Why3 a dit : gardez le contrôle en toute situation

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Arbitrary control dependence
- Context: static backward program slicing
- Definitions of control dependence

Danicic's algorithm
- Description
- Illustration
- Formalization in Coq

A new optimized algorithm
- Presentation
- Formalization in Why3
- Experiments

Conclusion

Plan

Plan

Definition

Static backward slicing (introduced by Weiser in 1981)

- simplifies a given program p but preserves the behavior w.r.t. a point of interest C (slicing criterion, typically a statement)
- removes irrelevant statements that do not impact C
- produces a simplified program q (slice)

Example: divisibility test

euclidean division of *a* by *b*

```
1 : quo = 0;
2 : r = a;
3 : while (b <= r) {
4 :     quo = quo + 1;
5 :     r = r - b;
    }
```

is the remainder equal to 0 ?

```
6 : if (r != 0) {
7 :     res = 0;
    } else {
8 :     res = 1;
    }
```

Original program p

?

control

data

Slice q w.r.t. line 8

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Original program *p*

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8 :     res = 1;
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```

Slice *q* w.r.t. line 8

control
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Plan

On a concrete structured language

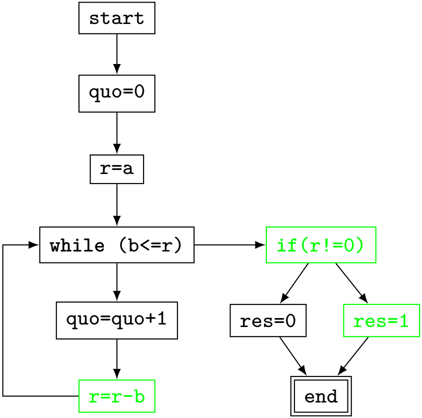
```
if (l: b) {
    ...
    l_then: stmt;
    ...
} else {
    ...
    l_else: stmt;
    ...
}
```

```
while (l: b) {
    ...
    l_body: stmt;
    ...
}
```

On a control flow graph

Using post-dominance (for ex. [Ferrante et al., 1987])

- v is control-dependent on u iff u has two children u_1 and u_2 such that u_1 is post-dominated by v, but not u_2



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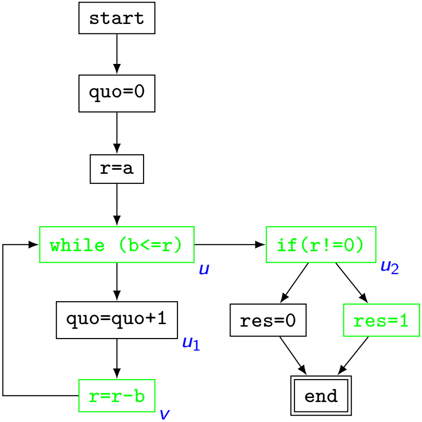
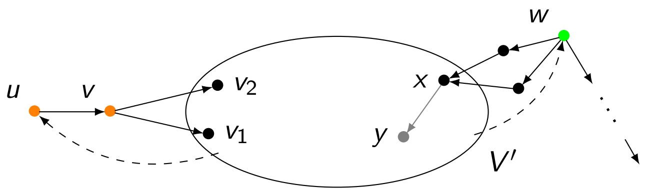


On a finite directed graph

- Elegant generalization of Danicic et al. in 2011
- A subset V' is closed under weak control dependence (or weakly control-closed) iff every node reachable from V' has at most one first-reachable node (observable) in V'.



$\text{obs}(x) = \{x\}$

$\text{obs}(w) = \{x\}$

$\text{obs}(u) = \{v_1, v_2\}$

$\text{obs}(v) = \{v_1, v_2\}$

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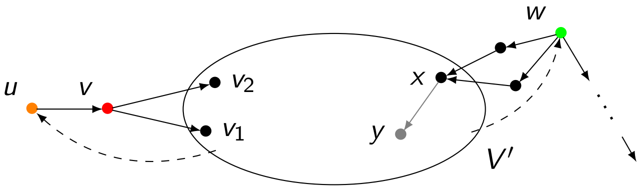
- A subset V' is closed under weak control dependence (or weakly control-closed) iff every node reachable from V' has at most one first-reachable node (observable) in V'.
- weak control-closure(V') = V' ∪ { all the vertices both reachable from V' and V'-weakly deciding }
- V'-weakly deciding = all the nodes giving rise to two non-trivial paths reaching V' that share no vertex except their origin.



