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**Production, scheduling, routing and inventory**
**Clustering and classification**

# GROUP METHODOLOGY IN PRODUCTION MANAGEMENT: A SOLUTION BASED ON A HIERARCHICAL CLASSIFICATION METHOD

## REPLY TO THE GARCIA–PROTH PROBLEM
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ISRAËL-CÉSAR LERMAN
*IRISA, Campus de Beaulieu, 35042 Rennes Cedex, France*

## SUMMARY

We consider the problem submitted by Garcia and Proth and we propose for it a solution obtained by simply applying our very general method of hierarchical classification based on the likelihood of the links, in the case of an incidence table of data. This application is done on each of the two sides of the incidence data table. In our approach, the common number $p$ of classes has not to be fixed arbitrarily, but is an observed possible result—provided by the notions of 'significant' levels and nodes of the classification tree—for the 'natural' association between two partitions. We compare our solution with the previous one.

KEY WORDS Group technology Production management Hierarchical and non-hierarchical clustering

## RECALL OF THE SETTING OF THE PROBLEM

A working process is a sequence of tasks which are performed in order to manufacture a part. We distinguish the type of a part and we say that two parts belong to the same part type if their working processes are the same.

We are looking for a pair of associated partitions, the first on the set of tasks and the second on the set of part types. A given class of tasks is called a production subsystem and a given class of part types is called a family. The numbers of classes of the two associated partitions are the same and the researched correspondence between the production subsystems and the families is one–one. These partitions must be determined in order to minimize the number of tasks performed in the production of part types which—respectively—do not belong to the families associated with the production subsystems including the concerned tasks.

This problem has been submitted by Garcia and Proth in a preceding issue of this journal.[1] They have introduced the following notations, that we adopt for reasons of conformity:

Let $X$ be the set of $n$ part types and let $Y$ be the set of $m$ tasks. We define the incidence matrix $A = \{a_{ij} \mid 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}$, with $n$ rows and $m$ columns, where

$$a_{ij} = \begin{cases} 1 \text{ if the working process of a part belonging to the part type } i \text{ includes the task } j. \\ 0 \text{ if not.} \end{cases} \quad (1)$$

The goal is to determine a pair $(\mathscr{X}, \mathscr{Y})$ of partitions on $X$ and $Y$, with the same number $p$

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of classes, denoted by

$$\mathcal{X} = \{X_1, X_2, \ldots, X_p\} \quad \text{and} \quad \mathcal{Y} = \{Y_1, Y_2, \ldots, Y_p\}$$

and such that the criterion

$$\sum\left\{a_{ij} \mid (i, j) \notin \bigcup_{1 \leqslant r \leqslant p} X_r \times Y_r\right\} \tag{2}$$

is minimum. More precisely, the quality of the solution is defined by the smallness of the value of

$$\sum\left\{u_i a_{ij} \mid (i, j) \notin \bigcup_{1 \leqslant r \leqslant p} X_r \times Y_r\right\} \tag{3}$$

where $u_i$ is an integer weight assigned to the part type $i$.

At first sight, the point is to find a non-hierarchical joint classification[2] with a common number of classes.

The solution submitted by Garcia and Proth starts with an initial partition on $X$ given by a specific version of the 'k-means' algorithm. The partition obtained, $\mathcal{X} = \{X_1, X_2, \ldots, X_r, \ldots, X_p\}$, with a fixed number of classes $p$, allows the beginning of an iterative process. The first step consists of defining a partition $\mathcal{Y} = \{Y_1, Y_2, \ldots, Y_r, \ldots, Y_p\}$, where $Y_r$ is composed of the elements $j$ for which the criterion

$$a_r(j) = \sum_{i \in X_r} u_i a_{ij} + \sum_{i \notin X_r} u_i(1 - a_{ij}) \tag{4}$$

is maximum. More precisely, $r$ is the first subscript maximizing $a_r(j)$. By interchanging the respective roles of the rows and the columns, $\mathcal{Y}$ leads—by an analogous procedure—to a new partition $\mathcal{X}^{(1)}$ which will give $\mathcal{Y}^{(1)}$ and so on. It is shown that the solution given by $(\mathcal{X}^{k+1}, \mathcal{Y}^{k+1})$ is—in a large sense—better than that given by $(\mathcal{X}^{k}, \mathcal{Y}^{k})$ with respect the criterion (to maximize):

$$\sum\left\{u_i a_{ij} \mid (i, j) \in \bigcup_{1 \leqslant r \leqslant p} X_r \times Y_r\right\} + \sum\left\{u_i(1 - a_{ij}) \mid (i, j) \notin \bigcup_{1 \leqslant r \leqslant p} X_r \times Y_r\right\} \tag{5}$$

which has also been considered by other authors.[3,4]

The process stops when two consecutive solutions lead to the same value of the criterion (5).

