provides additional computational power)
  - ⋆ Failure detectors
  - ⋆ Randomization
- Restrict the set of input vectors

The failure detector approach

- Given a problem: Find the weakest "assumptions" that has to added to an asynchronous system in order problems can be solved



Failure detectors

- Provide each process with a read-only local variable giving (possibly unreliable) information on failures
- Given a problem (object), give as few information as possible while allowing the object to be implemented
- According to the information on failures that is given, several "classes" of failure detectors can be defined

- Chandra T. and Toueg S., Unreliable failure detectors for reliable distributed systems. *Journal of the ACM*, 43(2):225-267, 1996

- Raynal M., *Communication and agreement abstractions for fault-tolerant asynchronous distributed systems.* Morgan & Claypool Pub., 251 pages, 2010

The weakest failure detector to solve consensus

- Ω: provides each process p_i with a read-only local variable $leader_i$ such that, after an unknown but finite time, the variables $leader_i$ of the non-crashed processes contain forever the same process identity of a non-crashed process
- Ω: weakest FD that allows consensus to be solved

- Chandra T.D., Hadzilacos V. and Toueg S., The Weakest Failure Detector for Solving Consensus. *Journal of the ACM*, 43(4):685-722, 1996

- Fernández A., Jiménez E., Raynal M., and Trédan G., A timing assumption and two t-resilient protocols for implementing an eventual leader service in asynchronous shared-memory systems. *Algorithmica*, 56(4):550-576, 2010

The notion of an indulgent distributed algorithm

- A distributed algorithm is indulgent with respect to a failure detector FD it uses to solve a problem Pb if
  - ⋆ it always guarantees the safety property defining Pb (i.e., whatever the correct/incorrect behavior of FD),
  - ⋆ and satisfies the liveness property associated with Pb at least when FD behaves correctly
- Hence, when the implementation of FD does not satisfies its specification, the algorithm may not terminate, but if it terminates its results are correct
- All Ω-based algorithms are indulgent
- Notions of stable vs unstable periods

- Guerraoui R., Indulgent algorithms. *Proc. 19th ACM Symposium on Principles of Distributed Computing (PODC'00)*, ACM Press, pp. 289-298, 2000

Randomization

- A classical way to break non-determinism
- Asynchronous round-based algorithms
- Requires to modify the termination property which becomes:

  The probability that a non-faulty process has decided by round r tends to 1, when the number of rounds tends to $+\infty$
- Notion of expected number of rounds to decide

- Ben-Or M., Another advantage of free choice: completely asynchronous agreement protocol. *Proc. 2d ACM Symposium on Principles of Distributed Computing (PODC'83)*, ACM Press, pp. 27-30, 1983

Restrict the set of input vectors

- Intuitively consider that an input vector "encodes" the value that has to be decided
- The consensus algorithm has to "decode" it
- To be possible the input vector has to satisfies some properties

- Friedman R., Mostéfaoui A., Rajsbaum S., and Raynal M., Asynchronous agreement and its relation with error-correcting codes. *IEEE Transactions on Computers*, 56(7):865-875, 2007

- Mostéfaoui A., Rajsbaum S., and Raynal M., Conditions on input vectors for consensus solvability in asynchronous distributed systems. *Journal of the ACM*, 50(6): 922-954, 2003

Examples of objects which can be solved in the basic read/write system

Snapshot object

- A snapshot object is an array of registers $A[1..n]$, where $A[i]$ can be written only by p_i
- It provides the processes with two operations
  - ⋆ $\forall i$: a write that allows p_i to write (only) in $A[i]$
  - ⋆ snapshot() can be invoked by any process and returns the value of the whole array
- These operations are atomic: each appears as if it been executed instantaneously at some point of the time line, between its start and its end events

- Afek Y., Attiya H., Dolev D., Gafni E., Merritt M., and Shavit N., Atomic snapshots of shared memory. *Journal of the ACM*, 40(4):873-890, 1993

One-shot M-renaming object (1)

- Each process p_i has an identity id_i taken from a large name space, whose size is N
- Initially a process knows only n and its initial identity id_i
- The aim is to allow processes to obtain new names in a new name space of size $M << N$
- The object provides processes with a single operation denoted `new_name`(id) where the input parameter is the identity of the invoking process; `new_name`$()$ returns a new name to the invoking process

- Attiya H., Bar-Noy A., Dolev D., Peleg D., and Reischuk R., Renaming in an asynchronous environment. *Journal of the ACM*, 37(3):524-548, 1990

One-shot M-renaming object (2)

- Validity. A new name is an integer in the set $[1..M]$.
- Agreement.
  No two processes obtain the same new name.
- Termination. If a process invokes `new_name`$()$ and does not crash, it eventually obtains a new name.