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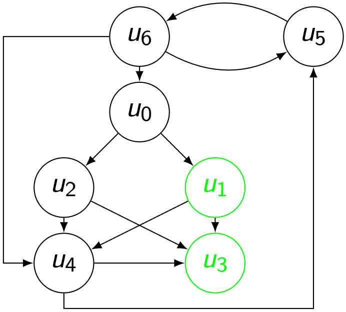
Running example

$V' = \{u_1, u_3\}$



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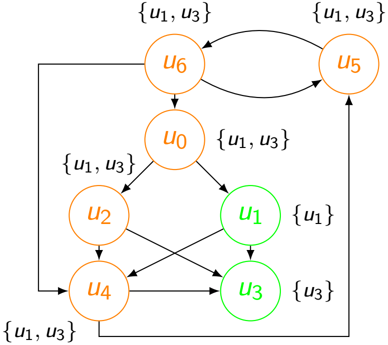

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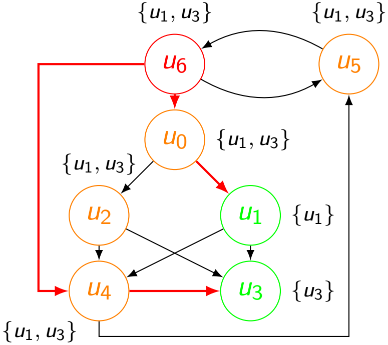

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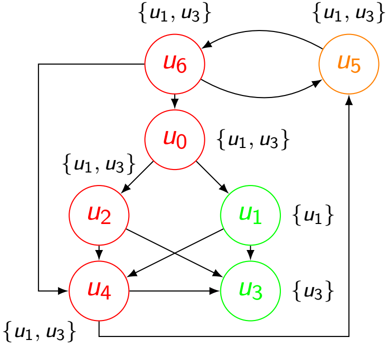

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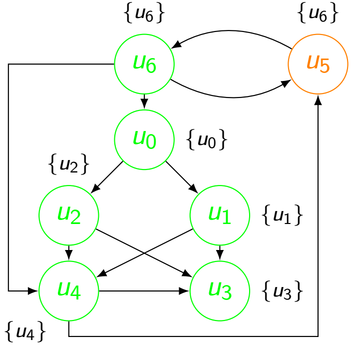

Closure: $\{u_0, u_1, u_2, u_3, u_4, u_6\}$

Plan

Plan

Idea

- Iterative algorithm
- Predicate $H(u, V')$ such that:
  - (H1) If $H(u, V')$ then u is V'-weakly deciding and reachable from V'
  - (H2) If there is no node u satisfying $H(u, V')$, then there is no V'-weakly deciding vertex reachable from V'

H's definition

$H(u, V')$: u is reachable from V', $|\text{obs}(u)| \geq 2$ and one of its children v satisfies $|\text{obs}(v)| = 1$.

Danicic's method to compute weak control closure

begin
  $W \leftarrow V'$;
  while *there exists a node* u *satisfying* $H(u, W)$ *in* V **do**
    choose such a node u;
    $W \leftarrow W \cup \{u\}$
  end
  return W
end

Key ideas:

- At each iteration, the weak control-closure of W is equal to the weak-control closure of V' (due to (H1)).
- At the end, W is weakly-control closed (due to (H2)).

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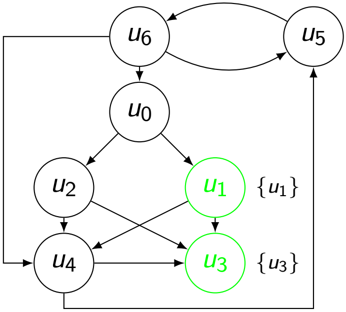
Danicic's algorithm on an example

$V' = \{u_1, u_3\}$



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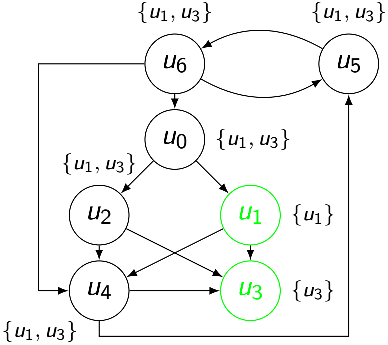