It must be noticed that this last criterion is different from that in (3), which evaluates the quality of the solution.

In fact the solution that we present will be based on a direct application of our hierarchical classification method. We have presented the framework of this method in an earlier article in this journal.[5]

In a few words, the method is applied separately on the sets $Y$ and $X$, respectively represented by the set of the columns and the set of the rows of the incidence table of data. What makes it possible to find a pair $(\mathcal{X}, \mathcal{Y})$ of associated partitions—as described above, with the same number $p$ of classes—are the notions of 'significant' levels and nodes of the classification tree. In our approach, the common number $p$ of classes has not to be fixed arbitrarily, but is an observed possible result for a 'natural' associaton between two partitions.

## THE CLASSIFICATION OF THE TASKS

### The similarity structure on the set of the tasks

According to our general presentation (cf. the second section of Reference 5), we are here facing an incidence table of data, where the set $Y$ of the tasks plays the role of the set of descriptive attributes (i.e. logical 0–1 variables). We will denote by $j$, $1 \leqslant j \leqslant m$, a current element of $Y$, and by $X_j$ the subset of the part types which use $j$ to be performed. More precisely, in $X_j$ each part type is duplicated accordingly to its weight.

Let us recall that in our approach, the formal expression of the association coefficient between variables is obtained with respect to a hypothesis of non-link (or no relation). Three fundamental forms of the hypothesis of non-link have been pointed out in Reference 4 (Chapter 2, Section IV.1) for the comparison between descriptive attributes. The associated random models, respectively denoted by $N_1$, $N_2$ and $N_3$, lead—for the random 'rough' index—to hypergeometric, binomial and Poisson distributions, respectively. We have tried the two indices associated with $N_1$ and $N_3$.

The first index may be put in the following form:

$$Q_1(j, j') = r_1(j, j')\sqrt{(n-1)} \tag{6}$$

where

$$r_1(j, j') = \frac{f(j \wedge j') - f(j)f(j')}{\sqrt{\{f(j)[1-f(j)]f(j')[1-f(j')]\}}} \tag{7}$$

is just Pearson's coefficient. $f(j)$ (resp. $f(j')$) is the proportion of weighted $i$ ($i \in X$) for which $a_{ij} = 1$ (resp. $a_{ij'} = 1$); in the same way $f(j \wedge j')$ is the proportion of weighted $i$ ($i \in X$) for which $a_{ij} \times a_{ij'} = 1$. These proportions take into account the weights $u_i$ ($i \in X$), since we—conceptually—work with the table of data where each row $i$ is duplicated accordingly to its weight $u_i$ ($i \in X$). More precisely,

$$\left.\begin{aligned} f(j) &= \Big(\sum_{1 \leqslant i \leqslant n} u_i a_{ij}\Big)\Big/\Big(\sum_{1 \leqslant i \leqslant n} u_i\Big) \\ f(j \wedge j') &= \Big(\sum_{1 \leqslant i \leqslant n} u_i a_{ij} a_{ij'}\Big)\Big/\Big(\sum_{1 \leqslant i \leqslant n} u_i\Big) \end{aligned}\right\} \tag{8}$$

The second coefficient, associated with $N_3$, may be written as follows:

$$Q_3(j, j') = r_3(j, j')\sqrt{n} \tag{9}$$

where

$$r_3(j, j') = \frac{f(j \wedge j') - f(j)f(j')}{\sqrt{[f(j)f(j')]}} \tag{10}$$

with the same notations as above.

Both indices give, by the 'likelihood link algorithm' (after global reduction and reference to a probability scale), the same tree directly associated with $N_1$ (see Figure 1).

This representation is reduced to the levels where a 'significant' node is detected (levels 5 and 8 marked by a star). A 'significant' node corresponds to a local maximum of the sequence of the values of the level's 'local statistic'.[4,5] But this representation is complete with respect to the whole of the information in the detailed tree, since each association of this last is laterally reproduced by the number of the level where it occurs. Thus, the two classes {1, 11} and {6, 9} are gathered at level 9.

Table I gives the distribution of the levels' statistics based on the 'preordonnance'.

