# MUESL 

Fortran implementation
Release 2.21.5

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## Contents

Introduction ..... 3
1 FML: Numerical Library ..... 4
1.1 Core Routines ..... 6
1.2 File Input/Output ..... 108
1.3 Data Analysis Functions ..... 120
1.4 Operators ..... 156
1.5 Elementary Math Functions ..... 195
1.6 Specialized Math Functions ..... 249
1.7 Elementary Matrix Manipulation Functions ..... 269
1.8 Matrix Functions ..... 316
1.9 Polynomial Functions ..... 367
1.10 Optimization and Function Functions ..... 405
1.11 Sparse Matrices ..... 438
2 FGL: Graphical Library ..... 460
2.1 Global graphic settings ..... 461
2.2 Window's and figure's management ..... 475
2.3 Figure properties ..... 485
2.4 Figure annotation - Low level graphic object's manipulation ..... 514
2.5 High level plotting routines - 2D ..... 523
2.6 Interactive routines ..... 556
Index ..... 574

## Introduction

This document describes in details the MUESLI Fortran library routines.
MUESLI is freely available at the following address: https://perso.univ-rennes1.fr/edouard. canot/muesli

More information can be found in the following documents:

- MUESLI Installation Guide
- MUESLI User's Guide
- MUESLI Inside

The source code of MUESLI comes with many examples which can also give some help.
MUESLI is split in two parts, which correspond to the following Fortran modules:

- Basically, FML (Fortran Muesli Library) contains all necessary materials to numerically work with a dynamic array (dynamic in size, type and structure), called mfArray; all available routines are described in the first section of this guide.
To work with FML, your Fortran source code must include the statement:

```
use fml
```

- FGL (Fortran Graphics Library) contains graphic routines which use the mfArray objects; these routines are described in the second section of this guide.
To work with FGL, your Fortran source code must include the statements:

```
use fml
use fgl
```

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## Typographic convention

In this guide, the colored brackets ('[' and ']') indicate some optional arguments or other things; similarly, the colored pipe ('I') specifies alternates in a choice. These symbols are used only in interface definition of the routines.

Coloring is also used to differentiate source code and execution output. Lines from source code will be always displayed in this color, whereas input/output in a terminal will be always displayed in this color.

For sake of clarity, array constructors use black brackets ' [' and ']' (Fortran 2003) instead of ' (/' and '/)' (Fortran 95).

## About the name

Muesli: loose mixture of mainly rolled oats and often also wheat flakes, together with various pieces of dried fruit, nuts, and seeds. There are many varieties, some of which also contain honey powder, spices, or chocolate. (from https://en.wikipedia.org/wiki/Muesli)

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## 1 FML: Numerical Library

FML contains all routines to work with matrices and to do some linear algebra.
Routines beginning with ' mf ' are functions, whereas those beginning with ' ms ' are subroutines. Functions are used to return only one object (e.g. one mfArray); subroutines can also return many objects, but always via the mfOut facility.

FML also defines some new derived types: mfArray, mfUnit, mf_Out, mfMatFactor, mfTriConnect, mfTetraConnect, mf_DE_Options, mf_NL_Options.

Besides, FML defines public constant entities:

- an integer parameter:

```
MF_DOUBLE
```

which defines the working precision in MUESLI. In the current version, it stores the kind of double precision reals. In all this document, when Fortran numerical kind is not specified, real and complex stands for real(kind=MF_DOUBLE) and complex (kind=MF_DOUBLE), respectively.

- some mfArray parameters:

MF_EMPTY, MF_COLON (and its alias MF_ALL), MF_END, MF_NO_ARG
which are, respectively, an empty mfArray, the ':' pseudo-operator (as in Fortran 90 or MATLAB, but specific to FML), an automatic index for pointing to the end of a dimension of an mfArray and a pseudo-keyword to tell that an argument is not present in mfOut.

- some real (kind=MF_DOUBLE) parameters:

```
MF_PI
MF_EPS
MF_INF
MF_NAN
MF_E
MF_REALMAX
MF_REALMIN
```

and some useful mathematical auxiliairy constants:

```
MF_BESSEL_J0_ROOTS(:)
MF_BESSEL_J1_ROOTS (:)
```

(30 first non-zero roots for both Bessel functions $\mathrm{J}_{0}$ and $\mathrm{J}_{1}$, stored in increasing order)

- one complex (kind=MF_DOUBLE) parameter:

MF_I

- and the character parameters:

```
MF_MUESLI_VERSION
MF_COMPILER_VERSION
```

which store, respectively, the MUESLI library version and the compiler vendor and version. The boolean mfIsVersion routine must be used in order to compare each of these parameters with a given version number. Besides, other character functions may be used to retrieve additional information: MF_LAPACK_VERSION() (the Blas/Lapack library version) and MF_COMPILATION_CONFIG() (the configuration used to compile the whole Muesli library: Debug or Optim).

MF_INF and MF_NAN contain special IEEE values (resp. Infinity and Not-a-Number).

FML also defines the following global variables:

- integer variables:

STDERR
STDIN
STDOUT
which can be modified either directly by the user, or via the msSetStdIO routine.

- logical variables: MF_NUMERICAL_CHECK which allows the user to tell Muesli to do additional checks.

The available routines have been arranged into sub-parts:
Core Routines
File Input/Output
Data Analysis Functions
Operators
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Optimization and Function Functions
Sparse Matrices.

### 1.1 Core Routines

```
mfArray
mf
=, msAssign
msSet, mfGet
mfCount
msDisplay
mfDisplayColumns
msFormat
msRelease
msAutoRelease
mfOut
mf_Out
msPointer, msFreePointer
mfNbPointers
msEquiv
mf_Int_List
mf_Real_List
mfIsEmpty
mfIsEqual, mfIsNotEqual
mfIsLogical
mfIsReal
mfIsComplex
mfIsNumeric
mfIsDense
mfIsSparse
mfIsScalar,mfIsVector,mfIsMatrix
mfIsRow, mfIsColumn
mfIsPerm
All, Any
Shape, mfShape
Size, mfSize
mfInt
mfDble
mfCmplx
mfGetMsgLevel, msSetMsgLevel
mfGetTrbLevel, msSetTrbLevel
msSetColoredMsg
msPrintColoredMsg
msSetTermColor
msMuesliTrace
msGetStdIO, msSetStdIO
msFlush
msPause
msSetTermWidth, mfGetTermWidth
msSetAutoFilling, mfGetAutoFilling
msInitArgs, msFreeArgs
msSetAsParameter
msReturnArray, mfIsTempoArray
msFlops, mfFlops
mfIsFlopsOk
automatic and dynamic array (derived type) mfArray conversion
mfArray assignment mfArray data modification and extraction true values count mfArray pretty print columns used in pretty print mfArray printing format MUESLI objects deallocation mfArray conditional deallocation groups output arguments set of output arguments (derived type) smart pointer between f 90 array and mfArray nb of f 90 pointers pointing to an mfArray smart pointer between mfArray and f90 array
list of integers (derived type)
list of reals (derived type)
mfArray empty test
mfArray equality test
mfArray boolean test
mfArray real test
mfArray complex test
mfArray numeric test
mfArray dense storage test
mfArray sparsity test
mfArray scalar, vector and matrix test
mfArray kind of vector test
mfArray permutation vector test
test on boolean mfArray
shape of an mfArray
size of an mfArray
scalar integer conversion
scalar real conversion
scalar complex conversion
message level tuning error traceback tuning colorize Muesli messages on terminal print a colored user message on terminal set color for printing on terminal
helper for debugging purpose usual logical unit inquiry and modification I/O flush
user pause or timing pause
set and get terminal character width set and get out-of-range filling by msSet mfArray arguments' protection and release data protection mark and check temporary mfArray
nb of floating-point operations inquire if flops is available
```

```
mfGetAutoComplex, msSetAutoComplex
msEnableFPE, msDisableFPE
mfGetRoundingMode, msSetRoundingMode
MF_NUMERICAL_CHECK
mfUnit
msUsePhysUnits
msSetPhysDim
msSetPhysUnitAbbrev
mfHasNoPhysDim
mfHaveSamePhysDim
msPrepHashes
msPrintHashes
msPostHashes
msPrepProgress
msPrintProgress
msPostProgress
mfReadLine
msReadHistoryFile
msWriteHistoryFile
msClearHistory
msAddEntryInHistory
msRemoveLastEntryInHistory
mfToLower
mfToUpper
msFindIOUnit
mfIsVersion
msRequMuesliVer
MF_MUESLI_VERSION
MF_COMPILER_VERSION
MF_COMPILATION_CONFIG
```

conversion to complex
run-time floating-point exceptions trapping
floating-point rounding mode
debugging additional checks
physical unit (derived type)
Physical units activation
set physical unit
set user physical unit abbrevation
dimensionless test
dimension equality test
hashes print preparation
hashes print
hashes print conclusion
percent progress preparation
percent progress
percent progress conclusion
read a line from terminal with editing and history facilities
read a 'readline' history file
write the 'readline' history in a file
clear the 'readline' history
add an entry in the 'readline' history
remove last entry in the 'readline' history
lowering string's case
uppering string's case
automatically find a free IO unit number
test on version strings
check for a minimum Muesli version
MUESLI version
COMPILER version
Configuration used to compile MUESLI

See also:
File Input/Output
Data Analysis Functions
Operators
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Optimization and Function Functions
Sparse Matrices
mfArray
automatic and dynamic array (derived type)

## Description:

This is the fundamental derived type in MUESLI. Declaration must be made as follows:

```
type(mfArray) :: A
```

This derived type may contain real or complex numerical values, using a dense or sparse storage. All numerical values are stored in double precision, i.e. use 8 bytes of memory.

Special data types are:

- permutation vector (integer vector)
- boolean values (in dense storage only)

All possible data types are sumarized in figure 1.


Figure 1: mfArray data types
The data type stored in a mfArray may be retrieved by the following inquiring functions: mfIsReal, mfIsComplex, mfIsDense, mfIsSparse, mfIsNumeric, mfIsPerm, mfIsLogical.

See also: mf, msRelease, msPointer, msEquiv, msSaveAscii

## Description:

This function initializes and returns a temporary mfArray from some common Fortran types: integer, real (single or double), complex (single or double), of rank 0 to 2 (scalar, vector or matrix).

For example:

```
mf( [ 1, 3, 5, 7, 11 ] )
mf( [ 1.3, 2.05, -5.1e-3 ] )
```

Whatever the argument type of mf (integer, single precision real, double precision real), all numerical numbers are stored in mfArrays as double precision.

By default, this function creates row vectors, so use the following syntax if you want a column vector (avoiding an unnecessary copy):

```
mf( [ 1, 3, 5, 7, 11 ], transpose=.true. )
```

This latter optional transposition works also for matrices. By default, the transposition is not used (see examples below).

See also: mfArray

## Example(s):

```
    real (kind=MF_DOUBLE) :: tab \((2,4)\)
    \(\operatorname{tab}(:,:)=\) reshape ( [(1.5d0*i,i=1,8)], [2,4] )
    A = mf ( tab )
    call msDisplay( A, "A" )
    At \(=m f(\) tab, transpose=.true. )
    call msDisplay( At, "At" )
```

output:
$\mathrm{A}=$

| 1.5000 | 4.5000 | 7.5000 | 10.5000 |
| :--- | :--- | :--- | :--- |
| 3.0000 | 6.0000 | 9.0000 | 12.0000 |

At $=$
$1.5000 \quad 3.0000$
$4.5000 \quad 6.0000$
$7.5000 \quad 9.0000$
$10.5000 \quad 12.0000$

## mfArray assignment

This Fortran operator has been overloaded in MUESLI to make working with mfArray easy.
For example, you can assign any numerical object to A, as simply as:

```
A = [ 1, 2, 3, 4, 5 ]
```

The right hand side ( $R H S$ ) can be of type boolean, integer, real (single or double) or complex (single or double), of rank 0 to 2 (scalar, vector or matrix), or of course mfArray.
As the allocation of data is automatic during assignment, you should free each mfArray as soon as possible (via the msRelease routine), in order to decrease the memory usage.

Sometimes, you could be warned that an assignment is inefficient, when the $R H S$ is a temporary mfArray. In such a case, you could use the msAssign routine instead.

Besides, MUESLI allows to assign a dense mfArray A to a boolean, an integer, a real or a complex. So, a statement like:

```
f90_obj = A
```

is valid under the following assumptions:

- shapes (of $L H S$ and $R H S$ ) must match;
- types must be consistent (i.e. numeric mfArrays can be assigned to integer or real f90 arrays whereas boolean mfArrays must be assigned to logical f90 arrays - mfArrays data types are defined in figure 1).

See also: msPointer, msEquiv

## msAssign

more efficient mfArray assignment

Interface:

```
subroutine msAssign( dest, source )
type(mfArray), intent(in) :: source
type(mfArray) :: dest
```

Description:
Ordinarily, you should use the simple $=$ form of assignment. The msAssign routine is intended to make it efficiently, when the $R H S$ is a temporary mfArray.

For example, you should write:

```
call msAssign( C, A + B )
```

which avoids a copy of the data contained in $\mathrm{A}+\mathrm{B}$ (an internal pointer is used instead). In comparison, the statement:

$$
C=A+B
$$

involves such a copy.
See also: =
msSet

## mfArray data modification

Generic Calling syntax:

```
call msSet( data, x, a1 [, a2] )
```


## Description:

This routine is used to modify some elements of the mfArray x . The indices of these elements are specified by a1 and a2 for array sections, or only by a1 for scattered elements in the whole array (in this case, a1 is a long-column-index vector).

If a2 is not present, the mfArray x may be a vector, because only one index is given to specify which elements have to be changed. Note that if the index a1 (or a2) is out of range, the shape of x is modified accordingly, i.e. the array is extended (see the remarks at the end of this section).

If a1 (or a2) is a vector, more than one element can be modified; moreover, this vector doesn't need to contain consecutive indices.

If the modification affects more than one element, and if data contains only one value, this latter value is spread across all the target elements.
data may be of type real, complex or mfArray. It doesn't need to be of same data type as $x$. A conversion is then made, and concerns either data or x , according to the type.

Usually a1 and a2 are integers (scalar or vector), but they may be of type mfArray (in this latter case, they cannot be matrices).

In the special case where a1 (or a2) is MF_COLON (or its alias MF_ALL), then all the specified row(s) (or column(s)) are simultaneously modified. If data is empty (i.e. equal to MF_EMPTY, but only for a dense mfArray $x$ ), then the specified row(s) or column(s) are deleted.
a1 and/or a2 may be also special integer sequences, written under the form

```
i1 .to. i2 [.by. i3]
or i1 .by. i3 .to. i2
[ ( i, i = i1, i2 [, i3] ) ]
```

(The operator named '.step.' is an alias for '.by.')
These special integer sequences can be combined as

```
    int_seq .and. int_seq
or int_seq .but. i1
```

The step i3 is taken equal to 1 if it is not present. Note that a1, a2, i1 and i2 may also be replaced by an expression using MF_END (e.g. MF_END-1) which is an automatic alias for the end value of the corresponding dimension. Be aware that arithmetic involving MF END can use only addition or substraction.

Finally, the mfArray $x$ cannot be temporary.

## Remarks:

- to initialize a whole mfArray with 0 or 1 , the following routines are available: mfZeros and mfOnes.
- when the mfArray x has a sparse structure, data may be a scalar or a vector (dense or sparse). In such a case, only whole columns of x may be modified. Modifying one element is however allowed, in so far as indices of this element are not out of range (see below, however).
- when the mfArray x has a dense structure, data cannot be sparse.
- not all combinaisons for the types of a1 and a2 are valid. Use preferently the same type for these two arguments.
- out-of-range indices are accepted, except when a long-column index is used. For the dense storage, the mfArray is reallocated to the right shape. A message of kind "info" is however emitted (see msSetMsgLevel). During the reallocation, the new elements not targeted are initialized by a predefined value (see below). For the sparse storage, the assignment is done only if the value of nzmax is sufficient (see msSpReAlloc), and, of course, no new elements are added, apart those explicitly targeted by the command.
- the predefined value used to fill the out-of-range indices is by default the $N a N$ (special IEEE number), but this can be changed by use of the msSetAutoFilling routine.

See also:
mfGet

## mfGet

## mfArray data extraction

## Generic Calling syntax:

```
mfGet( x, a1 [, a2] )
```


## Description:

This routine extracts some elements (described by the indices a1 and a2, or - when x is a matrix only by the long-column index a1) of the mfArray $x$ and returns them in an mfArray. Note that if the index a1 (or a2) is out of range, an error results and an empty mfArray is returned.

If a1 (or a2) is a vector, more than one element are extracted; moreover, this vector doesn't need to contain consecutive indices.

Usually a1 and a2 are integers (scalar or vector), but they may be of type real or boolean mfArray.
In the special case where a1 (resp. a2) is MF_COLON (or its alias MF_ALL), then all the specified columns (resp. rows) are extracted.
a1 and/or a2 may be special integer sequences, written under the form

```
    i1 .to. i2 [.by. i3]
or i1 .by. i3 .to. i2
[ ( i, i = i1, i2 [, i3] ) ]
```

(The operator named '. step.' is an alias for '. by.')
These special integer sequences can be combined as

```
    int_seq .and. int_seq
or int_seq .but. i1
```

The step i3 is taken equal to 1 if it is not present. Note that a1, a2, i1 and i2 may also be replaced by an expression using MF_END (e.g. MF_END-1) which is an automatic alias for the end value of the corresponding dimension. Be aware that arithmetic involving MF_END can use only addition or substraction.

Remarks: Sparse mfArrays are partially supported:

- this routine can extract a scalar value, a row or a column, or a submatrix; in this latter case, the submatrix must be contiguous;
- when extracting rows, the indices for these rows must be contiguous (this limitation doesn't hold when extracting columns);
- the above mentioned special integer sequences (built via the .to. and .by. operators) cannot be used (not yet) for indices a1 or a2;
- when extracting a single row or a single column, resulted mfArray has a dense structure.

See also: msSet

## mfCount

## Interface:

```
function mfCount( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```


## Description:

Returns the number of true values in the columns (or rows) of a boolean mfArray (dense or sparse).
If dim is present, A is always considered as a matrix, and the routine processes each column (if dim $=1$ ) or each row (if dim $=2$ ).

If dim is not present, it returns the number of true values in the whole array.
When out is a vector, it is dense.
The behavior of this routine is similar to the count Fortran 90 intrinsic function.