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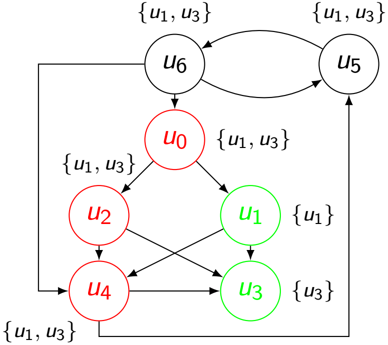

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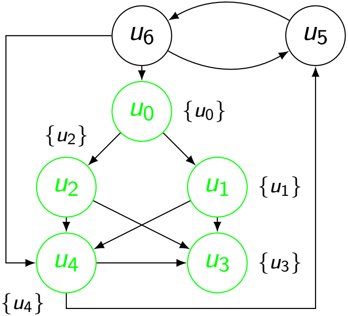

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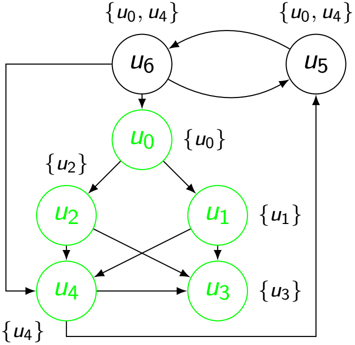

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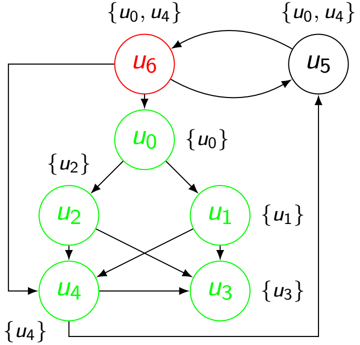

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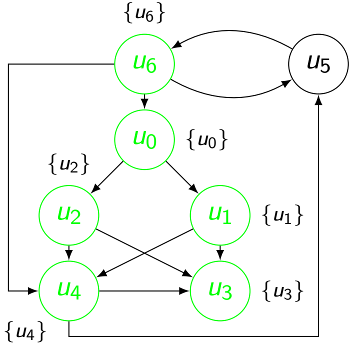

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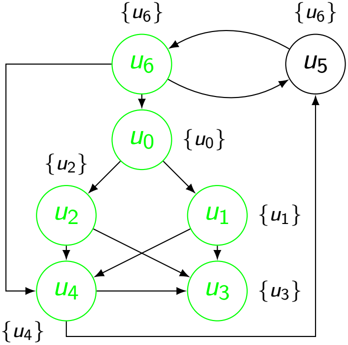

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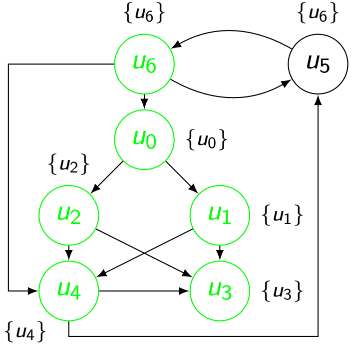

Closure: $\{u_0, u_1, u_2, u_3, u_4, u_6\}$

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A few words about the formalization in Coq

- A subset of Danicic's theory was formalized in Coq
- Danicic's algorithm was implemented and proved correct
- Size: 4000 loc of spec, 8000 loc of proof
- A Coq library à la OCamlgraph was missing

Limitations of Danicic's algorithm

A few small optimizations are possible:

- At each iteration, add all the nodes satisfying $H(u, W)$ instead of just one
- Weakening H: 2 and 1 are arbitrary, what is important is that $1 \leq |\mathrm{obs}(v)| < |\mathrm{obs}(u)|$.

More fundamentally, Danicic's algorithm does not take advantage of previous iterations to speed up the following ones.