Since we need a small number of classes, we only have to take into consideration the last levels of the tree. The global statistic indicates that the most significant partition is obtained

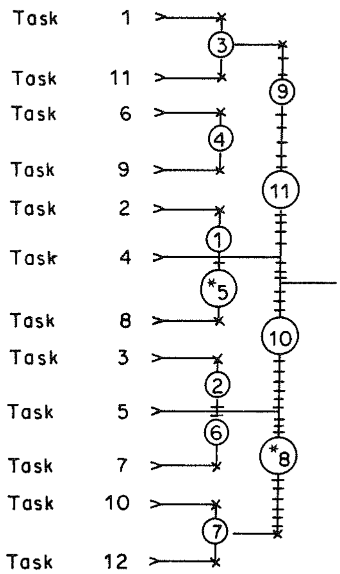


Figure 1. Tree representation

at level 9 with the three classes:

$$\{1, 6, 9, 11\},\ \{2, 4, 8\} \quad \text{and} \quad \{3, 5, 7, 10, 12\} \tag{11}$$

Intuitively speaking, this partition realizes the maximum of synthesis with such a small number of classes.

If we now examine the behaviour—over the last levels—of the local statistic $\theta_i$, a very significant node occurs at the eight level (*8) and corresponds to the aggregation of the two subclasses $\{3, 5, 7\}$ and $\{10, 12\}$. Henceforth, we have to consider the partition

$$\{\{1, 11\},\ \{6, 9\},\ \{2, 4, 8\},\ \{3, 5, 7, 10, 12\}\} \tag{12}$$

Finally—even though it is not accompanied by a significant node—we also have to take into consideration the classification which directly precedes that in (12) and for which the value of the 'global' statistic $\Sigma$ is the same (4·568). This partition into 5 classes is

$$\{\{1, 11\},\ \{6, 9\},\ \{2, 4, 8\},\ \{3, 5, 7\},\ \{10, 12\}\} \tag{13}$$

Table I

| | Level $i$ | Local statistic: $\tau_i$ | Global statistic: $\Sigma_i$ | $\theta_i = \Sigma_i - \Sigma_{(i-1)}$ |
|---|---|---|---|---|
| | 1 | 1.706 | 1.706 | 0·000 |
| | 2 | 1·706 | 2·394 | 0·688 |
| | 3 | 1·705 | 2·909 | 0·515 |
| | 4 | 1·595 | 3·279 | 0·370 |
| 1. Maximum | 5 | 2·351 | 3·970 | 0·692 |
| | 6 | 1·977 | 4·362 | 0·391 |
| | 7 | 1·523 | 4·568 | 0·207 |
| 2. Maximum | 8 | 1·976 | 4·567 | −0·001 |
| | 9 | 1·962 | 4·737 | 0·170 |
| | 10 | −1·301 | 2·130 | −2·607 |

Table II

| | | | | | | |
|---|---|---|---|---|---|---|
| $e$ | 1 | 9 | 12 | 6 | 7 | 10 |
| $V(e)$ | 0·54987 | 0·61031 | 0·64109 | 0·65092 | 0·65540 | 0·81368 |
| $e$ | 11 | 5 | 3 | 8 | 2 | 4 |
| $V(e)$ | 0·86349 | 0·94306 | 1·04969 | 1·10240 | 1·29210 | 1·54057 |

We will have to chose between the three partitions (11), (12) and (13), into 3, 4 and 5 classes, respectively. This choice will depend on the partitions that we will put in evidence on the set $X$ of the part types.

To this end, we consider the table (Table II) of the ordered values of the neutrality coefficient $V(e)$ that we assign to each element $e$ of the set $E$ to be classified. $V(e)$—which represents the observed variance of the proximities to $e$—indicates by its weakness the degree of neutrality of $e$, with respect to a classificatory goal. On the contrary, the bigger is the value of $V(e)$, the stronger is the intervention of $e$ in the 'leading' of the class where it appears.

With respect the above partition (12), this table leads to the following remarks:

1. The class $\{2, 4, 8\}$ is very typical; its elements have the highest values of $V(e)$.
2. The class $\{1, 11\}$ is led by the task 11, and its other element 1 is the most neutral. In the same way, the class $\{6, 9\}$ is led—but more slightly—by the task 9.
3. The class $\{3, 5, 7, 10, 12\}$ has two elements (tasks) 12 and 7 which are more perceptibly neutral than the others 3, 5 and 10.

## THE CLASSIFICATION OF THE PART TYPES

### Introduction

In the solution given by Garcia and Proth we need to start with an initial partition on $X$ or on $Y$. So, it is perfectly possible to begin with a partition of the smaller set (here $Y$). This partition may be in our case, one of (11), (12) or (13). The final two partitions retained on $X \times Y$, into three, four or five classes, will be those optimizing in the best way the criterion (8).

