QUAIL: A Quantitative Security Analyzer for Imperative Code*

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Abstract. Quantitative security analysis evaluates and compares how effectively a system protects its secret data. We introduce QUAIL, the first tool able to perform an arbitrary-precision quantitative analysis of the security of a system depending on private information. QUAIL builds a Markov Chain model of the system's behavior as observed by an attacker, and computes the correlation between the system's observable output and the behavior depending on the private information, obtaining the expected amount of bits of the secret that the attacker will infer by observing the system. QUAIL is able to evaluate the safety of randomized protocols depending on secret data, allowing to verify a security protocol's effectiveness. We experiment with a few examples and show that QUAIL's security analysis is more accurate and revealing than results of other tools.

1 Introduction

The Challenge. Qualitative analysis tools can verify the complete security of a protocol, i.e. that an attacker is unable to get any information on a secret by observing the system—a property known as *non-interference*. Non-interference holds when the system's output is independent from the value of the secret, so no information about the latter can be inferred from the former [20]. However, when non-interference does not hold, qualitative analysis cannot rank the security of a system: all unsafe systems are the same.

Quantitative analysis can be used to decide which of two alternative protocols is more secure. It can also asses security of systems that are insecure, but nevertheless useful, in the qualitative sense, such as a password authentication protocol, for which there is always a positive probability that an attacker will randomly guess the password. A quantitative analysis is challenging because it is not sufficient to find a counterexample to a specification to terminate. We need to analyze all possible behaviors of the system and quantify for each one the probability that it will happen and how much of the protocol's secret will be revealed. So far no tool was able to perform this analysis precisely.

Quantitative analysis with QUAIL. We use Quantified Information Flow to reduce the comparison of security of two systems to a computation of expected amount of information, in the information-theoretical sense, that an attacker would learn about the secret by observing a system's behavior. This expected amount of information is known as *information leakage* [9,15,8,12,21] of a system. It amounts to zero iff the system is

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non-interfering [15], else it represents the expected number of bits of the secret that the attacker is able to infer. The analysis generalizes naturally to more than two systems, hence allowing to decide which of them is less of a threat to the secrecy of the data.

To compute information leakage we use a stochastic model of a system as observed by the attacker. The model is obtained by resolving non-determinism in the system code, using the prior probability distribution over the secret values known to the attacker before an attack. Existing techniques represent this with a channel matrix from secret values to outputs [5]. They build a row of the channel matrix for each possible value of the secret, even if the system would behave in the same way for most of them. In contrast, we have proposed an automata based technique [3], using Markovian models. One state of a model represents an interval of values of the secret for which the system behaves in the same way, allowing for a much more compact and tractable representation.

We build a Markov chain representing the behavior observed by the attacker, then we hide the states that are not observable by the attacker, obtaining a smaller Markov chain—an *observable reduction*. Then we calculate the correlation between the output the attacker can observe and the behavior dependent on the secret, as it corresponds to the leakage. Since leakage in this case is mutual information, it can be computed by adding the entropy of the observable and secret-dependent views of the system and subtracting the entropy of the behavior depending on both. See [3] for details.

QUAIL (QUantitative Analyzer for Imperative Languages) implements this method. It is the first tool supporting arbitrary-precision quantitative evaluation of information leakage for randomized systems or protocols with secret data, including anonymity and authentication protocols. Systems are specified in a simple imperative modeling language further described on QUAILS website.

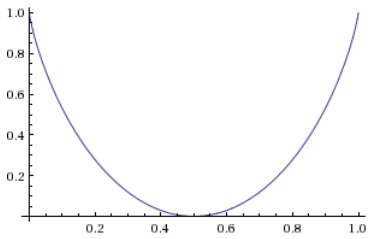

Fig. 1: Bit XOR leakage as a function of $\Pr(r = 1)$

QUAIL performs a white-box leakage analysis assuming that the attacker has knowledge of the system's code but no knowledge of the secret's value, and outputs the result and eventually information about the computation, including the Markov chains computed during the process.

Example. Consider a simple XOR operation example. Variable h stores a 1-bit secret. The protocol generates a random bit r, where $r = 1$ with probability p. It outputs the result of exclusive-or between values of h and r. The attacker knows p and can observe the output, so if $h = r$, but not the values of r or h.