Plan

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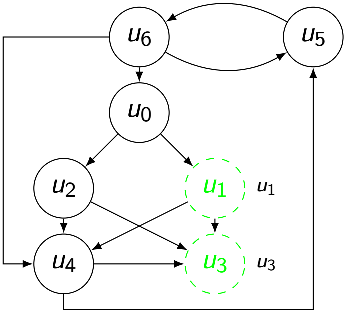
The optimized algorithm

- Again an iterative algorithm: start with $W = V'$ and make W grow
- Each vertex is labeled with a node in W which is a good candidate for an observable, but sometimes is not
- This labeling survives the iterations and can be reused
- At the end, W is the weak control-closure of V' and each node is labeled with its observable in the closure

The optimized algorithm on an example

$V' = \{u_1, u_3\}$



The optimized algorithm on an example

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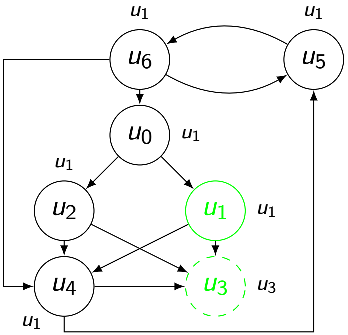
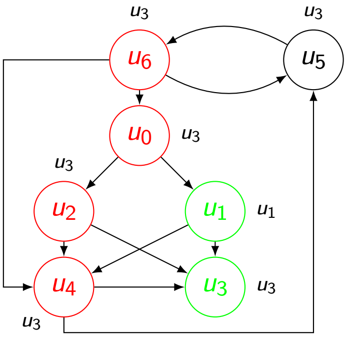