But—as we have previously expressed—our solution applies directly the same hierarchical method on the set $X$ of the part types. More precisely, each part type $i$ ($i \in X$) is weighted by its importance $u_i$. To take into account the weighting, we duplicate each $i$ $u_i$ times ($i \in X$). Henceforth, we have not to be surprised by the repetition of the leaf $i$ $u_i$ times ($i \in X$), in the representation tree (see Figure 2 below). But it would be perfectly possible to introduce in the computer program each $i$ as a class of size $u_i$ and to take into consideration the $u_i$ for the reactualization formula of the class's proximities.

### The similarity indices

We have tried the two indices corresponding to (7) and (10). More precisely, we define $r_1(i, i')$ and $r_3(i, i')$ by the means of the following formula:

$$r_1(i, i') = \frac{f(i \wedge i') - f(i)f(i')}{\sqrt{\{[f(i)[1 - f(i)]f(i')[1 - f(i')]\}}} \tag{14}$$

and

$$r_3(i, i') = \frac{f(i \wedge i') - f(i)f(i')}{[f(i)f(i')]} \tag{15}$$

Table II. Table of the variance of the proximities

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Item | 48 | 49 | 50 | 40 | 39 | 89 | 33 | 61 | 60 | 22 |
| | 0·61565 | 0·61565 | 0·61565 | 0·62135 | 0·62135 | 0·63917 | 0·71006 | 0·71810 | 0·71810 | 0·75319 |
| Item | 23 | 24 | 130 | 20 | 73 | 71 | 72 | 86 | 87 | 88 |
| | 0·75319 | 0·75319 | 0·75639 | 0·75639 | 0·76538 | 0·76538 | 0·76538 | 0·78833 | 0·76833 | 0·76833 |
| Item | 84 | 85 | 8 | 9 | 119 | 118 | 53 | 51 | 52 | 109 |
| | 0·76833 | 0·76833 | 0·76861 | 0·76861 | 0·76861 | 0·76861 | 0·78680 | 0·78680 | 0·78680 | 0·80753 |
| Item | 82 | 83 | 80 | 81 | 99 | 25 | 117 | 7 | 125 | 124 |
| | 0·81948 | 0·81948 | 0·81948 | 0·81948 | 0·83554 | 0·85464 | 0·85667 | 0·85667 | 0·86140 | 0·86140 |
| Item | 15 | 14 | 65 | 66 | 67 | 58 | 59 | 56 | 57 | 32 |
| | 0·86140 | 0·86140 | 0·86744 | 0·86744 | 0·86744 | 0·86926 | 0·86926 | 0·86926 | 0·86926 | 0·89217 |
| Item | 41 | 42 | 43 | 44 | 77 | 123 | 13 | 12 | 122 | 28 |
| | 0·89491 | 0·89491 | 0·89491 | 0·89491 | 0·90759 | 0·91046 | 0·91046 | 0·91046 | 0·91046 | 0·92525 |
| Item | 27 | 55 | 54 | 21 | 131 | 92 | 90 | 91 | 95 | 93 |
| | 0·92525 | 0·92673 | 0·92673 | 0·94209 | 0·94209 | 0·94567 | 0·94567 | 0·94567 | 0·95067 | 0·95067 |
| Item | 94 | 75 | 76 | 100 | 78 | 79 | 47 | 45 | 46 | 110 |
| | 0·95067 | 0·95832 | 0·95832 | 0·97107 | 0·99202 | 0·99202 | 1·01873 | 1·01873 | 1·01873 | 1·02869 |
| Item | 70 | 38 | 34 | 35 | 36 | 37 | 31 | 30 | 98 | 101 |
| | 1·03600 | 1·04347 | 1·04347 | 1·04347 | 1·04347 | 1·04347 | 1·05905 | 1·05905 | 1·10530 | 1·10530 |
| Item | 102 | 103 | 97 | 114 | 115 | 116 | 4 | 5 | 6 | 1 |
| | 1·10530 | 1·10530 | 1·10530 | 1·11247 | 1·11247 | 1·11247 | 1·11247 | 1·11247 | 1·11247 | 1·11480 |
| Item | 2 | 3 | 111 | 112 | 113 | 29 | 104 | 128 | 129 | 126 |
| | 1·11480 | 1·11480 | 1·11480 | 1·11480 | 1·11480 | 1·14368 | 1·14368 | 1·14974 | 1·14974 | 1·14974 |
| Item | 127 | 18 | 19 | 16 | 62 | 63 | 64 | 17 | 120 | 121 |
| | 1·14974 | 1·14974 | 1·14974 | 1·14974 | 1·14974 | 1·14974 | 1·14974 | 1·14974 | 1·19519 | 1·19519 |
| Item | 11 | 10 | 108 | 105 | 106 | 107 | 96 | 74 | 26 | 68 |
| | 1·19519 | 1·19519 | 1·30519 | 1·30519 | 1·30519 | 1·30519 | 1·47512 | 1·47512 | 1·47512 | 1·50442 |
| Item | 69 | | | | | | | | | |
| | 1·50442 | | | | | | | | | |