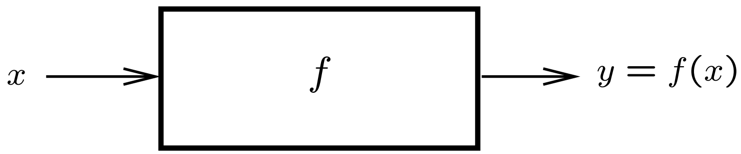
Let p be the number of processes that invoke `new_name`$()$.
$M = 2p - 1$ is a lower bound on the new name space

- Castañeda A. and Rajsbaum S., New combinatorial topology bounds for renaming: The upper bound. *Journal of the ACM*, 59(1), Article 3, 49 pages, 2012

- Herlihy M. and Shavit N., The topological structure of asynchronous computability. *Journal of the ACM*, 46(6):858-923, 1999

Sequential computing

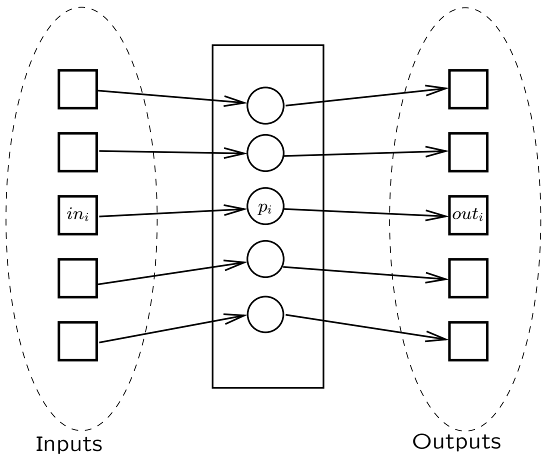
- Power and limit of sequential computing
- Central notion of a function: $y = f(x)$



- Notion of a computable function
- Several formalisms:
  Turing machine,
  Post system,
  Church's lambda calculus, etc.

The notion of a task in DC (1)

The DC counterpart of a function



The notion of a task in DC (2)

- A task T is a triple $(\mathcal{I}, \mathcal{O}, \Delta)$
  - $\star$ $\mathcal{I}$: set of input vectors (of size n)
  - $\star$ $\mathcal{O}$: set of output vectors (of size n)
  - $\star$ Δ: relation from $\mathcal{I}$ into $\mathcal{O}$: $\forall I \in \mathcal{I}: \Delta(I) \subseteq \mathcal{O}$
- $I[i]$: private input of p_i
- $O[i]$: private output of p_i
- $\forall I \in \mathcal{I}$:
  $\Delta(I) = \{$ output vectors that can be decided from I $\}$

Fundamental issues/results in asynchronous DC (1)

- Impossibility for a given process p_i to know if another process p_j has crashed or is only very slow (generates a lot of impossibilities)
- Due to the net effect of asynchrony and crashes the DC model is "weaker" than a Turing machine!
- There are Turing-computable functions that are not computable even in the presence of a single failure

  A lot of tasks cannot be solved in asynchronous crash-prone distributed systems while they can in a reliable distributed system

Fundamental issues/results in asynchronous DC (1)

- The question whether a task is 1-resilient computable can be reduced to a question of graph connectivity
- The question whether a task is computable in the presence of more failures:

  reducible to the question whether an associated geometric structure (called simplicial complex) has higher dimensional "holes", which is known to be undecidable
- Similar to oracles of classic computability, there are tasks which are computable only when given access to a distributed oracle for other tasks (leading to infinite hierarchies of tasks)

M. Herlihy, S. Rajsbaum, and M. Raynal, Power and limits of distributed computing shared memory models. *Theoretical Computer Science*, 509: 3-24 (2013)

A glance at synchronous systems

Complexity issues: gentle reminder

	Synchronous	Asynchronous
Failure-free	complexity	complexity
Crash-prone	complexity	computability

Asynchronous or synchronous failure-free systems

- Power = Turing machine
- Main issue: find the best solutions
- Example (cf. sorting pb)
  - ⋆ Leader election on a non-anonymous uni or bi-directional ring
  - ⋆ Message complexity: $O(n \log n)$
  - ⋆ Time complexity $O(\log n)$

- Dolev D., Klawe M., and Rodeh M., An $O(n \log n)$ unidirectional distributed algorithm for extrema finding in a circle. *Journal of Algorithms*, 3:245–260, 1982

