CVFP (Software Design and Formal Verification)
TD 4 : Bisimulations

Exercise 1 `Salad Niçoise`
Which of the different models soon drawn on the black board are bisimilar?

Exercise 2 `Maximal Bisimulation`

Show that if there exists a bisimilation $\mathcal{R}$ between $\mathcal{M}, w$ and $\mathcal{M}', w'$, then there exists a maximal bisimilation, i.e. a bisimilation $\mathcal{R}_m$ such that for every bisimilation $\mathcal{R}'$ between $\mathcal{M}, w$ and $\mathcal{M}', w'$, then $\mathcal{R}' \subseteq \mathcal{R}_m$.

Exercise 3 `Waiting in the Bakery`
Let's consider the following program:

```
procedure P_i
    loop
        non-critical section
        request (wait that y > 0 and do y := y − 1)
        critical section
        release (do y := y + 1)
    end loop
endProcedure
```

3.1. Give a control flow graph of this program.

3.2. Draw the KRIPKE model (also known as *transition-system*) of the system made up of two such processes P_1 and P_2 interleaving.

Let c_i be "The process P_i is in the critical section" and w_i be "The process P_i is waiting".

3.3. Are each of those formulae verified:

- $\forall G \neg(c_1 \wedge c_2)$ (safety),
- $\exists F c_1$,
- $\exists G \neg w_2$,
- $\exists GF w_1$,
- $\forall GF c_1$,
- $\forall G(w_1 \rightarrow F c_1)$ (liveness),
- $\forall(GF w_1 \rightarrow GF c_1)$,
- $\forall(GF w_1 \rightarrow \exists GF c_1)$.

Exercise 4 `The Theorem You've Always Wanted But Were Too Afraid To Ask`

Definition:
$\mathcal{M} = (W, R, V)$ is *image-finite* iff for all $w \in W$, $R(w)$ is finite.

Let's show the following theorem:

Theorem (Hennessy-Milner):
If $\mathcal{M}, w$ and $\mathcal{M}', w'$ satisfy the same modal formulas and $\mathcal{M}, \mathcal{M}'$ are image-finite, then $\mathcal{M}, w$ and $\mathcal{M}', w'$ are bissimilar.

We define the relation $\mathcal{B} \subseteq W \times W'$ as the modal equivalence, that is $u\mathcal{B}u'$ iff (for all modal formulas ϕ, $\mathcal{M}, u \models \phi$ iff $\mathcal{M}', u' \models \phi$).

4.1. Let u, u' such that $u\mathcal{B}u'$. Show that if $R(u)$ is not empty, then $R'(u')$ is not empty.

4.2. Show the theorem of HENNESSY-MILNER.

Exercise 5 `Positivity and Monoticity of a Formula`

A modal formula ϕ is *positive* (resp. *negative*) in p if p always appears under an even (resp. odd) number of negations.

A modal formula ϕ is *increasing* (resp. *decreasing*) in p if for all pointed frame $\mathcal{F}, w$ and valuation V such that $(\mathcal{F}, V), w \vDash \phi$, then for all valuation V' such that $V(p) \subseteq V'(p)$ (resp. $V'(p) \subseteq V(p)$), $(\mathcal{F}, V'), w \vDash \phi$.

5.1. Show that if a modal formula is positive in p, then it is increasing in p and that if it is negative in p, then it is decreasing in p.

5.2. Is the converse true?