If $p = 0.5$ the attacker cannot infer any information about h, the leakage is zero bits (non-interference). If $p = 0$ or $p = 1$ then she can determine precisely the value of h, and thus the leakage is 1 bit. This can be verified efficiently with language-based tools like APEX [10]. However, QUAIL is the only tool able to precisely compute the leakage for all possible values of p with arbitrary precision. Figure 1 shows that XOR protocol leaks more information as the value of r becomes more deterministic. For instance $p = 0.4$ is safer than $p = 0.8$.

2 QUAIL Implementation

The input model is specifed in QUAIL's imperative language designed to facilitate succinct and direct modeling of protocols, providing features such as arbitrary-size integer variables and arrays, random assignments, `while` and `for` loops, named constants and conditional statements. Figure 2 presents the input code for the bit XOR example.

For a given input code QUAIL builds an annotated Markov chain representing all possible executions of the protocol, then modifies it to encode the protocol when observed by the attacker whose aim is to discover the protocol's secret data. Finally, QUAIL extracts a model of the observable and secret-dependent behavior of the system, and computes the correlation between them, which is equivalent to the amount of bits of the secret that the attacker can infer by observing the system. We now discuss QUAIL implementation following the five steps of the method proposed in [3]:

Step 1: Preprocessing. QUAIL translates the input code into a simplified internal language. It rewrites conditional statements and loops (`if`, `for` and `while`) to conditional jumps (`if-goto`) and substitutes values for named constant references.

Step 2: Probabilistic symbolic execution. QUAIL performs a symbolic forward execution of the input program constructing its semantics as a finite Markov chain (a fully probabilistic transition system) with a single starting state. To this end, QUAIL needs to know the attacker's probability distributions over the secret variables. For each conditional branch, we compute the conditional probability of the guard being satisfied given the values of the public variables and the probability distributions over the secret variables. Then QUAIL generates two successor states, one for the case in which the guard is satisfied and one when not satisfied. This is the most time-consuming step, so QUAIL uses an on-the-fly optimization to avoid building internal states that would be removed in the next step. For instance, it does not generate new states for assignments to a non-observable public variable. Instead it changes the value of the variable in the current state.

Step 3: State hiding and model reduction. To represent what the attacker can examine, QUAIL reduces the Markov chain model by iteratively hiding all unobservable states. For the standard attacker, these are all the internal states, i.e. all the states except the initial and the output states. A state is hidden by creating transitions from its predecessors to its successors and removing it from the model. This operation normally eliminates more than 90% of the states of the Markov chain model, building its *observable reduction*. This operation also detects non-terminating loops and collapses then in a single non-termination state. States are equipped with a list of their predecessors and successors to quicken this step. An observable reduction looks like a probability distribution from the starting states to the output states, since all other states are hidden.

Step 4: Quotienting. Recall from Sect.1 that we have to quantify the correlation between the observable and secret-dependent views of the system. QUAIL relies on the notion of quotients to represent different views of the system and compute their correlation. A quotient is a Markov chain obtained by merging together states in the observable

```
1  observable int1 l; // bit l is the output
2  public int1 r; // bit r is random
3  secret int1 h; // bit h is the secret
4  random r:=randombit(0.5); // randomize r
5  if (h==r) then // calculate the XOR
6  assign l:=0;
7  else
8  assign l:=1;
9  fi
10 return; //terminate
```

```
1  observable int1 l;
2  public int1 r;
3  secret int1 h;
4  random (r):=randombit(0.5);
5  if ((h)==(r))
6  then goto 8;
7  else goto 10;
8  assign (l):=(0);
9  goto 11;
10 assign (l):=(1);
11 return;
```

Fig. 2: Bit XOR example: input code (on the left) and preprocessed code (on the right).

reduction that give the same value to some of the variables. QUAIL quotients the observable reduction separately three times to build three different views of the system. QUAIL uses the attacker model again to know which states are indistinguishable as they assign the same values to the observable variables. These states are merged in the *attacker's quotient*. Similarly, in the *secret's quotient* states are merged if they have the same possible values for the secret, while in the *joint quotient* states are merged if they both have the same values for the secret and cannot be discriminated by the attacker. Since information about the states' variables is not needed to compute entropy, quotients carry none, reducing time and memory required to compute them.