```
Example(s):
    x = mfMagic( 3 )
    call msDisplay( x, "x" )
    call msDisplay( mfCount(x>5.0d0), "mfCount( x > 5.0d0 )" )
    call msDisplay( mfCount(x>4.0d0,dim=1), "mfCount( x > 4.0d0, dim=1 )" )
    call msDisplay( mfCount(x>4.0dO,dim=2), "mfCount( x > 4.0d0, dim=2 )" )
output:
x =
    8 3 4
    1 5 9
    6 7 2
mfCount( x > 5.0d0 ) =
    4
mfCount( x > 5.0d0, dim=1 ) =
    2 1 1
mfCount( x > 4.0dO, dim=2 ) =
    1
    2
    2
```

First calling syntax:

```
call msDisplay( x1 [, string1, x2, string2, ...] &
    [, unit ] [, head ] [, tail ] )
```

This routine displays an mfArray object (and other object types, as described below) on the screen or terminal, in a pretty form which is column oriented. Not all the decimal digits are printed: their number depends on the format used.

The object x may also be: a Fortran type, integer, real (single or double), complex (single or double), of rank 0 to 2 (scalar, vector or matrix); a MUESLI derived type, mfUnit and mf_Int_List.

Only for mfArrays:

- up to four pairs ( x , string ) can be simultaneously displayed.
- if the optional integer argument head (resp. tail) is present, then only the first (resp. last) rows will be printed. These two arguments can be both present. This convenience is useful to see the begin and the end of a very long column vector. For the case of a long row vector, it will display the head first values, followed by the tail last values, but only if the width of your terminal is sufficient. Internal tests are made about consistency between head, tail and the shape of the array.

For mfArrays (not boolean) and mfUnits, the optional argument unit allows the user to specify another consistent (i.e. of same physical dimension) unit for printing. For example, a length registered in meter (S.I. unit) may be printed in millimeter. The use of the constant parameter SI_unit enforces the routine to display, if possible, the appropriate S.I. unit.

Only for mf_Int_Lists and vectors of mf_Int_List:

- only one pair ( $x$, string ) can be displayed at a time.
- an additional optional boolean argument (compact) may be used to print the integers rows more compactly.

Second calling syntax:
call msDisplay( $x$, string, legend1 [, legend2, ..., legend9 ] )
This latter syntax enables the insertion of legends at the head of each column. Note the maximum number possible, 9 , and also that it is restricted to real data in the x mfArray.
N.B.: When displaying an object having a great number of columns, this routine will print first the number of columns adapted to the width of your terminal. If the output is redirected to a file, you may want to change this width: then use the MF_COLUMNS environment variable (it is the terminal character width, like in the UNIX command resize, and not the matrix column's number).

See also: msFormat, mfDisplayColumns

Example(s): Many outputs from this routine are shown as examples in the MUESLI User Guide.
mfDisplayColumns
columns used in pretty print

Interface:
function mfDisplayColumns( A, unit ) result( out )
type(mfArray), intent(in) : : A
type(mfUnit), intent(in), optional :: unit
type(mfArray) : : out

Description:
mfDisplayColumns returns the number of elements printed by the routine msDisplay on each row.
See also: msDisplay

Calling syntax:

```
call msFormat( [ mantissa, exponent ] )
```

The optional argument mantissa may be "short" (5 digits printed), "long" (15 digits printed) or "hex" (hexadecimal output). By default, the short format is used.

The optional argument exponent may be:

- "auto" (automatic format: fixed or exponential format);
- "sci" (scientific format: exponent is always printed);
- "eng" (engineer format: if the mfArray to be displayed is not a scalar, a scale factor is always used and its exponent is moreover a multiple of three).

By default, the automatic format is used.
In the case where this routine is called without any argument, default values for both keywords are used.

See also: msDisplay

## Example(s):

```
    x = .t. mfSqrt( mf([ 2, 3, 5 ]) )
    call msDisplay( x, "x (short format)" )
    call msFormat( "long" )
    call msDisplay( x, "x (long format)" )
```

output:

```
x (short format) =
    1.4142
    1.7321
    2.2361
x (long format) =
    1.41421356237310
    1.73205080756888
    2.23606797749979
    x = [ 1.5, 2.6E1, 3.7E2, 4.8E4 ]
    call msFormat( exponent="sci" )
    call msDisplay( x, "x (scientific format)" )
    call msFormat( exponent="eng" )
    call msDisplay( x, "x (engineer format)" )
```

output:

```
x (scientific format) =
    1.5000E+00 2.6000E+01 3.7000E+02 4.8000E+04
x (engineer format) =
    1.0E+03 *
    0.0015 0.0260 0.3700 48.0000
```


## msRelease

When you have finished to work with an mfArray, you may deallocate its internal array by using:

```
call msRelease( x1 [, x2, x3, x4, x5, x6, x7 ] )
```

Up to seven mfArrays can be simultaneously released.
Remark: After execution of your program, some compilers or tools may report a warning about some memory leaks. Use of this routine for all of the automatic variables (described below) avoids such warnings.

This routine can be used to free the following derived types: mfArray, mfMatFactor, mfTriConnect, $m f$ TetraConnect, $m f$ _Int_List, $m f$ _NL_Options and mf_DE_Options (only one variable may be freed at a time, except for an mfArray object).

See also: msAutoRelease, msSetAsParameter, msExitFgl

## mfArray conditional deallocation

This routine is similar to msRelease:

```
call msAutoRelease( x1 [, x2, x3, x4, x5, x6, x7 ] )
```

but the deallocation is done only if the mfArray is temporary.
Is is used only in the context of a user-routine which takes arguments of type mfArray which can be temporary objects.

See also: msRelease
mfOut

This function is used to group output arguments in calls of some MUESLI subroutines. For example:

```
call msLU( mfOut(L,U,p), A )
```

This facility is only intended to clearly identify the output arguments from the input ones. It is reminiscent of the Matlab syntax:

$$
[\mathrm{L}, \mathrm{U}, \mathrm{p}]=\operatorname{lu}(\mathrm{A})
$$

Arguments of mfOut match always the following MUESLI derived types: mfArray or mfMatFactor.
Case of optional arguments: If some of these arguments are optional, they must be called in order, not by use of keyword as in Fortran 90 syntax. Moreover, for multiple optional arguments, one of them can be omitted by using the special mfArray MF_NO_ARG, as in the following example:

```
call msOdeSolve( mfOut( y, status, tolout, MF_NO_ARG, solve_log),
    deriv, t_span, y_0 )
```

Indeed, refering to the msOdeSolve routine, the yp and solve_log are both optional, and if the user wants to retrieve only the second one, he has to use MF_NO_ARG instead of yp.

Remarks: actually, the mfOut function returns an object of type mf_Out. You don't need to use this latter derived type unless you want to define yourself new routines which use mfOut to group output arguments.

See also: mfArray, mfMatFactor, mf_Out

## mf_Out

## Description:

This is a derived type in MUESLI. You need to employ this type only if you wish to define yourself subroutines whose output arguments are grouped with the mfOut function.

See also: mfArray, mfOut

# smart pointer between f90 array and mfArray 

Generic Interface:

```
subroutine msPointer( A, f90_ptr )
    type(mfArray) :: A
    real(kind=MF_DOUBLE), pointer :: f90_ptr(:[,:])
or complex(kind=MF_DOUBLE), pointer :: f90_ptr(:[,:])
```


## Description:

This routine is approximately equivalent to:

```
f90_ptr => A
```

and then, you can access the internal data of A via a classical Fortran array. This association is not always possible, because the types of A and f90_ptr must match. Ranks however doesn't need to match: you may have a rank- 1 array pointing to a rank- 2 array; in this latter case, we could see f 90 -ptr as a "long column pointer".

A cannot be temporary, because at any time it can be deleted, and then f90_ptr would become hangling.

## Warning:

During all the time when $f 90$ _ptr points to A , some internal properties of A are locked. So, it is strongly recommended to use the routine msFreePointer: (i) to nullify the pointer f90_ptr and (ii) to unlock properties of A. Be aware that
f90_ptr => null()
is not sufficient (this way will not unlock the internal mfArray properties of A).
See also: mfNbPointers, msFreePointer, msEquiv
msFreePointer
smart pointer release

Generic Interface:

```
subroutine msFreePointer( A, f90_ptr )
    type(mfArray) :: A
    real(kind=MF_DOUBLE), pointer :: f90_ptr(:[,:])
or complex(kind=MF_DOUBLE), pointer :: f90_ptr(:[,:])
```

Description:
This routine releases the link made by msPointer, and unlocks the internal properties of A.
See also: mfNbPointers, msPointer
mfNbPointers
nb of $f 90$ pointers pointing to an mfArray

Generic Interface:
function mfNbPointers( A ) result( n )
type(mfArray) :: A
integer :: n

Description:
This routine returns the number of Fortran 90 pointers pointing to A.
See also: msPointer, msFreePointer
msEquiv
smart pointer between mfArray and f90 array

## Generic Interface:

```
subroutine msEquiv( f90_array, A )
    real(kind=MF_DOUBLE), target :: f90_array(:[,:])
or complex(kind=MF_DOUBLE), target :: f90_array(:[,:])
    type(mfArray) :: A
```


## Description:

This routine is approximately equivalent to

```
A => f90_array
```

but the mfArray A becomes restricted, in the sense that you can modify the data but not the shape of your f90_array, nor its allocation status. As a consequence, you can use most of (but not all) MUESLI routines to work with your f90_array, via the mfArray A.

When f 90 _array is a rank- 1 array (here, renamed as $f 90$ _vector), the mfArray $A$ is by default a column vector. Use the optional argument new_shape to specify the new virtual dimensions of A, as follows:

```
call msEquiv( f90_vector, A, new_shape=[n1,n2] )
```

Of course, you must have consistance, i.e. the product $n_{1} n_{2}$ must be equal to the number of elements of f90_vector.

## Remark:

To release the link between A and f90_array, you must use the msRelease routine. Don't write

$$
\text { A } \Rightarrow \text { null ( }
$$

because usually A is not a pointer, it is just a derived type, i.e. a structure which includes the numerical data in a manner that is hidden to the user. Even if A has been declared as a pointer to an mfArray, this is not the good way to release properly A). If you forget to release this link, you could obtain an error when trying an assignment with A.

See also: msPointer

## Description:

This is a derived type in MUESLI. You may employ it for storing a list of integers of arbitrary length. It's definition is:

```
type, public :: mf_Int_List
    integer, allocatable :: list(:)
end type mf_Int_List
```

You must allocate yourself the internal component list, which is an ordinary Fortran array of integers. Also, you may declare and use an array of mf_Int_List, which is useful to work with a set of varying length lists (see the second example below).
msDisplay and msRelease routines may be used to, respectively, print and free this derived type. These two routines can be also used with an array of mf_Int_List.

## Example(s):

```
! type(mf_Int_List) :: int_list
allocate( int_list%list(7) )
int_list%list(:) = [ (i, i = 1, 7) ]
call msDisplay( int_list, "int_list")
! call msRelease( int_list )
```

output:

```
int_list =
1
    ! type(mf_Int_List) :: int_list_vec(5)
    n = 6
    allocate( int_list_vec(1)%list(n) )
    int_list_vec(1)%list(:) = [ (i, i = 1, n) ]
    n = 15
    allocate( int_list_vec(3)%list(n) )
    int_list_vec(3)%list(:) = [ (i, i = 100+1, 100+n) ]
    n = 9
    allocate( int_list_vec(5)%list(n) )
    int_list_vec(5)%list(:) = [ (i, i = 100000+1, 100000+n) ]
    call msDisplay( int_list_vec, "int_list_vec")
    ! call msRelease( int_list_vec)
```

output:

```
int_list_vec =
    1
    <EMPTY>
```



```
    <EMPTY>
    100001}10100002 100003 100004 100005 100006 100007 100008 100009 
```

mf_Real_List

## Description:

This is a derived type in MUESLI. You may employ it for storing a list of reals of arbitrary length. It's definition is:

```
type, public :: mf_Real_List
    integer, allocatable :: list(:)
end type mf_Real_List
```

You must allocate yourself the internal component list, which is an ordinary Fortran array of reals. Also, you may declare and use an array of mf_Real_List, which is useful to work with a set of varying length lists (see the second example below).
msDisplay and msRelease routines may be used to, respectively, print and free this derived type. These two routines can be also used with an array of mf_Real_List.

## Example(s):

```
! type(mf_Real_List) :: real_list
allocate( real_list%list(7) )
real_list%list(:) = [ (i+0.5d0, i = 1, 7) ]
call msDisplay( real_list, "real_list")
! call msRelease( real_list )
```

output:

```
int_list =
1.500}22.500 3.500 4.500 5.500 6.500 7.500 
    ! type(mf_Real_List) :: int_real_vec(5)
    n = 6
    allocate( real_list_vec(1)%list(n) )
    real_list_vec(1)%list(:) = [ (i+0.5d0, i = 1, n) ]
    n = 11
    allocate( real_list_vec(3)%list(n) )
    real_list_vec(3)%list(:) = [ (i+0.5d0, i = 10+1, 10+n) ]
    n = 9
    allocate( real_list_vec(5)%list(n) )
    real_list_vec(5)%list(:) = [ (i+0.5d0, i = 100+1, 100+n) ]
    call msDisplay( real_list_vec, "int_list_vec")
    ! call msRelease( real_list_vec )
```

output:

```
int_list_vec =
1.500 2.500 3.500 4.500 5.500 6.500
<EMPTY>
11.500 12.500 13.500 14.500 15.500 16.500
<EMPTY>
101.500 102.500 103.500 104.500 105.500 106.500 107.500
```

mfIsEmpty
mfArray empty test

Interface:

```
function mfIsEmpty( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Tests if the mfArray A is empty, i. e. has a null size (number of rows multiplied by nupmber of columns). Note that an sparse matrix (non empty) may have a number of nonzero elements (see mfNnz) that is zero.

See also: Shape, Size

## mfIsEqual

Interface:

```
function mfIsEqual( A1, A2 ) result( bool )
    type(mfArray), intent(in) :: A1, A2
    logical :: bool
```


## Description:

Tests if two mfArrays are numerically equal, whatever the structure is (i.e. sparse or dense, real or complex). The shapes can even mismatch, and can take the zero value.

Comparison between boolean mfArray are excluded: All combined with '. eqv.' should be used instead.
Remarks: This function returns a scalar logical, whereas '==' returns a boolean mfArray.
For example:

```
mfIsEqual( A1, A2 )
```

is equivalent to:

```
All( A1 == A2 )
```

but the former statement is much more efficient for sparse mfArrays.
See also: ==, All, mfAll, . eqv .

## mfIsNotEqual

Interface:

```
function mfIsNotEqual( A1, A2 ) result( bool )
    type(mfArray), intent(in) :: A1, A2
    logical :: bool
```

Description:
Returns the negation of mfIsEqual (A1, A2).
See also: mfIsEqual

## mfIsLogical

mfArray boolean test

Interface:

```
function mfIsLogical( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Returns '.true.' if the mfArray A is boolean, i.e. if A results from any boolean operation (' $==$ ', ' $>$ ', etc.) on mfArrays.

See also: ==

## mfIsReal

Interface:

```
function mfIsReal( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Returns '.true.' if the mfArray A is numeric, but not complex. The internal structure of A may be dense or sparse.

See also: mfIsNumeric, mfIsComplex

## mfIsComplex

Interface:

```
function mfIsComplex( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Returns '.true.' if the mfArray A is numeric and complex. The internal structure of A may be dense or sparse.

See also: mfIsNumeric, mfIsReal
mfIsNumeric
mfArray numeric test

Interface:

```
function mfIsNumeric( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Returns '.true.' if the mfArray A is numeric (real or complex). The internal structure of A may be dense or sparse.

See also: mfIsReal, mfIsComplex

## mfIsDense

Interface:

```
function mfIsDense( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```

Description:
Returns '.true.' if internal structure of A is dense. The data may be boolean, real or complex. See also: mfIsSparse

## mfIsSparse

Interface:

```
function mfIsSparse( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Returns '.true.' if internal structure of A is sparse. The data may be real or complex.
See also: mfIsDense

## mfIsScalar

Interface:

```
function mfIsScalar( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```

Description:
Returns '.true.' if A is a scalar.
See also: mfIsVector, mfIsMatrix
mfIsVector

Interface:
function mfIsVector( A ) result( bool )
type(mfArray), intent(in) :: A
logical :: bool

Description:
Returns '.true.' if A is a vector.
Remark: A scalar is not a vector, neither a matrix.
See also: mfIsScalar, mfIsMatrix, mfIsRow, mfIsColumn

Interface:
function mfIsMatrix( A ) result( bool )
type(mfArray), intent(in) :: A
logical :: bool

Description:
Returns '.true.' if A is a matrix.
Remark: A vector is not a matrix.
See also: mfIsScalar, mfIsVector
mfIsRow

Interface:

```
function mfIsRow( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```

Description:
Returns '.true.' if A is a row vector.
See also: mfIsVector, mfIsColumn
mfIsColumn
mfArray kind of vector test

Interface:

```
function mfIsColumn( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```

Description:
Returns '.true.' if A is a column vector.
See also: mfIsVector, mfIsRow

## mfIsPerm

mfArray permutation vector test

Interface:

```
function mfIsPerm( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Returns '.true.' if A has the type "permutation vector".
To make a real checking for a valid permutation, use mfCh ChPerm.
See also: mfIsLogical, mfIsNumeric

Interface:

```
function All( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Returns '.true.' if all elements of the mfArray A are $T R U E$ (therefore, A must be a boolean mfArray). See also: mfAll, Any

Any
test on boolean mfArray

Interface:

```
function Any( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```

Description:
Returns '.true.' if at least one element of the mfArray A is $T R U E$ (therefore, A must be a boolean mfArray).

See also: mfAny, All

Shape, mfShape
shape of an mfArray

First interface:

```
function Shape( A ) result( out )
    type(mfArray), intent(in) :: A
    integer :: out(2)
```


## Description:

Returns the shape of the mfArray A (dense or sparse); the returned integer array is always of rank 2, because A is a matrix.

Other interface:

```
function Shape( Qhouse ) result( out )
    type(mfMatFactor), intent(in) :: Qhouse
    integer :: out(2)
```


## Description:

Returns the shape of the mfMatFactor Qhouse, which comes from the QR decomposition of a sparse matrix; the returned integer array is always of rank 2, because an A is a matrix.