where $f(i)$ (resp. $f(i \wedge i')$) is the proportion of $j$ ($j \in Y$) for which $a_{ij} = 1$ (resp. $a_{ij}a_{i'j} = 1$). More concretely, $f(i)$ (resp. $f(i \wedge i')$) is the proportion of the necessary tasks in the working process of $i$ (resp. of $i$ and $i'$).

The best results that we present below (Table III) have been obtained with the association coefficient (14).

## The classification tree (Figure 2)

The behaviour of the 'global' statistic $\Sigma$—on the sequence of the tree levels—shows that the maximum is clearly reached at level 36 ($\Sigma_{36} = 56 \cdot 36$) (see Table IV). On the other hand, the node occuring at this level (*36) is 'significant' with respect to each of the two 'local' statistics: $\tau$ and $\theta$. So we present below the built classes at level 36.

The real taxonomic structure is into 5 classes that we label by (I), (II), (III), (IV) and (V):

(I): {1, 51, 5, 55, 16, 30, 13, 33, 42, 31, 45, 48, 6, 56, 20, 15, 47, 44, 37, 19}
(II): {7, 57, 17, 10, 60, 49, 34, 29, 23, 36, 8, 28, 58, 21}
(III): {25, 35, 27, 40, 43, 46, 38}

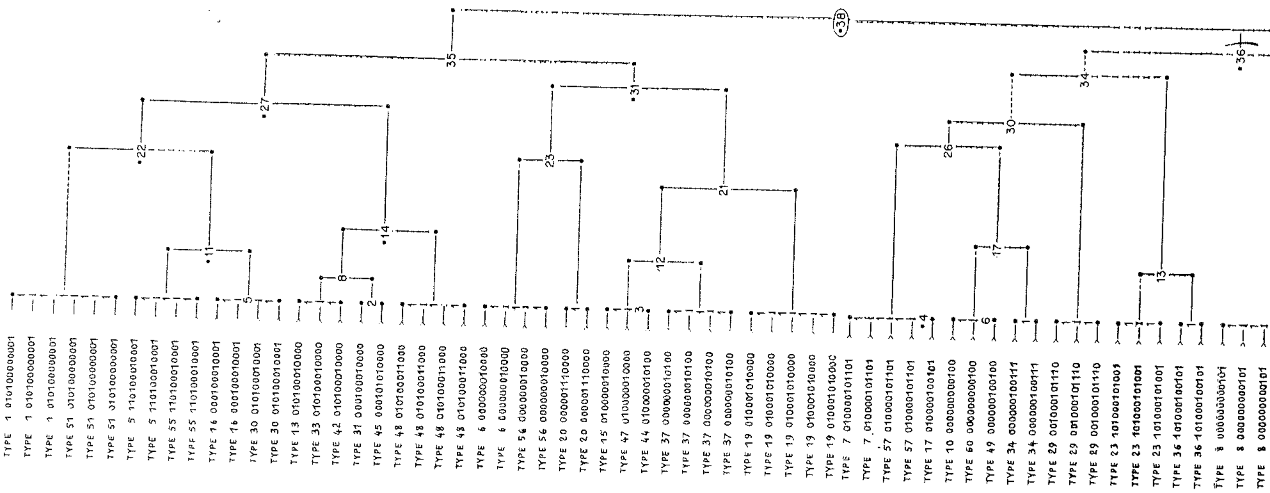
(IV) : {3, 53, 12, 50, 39, 4, 54, 32, 14, 22, 11, 18, 24}
(V) : {2, 52, 9, 59, 26, 41}.