- Higham L. and Przytycka T., A simple efficient algorithm for maximum finding on rings. *Information Processing Letters*, 58(6):319–324, 1996

Failure-prone synchronous systems

- Computability/Complexity results are similar to sequential computing
- Example: consensus problem:

Process failure model	Upper bound on t
crash failure	$t < n$
send omission failure	$t < n$
general omission failure	$t < n/2$
Byzantine failure	$t < n/3$

- In all cases: Lower bound on the number of rounds that the processes have to execute is $t + 1$

- Raynal M., *Fault-tolerant agreement in synchronous distributed systems.* Morgan & Claypool, 167 pages, 2010

Crash-prone synchr. systems with message adversaries

- Fully connected notwork
- Round-based computation
- A message adversary is a daemon which, at every round, is allowed to suppress messages
- No process knows in advance which are the links on which messages are suppressed during a round
- First introduced under the name mobile fault

- Santoro N. and Widmayer P., Time is not a healer. *Proc. 6th Annual Symposium on Theoretical Aspects of Computer Science (STACS'89)*, Springer LNCS 349, pp. 304-316, 1989.

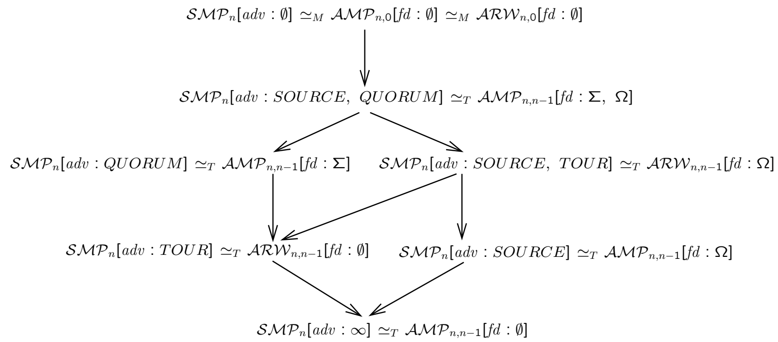
- Santoro N. and Widmayer P., Agreement in synchronous networks with ubiquitous faults. *Theoretical Computer Science*, 384(2-3): 232-249, 2007

The adversary TOUR

- The adversary TOUR is such that, in each round, and for each pair of processes (p_i, p_j), the adversary is allowed to suppress
  - ⋆ the message sent by p_i to p_j or the message sent by p_j to p_i
  - ⋆ but not both
- The synchronous message-passing model weakened by the message adversary TOUR and the asynchronous crash-prone read/write system model have the same computational power for distributed tasks

-Afek Y. and Gafni E., Asynchrony from synchrony. *Proc. Int'l Conference on Distributed Computing and Networking (ICDCN'13)*, Springer LNCS 7730, pp. 225-239, 2013

Message adversaries: a more global picture



- Raynal M. and Stainer J., Round-based synchrony weakened by message adversaries *vs* asynchrony enriched with failure detectors. *Proc. 33rd ACM Symposium on Principles of Distributed Computing (PODC '13)*, ACM Press, pp. 166-175, 2013

Conclusion

- The aim was to understand the power, subtleties and limits of crash-prone asynchronous distributed computing models
- A "Holy Grail" quest: have a view as clear as what we have in sequential computing wrt to computability, complexity, and languages hierarchy

The only slide to remember

Asynchrony and failures do modify

- Our view of synchronization
- The way synchronization has to be solved

The "other" only slide to remember!

Is there an end to the story?

Colorin colorado,

este cuento no se ha acabado ...

A few books on the topic

- Attiya H. and Welch J.L., *Distributed computing: fundamentals, simulations and advanced topics*,
Wiley-Interscience, 414 pages, 2004

- Herlihy M. and Shavit N., *The art of multiprocessor programming*.
Morgan Kaufmann, 508 pages, 2008

- Lynch N.A., *Distributed algorithms*. Morgan Kaufmann, 872 pages, 1996

- Raynal M., *Communication and agreement abstractions for fault-tolerant asynchronous distributed systems*.
Morgan & Claypool Pub., 251 pages, 2010

- Raynal M., *Fault-tolerant agreement in synchronous message-passing systems*.
Morgan & Claypool Publishers, 165 pages, 2010

- Raynal M., *Concurrent programming: algorithms, principles, and foundations*.
Springer, 530 pages, 2013

- Raynal M., *Distributed algorithms for message-passing systems*.
Springer, 515 pages, 2013

- Taubenfeld G., *Synchronization algorithms and concurrent programming*.
Pearson Education/Prentice Hall, 423 pages, 2006

Books