Remark: the mfShape routine is similar, but returns an mfArray instead.
See also: Size, mfMatFactor

## Size, mfSize

size of an mfArray

First interface:

```
function Size( A, idim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: idim
    integer :: out
```


## Description:

If idim is not present, returns the total number of elements of the mfArray $A$ (dense or sparse).
If idim is present ( 1 or 2 ), returns the specified dimension.

Other interface:

```
function Size( A, idim ) result( out )
    type(mfMatFactor), intent(in) :: A
    integer, intent(in), optional :: idim
    integer :: out
```


## Description:

If idim is not present, returns the total number of elements of the mfMatFactor $A$ (which comes from some decomposition of a sparse matrix).

If idim is present (1 or 2 ), returns the specified dimension.

## Remarks:

- the mfSize routine is similar, but returns an mfArray instead;
- for a sparse matrix, the size returned includes the zero elements: it is the logical size, not the physical size; so, for any storage kind, the size is always the product of the two dimensions returned by shape. Use the mfNnz routine to get the number of non zeros.

See also: Shape
mfInt

Interface:

```
function mfInt( A ) result( out )
    type(mfArray), intent(in) :: A
    integer :: out
```

Description:
This routine is a facility to convert a scalar mfArray A in an ordinary f90 integer.
A warning is emitted during conversion, in case of loss of precision.
See also: mfDble, mfCmplx
mfDble

Interface:

```
function mfDble( A ) result( out )
    type(mfArray), intent(in) :: A
    real(kind=MF_DOUBLE) :: out
```


## Description:

This routine is a facility to convert a scalar mfArray A in an ordinary f90 real.
If $A$ is complex, returns only the real part.
See also: mfInt, mfCmplx
mfCmplx
scalar complex conversion

Interface:

```
function mfCmplx( A ) result( out )
    type(mfArray), intent(in) :: A
    complex(kind=MF_DOUBLE) :: out
```

Description:
This routine is a facility to convert a scalar mfArray A in a complex.
See also: mfInt, mfDble
mfGetMsgLevel
message level tuning

Calling syntax:

```
level = mfGetMsgLevel( )
```

returns an integer, ranged from 0 to 3 , which is the current message level. See the msSetMsgLevel routine for a description of the message level.

See also: mfGetTrbLevel, msSetTrbLevel, msMuesliTrace

## msSetMsgLevel

## message level tuning

Calling syntax:
call msSetMsgLevel ( level )
where level is an integer, ranged from 0 to 3 , which specifies the message level, described as follows:

- message level $=3$ : all messages are printed (verbose mode)
- message level $=2$ : messages of kind 'ERROR' and 'Warning' are printed [default]
- message level $=1$ : only messages of kind 'ERROR' are printed
- message level $=0$ : nothing is printed (quiet mode)

There exists three kinds of messages emitted by the MUESLI library; they are tagged as "info", "Warning" and "ERROR". Only the last kind (the most important) stops temporarily the run-time execution via a "pause" statement, which requires the action of the user to resume the execution. The cause of these errors are generally due to a programming fault; this is the reason why the program stops.

As usual for $I E E E-754$ arithmetic, floating-point exceptions give $\operatorname{Inf}$ and $N a N$ values which quietly propagate. See msEnableFPE to change this default behavior.
N.B.: when message level is equal to 0 , programming errors (of kind "ERROR") kill the program. This may be useful when the executable must be launched via a batch system.

See also: mfGetMsgLevel, mfGetTrbLevel, msSetTrbLevel, msMuesliTrace

Calling syntax:

```
when = mfGetTrbLevel( )
```

returns a character (len=4), which is the current error traceback policy. See the msSetTrbLevel routine for a description of the error traceback policy.

See also: mfGetMsgLevel, msSetMsgLevel, msMuesliTrace

Calling syntax:
call msSetTrbLevel( when | level )
where when is a character (len=4) which specifies the error traceback policy, described as follows:

- when $=$ "all": error traceback is produced for all kind of messages (i.e. ERRORS, Warnings and infos)
- when $=$ "auto": error traceback is produced only for ERRORS [default]
- when $=$ "none": no error traceback

An alternative is to use the integer level argument, which can be equal to $0,1,2$ or 3 . An message level which is greater than or equal to level triggers the traceback. For example, when level is used with the value 2, only messages of kind 'ERROR' and 'Warning' produce a traceback.
N.B.: not all compilers are able to produce an error traceback on demand. See the MUESLI Inside document for more information.

See also: mfGetTrbLevel, mfGetMsgLevel, msSetMsgLevel, msMuesliTrace

## msSetColoredMsg

colorize messages on terminal

Calling syntax:

```
call msSetColoredMsg( "on" | "off" )
```

Description:
Colorizes all the Muesli messages written on the terminal. Four colours are used:

- red for the ERRORS;
- orange for the Warnings;
- yellow for the infos;
- grey-italic for the traceback.

Disabling the colors are useful before redirecting the program output to a file. Default is "on".
See also: mfGetMsgLevel, msSetMsgLevel, msMuesliTrace
msPrintColoredMsg
print a colored user message on terminal

Interface:
subroutine msPrintColoredMsg( fmt, msg, color )
character(len=*), intent(in) :: fmt, msg, color

Description:
Prints the string msg in this color and using format fmt.
Colors are limited to the following ones: "green", "red", "orange", "yellow" and "grey".
See also: msSetTermColor

## msSetTermColor

set color for printing on terminal

Interface:

```
subroutine msSetTermColor( color )
    character(len=*), intent(in) :: color
```


## Description:

Sets the color for printing user message with msPrintColoredMsg.
Colors are limited to the following ones: "green", "red", "orange", "yellow" and "grey".
To revert to the default color, set color to "normal".
msGetStdIO
usual logical unit inquiry

Interface:

```
subroutine msGetStdIO( stdin, stdout, stderr )
    integer, intent(out), optional :: stdin, stdout, stderr
```

Description:
Gets the Fortran unit for stdin (usually 5), stdout (usually 6 ) and stderr (usually 0 ).
See also: msSetStdIO

## msSetStdIO

usual logical unit modification

Interface:

```
subroutine msSetStdIO( stdin, stdout, stderr )
    integer, intent(in), optional :: stdin, stdout, stderr
```

Description:
Used to change the Fortran units.
See also: msGetStdIO
msFlush
I/O flush

Interface:

```
subroutine msFlush( unit )
    integer, intent(in) :: unit
```

Description:
Flushes the I/O unit unit.
msPause
user pause or timing pause

Calling syntax:
call msPause( [message] [, duration] )
If the optional argument message is present (character (len=*) ), this routine prints the message and waits for a [RETURN] from the user.

If the optional argument duration is present (real), this routine sleeps for the specified number of seconds. If this second argument is alone, the keyword duration= must appear in front of it.

## msSetTermWidth

set terminal character width

Syntax:
call msSetTermWidth( width | "auto" )

## Description:

Set manually the character width of the terminal, for pretty print with the msDisplay routine. The argument width must be an integer greater than 40.

If the character string "auto" is used as argument, then the character width of the terminal will be determined automatically, either from the MF_COLUMNS environment variable, or from the actual value (using the 'resize' shell command).

See also: mfGetTermWidth, msDisplay, mfDisplayColumns

## mfGetTermWidth

## get terminal character width

## Interface:

function mfGetTermWidth() result ( out )
integer :: out

Description:
Get the character width of the terminal. The returned value is actually an internal value used for pretty print in the msDisplay routine. This last value is obtained from:

- first, the value set manually via the msSetTermWidth routine;
- then, from the MF_COLUMNS environment variable;
- last, via the 'resize' Unix command; if this latter command is not available on your system, an information message is displayed and the default value 80 is used.

See also: msSetTermWidth, msDisplay, mfDisplayColumns

## msSetAutoFilling

set out-of-range filling by msSet

Syntax:
call msSetAutoFilling ( r )

Description:
Set manually the value of the real used to fill the elements of an mfArray by msSet when the indices are out-of-range.

Default value is the special IEEE number $N a N$, which means Not-a-Number.
See also: mfGetAutoFilling
mfGetAutoFilling
get out-of-range filling by msSet

Interface:

```
function mfGetAutoFilling() result ( r )
    real(kind=MF_DOUBLE) :: r
```


## Description:

Get the value of the real used to fill the elements of an mfArray by msSet when the indices are out-ofrange.

Default value is the special $I E E E$ number $N a N$, which means $N o t-a-N u m b e r$, but this number may be changed by use of msSetAutoFilling.

## msInitArgs

## mfArray arguments' protection

## Interface:

```
subroutine msInitArgs( x1, &
    x2, x3, x4, x5, x6, x7 )
    type(mfArray) :: x1
    type(mfArray), optional :: x2, x3, x4, x5, x6, x7
```

Description:
Inside a user-defined routine, this routine protects some mfArray arguments against a (library) deallocation, in case where they are temporary.

This routine should always be paired with the other routine msFreeArgs.
msFreeArgs

## Interface:

```
subroutine msFreeArgs( x1, &
x2, x3, x4, x5, x6, x7 )
type(mfArray) :: x1
type(mfArray), optional :: x2, x3, x4, x5, x6, x7
```

Description:
Inside a user-defined routine, this routine ends the argument's protection set by msInitArgs.
This routine should always be paired with the other routine msInitArgs.

## msSetAsParameter

## data protection

## Interface:

```
subroutine msSetAsParameter( x1, x2, x3, x4, x5, x6, x7, param )
    type(mfArray) :: x1
    type(mfArray), optional :: x2, x3, x4, x5, x6, x7
    logical :: param
```

Description:
When used with "param=.true.", protects the data (and type) inside each mfArrays. Lets the user creating pseudo parameters mfArrays, because the Fortran attribute parameter cannot be used in any circumstances.

Up to seven mfArrays can be simultaneously set.
Before freeing an mfArray via the msRelease routine, the user must apply msSetAsParameter with "param=.false.".
Of course, by default, an mfArray is not data-protected.
msReturnArray
temporary mfArray mark

Interface:

```
subroutine msReturnArray( A )
```

    type(mfArray) :: A
    
## Description:

At the end of a user-defined function returning an mfArray, marks this object as temporary.
Avoids memory leaks during the execution of your program.
See also: msInitArgs, msFreeArgs, mf IsTempoArray
mfIsTempoArray

Interface:
function mfIsTempoArray( A ) result( bool )
type(mfArray), intent(in) : : A
logical :: bool

Description:
Check if the mfArray A is marked as temporary.
See also: msInitArgs, msFreeArgs, msReturnArray
msFlops
nb of floating-point operations

## Interface:

subroutine msFlops( count )
integer*8, intent(out) :: count

## Description:

Returns the number of floating-point operations made by the processor, since the last initialization. Initialization must be made by:
call msFlops( init=0 )
The other call:
call msFlops ( count )
returns the flops count since the initialization.
count $\geq 0$, except if the program has been linked with the dummy version of PAPI (dummy_papi.o), or if the OS doesn't support hardware counting (e.g. the linux kernel has not been patched for use of the PERFCTR library); in the latter case, msFlops always returns the value -1 .

Remarks: requires a specific version of the PAPI library (cf. in papi/version)
See also: mfFlops, mfIsFlops0k

## mfFlops

## nb of floating-point operations

Calling syntax:

```
count = mfFlops( )
```

Description:
Returns the number of floating-point operations made by the processor, since the last initialization, in the mfArray count.

Initialization must be made by:
call msFlops( init=0 )
count $\geq 0$, except if the program has been linked with the dummy version of PAPI (dummy_papi.o), or if the OS doesn't support hardware counting (e.g. the linux kernel has not been patched for use of the PERFCTR library); in the latter case, mfFlops always returns the value -1 .

Remarks: requires a specific version of the PAPI library (cf. in papi/version)
See also: msFlops, mfIsFlopsOk
mfIsFlopsOk
inquire if flops is available

Calling syntax:

```
bool = mfIsFlopsOk( )
```

Description:
Returns a logical revealing the availability of the mfFlops routine.
See also: mfFlops, msFlops

## msSetAutoComplex

set auto conversion to complex

Syntax:

```
call msSetAutoComplex( .true. | .false.)
```

Description:
By default, real mfArrays are automatically converted in complex when operation like $\sqrt{-1}$ are made.
When calling this routine with argument equal to .false., conversion to complex is not done, then resulting to a special $I E E E$ value (Inf or $N a N$, according to the operation used).

See also: mfGetAutoComplex
mfGetAutoComplex
get auto conversion to complex

Interface:
function mfGetAutoComplex() result( bool )
logical, intent(in) :: bool

Description:
Returns a boolean specifying whether MUESLI convert real to complex for some operation (like $\sqrt{-1}$ ).

See also: msSetAutoComplex
msEnableFPE

# run-time floating-point exceptions trapping 

## Interface:

```
subroutine msEnableFPE( exception, full_trapping )
character(len=*), intent(in) :: exception
logical, intent(in), optional :: full_trapping
```


## Description:

Enables, at run-time, the trapping for a given floating-point exception, i.e. the program stops when the specified exception(s) is (are) encountered.
exception may be one of: "overflow", "zero_divide", "invalid", "underflow" or "usual_exceptions".
"usual_exceptions" is a shortcut for the main three traditional exceptions: "overflow", "zero_divide", "invalid".
full_trapping is an optional argument which allows the trapping inside the MUESLI library sources (default is .false.). Note that this features is usually reserved to the MUESLI developpers.

Remark: The call may be located anywhere is the program. Usually, it is added at the beginning of the program. However, exceptions' trapping may be added or removed later on.

See also: msDisableFPE

## msDisableFPE

## run-time floating-point exceptions trapping

## Interface

```
subroutine msDisableFPE( exception )
character(len=*), intent(in) :: exception
```

Description:
Disables, at run-time, the trapping for a given floating-point exception, i.e. the program doesn't stop when the specified exception(s) is (are) encountered, but produces accordingly $\operatorname{Infs}$ and $N a N s$
exception may be one of: "overflow", "zero_divide", "invalid", "underflow" or "usual_exceptions".
"usual_exceptions" is a shortcut for the main three traditional exceptions: "overflow", "zero_divide", "invalid".

See also: msEnableFPE

## mfGetRoundingMode

## floating-point rounding mode

Calling syntax:
rounding_mode $=\mathrm{mfGetRoundingMode}($ )

Description:
Returns the floating-point rounding mode in a character string. The IEEE-754 rounding mode may be equal to:

- "to_zero": IEEE-754 rounding to zero
- "nearest": IEEE-754 rounding to nearest
- "up": IEEE-754 rounding to pos. infinity
- "down": IEEE-754 rounding to neg. infinity

See also: msSetRoundingMode

## msSetRoundingMode

## floating-point rounding mode

## Interface:

```
subroutine msSetRoundingMode( rounding_mode )
character(len=*) :: rounding_mode
```


## Description:

Set the floating-point rounding mode to the specified mode. The IEEE-754 rounding mode may be equal to:

- "to_zero": IEEE-754 rounding to zero
- "nearest": IEEE-754 rounding to nearest
- "up": IEEE-754 rounding to pos. infinity
- "down": IEEE-754 rounding to neg. infinity

Warning: The ISO/IEC TR 15580:1998(E) Technical Report, concerning the behavior of Fortran programs, specifies (Section 2.4): "In a procedure, the processor ensures that the flags for rounding have the same values on return as on entry." In the current MUESLI implementation, the rounding mode is set via a C routine, and can violate the preceding requirement; then the use of the msSetRoundingMode routine is left to the programmer responsability.

See also: mfGetRoundingMode

## MF_NUMERICAL_CHECK

debugging additional checks

## Interface:

logical :: MF_NUMERICAL_CHECK

Description:
This logical global variable is used by the library to do additional checks (validity of certain results, presence of $N a N$ and, if required, precision of the jacobian matrix provided by the user, ...).

When Muesli is compiled in DEBUG mode (usually by the developpers of Muesli), this variable is initially set to TRUE. Otherwise, for ordinary users of the library (which certainly compile it in OPTIM mode), it is initially set to $F A L S E$.

For debugging purpose, you can set its value temporarily to $T R U E$, especially when using the ODE/DAE solvers (msOdesolve, msDaesolve), or the minimization routines (mf/msLsqNonLin).

See also: mf DE_Options
mfUnit

## physical unit (derived type)

## Description:

The Fortran derived type mfUnit (hidden) contains some usual physical units and constants. It can be used to initialize the physical unit of an mfArray, via the operators ' $*$ ' and ' $=$ '.

No declarations has to be done. Instead, the program may refer to some predefined mfUnits, as e.g.:

```
y = 0.25d0*u_kg
v = 0.10d0*c_speed_of_light
```

Remarks:

- see the MUESLI User Guide document for a complete list of the available physical units, physical constants and multipliers.
- physical units begin with 'u_', multipliers begin with 'm-', whereas physical constants begin with 'c_'.