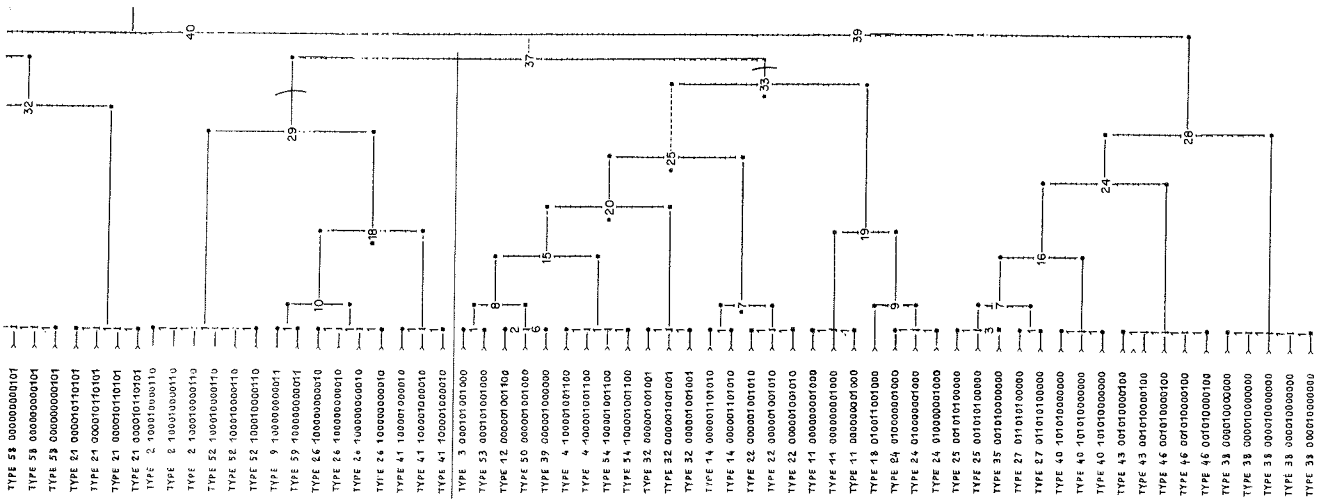
It is possible to retain this partition with 5 classes by associating it with those 5 classes obtained on the set of tasks (cf. (13)). It is easy to establish, by the crossing table in Figure 3, the one-one correspondence between the set of 5 classes on $X$ and the set of 5 classes on $Y$.

Nevertheless 5 is not the only possible number of classes of the two corresponding partitions on $X$ and on $Y$. Others numbers, such as 4 and 3, may be tried in order to improve the quality of the solution, with respect to the management problem, defined by the smallness of the criterion (3). Effectively, this last—that is not a taxonomic criterion—has not been directly

Table IV

| | Level $i$ | Local statistic: $\tau_i$ | Global statistic: $\Sigma_i$ | $\theta_i = \Sigma_i - \Sigma_{(i-1)}$ |
|---|---|---|---|---|
| | 1 | 24·332 | 24·322 | 0·000 |
| | 2 | 2·406 | 24·435 | 0·113 |
| | 3 | 3·386 | 24·657 | 0·222 |
| 1. Maximum | 4 | 3·460 | 24·887 | 0·230 |
| | 5 | 3·431 | 25·112 | 0·224 |
| | 6 | 3·190 | 25·301 | 0·189 |
| 2. Maximum | 7 | 5·880 | 25·943 | 0·642 |
| | 8 | 5·548 | 26·493 | 0·550 |
| | 9 | 2·773 | 26·629 | 0·136 |
| | 10 | 4·821 | 27·041 | 0·412 |
| 3. Maximum | 11 | 6·837 | 27·854 | 0·813 |
| | 12 | 4·729 | 28·199 | 0·344 |
| | 13 | 4·046 | 28·470 | 0·272 |
| 4. Maximum | 14 | 7·245 | 29·334 | 0·864 |
| | 15 | 6·574 | 29·997 | 0·663 |
| | 16 | 5·720 | 30·489 | 0·491 |
| | 17 | 3·558 | 30·675 | 0·187 |
| 5. Maximum | 18 | 7·118 | 31·451 | 0·776 |
| | 19 | 5·520 | 31·902 | 0·451 |
| 6. Maximum | 20 | 7·628 | 32·714 | 0·812 |
| | 21 | 7·446 | 33·398 | 0·683 |
| 7. Maximum | 22 | 11·725 | 35·321 | 1·923 |
| | 23 | 4·223 | 35·547 | 0·226 |
| | 24 | 9·943 | 36·814 | 1·267 |
| 8. Maximum | 25 | 10·924 | 38·200 | 1·386 |
| | 26 | 7·225 | 38·796 | 0·597 |
| 9. Maximum | 27 | 16·637 | 41·935 | 3·138 |
| | 28 | 12·497 | 43·612 | 1·677 |
| | 29 | 11·396 | 44·962 | 1·350 |
| | 30 | 7·392 | 45·429 | 0·467 |
| 10. Maximum | 31 | 13·019 | 47·154 | 1·725 |
| | 32 | 10·833 | 48·312 | 1·158 |
| 11. Maximum | 33 | 13·979 | 49·764 | 1·452 |
| | 34 | 9·123 | 50·225 | 0·461 |
| | 35 | 20·253 | 52·263 | 2·039 |
| 12. Maximum | 36 | 22·150 | 56·360 | 4·096 |
| | 37 | 3·652 | 51·654 | −4·706 |
| 13. Maximum | 38 | 4·668 | 38·974 | −12·679 |
| | 39 | −1·514 | 32·431 | −6·544 |