Step 5: Entropy and leakage computation. The *information leakage* can be computed as the sum of the entropies of the attacker's and secret's quotients minus the entropy of the joint quotient [3]. The three entropy computations are independent and can be parallelized. QUAIL outputs the leakage with the desired amount of significant digits and the running time in milliseconds. If requested, QUAIL plots the Markov chain models using Graphviz.

3 On Using QUAIL

QUAIL is freely available from https://project.inria.fr/quail, including source code, binaries and example files. We demonstrate usage of QUAIL to analyze the bit XOR example. Let bit_xor.quail be the file containing the input shown in Fig. 2. The command

```
quail bit_xor.quail -p 2 -v 0
```

executes QUAIL with precision limited to 2 digits (`-p 2`), suppressing all output except the leakage result (`-v 0`). In response QUAIL generates a file bit_xor.quail.pp with the preprocessed code shown in Fig. 2, analyzes it and finally answers `0.0` showing that in this case the protocol leaks no information (so non-interference). For different probability of the random bit r in line 4 QUAIL obtains a different leakage (cf. Fig. 1). For instance, for $p = 0.8$ the leakage is $\sim$0.27807 bits.

4 Comparison with Other Tools

QUAIL precisely evaluates the value of leakage of the input code. This not only allows proving non-interference (absence of leakage) but also enables comparing relative safety of similar protocols. This is particularly important for protocols that exhibit

Table 1: QUAIL analysis of the leakage in an authentication program

Password length	2	32	64	500
Leakage	$8.11 \cdot 10^{-1}$	$7.78 \cdot 10^{-9}$	$3.54 \cdot 10^{-18}$	$1.52 \cdot 10^{-148}$

inherent leakage, such as authentication protocols. For instance, with a simple password authentication, the user inputs a password and is granted access privilege if the password corresponds to the secret stored in the system. The chance of an attacker guessing a password is always positive (although it depends on the password's length). Also, even if the attacker gets rejected she learns something about the secret—the fact that the attempted value was not correct. QUAIL can quantify the precise leakage as a function of the bit length of the password, as shown in Table 1.

Existing *qualitative* tools can establish whether a protocol is completely secure or not, i.e. whether it respects non-interference. They cannot discriminate protocols that allow acceptable and unacceptable violations of non-interference. APEX [10] is an analyzer for probabilistic programs that can check programs equivalence, while PRISM [14] is a probabilistic model-checker. With these tools authentication protocols will always be flagged as unsafe, and a comparison between them is impossible.

QUAIL can be used also to analyze anonymity protocols, like the grade protocol and the dining cryptographers [6]. The interested reader can find discussion and input code for these examples on the QUAIL website. These protocols provide full anonymity on the condition that some random data is generated with a uniform probability distribution; their effectiveness in these cases can be efficiently verified with the qualitative tools above. If the probability distribution over the random data is not uniform some private data is leaked, and QUAIL is again the only tool that can quantify this leakage. Presently, the models for these protocols tend to grow exponentially, so the analysis becomes time-consuming already for about 6–7 agents.

Qualitative tools and technique are more closely related to QUAIL; we present some of them and discuss the main differences. It is worth noting that most of them either do not work for analyzing probabilistic programs [1,11,13,17] or are based on a channel matrix with an impractical number of lines [4,7].

JPF-QIF [19] is a tool that computes an upper bound of the leakage by estimating the number of possible outputs of the system. JPG-QIF is much less precise than QUAIL, and it is not able for instance to prove that the security of an authentication increases by increasing the password size.

McCamant and Ernst [16] and Newsome, McCamant and Song [18] propose quantitative extensions of taint analysis. This approach, while feasible even for large programs, still does not allow to analyze probabilistic programs, making it unsuitable for security protocols.

Bérard et al. propose a quantification of information leakage based on mutual information, though they name it restrictive probabilistic opacity [2] and do not refer to some of the core papers of the subject, like the works of Clark, Hunt and Malacaria [8,9]. The approach tries to quantify leakage on probabilistic models, and is thus phylosophically close to ours. They compute mutual information as the expected difference between

prior and posterior entropy, and since the latter depends on all possible values of the secret we expect that an eventual implementation would be in general very inefficient compared to the QUAIL quotient-based approach.