After propagation of u_1

The optimized algorithm on an example

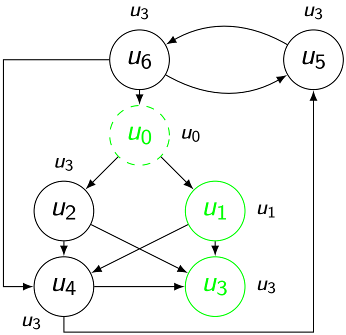
$V' = \{u_1, u_3\}$



Propagation of u_3

The optimized algorithm on an example

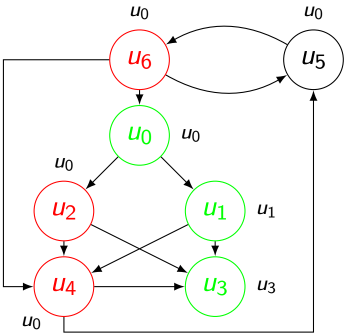
$V' = \{u_1, u_3\}$



After propagation of u_3

The optimized algorithm on an example

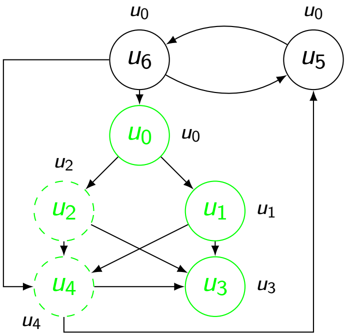
$V' = \{u_1, u_3\}$



Propagation of u_0

The optimized algorithm on an example

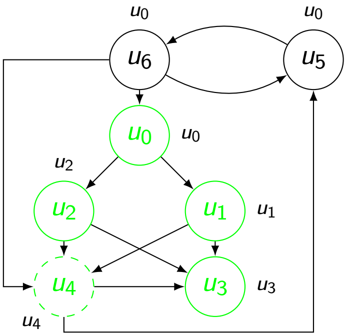
$V' = \{u_1, u_3\}$



After propagation of u_0

The optimized algorithm on an example

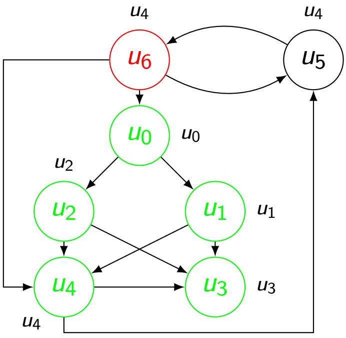
$V' = \{u_1, u_3\}$



After propagation of u_2

The optimized algorithm on an example

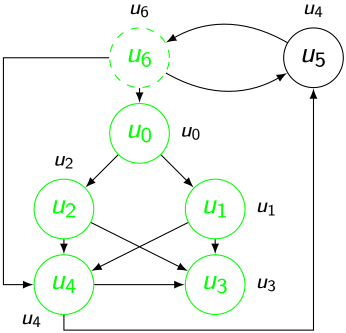
$V' = \{u_1, u_3\}$



Propagation of u_4

The optimized algorithm on an example

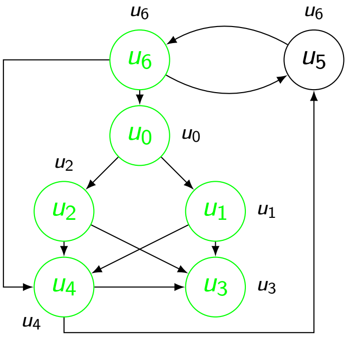
$V' = \{u_1, u_3\}$



After propagation of u_4

The optimized algorithm on an example

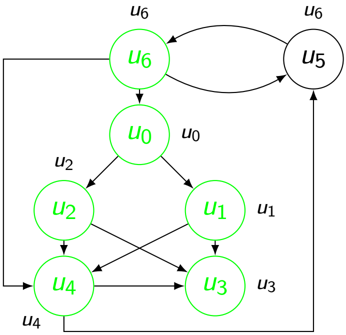
$V' = \{u_1, u_3\}$



After propagation of u_6

The optimized algorithm on an example

$V' = \{u_1, u_3\}$



Closure: $\{u_0, u_1, u_2, u_3, u_4, u_6\}$

Plan

A few words about the formalization in Why3

The Why3 development is split into two parts:

- a small part of weak control dependence's theory (80 loc)
  - everything proved
  - except one lemma is admitted (but is proved in the Coq formalization)
- the new algorithm (250 loc)
  - split into 4 functions
  - a lot of proofs are automatic
  - the preservations of the main invariants were done in Coq

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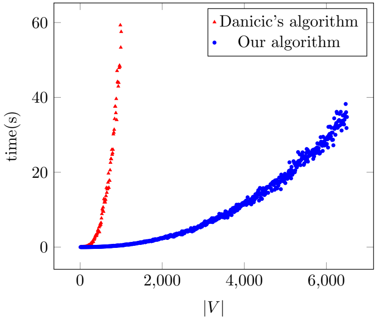
Experiments

- Both algorithms were implemented in OCaml using OCamlgraph
- They were run on randomly generated graphs
- Checked against the Coq extraction on small graphs



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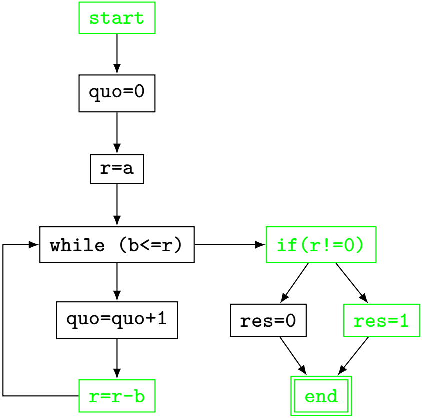
Conclusion:

- Formalization in Coq of an elegant theory of control dependence on finite directed graphs and of an algorithm computing closure under control dependence (Danicic et al., 2011)
- Design of an optimization of this algorithm
- Proof in Why3 of this new algorithm
- Experiments confirm the new algorithm outperforms Danicic's method

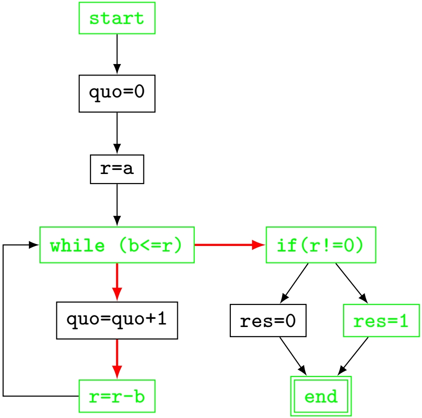
Future work:

- Integrate this work in a theory of program slicing
- Weak control dependence -> strong control dependence

Weak control-closure on euclidean division



Weak control-closure on euclidean division



Graph theory

	Alt-Ergo (1.30)	CVC4 (1.5)	Coq (8.6.1)	Eprover (2.0)	Z3 (4.5.0)
Number	10	14	4	6	0
Min time (s)	0	0,02	0,27	0,01	0
Max time (s)	0,01	0,67	0,37	0,44	0
Avg time (s)	0,01	0,083	0,3	0,093	N/A

+ 1 axiom (but proved in the Coq formalization)

Algorithm

	Alt-Ergo (1.30)	CVC4 (1.5)	Coq (8.6.1)	Eprover (2.0)	Z3 (4.5.0)
Number	233	12	4	4	2
Min time (s)	0,01	0,08	0,32	0,08	0,34
Max time (s)	3,96	0,83	0,76	2,35	3,18
Avg time (s)	0,18	0,46	0,48	0,72	1,76