See also: *, =, msUsePhysUnits, msSetPhysDim, mfHasNoPhysDim, mfHaveSamePhysDim
msUsePhysUnits
Physical units activation

Interface:
subroutine msUsePhysUnits( mode )
character(len=*), intent(in) :: mode

Description:
Activates or de-activates the computation of physical units; mode must be "on" or "off".
Default is "off".
See also: mfUnit, msSetPhysDim

## msSetPhysDim

set physical unit

The first interface:

```
subroutine msSetPhysDim( A,\&
Mass, Length, Time, &
Temp, Electr_Intens, Mole, Lumin )
type(mfArray), intent(in out) :: A
real(kind=MF_DOUBLE), intent(in), optional :: Mass, &
Length, &
Time, &
Temp, &
Electr_Intens, &
Mole, &
Lumin
```

is used for setting the physical unit of the mfArray A, from some basic physical quantities.
The second interface:

```
subroutine msSetPhysDim( A, B )
    type(mfArray), intent(in out) :: A
    type(mfArray), intent(in) :: B
```

allows the user to set the physical unit of the mfArray A, by copying that of the mfArray B.
See also: mfUnit, msUsePhysUnits, mfHasNoPhysDim, mfHaveSamePhysDim
msSetPhysUnitAbbrev
set user physical unit abbrevation

Interface:

```
subroutine msSetPhysUnitAbbrev( user_unit, abbrev )
    type(mfUnit) :: user_unit
    character(len=*) :: abbrev
```

is used for setting the abbrevation of a user physical unit.
See also: mfUnit, msUsePhysUnits,
mfHasNoPhysDim

Interface:

```
function mfHasNoPhysDim( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```

Description:
Tests if the mfArray A is dimensionless.
See also: mfUnit, msUsePhysUnits, msSetPhysDim mfHaveSamePhysDim
mfHaveSamePhysDim

Interface:

```
function mfHaveSamePhysDim( A, B ) result( bool )
    type(mfArray), intent(in) :: A, B
    logical :: bool
```

Description:
Tests if the mfArrays A and B have the same physical dimensions.
See also: mfUnit, msUsePhysUnits, msSetPhysDim, mfHasNoPhysDim
msPrepHashes
hashes print preparation

Calling syntax:

```
call msPrepHashes( start, end )
```

Description:
Prepares data before calls to msPrintHashes.
start and end are numerical values used to monitor the progress bar. They must be both of integer or real (double) type.

See also: msPrintHashes, msPostHashes

## msPrintHashes

Calling syntax:

```
call msPrintHashes( val )
```


## Description:

Makes on the terminal a progress bar by printing hashes (\#).
Usually, the call is located inside a loop; val is the numerical value (either integer or real double) monitoring the progression. It's numerical extremum values have to be passed as arguments of msPrepHashes.

See also: msPostHashes, msPrintProgress

```
Example(s):
    integer :: i_start, i_end, i
    call msPrepHashes( i_start, i_end )
    do i = i_start, i_end
    <some computation>
    call msPrintHashes( i )
end do
call msPostHashes( )
or:
double precision :: time_start, time_end, time
call msPrepHashes( time_start, time_end )
time = time_start
do while( time <= time_end)
    <some computation>
    ...
    call msPrintHashes( time )
    time = <new value of time, increasing but may be not linear with iterations>
end do
call msPostHashes( )
```

msPostHashes
hashes print conclusion

Calling syntax:
subroutine msPostHashes()

Description:
Should be added after the loop containing the call to msPrintHashes.
See also: msPrepHashes, msPrintHashes

Calling syntax:

```
call msPrepProgress( start, end [, disp_times, estimator] )
```


## Description:

Prepares data before calls to msPrintProgress.
start and end are numerical values used to monitor the progress bar. They must be both of integer or real (double) type.

If the logical optional argument disp_times is set to .false., time values (left time and estimated remaining time - wall clock time is used, not CPU time) are not displayed (default is TRUE).

The string optional argument estimator prescribes the method to be used to estimate the remaining time. Default is "GLE" (Global Linear Estimator), which corresponds to a fast method but with the assumption of a linear behavior of the cost along all the iterations. The other available method is "LPE" (Local Power Estimator), which involves a Least Square problem, and assumes that the behavior of the cost is under a power form; this latter method is more expensive but should work well with a larger class of problems than "GLE".

See also: msPrintProgress, msPostProgress
msPrintProgress

Calling syntax:

```
call msPrintProgress( val )
```

Description:
Print on STDOUT the percentage of the work already done. According to the setting used during initialization via msPrepProgress, the estimated remaining time may also be displayed.

Usually, the call is located inside a loop (at the end); val is the numerical value (either integer or real double) monitoring the progression. It's numerical extremum values have to be passed as arguments of msPrepProgress.

See also: msPostProgress, msPrintHashes

Example(s):
integer :: i_start, i_end, i
call msPrepProgress( i_start, i_end )
do i = i_start, i_end
...
<some computation>
call msPrintProgress( i )
end do
call msPostProgress( )
or:
double precision :: time_start, time_end, time
call msPrepProgress( time_start, time_end )
time = time_start
do while( time <= time_end)
...
<some computation>
call msPrintProgress ( time )
time = <new value of time, increasing but may be not linear with iterations>
end do
call msPostProgress( )
msPostProgress
percent progress conclusion

Interface:
subroutine msPostProgress()

Description:
Should be added after the loop containing the call to msPrintProgress.
See also: msPrepProgress, msPrintProgress
mfToLower

Calling syntax:

```
s2 = mfToLower( s1 )
```

Description:
Returns the input strings s1 with each letter converted to lower case.
See also: mfToUpper
mfToUpper

Calling syntax:

```
s2 = mfToUpper( s1 )
```

Description:
Returns the input strings s1 with each letter converted to upper case.
See also: mfToLower
msFindIOUnit
automatically find a free IO unit number

Interface:

```
subroutine msFindIOUnit( unit )
    integer, intent(out) :: unit
```

Description:
Returns an integer IO unit which can be used for opening a file.

## mfIsVersion

## test on version strings

Calling syntax:

```
bool = mfIsVersion( v_1, op, v_2 )
```


## Description:

Compares the two version strings v_1 and v_2 with the operator op ("==", ">", ">=", "<", "<="). The version string is a character string, of size at most 8 , matching the rule $a . b . c$, where $a, b$ and $c$ are integers ranged from 0 to 99 .

This function returns a logical and is usually applied to strings as MF_MUESLI_VERSION or MF_LAPACK_VERSION.

See also: msRequMuesliVer
msRequMuesliVer
check for a minimum Muesli version

## Interface:

```
subroutine msRequMuesliVer( version )
    character(len=*) :: version
```


## Description:

Checks that the current Muesli version used at run-time is at least equal to that provided in the "version" argument. This argument must be a string of the form a.b.c, where $a$, $b$ and $c$ are integers ranged from 0 to 99 .

See also: MF_MUESLI_VERSION, mfIsVersion

Calling syntax:

```
    string = MF_COMPILER_VERSION
```

returns the compiler vendor and version used during the compilation of the MUESLI library.
See also: MF_MUESLI_VERSION, MF_COMPILATION_CONFIG, MF_LAPACK_VERSION

Calling syntax:

```
    string = MF_COMPILATION_CONFIG()
```

returns either "Debug" or "Optim".
Note the parenthesis used after the name of the variable, because it is implemented as a function. See also: MF_MUESLI_VERSION, MF_COMPILER_VERSION, MF_LAPACK_VERSION

## MF_MUESLI_VERSION

Calling syntax:

```
string = MF_MUESLI_VERSION
```

returns the MUESLI version used during the link of the executable.
The returned string may be used by the mfIsVersion boolean function.
See also: MF_COMPILER_VERSION, MF_COMPILATION_CONFIG, MF_LAPACK_VERSION

## Interface:

```
subroutine msMuesliTrace( pause )
    character(len=*), intent(in) :: pause
```

Description: Basically, this subroutine helps the user by printing, if available, a traceback of the program; optionally, it can make a pause (only if the pause argument is equal to "yes").

Contrary to the internal traceback routine, the behavior of this routine depends only on the value of the message level global MUESLI property.

Remark: In order to be useful, the use of this routine requires that the Muesli library has been compiled with debugging turned $O n$. This is the case if the library is used in the $D E B U G$ mode. On the contrary, it suffices that the user adds the appropriate option (e.g. -g for most of compilers) for its own source files.

See also: mfGetMsgLeve1, msSetMsgLeve1

## mfReadLine reads a line from terminal with editing and history facilities

## Interface:

```
function mfReadLine( prompt ) result( string )
        character(len=*), intent(in), optional :: prompt
        character(len=1024) :: res
```

Description: This function prompts the string prompt and waits the user to input a character string, using the readline library. As a result, the user can navigate in an history, recall any previous input and modify it.

It returns the string string, which is automatically added in the history only if it differs from the last entry.

The history can be saved with the use of the msWriteHistoryFile routine.
A small example of call can be found in the Muesli User's Guide.
See also: msReadHistoryFile, msClearHistory, msAddEntryInHistory,
msRemoveLastEntryInHistory
msReadHistoryFile
read a 'readline' history file

Interface:
subroutine msReadHistoryFile( filename )
character(len=*), intent(in), optional :: filename

Description: Reads an 'history' file previously stored by the msWriteHistoryFile routine.
If the optional filename is not present, 'history' is read from the file: /.mfreadline.history
See also: mfReadLine, msClearHistory, msAddEntryInHistory, msRemoveLastEntryInHistory
msWriteHistoryFile
write a 'readline' history file

Interface:
subroutine msWriteHistoryFile( filename )
character(len=*), intent(in), optional :: filename

Description: Writes an 'history' file in filename. This 'history' contains all entries read by the mfReadLine routine.
See also: msReadHistoryFile, msClearHistory, msAddEntryInHistory, msRemoveLastEntryInHistory
msClearHistory

clear the 'readline' history

Interface:
subroutine msClearHistory( )

Description: Clears all entries read by the mfReadLine routine.
See also: msReadHistoryFile, msWriteHistoryFile, msAddEntryInHistory, msRemoveLastEntryInHistory
msAddEntryInHistory
add an entry in the 'readline' history

Interface:

```
subroutine msAddEntryInHistory( string )
    character(len=*) :: string
```

Description: Adds the entry specified by the string argument in the 'readline' history.
See also: mfReadLine, msReadHistoryFile, msWriteHistoryFile, msClearHistory, msRemoveLastEntryInHistory
msRemoveLastEntryInHistory
remove last entry in the 'readline' history

Interface:
subroutine msRemoveLastEntryInHistory ( )

Description: Remove last entry read by the mfReadLine routine.
See also: msReadHistoryFile, msWriteHistoryFile, msClearHistory, msAddEntryInHistory

### 1.2 File Input/Output

| msSaveAscii | ASCII file saving |
| :---: | :---: |
| mfLoadAscii | ASCII file loading |
| msSaveSparse | Sparse ASCII file saving |
| mfLoadSparse | Sparse ASCII file loading |
| msLoadSparse | Sparse ASCII file (with Right-Hand-Side) loading |
| msSave | Binary file saving |
| mfLoad | Binary file loading |
| mfLoadTriConnect | Connectivity Binary file loading |
| msSaveHDF5 | HDF5 file saving |
| mfLoadHDF5 | HDF5 file loading |
| msMedit | graphic mfArray editor |

See also:
Core Routines
Data Analysis Functions
Operators
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Optimization and Function Functions
Sparse Matrices

## Interface:

```
subroutine msSaveAscii( filename, A, append, format )
character(len=*), intent(in) :: filename
type(mfArray), intent(in) :: A
logical, intent(in), optional :: append
character(len=*), intent(in), optional :: format
```


## Description:

Saves under ASCII form the dense mfArray A in the file named filename.
A is saved via a full 2 D format, i.e. the number of lines of filename is exactly the number of rows of A. Special IEEE values ( $N a N$ and $\operatorname{Infs}$ ) are written also in ASCII form (using the strings "NaN", "Infinity" and "-Infinity").

If the boolean append is present and $T R U E$ then the data is written at the end of the (previously written) file. The default is to overwrite a pre-existing file.

If the character string format is present, it specifies the Fortran format used during the write of individual numbers (for example "ES13.6"). It must be of course a non-empty valid format for writing numerical real values. As usual, if the format is not sufficient to write the required number of digits, stars will be printed out instead (for example, "F3.0" is the shortest format if your are sure that all numbers in the mfArray are integers; and even "F2.0" if they are all positive).
msSaveAscii doesn't store the internal properties of A, so writing a boolean mfArray converts it in real. Another limitation is that only the real part of complex numbers is written; to save the imaginary part, use:

```
call msSaveAscii( filename, mfImag(A) )
```


## Remarks.

- if A contains a permutation, it will be written as a column vector, using integers (the optional argument format will be discarded). However, reading again the same file with mfLoadAscii will convert it into real numbers, as ordinary matrices; msSave is therefore recommended to save a permutation vector.
- to avoid loss of precision during the read of filename, a sufficient number of decimal digits is used to print each real when the optional argument format is not present.
- do not use the optional arguments append and format if you plan to read the file with mfLoadAscii.
- to save sparse matrices, use the msSaveSparse routine.

See also: mfLoadAscii, msSave, msSaveHDF5
mfLoadAscii

## Interface:

```
function mfLoadAscii( filename,
    &
    ieee, comment, rect, row_max, col_max ) result( out )
    character(len=*), intent(in) :: filename
    logical, intent(in), optional :: ieee, rect
    character(len=1), intent(in), optional :: comment
    integer, intent(in), optional :: row_max, col_max
    type(mfArray) :: out
```


## Description:

Reads data from an ASCII file named filename and copy it in an mfArray. Inside the file, numerical data are separated by one or more space(s) and tabulation(s). Usually, this file has been created by the msSaveAscii routine but actually, nearly all kinds of ASCII file may be read by mfLoadAscii: see below.

If ieee is present and equal to .true., this routine is able to read special IEEE values, such as $N a N$ and Inf (but this feature makes the read not very efficient for some compilers). The default behavior is to rise an error if any non numeric value is found.

If comment is present, all lines beginning (at any position) by this character are skipped (of course, this character must be different from: a space, a tabulation, a carriage-return, a digit, the signs + and - , the letters I and N which begin Inf and NaN); comments after data are also ignored. Empty lines are treated as commented. The default behavior assumes that the file contains only numerical values and no empty lines.

If rect is present and equal to .false., the file read may contain data which are not under a rectangular layout. In this latter case, the routine returns a column vector mfArray. By default, this routine expects a rectangular layout, i.e. the same number of reals stored in each row.

For the standard case only (i.e., rect is equal to .true., or not present), the optional arguments row_max and col_max allow the user to read partially the data file. These two arguments are not dependent (and do not need to be both present). Note also that the whole data file may have a non-rectangular layout: it is sufficient that (row_max, col_max) covers a rectangular subpart of the data file.

Remark: DOS or UNIX text files are supported.
See also: msSaveAscii, mfLoad, mfLoadHDF5

## msSaveSparse

## Sparse ASCII file saving

## Interface:

```
subroutine msSaveSparse( filename, A, format )
character(len=*), intent(in) :: filename
type(mfArray), intent(in) :: A
character(len=3), intent(in), optional :: format
```


## Description:

Saves the sparse matrix mfArray A in the file filename, using the (optionally) specified ASCII format:

- "CSC": Compact Sparse Column (default)

The first line contains exactly five items, which are the numbers of rows and columns (nrow and ncol), an integer ( 1 or 0 ) specifying whether the entries are row-sorted or not in each colums, the tag "CSC" and the type of data ("real" or "complex");
the following line(s) contains the integer vector of the pointers to the columns;
the following line(s) contains the integer vector of the row indices;
the remaining line(s) contains the floating-point values.

- "CSR": Compact Sparse Row

Very close to the previous format, except that row and column role are exchanged.

- "HBO": Harwell-Boeing

A commonly used format for sparse matrices, described at: http://math.nist.gov/ MatrixMarket/formats.html\#hb

- "MTX": Matrix Market (also called "Coordinates format", or "COD")

A commonly used format for sparse matrices, described at: MatrixMarket/formats.html\#MMformat

If format is not present, this routine uses the "CSC" format.
The routine always overwrites a pre-existing file.

See also: mfLoadSparse

## Interface:

```
function mfLoadSparse( filename, format, duplicated_entries ) result( out )
character(len=*), intent(in) :: filename
character(len=3), intent(in), optional :: format
character(len=*), intent(in), optional :: duplicated_entries
type(mfArray) :: out
```


## Description:

Reads the file filename and copy the sparse matrix it contains in an mfArray.
When reading the file, it uses the (optionally) specified ASCII format:

- "CSC": Compact Sparse Column
- "CSR": Compact Sparse Row
- "HBO": Harwell-Boeing (without Right Hand Side; on the contrary, use msLoadSparse instead)
- "MTX": Matrix Market (also called "Coordinates format", or "COO")

If format is not present, this routine tries to auto-detect the used format.
The description of these different formats can be found at the msSaveSparse entry.
After reading the matrix:

- eventual duplicated entries are treated according to the optional character argument duplicated_entries. When this argument is present and equal to "ignored", duplicated entries are ignored; when it is equal to "added" (default value), duplicated entries are added; when it is equal to "replaced", last entries found overwrite previous one.
- entries containing a null value are removed.

See also: msLoadSparse, msSaveSparse, mfSpImport

msLoadSparse<br>Sparse ASCII file (with Right-Hand-Side) loading

Calling syntax:

```
call msLoadSparse( mfOut(A[,RHS]), filename [, format, duplicated_entries] )
```

Description:
Similar to mfLoadSparse but allows the user to retrieve a Right-Hand-Side (HBO format only) together with the sparse matrix.

The input arguments are described in the mfLoadSparse description.
A and RHS are both mfArrays and, after reading the file, contain respectively the sparse matrix and the Right-Hand-Side (dense or sparse, according to the file content).

See also: mfLoadSparse, msSaveSparse, mfSpImport
msSave

Binary file saving

## Interface:

```
    subroutine msSave( filename, A, compressed )
        character(len=*), intent(in) :: filename
        type(mfArray), intent(in) :: A
or type(mfTriConnect), intent(in) :: A
        logical, optional :: compressed
```


## Description:

Saves under a binary form the mfArray (or mfTriConnect) A in the file named filename. The MUESLI Binary Format (MBF) used is specific to the current library. Any filename extension is supported; the ".mbf" used in the MUESLI examples just helps to remind that it is a MUESLI binary file.

If compressed is present and true, or if the specified filename ends with the extension ". gz", then msSave directly stores the file in the 'gzip' compressed format.

## Remarks:

- to read the created file, you must use mfLoad (or mfLoadTriConnect), because a special format is used to organize data inside it. MUESLI add a special tag to help with compatibility between versions.
- msSave takes care of endianness of your architecture, so read/write this kind of file should be possible across big endian and little endian machines.
- msSave stores all the internal properties of A, including its dynamic type, structure and data, hidden matrix properties and physical units; so mfLoad (or mfLoadTriConnect) should restore exactly the same mfArray (but for sparse structure, internal arrays will have the minimal size).
- msSave always overwrites a pre-existing file.

See also: msSaveAscii, msSaveHDF5

## mfLoad

Interface:

```
function mfLoad( filename ) result( out )
    character(len=*), intent(in) :: filename
    type(mfArray) :: out
```

Description:
Reads a matrix previously stored in the file named filename (MUESLI Binary Format) by the routine msSave, and copy it in an mfArray.

This routine is able to read directly a compressed gzipped file, as stored by msSave.
See also: mfLoadAscii
mfLoadTriConnect

Interface:
function mfLoadTriConnect( filename ) result( out )
character(len=*), intent(in) :: filename
type(mfTriConnect) : : out

Description:
Reads a connectivity structure previously stored in the file named filename (MUESLI Binary Format) by the routine msSave, and copy it in an mfArray.

See also: mfLoadAscii

## Interface:

```
subroutine msSaveHDF5( filename, A, &
name, file_access, mfarray_overwrite, status )
character(len=*), intent(in) :: filename
type(mfArray), intent(in) :: A
character(len=*), intent(in), optional :: name
character(len=*), intent(in), optional :: file_access
logical, intent(in), optional :: mfarray_overwrite
integer, intent(out), optional :: status
```


## Description:

By default (or if file_access is present and equal to "trunc"), this routine creates a new HDF5 file (or overwrites a pre-existing one) and writes the mfArray A (dense or sparse) in it.

If file_access is present and equal to "append", it tries to add in the existing HDF5 file a new group for $A$. In the case where a group with the same name already exists, the group will be overwritten only if mfarray_overwrite is present and equal to true; on the contrary, it will return a non zero value in the status flag (if this later optional argument is present).

The mfArray A is written as one group (named from the optional argument name, or "untitled mfArray" by default), with all its MUESLI properties, stored as group attributes.

The created file can be read in MUESLI by the routine mfLoadHDF5; moreover, it can be browsed by a specific viewer, as 'hdfview' for example.

Remark: No specific extension is required for the "filename", nevertheless ".h5" or ".hdf5" are often used.

See also: msSave, msSaveAscii

## mfLoadHDF5

## Interface:

```
function mfLoadHDF5( filename, name ) result( out )
    character(len=*), intent(in) :: filename
    character(len=*), intent(in), optional :: name
    type(mfArray) :: out
```

Description:
Reads from an HDF5 file and tries to find an mfArray named from the optional argument name. If this latter in not present, it tries to get the first group which has an attribute named MF_MUESLI_VERSION. It doesn't enter in any group: the searched mfArray must be located just under the root structure ("/") of the HDF5 file.

The file filename must have been created by msSaveHDF5.
See also: mfLoad, mfLoadAscii

## msMedit

## Interface:

```
subroutine msMedit( A )
```

type(mfArray), intent(in out) :: A

## Description:

This routine allows the user to interactively modify a real mfArray, with a small graphical Matrix-editor.
Actually, it launch another X11 (Qt4-based) application, named 'meditor' (spreadsheet-like), which displays a table containing initially the matrix A. The user can modify the values and extend the shape of the matrix. When the window is closed, new data are automatically saved in the mfArray A.

Special IEEE values can be entered in cells, by typing their ASCII names, $N a N$ and Inf ('meditor' is not case-sensitive). Bad entries are ignored.

To extend the matrix, click anywhere outside the matrix and enter a value (empty cells of the new matrix will be filled with 0 ).

## Remarks:

- complex numeric matrices, or containing boolean values, are not accepted;
- if A is a sparse matrix, it is first converted to a dense one before display, but the new modified matrix will be converted back to sparse storage;
- the run of the 'meditor' tool blocks the calling program till it's window is closed by the user;
- the size of A is limited to $200 \times 200$ : it can be changed in the file 'meditor.cpp' if needed (of course, the 'meditor' tool has to be compiled again).


### 1.3 Data Analysis Functions

```
mfMax, msMax
mfMin, msMin
mfSum
mfExtrema
mfProd
mfSort, msSort
mfIsSorted
mfSortRows
mfDiff
mfGradient
msGradient
mfMean
mfMedian
mfVar
mfStd
mfRMS
msHist
mfMoments
mfSmooth
mfFFT, mfInvFFT
mfFFT2, mfInvFFT2
mfFourierCos, mfInvFourierCos
mfFourierSin, mfInvFourierSin
mfFourierLeg, mfInvFourierLeg
```

mfArray max
mfArray min
columns sum
data extrema
columns product
columns/rows sort
vector sort test
sort whole rows of a matrix
difference and approximate derivative approximate 1D gradient approximate 2 D gradient
columns mean
columns median
columns variance
columns standard deviation
root mean square
data histogram
few first moments of a distribution
smoothing of vector values
discrete Fast Fourier transformation 2D discrete Fast Fourier transformation discrete Fourier cosine transformation discrete Fourier sine transformation discrete Fourier-Legendre transformation

See also:
Core Routines
File Input/Output
Operators
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Optimization and Function Functions
Sparse Matrices
mfMax
mf Max has two forms. The first one is used to compute the maximum value of the columns (default behavior) or of the rows of an mfArray. The second one compares two matrices.

## First interface:

```
function mfMax( A, dim, nanflag ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    character(len=*), intent(in), optional :: nanflag
    type(mfArray) :: out
```

If dim is present, A is always considered as a matrix, and the returned vector contains the maximum of each column (if dim $=1$ ) or the maximum of each row (if $\operatorname{dim}=2$ ).

If dim is not present: if A is a vector, mfMax returns the largest element; otherwise it returns the max of each columns.

If the string nanflag is present, it must be equal to either "omitNaN", or "includeNaN" (practically, the argument is not case sensitive). By default, NaN values are discarded (see examples below), so "omitNaN" is implied.

For a sparse mfArray A, the result is exactly the same as for the full corresponding matrix. In all cases (sparse or dense), it must contain real values.

## Second interface:

```
function mfMax( A, B, nanflag ) result( out )
    type(mfArray), intent(in) :: A
    type(mfArray), intent(in) :: B
or real(kind=MF_DOUBLE), intent(in) :: B
    character(len=*), intent(in), optional :: nanflag
    type(mfArray) :: out
```

When the two arguments A and B are mfArrays (sparse storage is allowed), they must have the same shape and must contain real values; in this case, the max operation is applied element-wise. Otherwise, the second argument $B$ may be a scalar real.

The optional argument nanflag has the same meaning as in the first interface.
See also: mfMin, msMax, mfExtrema

Example(s):

```
x = [ 1.0dO, MF_NAN, 5.0d0, 2.0dO, 4.0dO ]
call msDisplay( mfMax(x), "mfMax(x)" )
```

output:

```
mfMax(x) =
```

5.0000
$\mathrm{x}=$ [ 1.0d0, MF_NAN, 5.0d0, 2.0d0, 4.0d0 ]
call msDisplay ( mfMax (x, nanflag='includeNaN'), "mfMax(x,nanflag='includeNaN')" )
output:

```
mfMax(x,nanflag='includeNaN') =
```

NaN
$\mathrm{x}=[\mathrm{MF}$ _NAN, MF_NAN ]
call msDisplay( $\operatorname{mfMax}(x), \quad " m f \operatorname{Max}(x) ")$
output:

```
mfMax(x) =
```

NaN
msMax

## mfArray max

Calling syntax:

```
call msMax( mfOut(v,i), A [, dim, nanflag] )
```


## Description:

Computes the maximum value of the columns (default behavior) or of the rows of the mfArray A (real values only) and shows where this maximum occurs.

If dim is present, A is always considered as a matrix, and the vector v contains the maximum of each column (if $\operatorname{dim}=1$ ) or the maximum of each row (if dim $=2$ ). indices of these maxima are returned in the vector $i$.

If dim is not present: if A is a vector, msMax returns the largest element in v and the corresponding index in $i$; otherwise it returns the max of each columns in $v$ and the corresponding indices in $i$.

The optional argument nanflag has the same meaning as in the mfMax routine.
For a sparse mfArray A, the result is exactly the same as for the full corresponding matrix. In all cases (sparse or dense), it must contain real values.

See also: mfOut, msMin, mfExtrema

## Example(s):

```
x = [ 1.0dO, MF_NAN, 5.OdO, 2.OdO, 4.0dO ]
call msDisplay( x, "x" )
call msMax( mfOut(a,i), x )
call msDisplay( a, "max" )
call msDisplay( i, "for index" )
```

output:

```
x =
1.0000 NaN \(5.0000 \quad 2.0000 \quad 4.0000\)
max =
    5
for index =
    3
    call msMax( mfOut(a,i), x, nanflag='includeNaN' )
    call msDisplay( a, "max" )
    call msDisplay( i, "for index" )
```

output:
$\max =$
NaN
for index =
2
mfMin

## mfArray min

mfMin has two forms. The first one is used to compute the minimum value of the columns (default behavior) or of the row of an mfArray. The second one compares two matrices.

## First interface:

```
function mfMin( A, dim, nanflag ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    character(len=*), intent(in), optional :: nanflag
    type(mfArray) :: out
```

If dim is present, $A$ is always considered as a matrix, and the returned vector contains the minimum of each column (if $\operatorname{dim}=1$ ) or the minimum of each row (if $\operatorname{dim}=2$ ).

If dim is not present: if $A$ is a vector, mfMin returns the smallest element; otherwise it returns the min of each columns.

If the string nanflag is present, it must be equal to either "omitNaN", or "includeNaN" (practically, the argument is not case sensitive). By default, NaN values are discarded (see examples below), so "omitNaN" is implied.

For a sparse mfArray A, the result is exactly the same as for the full corresponding matrix. In all cases (sparse or dense), it must contain real values.

## Second interface:

```
function mfMin( A, B, nanflag ) result( out )
    type(mfArray), intent(in) :: A
    type(mfArray), intent(in) :: B
or real(kind=MF_DOUBLE), intent(in) :: B
    character(len=*), intent(in), optional :: nanflag
    type(mfArray) :: out
```

When the two arguments A and B are mfArrays (sparse storage is allowed), they must have the same shape and must contain real values; in this case, the min operation is applied element-wise. Otherwise, the second argument $B$ may be a scalar real.

The optional argument nanflag has the same meaning as in the first interface.
See also: mfMax, msMin, mfExtrema

Example(s):

```
x = [ 1.0dO, MF_NAN, 5.0d0, 2.0dO, 4.0dO ]
call msDisplay( mfMin(x), "mfMin(x)" )
```

output:

```
mfMin(x) =
```

1.0000

$$
\cdots / \ldots
$$

$\mathrm{x}=$ [ 1.0d0, MF_NAN, 5.0d0, 2.0d0, 4.0d0 ]
call msDisplay ( mfMin(x,nanflag='includeNaN'), "mfMin(x,nanflag='includeNaN')" )
output:

```
mfMin(x,nanflag='includeNaN') =
```

NaN
$\mathrm{x}=[\mathrm{MF}$ _NAN, MF_NAN ]
call msDisplay( mfMin(x), "mfMin(x)" )
output:

```
mfMin(x) =
```

NaN
msMin

Calling syntax:

```
call msMin( mfOut(v,i), A [, dim, nanflag] )
```


## Description:

Computes the minimum value of the columns (default behavior) or of the rows of the mfArray A (real values only) and shows where this minimum occurs.

If dim is present, A is always considered as a matrix, and the vector v contains the minimum of each column (if dim $=1$ ) or the minimum of each row (if dim $=2$ ). indices of these minima are returned in the vector $i$.

If dim is not present: if A is a vector, mfMax returns the smallest element in v and the corresponding index in $i$; otherwise it returns the min of each columns in $v$ and the corresponding indices in $i$.

The optional argument nanflag has the same meaning as in the mfMin routine.
For a sparse mfArray A, the result is exactly the same as for the full corresponding matrix. In all cases (sparse or dense), it must contain real values.

See also: mfOut, msMax, mfExtrema

## Example(s):

```
x = [ 1.0dO, MF_NAN, 5.OdO, 2.OdO, 4.0dO ]
call msDisplay( x, "x" )
call msMin( mfOut(a,i), x )
call msDisplay( a, "min" )
call msDisplay( i, "for index" )
```

output:

```
x =
    1.0000 NaN 5.0000 2.0000 4.0000
min =
    1
for index =
    1
    call msMin( mfOut(a,i), x, nanflag='includeNaN' )
    call msDisplay( a, "min" )
    call msDisplay( i, "for index" )
```

output:
$\min =$
NaN
for index $=$
2

## mfExtrema

## data extrema

## Interface:

```
function mfExtrema( A, nanflag ) result( out )
    type(mfArray), intent(in) :: A
    character(len=*), intent(in), optional :: nanflag
    type(mfArray) :: out
```

Description:
Returns in an mfArray the two extreme values of an mfArray, in ascent order, i.e. the couple (min, max).

The optional argument nanflag has the same meaning as in the mfMin and mfMax routines.
See also: msMin, msMax

```
Example(s):
    x = [ 5, 9, 3, 6, 0, 2 ]
    call msDisplay( x, "x" )
    call msDisplay( mfExtrema(x), "extrema" )
output:
    x =
    5 [rlllll
extrema =
    0 9
    x = [ 5, 9, MF_NAN, 6, 0, 2 ]
    call msDisplay( x, "x" )
    call msDisplay( mfExtrema(x,nanflag='includeNaN'), "extrema (include NaN)" )
output:
x =
    5 9 NaN 6 % 0 % 2
extrema (include NaN) =
    NaN NaN
```

mfSum

Interface:

```
function mfSum( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```


## Description:

Computes the sum of the columns (or rows) of an mfArray.
If dim is present, A is always considered as a matrix, and the output contains the sum of each column (if $\operatorname{dim}=1$ ) or the sum of each row (if $\operatorname{dim}=2$ ).

If dim is not present: if $A$ is a vector, mfSum returns the sum of all elements; otherwise it returns the sum of each columns.

See also: mfProd
mfProd

## Interface:

```
function mfProd( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```


## Description:

Computes the product of the columns (or rows) of an mfArray. The behavior of this routine is similar to the product Fortran 90 intrinsic function.

If dim is present, A is always considered as a matrix, and the output contains the product of each column (if $\operatorname{dim}=1$ ) or the product of each row (if $\operatorname{dim}=2$ ).

If dim is not present: if $A$ is a vector, mfSum returns the product of all elements; otherwise it returns the product of each columns.

Remarks: this routine cannot be applied to a boolean mfArray.
Warning: this function cannot be applied to sparse matrices.
See also: mfSum
mfSort
columns sort

## Interface:

```
function mfSort( A, dim, mode ) result( out )
type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
character(len=*), intent(in), optional :: mode
type(mfArray) :: out
```


## Description:

If dim is not present, sorts the values of the vector A , and returns the sorted vector in an mfArray.
If dim is present, A may be a matrix. In this case, the sort is applied to the columns if dim is equal to 1 , and to the rows if dim is equal to 2 .

## Remarks:

- this routine use a quick sort algorithm. By default, or if the optional argument mode is equal to "ascending" values are sorted in increasing order; on the contrary, if the optional argument mode is equal to "descending" values are sorted in decreasing order.
- currently, only rank- 1 mfArrays (i.e. vectors) may contain $N a N$ values. These $N a N$ s are moved to the tail of the sorted vector.
- see msSort for an 'in-place' version.

Warning: If the mfArray A is a matrix, it must be real; for a complex vector mfArray, the sort is applied first on the module values, then on the phase-angle.

See also: msSort, mfSortRows

```
Example(s):
    x = mf( [ 1.0dO, MF_NAN, 5.OdO, MF_NAN, 2.0dO, 4.0d0 ] )
    call msDisplay( x, "x" )
    call msDisplay( mfSort(x), "mfSort(x)" )
    call msDisplay( mfSort(x,'descending'), "mfSort(x,'descending')" )
```

output:
$\mathrm{x}=$
$\begin{array}{lllll}1.0000 & \text { NaN } 5.0000 \text { NaN } 2.00004 .0000\end{array}$
$\operatorname{mfSort}(x)=$

| 1.0000 | 2.0000 | 4.0000 | 5.0000 | NaN | NaN |
| :--- | :--- | :--- | :--- | :--- | :--- |

mfSort( $x$,'descending') $=$

| 5.0000 | 4.0000 | 2.0000 | 1.0000 | $\mathrm{NaN} \quad$ NaN |
| :--- | :--- | :--- | :--- | :--- |

msSort
columns sort

First calling syntax:
call msSort ( A [, dim, mode ] )
This is the 'in-place' version of mfSort.

Second calling syntax:

```
call msSort ( mfOut(v,i), A [, dim, mode ] )
```

The use of this subroutine is similar to mfSort but the result is returned in the mfArray v ; in addition, indices of the sort are stored in the vector i.

See also: mfOut,mfSortRows

```
Example(s):
    x = mfMagic(3)
    call msDisplay( x, "x" )
    call msSort( mfOut(a,i), x, dim=1 )
    call msDisplay( a, "ascending column sort" )
    call msDisplay( i, "with indices" )
    call msSort( mfOut(a,i), x, dim=2, mode="descending" )
    call msDisplay( a, "descending row sort" )
    call msDisplay( i, "with indices" )
```

output:
$\mathrm{x}=$
$8 \quad 3 \quad 4$
159
$\begin{array}{ll}6 & 7\end{array}$
ascending column sort $=$
132
$6 \quad 5 \quad 4$
$8 \quad 7 \quad 9$
with indices =
213
$3 \quad 2 \quad 1$
132
descending row sort =
$8 \quad 4 \quad 3$
$9 \quad 5 \quad 1$
with indices =

| 1 | 3 | 2 |
| :--- | :--- | :--- |
| 3 | 2 | 1 |
| 2 | 1 | 3 |

## mfIsSorted

Interface:

```
bool = mfIsSorted( v, mode )
```


## Description:

Checks that the mfArray vector v is sorted, according the mode, which can be equal to "ascend", "descend" or "either".

If the optional argument mode is not present, then an ascending sort is presumed.
This function returns a logical.
Remarks:

- currently, only rank-1 mfArrays (i.e. vectors) can be checked.
- the vector v may contain $N a N$ values. The vector is considered as sorted if all $N a N$ s are located at the end.
- for a complex vector mfArray, sorting concerns first the module, then the phase-angle.

See also: mfSort

## mfSortRows

## sort whole rows of a matrix

## Interface

```
function mfSortRows( A, col, mode ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: col
    character(len=*), intent(in), optional :: mode
    type(mfArray) :: out
```


## Description:

If col is not present, sorts the rows of the matrix A according to the first column, and returns the result in an mfArray.

If col is present, the rows of A are sorted according to the column number col.

## Remarks:

- this routine use a quick sort algorithm. By default, or if the optional argument mode is equal to "ascending" values are sorted in increasing order; on the contrary, if the optional argument mode is equal to "descending" values are sorted in decreasing order.
- currently, the matrix A should not contain $N a N$ values.

Warning: The mfArray A must be real.
See also: mfSort, msSort

## Example(s):

```
\(\mathrm{x}=\mathrm{mf}\) Magic \((3)\)
call msDisplay( x, "x" )
call msDisplay( mfSortRows(x,3), "mfSortRows(x,3)" )
call msDisplay( mfSortRows(x,1,"descending"), "mfSortRows(x,1,'descending')" )
```

output:
$\mathrm{x}=$
$8 \quad 3 \quad 4$
$1 \quad 5 \quad 9$
$6 \quad 7 \quad 2$
mfSortRows $(x, 3)=$
$\begin{array}{ll}6 & 7\end{array}$
$8 \quad 3 \quad 4$
159
mfSortRows(x,1,'descending') =
$6 \quad 7 \quad 2$
159
$8 \quad 3 \quad 4$
mfDiff

## difference and approximate derivative

## Interface

function mfDiff( A, dim ) result( out )

```
type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```


## Description:

Computes the difference of two consecutive values of the columns (or rows) of an mfArray A.
If dim is present, A is always considered as a matrix, and the routine computes the difference of two consecutive values for each column (if $\operatorname{dim}=1$ ) or for each row (if $\operatorname{dim}=2$ ).