References

1. Michael Backes, Boris Köpf, and Andrey Rybalchenko. Automatic discovery and quantification of information leaks. In *IEEE Symposium on Security and Privacy*, 2009.
2. Béatrice Bérard, John Mullins, and Mathieu Sassolas. Quantifying opacity. In G. Ciardo and R. Segala, editors, *QEST'10*. IEEE Computer Society, September 2010.
3. Fabrizio Biondi, Axel Legay, Pasquale Malacaria, and Andrzej Wasowski. Quantifying information leakage of randomized protocols. In Roberto Giacobazzi, Josh Berdine, and Isabella Mastroeni, editors, *VMCAI*. Springer, 2013.
4. Konstantinos Chatzikokolakis, Tom Chothia, and Apratim Guha. Statistical measurement of information leakage. In Javier Esparza and Rupak Majumdar, editors, *TACAS*, 2010.
5. Konstantinos Chatzikokolakis, Catuscia Palamidessi, and Prakash Panangaden. Anonymity protocols as noisy channels. In *Information and Computation*. Springer, 2006.
6. David Chaum. The dining cryptographers problem: Unconditional sender and recipient untraceability. *Journal of Cryptology*, 1:65–75, 1988.
7. Tom Chothia and Apratim Guha. A statistical test for information leaks using continuous mutual information. In *CSF*, pages 177–190. IEEE Computer Society, 2011.
8. David Clark, Sebastian Hunt, and Pasquale Malacaria. Quantitative analysis of the leakage of confidential data. *Electr. Notes Theor. Comput. Sci.*, 59(3):238–251, 2001.
9. David Clark, Sebastian Hunt, and Pasquale Malacaria. A static analysis for quantifying information flow in a simple imperative language. *Journal of Computer Security*, 15, 2007.
10. S. Kiefer, A. S. Murawski, J. Ouaknine, B. Wachter, and J. Worrell. Apex: An analyzer for open probabilistic programs. In *Proc. CAV'12*, LNCS. Springer, 2012.
11. Vladimir Klebanov. Precise quantitative information flow analysis using symbolic model counting. In Fabio Martinelli and Flemming Nielson, editors, *QASA*, 2012.
12. Boris Köpf and David A. Basin. An information-theoretic model for adaptive side-channel attacks. In *ACM Conference on Computer and Communications Security*, 2007.
13. Boris Köpf, Laurent Mauborgne, and Martín Ochoa. Automatic quantification of cache side-channels. In P. Madhusudan and Sanjit A. Seshia, editors, *CAV*, 2012.
14. M. Kwiatkowska, G. Norman, and D. Parker. PRISM 4.0: Verification of probabilistic real-time systems. In *Proc. CAV'11*, volume 6806 of *LNCS*. Springer, 2011.
15. Pasquale Malacaria. Algebraic foundations for information theoretical, probabilistic and guessability measures of information flow. *CoRR*, abs/1101.3453, 2011.
16. Stephen McCamant and Michael D. Ernst. Quantitative information flow as network flow capacity. In Rajiv Gupta and Saman P. Amarasinghe, editors, *PLDI*. ACM, 2008.
17. Chunyan Mu and David Clark. A tool: Quantitative analyser for programs. In *QEST*. IEEE Computer Society, 2011.
18. James Newsome, Stephen McCamant, and Dawn Song. Measuring channel capacity to distinguish undue influence. In S. Chong and D. A. Naumann, editors, *PLAS*. ACM, 2009.
19. Quoc-Sang Phan, Pasquale Malacaria, Oksana Tkachuk, and Corina S. Pasareanu. Symbolic quantitative information flow. *ACM SIGSOFT Software Engineering Notes*, 37(6):1–5, 2012.
20. Andrei Sabelfeld and Andrew C. Myers. Language-based information-flow security. *IEEE Journal on Selected Areas in Communications*, 21(1):5–19, 2003.
21. Geoffrey Smith. On the foundations of quantitative information flow. In Luca de Alfaro, editor, *FOSSACS*, volume 5504 of *LNCS*, pages 288–302. Springer, 2009.