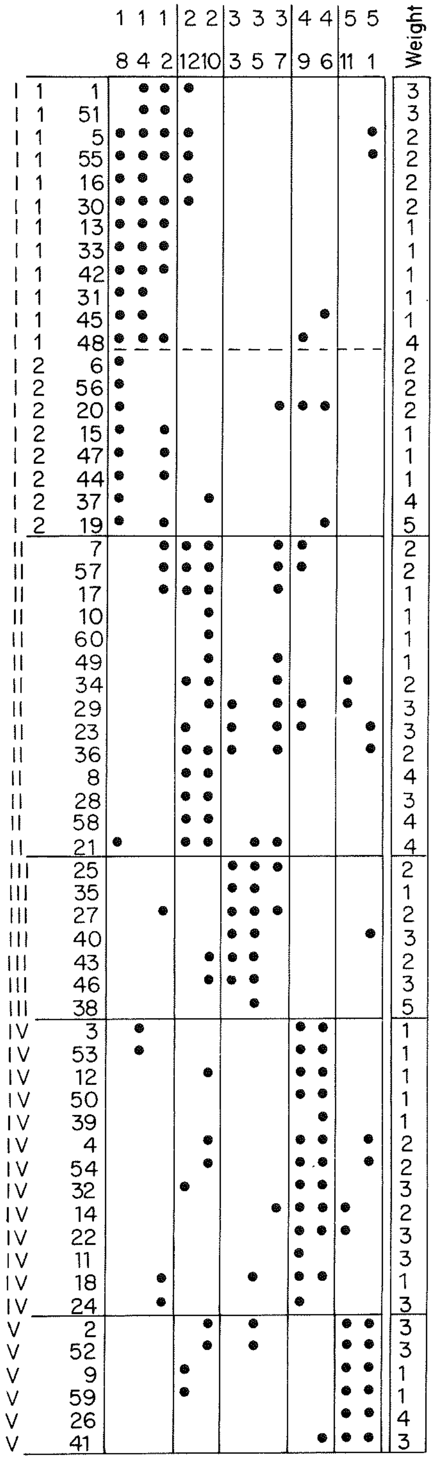


Figure 3.

optimized. We will resolve the question of the best choice—with respect to the application of our hierarchical method on the two sides of the data table—by considering the crossing table of Figure 3.

## THE CROSSING TABLE: DISCUSSION

It is relatively easy to establish—by counting the frequencies of ones in the incidence data table—the correspondence between the 5 classes of the above partition on $X$ and those of the partition (13) on $Y$. The association is the following:

(I) ⟷ {2, 4, 8}
(II) ⟷ {10, 12}
(III) ⟷ {3, 5, 7}
(IV) ⟷ {6, 9}
(V) ⟷ {1, 11}

Let us compare this result with those obtained by Garcia and Proth defined by a pair of partitions on $(X, Y)$ with the same number, 5, of classes (cf. Figure 2 of Reference 1).

Our partition of $Y$ is very slightly different. The only difference concerns item 7 which is moved from the class {7, 10, 12} to the class {3, 5}, in order to obtain our partition.

Similarly, the difference is not perceptible concerning the two partitions obtained on $X$. The five classes obtained in the previous solution are

① : ⟨1,5,6,13,15,16,19,30,31,33,42,44,45,47,48,51,55,56⟩
② : {34,36,37,7,21,23,28,29,49,8,10,17,57,58,60}
③ : {11,12,18,4,32,50,20,53,54,14,22,3,39,24}
④ {38,46,35,40,27,43,25}
⑤ : {2,52,9,59,41}

The correspondence between our classes and their classes can be determined from the association between the classes of the two very close partitions on $Y$. Thus, the correspondence is the following:

① ⟷ (I)
② ⟷ (II)
③ ⟷ (III)
④ ⟷ (IV)
⑤ ⟷ (V)

The distinction between the two partitions on $X$ only concerns the assignment of the part types 37 and 20, which belong to classes (2) and (3), respectively, in the previous solution and which are included in class (I) of our solution.