If dim is not present: if A is a vector, then mfDiff computes the difference of two consecutive values in the natural way; otherwise the same process is applied for each columns.

Remark: this function cannot be applied to sparse matrices.
See also: mfGradient, msGradient

Example(s):

```
    x = mf( [ 1, 2, 3, 2, 1 ] )
    call msDisplay( x, "x" )
    call msDisplay( mfDiff(x), "mfDiff(x)" )
```

output:

```
x =
    1 2 3 2 1
mfDiff(x) =
    1 
```

mfGradient

## approximate 1D gradient

Calling syntax:

```
D = mfGradient( F, h [, dim, location] )
```


## Description:

Computes the gradient D of the rank-1 mfArray F.
If dim is present, F is always considered as a matrix, and the routine computes the gradient for each column (if $\operatorname{dim}=1$ ) or for each row (if $\operatorname{dim}=2$ ).

If dim is not present: if $F$ is a vector, then mfGradient computes the gradient in the natural way; otherwise the same process is applied for each columns.

The data is supposed to be equally spaced with $h$ as increment. $h$ may be a double precision real or a scalar mfArray; it may have negative values.

If the optional argument location (character string) is equal to "centered" then the gradient is computed on an internal centered, staggered grid; otherwise it is computed at the same points as F.

Remark: The gradient is computed using second order formulæ.
See also: msGradient, mfDiff, mfOut

## Example(s):

```
integer :: nx = 5
real(kind=MF_DOUBLE) :: lx = 1.0d0, hx
x = mfLinSpace(0.0d0,lx,nx)
F = x**2
call msDisplay( F, "F" )
hx = lx/(nx-1)
Fx = mfGradient( F, hx )
call msDisplay( Fx, "Fx = grad_x(F)" )
```

output:

```
F =
    0.0000
            0.0625
            0.2500
        0.5625
    1.0000
Fx = grad_x(F) =
    0.0000 0.5000 1.0000 1.5000
                                    2 . 0 0 0 0
    fx = mfGradient( F, hx, "centered" )
    call msDisplay( fx, "grad_x(F)" )
```

output:

```
grad_x(z) =
    0.2500 0.7500 1.2500 1.7500
```

msGradient

## approximate 2 D gradient

Calling syntax:

```
call msGradient( mfOut( Fi, Fj ), F, hi, hj [, location] )
```


## Description:

Computes the gradient of the rank- 2 mfArray F , along the two dimensions. The data are supposed to be equally spaced in both direction, with hi and hj as increments, which may have negative values.

As a result, Fi is the gradient of F with respect to increasing indices $i$ (i.e. along the rows) and Fj is the gradient of F with respect to increasing indices $j$ (i.e. along the columns).

If the optional argument location (character string) is equal to "centered" then the two components of the gradient are computed on an internal centered, staggered grid; otherwise they are computed at the same points as $F$.

## Remarks:

- this function cannot be applied to sparse matrices. For all cases and all points, the gradient is computed using second order formulæ.
- if the data matrix F is associated to coordinates $X$ and $Y$, hi and hj may be chosen to give directly the two components $F_{x}$ and $F_{y}$ (see the Muesli User Guide, section entitled "Data Analysis with mfArrays").

See also: mfGradient, mfDiff, mfOut

## Example(s):

```
integer :: nx = 5, ny = 5
real(kind=MF_DOUBLE) :: lx = 1.0dO, ly = 2.0d0, hx, hy
call msMeshGrid( mfOut(x,y), mfLinSpace(0.0d0,lx,nx),
            .t. mfLinSpace(ly,0.0d0,ny) )
call msDisplay( x, "x", y, "y" )
F = (2*x+y)**2
call msDisplay( F, "F" )
hx = lx/(nx-1)
hy = -ly/(ny-1)
call msGradient( mfOut(Fy,Fx), F, hy, hx )
call msDisplay( Fx, "Fx = grad_x(F)", Fy, "Fy = grad_y(F)" )
```

output:

```
x =
\begin{tabular}{lllll}
0.0000 & 0.2500 & 0.5000 & 0.7500 & 1.0000 \\
0.0000 & 0.2500 & 0.5000 & 0.7500 & 1.0000 \\
0.0000 & 0.2500 & 0.5000 & 0.7500 & 1.0000 \\
0.0000 & 0.2500 & 0.5000 & 0.7500 & 1.0000 \\
0.0000 & 0.2500 & 0.5000 & 0.7500 & 1.0000
\end{tabular}
```

| $\mathrm{y}=$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2.0000 |  | 2.0000 | 2.0000 | 2.0000 | 2.0000 |
| 1.5000 |  | 1.5000 | 1.5000 | 1.5000 | 1.5000 |
| 1.0000 |  | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 0.5000 |  | 0.5000 | 0.5000 | 0.5000 | 0.5000 |
| 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $F=$ |  |  |  |  |  |
| 4.0000 |  | 6.2500 | 9.0000 | 12.2500 | 16.0000 |
| 2.2500 |  | 4.0000 | 6.2500 | 9.0000 | 12.2500 |
| 1.0000 |  | 2.2500 | 4.0000 | 6.2500 | 9.0000 |
| 0.2500 |  | 1.0000 | 2.2500 | 4.0000 | 6.2500 |
| 0.0000 |  | 0.2500 | 1.0000 | 2.2500 | 4.0000 |
| $\mathrm{Fx}=\operatorname{grad}_{-} \mathrm{x}(\mathrm{F})=$ |  |  |  |  |  |
| 8 | 10 | 12 | 1416 |  |  |
| 6 | 8 | 10 | 1214 |  |  |
| 4 | 6 | 8 | 1012 |  |  |
| 2 | 4 | 6 | 810 |  |  |
| 0 | 2 | 4 | 68 |  |  |

Fy = grad_y (F) =

| 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- |
| 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 |
| 1 | 2 | 3 | 4 | 5 |
| 0 | 1 | 2 | 3 | 4 |

call msGradient( mfOut(Fy,Fx), F, hy, hx, "centered" )
call msDisplay ( Fx, "Fx = grad_x(F)", Fy, "Fy = grad_y(F)" )
output:

| $F \mathrm{x}=$ | $\operatorname{grad} \mathrm{x}(\mathrm{F})=$ |  |  |
| ---: | :--- | ---: | :--- |
| 8 | 10 | 12 | 14 |
| 6 | 8 | 10 | 12 |
| 4 | 6 | 8 | 10 |
| 2 | 4 | 6 | 8 |

Fy $=\operatorname{grad}_{-y}(F)=$

| 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- |
| 3 | 4 | 5 | 6 |
| 2 | 3 | 4 | 5 |
| 1 | 2 | 3 | 4 |

## mfMean

## Interface:

```
function mfMean( A, dim ) result( out )
type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```


## Description:

Computes the mean of the columns (or rows) of an mfArray, as $\frac{1}{N} \sum_{i=1}^{N} a_{i}$.
If dim is present, A is always considered as a matrix, and the output contains the mean of each column (if $\operatorname{dim}=1$ ) or the mean of each row (if $\operatorname{dim}=2$ ).

If dim is not present: if A is a vector, mfMean returns the mean of all elements; otherwise it returns the mean of each columns.

Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray A doesn't contain any $N a N$ values.

See also: mfMedian, mfVar, mfStd, mfRMS, mfMoments

## mfMedian

## Interface:

```
function mfMedian( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```


## Description:

Computes the median of the columns (or rows) of an mfArray. The median value of a vector is, after sorting this vector, the value located in the middle.

If dim is present, A is always considered as a matrix, and the output contains the median of each column (if $\operatorname{dim}=1$ ) or the median of each row (if $\operatorname{dim}=2$ ).

If dim is not present: if A is a vector, mfMedian returns the median of all elements; otherwise it returns the median of each columns.

Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray A doesn't contain any $N a N$ values.

See also: mfMean, mfVar, mfStd, mfMoments
$m f V a r$
columns variance

## Interface:

```
function mfVar( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
        type(mfArray) :: out
```


## Description:

Computes the variance of the columns (or rows) of an mfArray, as $\frac{1}{N} \sum_{i=1}^{N}\left(a_{i}-\bar{a}\right)^{2}$, with $\bar{a}=\frac{1}{N} \sum_{i=1}^{N} a_{i}$.
If dim is present, A is always considered as a matrix, and the output contains the variance of each column (if $\operatorname{dim}=1$ ) or the variance of each row (if dim $=2$ ).

If dim is not present: if $A$ is a vector, mfVar returns the variance of all elements; otherwise it returns the variance of each columns.

Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray A doesn't contain any $N a N$ values.

See also: mfMean, mfMedian, mfStd, mfMoments

## mfStd

## columns standard deviation

## Interface:

```
function mfStd( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```


## Description:

Computes the standard deviation of the columns (or rows) of an mfArray. The standard deviation is the square root of the variance.

If dim is present, A is always considered as a matrix, and the output contains the standard deviation of each column (if dim $=1$ ) or the standard deviation of each row (if dim $=2$ ).

If dim is not present: if A is a vector, mfStd returns the standard deviation of all elements; otherwise it returns the standard deviation of each columns.

## Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray A doesn't contain any $N a N$ values.

See also: mfMean, mfRMS, mfMedian, mfVar, mfMoments
mfRMS

## Interface:

```
function mfRMS( A, dim ) result( out )
type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```


## Description:

Computes the Root-Mean-Square of the columns (or rows) of an mfArray.
The RMS is defined by:

$$
a_{R M S}=\sqrt{\frac{1}{N} \sum_{i=1}^{N} a_{i}^{2}}
$$

where $N$ is the number of elements $x_{i}$. The following relation holds:

$$
a_{R M S}^{2}=\bar{a}^{2}+\sigma_{a}^{2}
$$

so the RMS of a signal is synonym to its standard deviation only when the mean is zero.
If dim is present, A is always considered as a matrix, and the output contains the mean of each column (if $\operatorname{dim}=1$ ) or the RMS of each row (if dim $=2$ ).

If dim is not present: if $A$ is a vector, mfRMS returns the RMS of all elements; otherwise it returns the RMS of each columns.

## Warning:

- this function can be applied only to real, dense matrices;
- the user must take care that the mfArray A doesn't contain any $N a N$ values.

See also: mfMean, mfMedian, mfVar, mfStd, mfMoments
msHist

## data histogram

Calling syntax:

```
call msHist( mfOut( num [, x_bin ] ), x, x_min, x_max, n_bin )
```

computes a data histogram.
The vector mfArray $x$ contains the data. The histogram is computed for data ranged from x_min to $x \_m a x$ using a number of bins equal to n_bin.

The output mfArray num is a vector of the bins values (length equal to n_bin).
If present, the output mfArray x_bin is a vector containing the abscissas of the bins (length equal to n_bin+1).

Note: to process discrete data, it is recommended to choose x _min as the minimum integer value minus $1 / 2$, and x max as the maximum integer value plus $1 / 2$ (see example below).

See also: mf/msPlotHist, msCumulHist, mfOut

```
Example(s):
    x = .t. mf( [ 12, 11, 8, 10, 6, 8, 9, 10, 12, 9, 10, 8, 6, 10 ] )
    call msHist( mfOut( y, z ), x, 6.0d0-0.5d0, 12.0d0+0.5d0, 7 )
    call msDisplay( y, "y (hist)", z, "z (x_bin)" )
```

output:
y (hist) =
2
0
3
2
4
1
2
$z\left(x \_b i n\right)=$
5.5000
6.5000
7.5000
8.5000
9.5000
10.5000
11.5000
12.5000

## mfMoments

## few first moments of a distribution

## Interface:

function mfMoments( v ) result( out )
type(mfArray), intent(in) :: v type(mfArray) :: out

## Description:

Computes some moments of the data distribution provided in the vector mfArray v .
It returns a vector mfArray of length 6 , containing in order:

- the mean value of data
- the average deviation
- the standard deviation
- the variance
- the skewness
- the kurtosis

Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray v doesn't contain any $N a N$ values.

See also: mfMean, mfMedian, mfVar, mfStd

## mfSmooth

Interface:

```
function mfSmooth( v [, span ] ) result( out )
    type(mfArray), intent(in) :: v
    integer, intent(in), optional :: span
    type(mfArray) :: out
```

Description:
Applies the moving average method to the vector mfArray v. It is equivalent to a lowpass filter.
The width of the sliding window is span which must be a non negative integer. The span value should not be greater than the number of elements in v . The default value is 5 .

See also: mfSpline
mfFFT

## discrete Fast Fourier transformation

## Interface:

```
function mfFFT( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```

Description:
Applies a Fast Fourier transformation to the columns (or rows) of an mfArray, real or complex. The output mfArray is always complex.

If dim is present, A is always considered as a matrix, and the output contains the transformation coefficients of each column (if dim $=1$ ) or the transformation of each row (if dim $=2$ ).

If dim is not present: if A is a vector, mfFFT returns the transformation of all elements; otherwise it returns the transformation of each columns.

Warning: this function cannot be applied to sparse matrices.
Remarks: abscissa are supposed to be equally spaced.
See also: mfInvFFT, mfFFT2, mfFourierCos, mfFourierSin, mfFourierLeg
mfInvFFT
inverse Fast Fourier transformation

Interface:
function $m f \operatorname{InvFFT}(\mathrm{~A}, \operatorname{dim})$ result( out )

```
type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

Description:
This is the inverse tranformation of mfFFT. So, for any vector v, we have:
$\mathrm{v}=\mathrm{mfInvFFT}(\mathrm{mfFFT}(\mathrm{v}))$
See also: mfFFT, mfFFT2, mfFourierCos, mfFourierSin, mfFourierLeg
mfFFT2

## 2D discrete Fast Fourier transformation

Interface:
function $m f F F T 2(A)$ result ( out )
type(mfArray), intent(in) :: A
type(mfArray) :: out

Description:
Applies a two dimensional Fast Fourier transformation to the mfArray A, real or complex. The output mfArray is always complex.

Warning: this function cannot be applied to sparse matrices.
Remarks: abscissa are supposed to be equally spaced for both dimensions.
See also: mfInvFFT2, mfFFT, mfFourierCos, mfFourierSin, mfFourierLeg
mfInvFFT2
inverse 2D Fast Fourier transformation

Interface:

```
function mfInvFFT2( A ) result( out )
    type(mfArray), intent(in) :: A
    type(mfArray) :: out
```

Description:
This is the inverse tranformation of mfFFT2. So, for any complex matrix A, we have:
$\mathrm{A}=\mathrm{mfInvFFT}(\operatorname{mfFFT}(\mathrm{A}))$
See also: mfFFT2, mfFFT, mfFourierCos, mfFourierSin, mfFourierLeg

## mfFourierCos

## discrete Fourier cosine transformation

## Interface:

```
function mfFourierCos( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```


## Description:

Applies a discrete Fourier cosine transformation to the columns (or rows) of an mfArray.
If dim is present, A is always considered as a matrix, and the output contains the transformation coefficients of each column (if dim $=1$ ) or the transformation of each row (if dim $=2$ ).

If dim is not present: if $A$ is a vector, mfFourierCos returns the transformation of all elements; otherwise it returns the transformation of each columns.

Warning: this function cannot be applied to sparse matrices. Moreover, the array must contain real values.

Remarks:

- abscissa are supposed to be equally spaced;
- actually, data are already considered symmetric on the interval $[-n+1, n+1]$; therefore, there doesn't exist any constraint on the value of the data vector.

See also: mfInvFourierCos, mfFFT, mfFFT2, mfFourierSin, mfFourierLeg
mfInvFourierCos
inverse Fourier cosine transformation

Interface:

```
function mfInvFourierCos( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```

Description:
This is the inverse tranformation of mfFourierCos. So, for any vector v, we have:
$\mathrm{v}=\mathrm{mf} \operatorname{InvFourierCos(mfFourierCos(v))}$
See also: mfFourierCos, mfFFT, mfFFT2, mfFourierSin, mfFourierLeg

## Interface:

```
function mfFourierSin( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```


## Description:

Applies a discrete Fourier sine transformation to the columns (or rows) of an mfArray.
If dim is present, $A$ is always considered as a matrix, and the output contains the transformation coefficients of each column (if dim $=1$ ) or the transformation of each row (if dim $=2$ ).

If dim is not present: if $A$ is a vector, mfFourierSin returns the transformation of all elements; otherwise it returns the transformation of each columns.

Warning: this function cannot be applied to sparse matrices. Moreover, the array must contain real values.

## Remarks:

- abscissa are supposed to be equally spaced;
- actually, data are already considered antisymmetric on the interval $[-n+1, n+1]$; therefore, there exists some constraints: the conditions $y(1)=y(n+1)=0$ are required.

See also: mfInvFourierSin, mfFFT, mfFFT2, mfFourierCos, mfFourierLeg
mfInvFourierSin
inverse Fourier sine transformation

Interface:

```
function mfInvFourierSin( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```

Description:
This is the inverse tranformation of mfFourierSin. So, for any vector v, we have:
$\mathrm{v}=\mathrm{mfInvFourierSin}(\mathrm{mfFourierSin}(\mathrm{v})$ )
See also: mfFourierSin, mfFFT, mfFFT2, mfFourierCos, mfFourierLeg

## mfFourierLeg

## Fourier-Legendre transform

## Interface:

```
function mfFourierLeg( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```


## Description:

Applies a discrete Fourier-Legendre transformation to the columns (or rows) of an mfArray.
This transformation is also called "discrete spherical Fourier transform".
If dim is present, $A$ is always considered as a matrix, and the output contains the transformation coefficients of each column (if dim $=1$ ) or the transformation of each row (if dim $=2$ ).

If dim is not present: if A is a vector, mfFourierLeg returns the transformation of all elements; otherwise it returns the transformation of each columns.

Warning: this function cannot be applied to sparse matrices. Moreover, the array must contain real values.

Remarks:

- abscissa are supposed to be spanned over $[-1,1]$ like a $\cos (\theta)$ function, with $\theta$ equally spaced;
- actually, data are already considered symmetric on the interval $[-n+1, n+1]$; therefore, there doesn't exist any constraint on the value of the data vector.

See also: mfInvFourierLeg, mfLegendre, mfFFT, mfFFT2, mfFourierSin, mfFourierCos
$m f$ InvFourierLeg

## inverse Fourier-Legendre transform

Interface:

```
function mfInvFourierLeg( A, dim ) result( out )
```

```
type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

Description:
This is the inverse tranformation of mfFourierLeg. So, for any vector v, we have:
$\mathrm{v}=\mathrm{mfInvFourierLeg(mfFourierLeg(v))}$
See also: mfFourierLeg, mfLegendre, mfFFT, mfFFT2, mfFourierSin, mfFourierCos

### 1.4 Operators

| + | mfArray sum |
| :---: | :---: |
| - | mfArray difference |
| mfMul, .x. | mfArray product |
| mfCross | cross-product of two vectors |
| mfKron | Kronecker tensor product |
| * | mfArray element-wise product |
| / | mfArray element-wise quotient |
| ** | mfArray element-wise exponentiation |
| .t. | mfArray transposition |
| .h. | mfArray conjugate transposition |
| .vc. | vertical concatenation |
| .hc. | horizontal concatenation |
| msHorizConcat | sparse matrix horizontal concatenation |
| mfColPerm, msColPerm | columns permutation |
| mfRowPerm, msRowPerm | rows permutation |
| mfColScale, msColScale | columns scaling |
| mfRowScale, msRowScale | rows scaling |
| mf InvPerm | inverse of a permutation |
| >=, > | mfArray relational operator |
| <=, < | mfArray relational operator |
| $==$, /= | mfArray relational operator |
| $m f A l l, m f A n y$ | mfArray boolean operator |
| mfColon | mfArray sequence constructor |
| .not., .and., or. | mfArray logical operator |
| .eqv., .neqv. | mfArray logical operator |
| mfIsMember | elements members of a set |
| $m f$ Intersect | common elements of two sets |
| mfUnion | all elements of two sets |
| mfUnique | unique elements of a set |

See also:
Core Routines
File Input/Output
Data Analysis Functions
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Optimization and Function Functions
Sparse Matrices

Calling syntax:

$$
C=A+B
$$

## Description:

Computes the sum of two mfArrays of same shape.
One of them may be a scalar of type real (single or double) or complex; in such a case, if the mfArray is not a scalar, then the operation is made on all elements.

