CVFP (Software Design and Formal Verification)
TD 1: Introduction to Software Design

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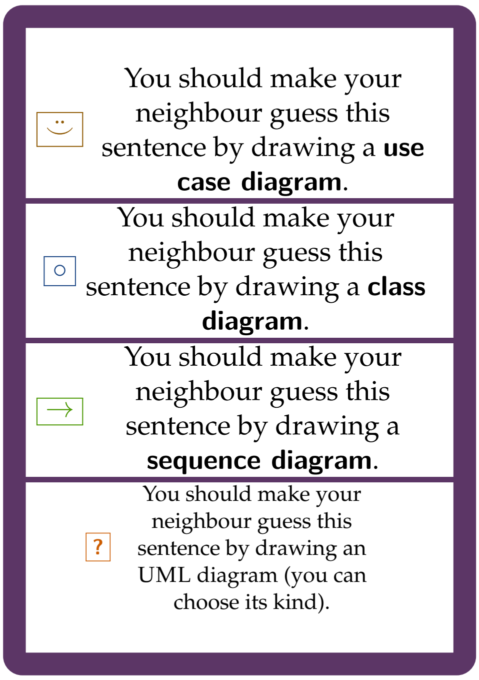
Exercise 1 `As Usually In The Beginning Of The Year, We Need To Know More About Your Parents.`

1.1. Design an UML object diagram representing your family tree.

1.2. Infer from it a class diagram, on which you'll annotate the various multiplicities.

Exercise 2 `Bingo!`

Let's apply UML while playing some Pictionary game. You should receive soon a card with four sentences your neighbour is going to guess using UML diagrams:



Exercise 3 `Morning Cross-Words`

A *context* $\mathcal{C}$ is a formula containing a (unique) 0-arity predicate $\square$ named *hole*. An *instantiation* $\mathcal{C}(P)$ of such a context by a formula P is the formula $\mathcal{C}\left[{}^{P}/_{\square}\right]$, where the hole has been replaced by P.

Link every formula with a context so that every resulting formula is valid.

$\exists x, \mathcal{P}(x)$	●	●	$\square \to \exists x, \mathcal{P}(x)$
$\forall x, (\mathcal{P}(f(x)) \land \neg\mathcal{P}(f(x)))$	○	○	$\exists y, \square$
$\forall x, (\mathcal{Q}(y,y) \to \mathcal{Q}(x,x))$	○	○	$\exists y, \square \to \forall y, \exists x, \mathcal{Q}(y,x)$
$\forall x, \mathcal{P}(x)$	○	○	$\mathcal{P}(y) \to (\square \to \forall x, \mathcal{P}(x)) \to \mathcal{P}(f(f(y)))$
$\forall x, \mathcal{Q}(x,y)$	○	○	$\forall x, ((\mathcal{P}(x) \to \square) \to \mathcal{P}(x)) \to \mathcal{P}(x)$
$\forall x, (\mathcal{P}(x) \to \mathcal{P}(f(x)))$	○	○	$\neg\square \to \exists x, \neg\mathcal{P}(x)$

Exercise 4 `Let's Decipher Formulae!`

For each formula, give a model if it's satisfiable or (informally) explain why its negation is valid.

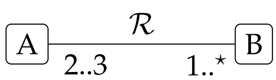
- $\forall x, \neg\mathcal{P}(x) \to \mathcal{P}(x)$
- $\forall x, \forall y, \forall z, (\mathcal{Q}(x,y) \to \mathcal{Q}(x,z) \to \mathcal{R}(y,z)) \land \exists x, \exists y, \neg\mathcal{R}(x,y) \land \forall x, \exists y, \mathcal{Q}(x,y)$
- $\forall x, \forall y, (\mathcal{Q}(x,y) \to \mathcal{Q}(y,x) \to \mathcal{R}(x,y))$
  $\land \forall x, \forall y, (\mathcal{Q}(x,y) \to \mathcal{Q}(y,x))$
  $\land \forall x, (\mathcal{Q}(x,x) \land \mathcal{R}(x,x))$
  $\land \exists x, \exists y, (\neg\mathcal{R}(x,y) \land \mathcal{Q}(x,y))$

Exercise 5 `Never Forget That UML Diagrams Are In Fact Formulae.`

5.1. Given a formula $\phi(x)$ where x is the only free-variable in $\phi(x)$, give a formula meaning that there are between 2 and 5 elements x such that $\phi(x)$.

► *We can construct such a formula by splitting it into a part saying that there are at least* 2 *such elements and a part saying that there are at most* 5.

5.2. Translate the following class diagram into a first-order formula.



5.3. Translate the class diagram designed in Exercise 1 into a first-order formula.

5.4. ($\star$) Play the FIRST-ORDER FORMULAE PICTIONARY and try to succeed in at least one sentence without cheating.