Let us now calculate the values of the criterion for both solutions with 5 classes. We will give in fact the value of the ratio

$$\frac{\sum \left\{u_i a_{ij} \mid (i, j) \notin \bigcup_{1 \leqslant r \leqslant p} X_r \times Y_r\right\}}{\sum \{u_i a_{ij} \mid (i, j) \in X \times Y\}} \tag{16}$$

In our solution, the Cartesian products $\{X_r \times Y_r \mid 1 \leqslant r \leqslant p\}$ are

$$(\mathrm{I}) \times \{2,4,8\},\ (\mathrm{II}) \times \{10,12\},\ (\mathrm{III}) \times \{3,5,7\},\ (\mathrm{IV}) \times \{6,9\},\ (\mathrm{V}) \times \{1,11\} \tag{17}$$

The value of the above criterion (16) is

$$(35 + 61 + 10 + 26 + 17)/1584 = 0 \cdot 0940$$

In the solution of Garcia and Proth, the Cartesian products $\{X_r \times Y_r / 1 \leqslant r \leqslant p\}$ are

$$① \times \{2,4,8\},\ ② \times \{7,10,12\},\ ③ \times \{6,9\},\ ④ \times \{3,5\},\ ⑤ \times \{1,11\} \tag{18}$$

The value of the criterion is

$$(25 + 45 + 30 + 12 + 17)/1584 = 0 \cdot 0814$$

We cannot decide if the difference between the two values of the criterion for the two solutions is really significant. This difference is mainly due to the association of task 7 with $\{3,5\}$ instead of with $\{10,12\}$. In any case we emphasize the fact that our solution is very rapidly obtained *simply by an application* of our hierarchical classification method in this particular case. Henceforth, it is possible—with this fixed number of classes—to improve our solution by applying a simple non-hierarchical classification algorithm, for example the 'transfer' algorithm (Reference 7 and Reference 4, chapter 2), with respect to the criterion (16).

On the contrary, the method of Garcia and Proth is adjusted to resolve the specific problem.

In any way a solution with 4 classes will appear perceptibly better than those with 5 classes.

The weighted $X$ may be considered as a set of 132 elements. Then the associations of the last levels are more stable in the classification tree on $Y$, than in that on $X$, since the latter is described by only 12 elements. The partition of $Y$ into 4 classes is (cf. (12)).

$$\{\{2,4,8\},\ \{3,5,7,10,12\},\ \{6,9\},\ \{1,11\}\}$$

The corresponding partition on $X$ is, with our preceding notations,

$$\{(\mathrm{I}), (\mathrm{II})\ \&\ (\mathrm{III}), (\mathrm{IV}), (\mathrm{V})\}$$

The value of the criterion becomes

$$(35 + 34 + 26 + 17)/1584 = 0 \cdot 0707$$

It is of course possible to consider in the solution (5) the union of the two classes $\{7,10,12\}$ and $\{3,5\}$ and then that of the two classes ② and ④, replacing the Cartesian products (18) by

$$① \times \{2,4,8\},\ ② \cup ④ \times \{3,5,7,10,12\},\ ③ \times \{6,9\},\ ⑤ \times \{1,11\} \tag{19}$$

In this case the value of the criterion attains 0·068 which is slightly different from 0·0707. On the other hand, we have not in the previous method *any indication* of the relevance of the association between $\{3,5\}$ and $\{7,10,12\}$.

We have mentioned above that the main difference between the two solutions in the case of 5 classes, concerns the position of the item 7. Task 7 has been linked in our method to task 3 and not to tasks 10 or 12. If we examine this particular situation, we have

$$s(7,3) = 12 \quad \text{and} \quad n(3) = 21$$
$$s(7,10) = 17 \quad \text{and} \quad n(10) = 46$$
$$s(7,12) = 16 \quad \text{and} \quad n(12) = 43$$

but,

$$s(7 \wedge 3)/n(3) = 12/21 = 0{\cdot}571$$
$$s(7 \wedge 10)/n(10) = 17/46 = 0{\cdot}370$$
$$s(7 \wedge 12)/n(12) = 16/43 = 0{\cdot}372$$

Henceforth, many taxonomic methods would make the same choice as ours. Once again, we can emphasize the fact that the quality criterion (3) has a dissymetric nature from the taxonomic point of view.

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