Remarks: A and B may have a dense or sparse structure.
See also: -

Calling syntax:

$$
\mathrm{C}=\mathrm{A}-\mathrm{B}
$$

## Description:

Computes the difference of two mfArrays of same shape.
One of them may be a scalar of type real (single or double) or complex; in such a case, if the mfArray is not a scalar, then the operation is made on all elements.

Remarks: A and B may have a dense or sparse structure.
See also: +
mfMul, .x.

Interface:
function mfMul( A, B, transp ) result( out )

```
type(mfArray), intent(in) :: A, B
integer, intent(in), optional :: transp
type(mfArray) :: out
```

A .x. B is a shortcut for writing: mfMul(A, B )

Description:
Computes the matrix product $A * B$ (which is not commutative).
When the optional argument transp is present, it computes $A^{\prime} * B(\operatorname{transp}=1)$ or $A * B^{\prime}(\operatorname{transp}=2)$. This latter option is valid only for dense mfArrays.

The shape of the two matrices must respect the classical matrix-product rule, according to the value of transp.

Remarks: the element-wise operation is done with the operator ' $*$ '.
This routine can be applied to dense or sparse mfArrays of any type (real or complex).
See also: mfCross
mfCross
cross-product of two vectors

Interface:

```
function mfCross( u, v ) result( out )
    type(mfArray), intent(in) :: u, v
    type(mfArray) :: out
```

Description:
Computes the cross-product of two vectors in $R^{3}$. See also: mfMul
mfKron

Interface:

```
function mfKron( A, B ) result( out )
    type(mfArray), intent(in) :: A, B
    type(mfArray) :: out
```

Description:
Computes the Kronecker product of A and B. If the shape of A is $(m, n)$ and the shape of B is $(p, q)$, then the shape of the Kronecker product will be ( $m p, n q$ ).

The mfArrays A and B may be dense or sparse.
Remark: Currently, only real mfArrays are accepted.
See also: mfMul

## mfArray element-wise product

Calling syntax:

$$
\mathrm{C}=\mathrm{A} * \mathrm{~B}
$$

## Description:

Computes the product (element-wise) of two mfArrays of same shape.
One of them may be a scalar of type real (single or double) or complex; in such a case, if the mfArray is not a scalar, then the operation is made on all elements.

Only for this routine, B may be of type mfUnit. This feature allows the user to change the physical unit of the mfArray $A$.

Remarks: A and B may have a dense or sparse structure.
See also: mfMul, /

Calling syntax:

$$
\mathrm{C}=\mathrm{A} / \mathrm{B}
$$

## Description:

Computes the division (element-wise) of two mfArrays of same shape.
One of them may be a scalar of type real (single or double) or complex; in such a case, if the mfArray is not a scalar, then the operation is made on all elements.

Remarks: A and B may have a dense or sparse structure.
See also: mfLDiv, mfRDiv, *

## mfArray element-wise exponentiation

Calling syntax:

$$
\mathrm{C}=\mathrm{A} * * \mathrm{i}
$$

Description:
Computes the power (element-wise) of the mfArray A to the exponent i, which must be a scalar, of type integer or real.

Remarks: A may have a dense or sparse structure.

## .t.

## mfArray transposition

Calling syntax:

$$
\mathrm{C}=. \mathrm{t} . \mathrm{A}
$$

## Description:

The prefixed operator '.t.' transposes the mfArray A.
A warning is emitted if this routine is applied to a complex mfArray (in this case, the operator '.h.' should be used instead).

Remarks: A may have a dense or sparse structure. This operator can also be applied to integer or real arrays, of rank 1 or 2 (i.e. vector or matrix); in all cases it returns an mfArray.

## .h.

mfArray conjugate transposition

Calling syntax:

$$
\mathrm{C}=\mathrm{h} . \mathrm{A}
$$

## Description:

The prefixed operator '. h .' returns the conjugate transpose of the mfArray A.
Remarks: A may have a dense or sparse structure. This operator should be applied to complex arrays, of rank 1 or 2 (i.e. vector or matrix); in all cases it returns an mfArray. If A is a real mfArray, it is converted in a complex mfArray.
See also: .t.
.vc.
vertical concatenation

Calling syntax:
C = A .vc. B

Description:
Vertical concatenation of mfArrays (dense and/or sparse).
$A$ and $B$ must have the same number of columns.
One of the two mfArrays can be empty: it enables the construction of a matrix inside a loop, from an empty one.

Remark: This operator also accepts usual Fortran 90 1-rank numeric array (i.e. either real or complex). See also: .hc.
.hc.
horizontal concatenation

Calling syntax:
C = A .hc. B

Description:
Horizontal concatenation of mfArrays (dense and/or sparse).
$A$ and $B$ must have the same number of rows.
One of the two mfArrays can be empty: it enables the construction of a matrix inside a loop, from an empty one.

Remarks:

- when working with sparse mfArrays, the subroutine msHorizConcat is much more efficient.
- this operator also accepts usual Fortran 90 1-rank numeric array (i.e. either real or complex).

See also: msHorizConcat, .vc.
msHorizConcat

## sparse matrix horizontal concatenation

## Interface:

subroutine msHorizConcat( A, data)
type(mfArray), intent(in out) :: A
type(mfArray), intent(in) :: data

## Description:

This subroutine version of operator (.hc.) is intended to be more efficient, because: (i) modifications of A are made "in place" and (ii) allocation of space is anticipated and over-dimensioned.

A can be empty: it enables the construction of a matrix inside a loop, from an empty one.
The mfArrays A must be sparse; the mfArray data must be sparse if it is a matrix, but can be dense if it is a vector.

Restrictions: the two mfArrays must have the same data type.
Remarks: use operator(.hc.) instead of for dense matrices.
See also: .hc., .vc.
mf/msColPerm,
columns permutation

First calling syntax:

```
B = mfColPerm( A, p )
```

Description:
Returns the columns permutation of the mfArray A (may be of any type, dense or sparse), using the permutation p : the $j^{t h}$ column of B is the $p(j)^{t h}$ column of A .

In Matlab syntax or Fortran 90 notation, it computes $B=A(:, p)$.
$p$ is either an ordinary vector mfArray, containing the (integer) indices of the permutation, or a true permutation mfArray (see mfPerm).

The second calling syntax:
call msColPerm ( A, p )
applies the same operation, but in-place.
Remark: This routine is much more efficient than computing a column's permutation by calling $m f G e t\left(A, M F \_A L L, p\right)$.

See also: mf/msRowPerm, mf/msColScale, mf/msRowScale

## mf/msRowPerm,

rows permutation

First calling syntax:

```
B = mfRowPerm( A, p )
```

Description:
Returns the rows permutation of the mfArray A (may be of any type, dense or sparse), using the permutation p: the $i^{t h}$ row of B is the $p(i)^{t h}$ row of A .

In Matlab syntax or Fortran 90 notation, it computes $B=A(p,:)$.
$p$ is either an ordinary vector mfArray, containing the (integer) indices of the permutation, or a true permutation mfArray (see mfPerm).

The second calling syntax:
call msRowPerm ( A, p )
applies the same operation, but in-place.
Remark: This routine is much more efficient than computing a row's permutation by calling mfGet (A, p, MF_ALL).

See also: mf/msColPerm, mf/msColScale, mf/msRowScale
mf/msColScale,
columns scaling

First calling syntax:

```
B = mfColScale( A, s )
```

Description:
Returns the columns scaling of the mfArray A (must be numeric, dense or sparse), using the dense vector s : the $j^{t h}$ column of B is the $j^{\text {th }}$ column of A multiplied by $s(j)$.

In Matlab syntax, it computes $B=A * \operatorname{diag}(s)$.
Restrictions: the vector s must be a real mfArray.

The second calling syntax:

```
call msColScale( A, s )
```

applies the same operation, but in-place.
See also: mf/msColPerm, mf/msRowPerm, mf/msRowScale

## mf/msRowScale,

First calling syntax:

```
B = mfRowScale( A, s )
```


## Description:

Returns the rows scaling of the mfArray A (must be numeric, dense or sparse), using the dense vector s : the $i^{\text {th }}$ row of B is the $i^{t h}$ row of A multiplied by $s(i)$.

In Matlab syntax, it computes $B=\operatorname{diag}(s) * A$.
Restrictions: the vector s must be a real mfArray.

The second calling syntax:
call msRowScale( A, s )
applies the same operation, but in-place.
See also: mf/msColPerm, mf/msRowPerm, mf/msColScale
mfInvPerm,
inverse of a permutation

Calling syntax:

```
p_inv = mfInvPerm( p )
```

Description:
Inverts the permutation p .
See also: $\mathrm{mf} / \mathrm{msColPerm}, \mathrm{mf} / \mathrm{msRowPerm}$

Calling syntax:

$$
C=A>=B
$$

Description:
Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.
A and B must have the same shape and must be, of course, of type real.
$B$ may be a scalar, real or even integer.
Remarks: a boolean mfArray contains $T R U E$ and $F A L S E$ values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '. and.', '. or .', etc.).
See also: >, <=, <, ==, /=

Calling syntax:

$$
\mathrm{C}=\mathrm{A}>\mathrm{B}
$$

Description:
Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.
A and B must have the same shape and must be, of course, of type real.
$B$ may be a scalar, real or even integer.
Remarks: a boolean mfArray contains $T R U E$ and $F A L S E$ values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '. and.', '. or .', etc.).
See also: >=, <=, <, ==, /=

Calling syntax:

$$
C=A<=B
$$

Description:
Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.
A and B must have the same shape and must be, of course, of type real.
$B$ may be a scalar, real or even integer.
Remarks: a boolean mfArray contains $T R U E$ and $F A L S E$ values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '. and.', '. or .', etc.).
See also: <, >=, >, ==, /=
$<$
mfArray relational operator

Calling syntax:

$$
C=A<B
$$

Description:
Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.
A and B must have the same shape and must be, of course, of type real.
$B$ may be a scalar, real or even integer.
Remarks: a boolean mfArray contains $T R U E$ and $F A L S E$ values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '. and.', '. or .', etc.).
See also: <=, >=, >, ==, /=

Calling syntax:

$$
C=A=B
$$

Description:
Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.
A and B must have the same shape and must be, of course, of type real.
$B$ may be a scalar, real or even integer.
Remarks: a boolean mfArray contains $T R U E$ and $F A L S E$ values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '. and.', '. or .', etc.).
See also: /=, mfIsEqual, <=, <, >=, >
/=
mfArray relational operator

Calling syntax:

$$
\mathrm{C}=\mathrm{A} /=\mathrm{B}
$$

Description:
Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.
A and B must have the same shape and must be, of course, of type real.
$B$ may be a scalar, real or even integer.
Remarks: a boolean mfArray contains $T R U E$ and $F A L S E$ values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '. and.', '. or .', etc.).
See also: ==, mfIsEqual, <=, <, >=, >
mfAll
mfArray boolean operator

Interface:

```
function mfAll( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```

Description:
If A is a vector mfArray, returns a scalar boolean mfArray which is TRUE (i.e. 1) if all elements are TRUE.

If $A$ is a matrix, the routine works down the columns, returning a row vector mfArray.
If dim is present, it indicates on which dimension the routine must be applied.
See also: All, mfAny

## mfAny

## mfArray boolean operator

Interface:

```
function mfAny( A, dim ) result( out )
    type(mfArray), intent(in) :: A
    integer, intent(in), optional :: dim
    type(mfArray) :: out
```

Description:
If A is a vector mfArray, returns a scalar boolean mfArray which is TRUE (i.e. 1) if at least one element is TRUE.

If $A$ is a matrix, the routine works down the columns, returning a row vector mfArray.
If dim is present, it indicates on which dimension the routine must be applied.
See also: Any, mfAll
mfColon

## mfArray sequence constructor

## Interface:

```
A = mfColon( start, end [, step, tol ] )
```


## Description:

Returns an mfArray containing a sequence: a list of reals equally spaced, from start to end, with the specified step.

By default (i.e. when step is not present), increment is unity.
Arguments start, end and step may be of type integer or real (single or double precision), but all of same kind and precision.

Because of the rounding errors, the last number of the sequence seldom matches to the specified value of end, especially when step is not an integer. To avoid this unrequired behavior, the tolerance tol can be used to round the resulting numbers in the sequence to a relative precision equal to tol. Therefore, the tolerance tol should be present only in the case where the first three arguments are reals, all of them having the same precision; moreover, it should be a negative power of ten, e.g. 0.001.

When step is not present, the value of end must be greater than that of start. When step is present, its sign must be the same as that of end-start. step cannot have a zero value.

See also: mfLinSpace, mfLogSpace

## Example(s):

```
    call msDisplay( mfColon(1,15,step=3), "mfColon(1,15,step=3)" )
```

output:

```
mfColon(1,15,step=3) =
\begin{tabular}{lllll}
1 & 4 & 7 & 10 & 13
\end{tabular}
```

call msDisplay( mfColon(1.0,3.0,step=0.3333,tol=0.001),
"mfColon(1.0,3.0,step=0.3333,tol=0.001)" )
output:

```
mfColon(1.0,3.0,step=0.3333,tol=0.001) =
\begin{tabular}{lllllll}
1.0000 & 1.3330 & 1.6670 & 2.0000 & 2.3330 & 2.6670 & 3.0000
\end{tabular}
```

```
call msDisplay( mfColon( 0.1, 1.3, step=0.2 ),
    "mfColon( 0.1, 1.3, step=0.2 )" )
```

output:

```
mfColon( 0.1, 1.3, step=0.2 ) =
\begin{tabular}{llllll}
0.1000 & 0.3000 & 0.5000 & 0.7000 & 0.9000 & 1.1000
\end{tabular}
```

call msDisplay( mfColon( 0.1, 1.3, step=0.2, tol=0.0001 ), \& "mfColon( 0.1, 1.3, step=0.2, tol=0.0001 )" )
output:

```
mfColon( 0.1, 1.3, step=0.2, tol=0.0001 ) =
\begin{tabular}{lllllll}
0.1000 & 0.3000 & 0.5000 & 0.7000 & 0.9000 & 1.1000 & 1.3000
\end{tabular}
```

.not.
mfArray logical operator

Calling syntax:

$$
\mathrm{C}=. \text { not } . \mathrm{A}
$$

Description:
Takes the negation (element-wise) of a boolean mfArray.
Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .and., .or., .eqv., .neqv.
. and .
mfArray logical operator

Calling syntax:

$$
\mathrm{C}=\mathrm{A} . \text { and. } \mathrm{B}
$$

Description:
Composes (element-wise) two boolean mfArrays.
Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .not., .or., .eqv., .neqv.
.or .

Calling syntax:

$$
C=A \text {.or. } B
$$

Description:
Composes (element-wise) two boolean mfArrays.
Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .not., .and., .eqv., .neqv.
. eqv.
mfArray logical operator

Calling syntax:

$$
C=A \cdot \text { eqv. } B
$$

Description:
Composes (element-wise) two boolean mfArrays.
Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .not., .and., .or., .neqv.
.neqv.

Calling syntax:

$$
C=A . n e q v . B
$$

Description:
Composes (element-wise) two boolean mfArrays.
Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .not., .and., .or., .eqv

## mfIsMember

## elements members of a set

Calling syntax:

```
E = mfIsMember( A, set [, tol ] )
```


## Description:

Returns a boolean mfArray which shows which elements of the mfArray A are member of the mfArray set.

E has the same shape as A.
The tol optional argument is the tolerance used in the comparison test. By default, tolerance is zero, i.e. a strict equality is used to discard elements.

Remark: mfArrays A and set must be real with a dense structure.
See also: mfIntersect, mfUnion, mfUnique

## Example(s):

```
a = mf([ 1, 2, 3 ]) .vc. mf([ 4, 5, 6 ])
set = [ 0, 12, 8, 6, 2, 10, 4 ]
call msDisplay( a, "a", set, "set" )
call msDisplay( mfIsMember(a,set), "mfIsMember(a,set)" )
output:
```

$\mathrm{a}=$
123
$4 \quad 5 \quad 6$
set $=$
$\begin{array}{lllllll}0 & 12 & 8 & 6 & 2 & 10 & 4\end{array}$
mfIsMember $(\mathrm{a}, \mathrm{set})=$

| $F$ | $T$ | $F$ |
| :--- | :--- | :--- |
| $T$ | $F$ | $T$ |

