Proposition 6 *The call dpll(ϵ, φ) terminates.*

PROOF.
Let us give a proof by contradiction. Suppose that the *dpll*(ϵ, φ) does not terminate. We exhibit a strictly increasing sequence $(x^{(t)})_t$ of elements in a finite set with a strict total order. This leads to a contradiction.

Definition of $(x^{(t)})_t$. Given a partial valuation ν (where assigned litterals have levels), let $k(\nu, i)$ be the number of assigned (chosen or deduced) literals at level i in ν. Let us consider the t^{th} call of *dpll*. Let

$$x^{(t)} = (k(\nu, 0), \dots, k(\nu, n)) \quad \in \quad \{0, \dots, n\}^{n+1}$$

where ν is the valuation at t^{th} call of *dpll* after saturation by unit propagation.

Evolution of $x^{(t)}$. There are two reasons for a $t + 1^{th}$ call of *dpll*:

- Either we chose a new literal. For instance:

before	after
$\neg p$ q r $\neg s$ t u v w a	$\neg p$ q r $\neg s$ t u v w a e

In the example:

$$\begin{array}{lcl} x^{(t)} & = & (0, \; 3, \; 2, \; 1, \; 1, \; 2, \; 0, \; 0, \dots); \\ x^{(t+1)} & = & (0, \; 3, \; 2, \; 1, \; 1, \; 2, \; \mathbf{1}, \; 0, \dots). \end{array}$$

- Or we performed backjumping. For instance:

before	after
$\neg p$ q r $\neg s$ t u v w a e b	$\neg p$ q r $\neg s$ t [for sure a] ...

In the example:

$$\begin{array}{lcl} x^{(t)} = & (0, \; 3, \; 2, & 1, \; 1, 2, 2, 0, \dots \;); \\ x^{(t+1)} = & (0, \; 3, \; > \mathbf{2}, & 0, \; 0, \dots \;). \end{array}$$

Lexicographical order. That is why we introduce the lexicographic order $>$ on $\{0, \dots, n\}^{n+1}$. Given $x, y \in \{0, \dots, n\}^{n+1}$, we have $x > y$ iff there exists an integer $i \in \{0, \dots, n\}$ such that $x_i > y_i$ and for all $j < i$, we have $x_j = y_j$.

Conclusion. The sequence $(x^{(t)})_t$ is strictly increasing and takes only a finite number of values. Contradiction.

■