```
mfIntersect
```

common elements of two sets

Calling syntax:

```
E = mfIntersect( A, B [, tol ] )
```

Description:
Returns an mfArray containing all elements common to the mfArrays A and B. Duplicated elements are discarded; the result is a sorted row vector.

The tol optional argument is the tolerance used in the equality test to discard duplicated elements, if any. By default, tolerance is zero, i.e. a strict equality is used to discard elements.

Remark: mfArrays A and B must be both real with a dense structure.
See also: mfIsMember, mfUnion, mfUnique

## Example(s):

```
A = reshape( [ 9, 7, 20, 4, 1, 5, 9, 7, 21, 1, 5, 25 ], [ 3, 4 ] )
B = reshape( [ 25, 15, 8, 8, 6, 5, 5, 4, 20, 10 ], [ 5, 2 ] )
call msDisplay( A, "A", B, "B" )
call msDisplay( mfIntersect(A,B), "mfIntersect(A,B)" )
```

output:

```
A =
    9 4 9 1
    7
    20 5 21 25
B =
    25 5
    15 5
    8 4
    8 20
    6 10
mfIntersect(A,B) =
    4 5 20 25
```

mfUnion
all elements of two sets

Calling syntax:

```
E = mfUnion( A, B [, tol ] )
```


## Description:

Returns an mfArray containing all elements of to the mfArrays A and B. Duplicated elements are discarded; the result is a sorted row vector.

The tol optional argument is the tolerance used in the equality test to discard duplicated elements, if any. By default, tolerance is zero, i.e. a strict equality is used to discard elements.

Remark: mfArrays A and B must be both real with a dense structure.
See also: mfIsMember, mfIntersect, mfUnique

## Example(s):

```
A = reshape( [ 9, 7, 20, 4, 1, 5, 9, 7, 21, 1, 5, 25 ], [ 3, 4 ] )
B = reshape( [ 25, 15, 8, 8, 6, 5, 5, 4, 20, 10 ], [ 5, 2 ] )
call msDisplay( A, "A", B, "B" )
call msDisplay( mfIntersect(A,B), "mfIntersect(A,B)" )
```

output:
$\mathrm{A}=$

| 9 | 4 | 9 | 1 |
| ---: | ---: | ---: | ---: |
| 7 | 1 | 7 | 5 |
| 20 | 5 | 21 | 25 |

$B=$
$25 \quad 5$
$15 \quad 5$
84

820
610
mfIntersect $(A, B)=$

| 1 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 21 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

mfUnique
unique elements of a set

## Interface:

```
function mfUnique( A, order, occurrence, tol ) result( out )
type(mfArray) :: A
real(kind=MF_DOUBLE), intent(in), optional :: tol
type(mfArray) :: out
```


## Description:

Returns the unique elements of the mfArray A. A must be a vector (either row or column).
By default, the returned elements are sorted in increasing order. However, their layout may be changed by using the following optional parameters:

- the order optional argument (= "asc", "des" or "no") may be used to specify the ordering.
- if order="no", no ordering is applied to the resulting elements but it is required to add the other occurrence optional argument (value may be "first" or "last") to specify if the selected element is the first one or the last one.

The tol optional argument is the tolerance used in the equality test to discard duplicated elements, if any. By default, tolerance is zero, i.e. a strict equality is used to discard elements.

Remark: the mfArray A must be real with a dense structure.
See also: mfIsMember, mfUnion, mfIntersect

## Example(s):

```
a = [ 2, 4, 6, 8, 9, 8, 7, 1, 5, 3, 6, 2, 4 ]
call msDisplay( a, "a" )
call msDisplay( mfUnique(a), "mfUnique(a)" )
call msDisplay( mfUnique(a,order="des"), 'mfUnique(a,order="des")' )
call msDisplay( mfUnique(a,order="no",occurrence="first"), &
    'mfUnique(a,order="no",occurrence="first")' )
call msDisplay( mfUnique(a,order="no",occurrence="last"),
    'mfUnique(a,order="no",occurrence="last")' )
```

output:

```
a =
```

| 2 | 4 | 6 | 8 | 9 | 8 | 7 | 1 | 5 | 3 | 6 | 2 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

mfUnique (a) =
$\begin{array}{lllllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\end{array}$
mfUnique(a,order="des") =
$\begin{array}{lllllllll}9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1\end{array}$

```
mfUnique(a,order="no",occurrence="first") =
    2 
mfUnique(a,order="no",occurrence="last") =
\begin{tabular}{lllllllll}
9 & 8 & 7 & 1 & 5 & 3 & 6 & 2 & 4
\end{tabular}
```


### 1.5 Elementary Math Functions

```
mfACos
mfACosh
mfACot
mfACoth
mfACsc
mfACsch
mfASec
mfASech
mfASin
mfASinh
mfATan
mfATan2
mfATanh
mfCos
mfCosh hyperbolic cosine
mfCot cotangent
mfCoth hyperbolic cotangent
mfCsc cosecant
mfCsch hyperbolic cosecant
mfSec
mfSech hyperbolic secant
mfSin sine
mfSinh hyperbolic sine
mfTan tangent
mfTanh hyperbolic tangent
mfSqrt square root
mfExp exponential
mfExpm1 exponential minus one
mfLog natural logarithm
mfLog1p natural logarithm of 1+x
mfLog10 base 10 logarithm
mf/msLog2 base 2 logarithm
mfPow10 base 10 power
mfPow2 base 2 power
mfFun, mfFun2 general function (one or two arguments)
mfGridFun
mfAbs absolute value
mfAngle phase angle
mfComplex complex conversion
mfConj complex conjugate
mfImag complex imaginary part
mfReal complex real part
mfHypot hypotenuse
mfRound round towards nearest integer
mfCeil round towards plus infinity
mfFix round towards zero
mfFloor round towards minus infinity
mfMod modulus after division
mfRem remainder after division
mfSign signum function
mfCSign complex signum function
```


## See also:

Core Routines
File Input/Output
Data Analysis Functions
Operators
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Optimization and Function Functions
Sparse Matrices
mfACos
inverse cosine

Calling syntax:

```
C = mfACos( A )
```

Description:
Returns the inverse cosine (element-wise) of the mfArray A (numerical dense only).
See also: mfCos, mfACosh, mfCosh
mfACosh
inverse hyperbolic cosine

Calling syntax:

```
C = mfACosh( A )
```

Description:
Returns the inverse hyperbolic cosine (element-wise) of the mfArray A (numerical dense only).
See also: mfCos, mfACos, mfCosh
mfACot

Calling syntax:

```
C = mfACot( A )
```

Description:
Returns the inverse cotangent (element-wise) of the mfArray A (numerical dense only).
See also: $\mathrm{mfCot}, \mathrm{mfACoth}, \mathrm{mfCoth}$
mfACoth
inverse hyperbolic cotangent

Calling syntax:

```
C = mfACoth( A )
```

Description:
Returns the inverse hyperbolic cotangent (element-wise) of the mfArray A (numerical dense only).
See also: mfCot, mfACot, mfCoth
mfACsc
inverse cosecant

Calling syntax:

```
C = mfACsc( A )
```

Description:
Returns the inverse cosecant (element-wise) of the mfArray A (numerical dense only).
See also: mfCsc, mfACsch, mfCsch

## mfACsch

inverse hyperbolic cosecant

Calling syntax:

```
C = mfACsch( A )
```

Description:
Returns the inverse hyperbolic cosecant (element-wise) of the mfArray A (numerical dense only).
See also: mfCsc, mfACsc, mfCsch

## mfASec

inverse secant

Calling syntax:

```
C = mfASec( A )
```

Description:
Returns the inverse secant (element-wise) of the mfArray A (numerical dense only).
See also: mfSec, mfASech, mfSech

## mfASech

inverse hyperbolic secant

Calling syntax:

```
C = mfASech( A )
```

Description:
Returns the inverse hyperbolic secant (element-wise) of the mfArray A (numerical dense only).
See also: mfSec, mfASec, mfSech

## mfASin

Calling syntax:

```
C = mfASin( A )
```


## Description:

Returns the inverse sine (element-wise) of the mfArray A (numerical dense only).
See also: mfSin, mfASinh, mfSinh

## mfASinh

Calling syntax:

```
C = mfASinh( A )
```

Description:
Returns the inverse hyperbolic sine (element-wise) of the mfArray A (numerical dense only).
See also: mfSin, mfASin, mfSinh
mfATan

Calling syntax:

```
C = mfATan( A )
```

Description:
Returns the inverse tangent (element-wise) of the mfArray A (numerical dense only).
See also: mfATan2, mfTan, mfATanh, mfTanh
mfATan2
four quadrant inverse tangent

Calling syntax:

```
C = mfATan2( Y, X )
```

Description:
Returns the four quadrant inverse tangent (element-wise) of the quotient $Y / X$, and the result is in ( $-\pi, \pi$ ].
$X$ and $Y$ must be numerical, dense mfArrays.
See also: mfATan, mfTan, mfATanh, mfTanh
mfATanh

Calling syntax:

```
C = mfATanh( A )
```

Description:
Returns the inverse hyperbolic tangent (element-wise) of the mfArray A (numerical dense only). See also: mfTan, mfATan, mfTanh

## mfCos

cosine

Calling syntax:

```
C = mfCos( A )
```

Description:
Returns the cosine (element-wise) of the mfArray A (numerical dense only).
See also: mfACos, mfACosh, mfCosh

## mfCosh

hyperbolic cosine

Calling syntax:

```
C = mfCosh( A )
```

Description:
Returns the hyperbolic cosine (element-wise) of the mfArray A (numerical dense only).
See also: mfCos, mfACos, mfACosh
mfCot

Calling syntax:

```
C = mfCot( A )
```

Description:
Returns the cotangent (element-wise) of the mfArray A (numerical dense only).
See also: mfACot, mfACoth, mfCoth
mfCoth
hyperbolic cotangent

Calling syntax:

```
C = mfCoth( A )
```

Description:
Returns the hyperbolic cotangent (element-wise) of the mfArray A (numerical dense only).
See also: $\mathrm{mfCot}, \mathrm{mfACot}, \mathrm{mfACoth}$
mfCsc

Calling syntax:

```
C = mfCsc( A )
```

Description:
Returns the cosecant (element-wise) of the mfArray A (numerical dense only).
See also: mfACsc, mfACsch, mfCsch
mfCsch
hyperbolic cosecant

Calling syntax:

```
C = mfCsch( A )
```

Description:
Returns the hyperbolic cosecant (element-wise) of the mfArray A (numerical dense only).
See also: mfCsc, mfACsc, mfACsch
mfSec

Calling syntax:

```
C = mfSec( A )
```

Description:
Returns the secant (element-wise) of the mfArray A (numerical dense only).
See also: mfASec, mfASech, mfSech
mfSech
hyperbolic secant

Calling syntax:

```
C = mfSech( A )
```

Description:
Returns the hyperbolic secant (element-wise) of the mfArray A (numerical dense only).
See also: mfASech, mfASec, mfSec
mfSin

Calling syntax:

```
C = mfSin( A )
```

Description:
Returns the sine (element-wise) of the mfArray A (numerical dense only).
See also: mfASin, mfASinh, mfSinh

## mfSinh

hyperbolic sine

Calling syntax:

```
C = mfSinh( A )
```


## Description:

Returns the hyperbolic sine (element-wise) of the mfArray A (numerical dense only).
See also: mfASin, mfASinh, mfSin
$m f T a n$

Calling syntax:

```
C = mfTan( A )
```

Description:
Returns the tangent (element-wise) of the mfArray A (numerical dense only).
See also: mfATan, mfATanh, mfTanh
mfTanh
hyperbolic tangent

Calling syntax:

```
C = mfTanh( A )
```

Description:
Returns the hyperbolic tangent (element-wise) of the mfArray A (numerical dense only).
See also: mfATan, mfATanh, mfTan
mfSqrt

Calling syntax:

```
C = mfSqrt( A )
```

Description:
Returns the square root (element-wise) of the mfArray A (dense or sparse, real or complex).
The returned mfArray may be of type complex if any element has a negative value.
See also: mfSqrtm
$m f E x p$

Calling syntax:

```
C = mfExp( A )
```

Description:
Returns the exponential (element-wise) of the mfArray A (numerical dense only).
See also: mfLog, mfLog1p, mfExpm1, mfExpm

## mfExpm1

Calling syntax:

```
C = mfExpm1( A )
```

Description:
Returns the value of $\exp (x)-1$, for each $x$ element of the $m f$ Array A. This operation is made element-wise. See also: mfExp, mfLog, mfLog1p, mfExpm

## mfLog

Calling syntax:

```
C = mfLog( A )
```

Description:
Returns the natural logarithm (element-wise) of the mfArray A (numerical dense only).
See also: mfLog1p, mfExp, mfExpm1, mfLogm
$m f \log 1 p$
natural logarithm of $1+x$

Calling syntax:

```
C = mfLog1p( A )
```


## Description:

Returns the value of $\log (1+x)$, for each $x$ element of the mfArray A (numerical dense only). This operation is made element-wise.

See also: mfLog, mfExp, mfExpm1, mfExpm
$m f L o g 10$
base 10 logarithm

Calling syntax:

```
C = mfLog10( A )
```

Description:
Returns the base 10 logarithm (element-wise) of the mfArray A (numerical dense only).
See also: mfLog, mf/msLog2

## mf/msLog2

base 2 logarithm

Calling syntax:

```
C = mfLog2( A )
```

Description:
Returns the base 2 logarithm (element-wise) of the mfArray A (numerical dense only).
The subroutine version returns two mfArrays:

```
call msLog2( mfOut(M,E), A )
```

The mantissa $M$ has its element ranged in $[0.5,1[$, whereas elements of $E$ are integer exponents.
Each element of A verifies: $a=m 2^{e}$
See also: mfPow2, mfLog, mfLog10, mfOut
mfPow10
base 10 power

Calling syntax:

```
C = mfPow10( A )
```

Description:
Returns the base 10 power (element-wise) of the mfArray A (numerical dense only).
See also: mfLog10
mfPow2
base 2 power

First usage:

```
C = mfPow2( A )
```

Description:
Returns the base 2 power (element-wise) of the mfArray A (numerical dense only).

Second usage:

```
C = mfPow2( M, E )
```


## Description:

Returns numbers which write: $m 2^{e}$, for each (real) $m$ of M and each (integer) $e$ of $\mathrm{E} . \mathrm{M}$ and E are two mf Arrays.

See also: mfLog2

## mfFun

Usage:

```
C = mfFun( A, fun )
```


## Description:

Apply the user-defined function (element-wise) fun to the mfArray A (numerical dense only).
fun is a real or complex function having the following interface:

```
    real(kind=MF_DOUBLE) function fun( x1 )
        real(kind=MF_DOUBLE), intent(in) :: x1
    end function
or
    complex(kind=MF_DOUBLE) function fun( x1 )
        complex(kind=MF_DOUBLE), intent(in) :: x1
    end function
```

See also: mfFunm, mfFun2

## mfFun2

Usage:

```
C = mfFun2( A, B, fun )
```


## Description:

Apply the user-defined function (element-wise) fun to the mfArrays A and B (numerical dense only).
fun is a real or complex function having the following interface:

```
    real(kind=MF_DOUBLE) function fun( x1, x2 )
        real(kind=MF_DOUBLE), intent(in) :: x1, x2
    end function
or
    complex(kind=MF_DOUBLE) function fun( x1, x2 )
        complex(kind=MF_DOUBLE), intent(in) :: x1, x2
    end function
```

See also: mfFun, mfGridFun

## mfGridFun

Usage:

```
Z = mfGridFun( v_x, v_y, fun )
```

applies the 2-argument function fun to the couple of coordinate matrices ( $X, Y$ ) generated by the vectors v_x and v_y, without building $X$ and $Y$ (see msMeshGrid about contraints concerning v_x and v_y). It aims at economize memory for large grids.
fun is either a character string giving the name of a $2-\arg$ function (as complex, hypot, atan2, ...), or the name of a user defined function. In this latter case, the user function must have the following interface:

```
real(kind=MF_DOUBLE) function fun( x1, x2 )
    real(kind=MF_DOUBLE), intent(in) :: x1, x2
end function
```

See also: mfFun2

## mfAbs

## absolute value

Calling syntax:

```
C = mfAbs( A )
```

Description:
Returns the absolute value (or the module for complex numbers) of the mfArray A (dense or sparse). This operation is made element-wise.

See also: mfAngle

## mfAngle

Calling syntax:

```
C = mfAngle( A )
```


## Description:

Returns the phase angle (element-wise), in radians of the complex mfArray A (dense or sparse).
For real values, this function returns 0 or $\pi$. Moreover, it returns 0 when argument is null (real or complex), so maintains the sparsity of the mfArray.

See also: mfAbs
mfComplex
complex conversion

Calling syntax:

```
C = mfComplex( A [, B] )
```

Description:
Converts a real mfArray A (dense or sparse) in a complex one.
If the optional argument mfArray B is present, builds a complex mfArray whose real part is A and the imaginary part is B. In this latter case, A and B must be both real (dense or sparse).

See also: mfReal, mfImag

## mfConj

complex conjugate

Calling syntax:

```
C = mfConj( A )
```

Description:
Returns the conjugate (element-wise) of the complex mfArray A (dense or sparse).
A may be real, but the returned mfArray is complex.
See also: .h.
mfImag
complex imaginary part

Calling syntax:

```
C = mfImag( A )
```

Description:
Returns the imaginary part (element-wise) of the complex mfArray A (dense or sparse).
See also: mfReal
mfReal

Calling syntax:

```
C = mfReal( A )
```

Description:
Returns the real part (element-wise) of the complex mfArray A (dense or sparse).
See also: mfImag

## mfHypot

Calling syntax:

```
C = mfHypot( A, B )
```

Description:
Returns the hypotenuse of a right rectangle whose sides (mfArrays A and B, in dense format) are provided. This operation is made element-wise.

Remarks: the computation is done without overflow or underflow.
mfRound
round towards nearest integer

Calling syntax:

```
C = mfRound( A )
```

Description:
Rounds towards nearest integer each element of the mfArray A (dense or sparse). The behavior of this routine is similar to the anint Fortran 90 intrinsic function.

See also: mfCeil, mfFix, mfFloor
mfCeil
round towards plus infinity

Calling syntax:

```
C = mfCeil( A )
```

Description:
Rounds towards plus infinity each element of the mfArray A (dense or sparse).
See also: mfFix, mfFloor, mfRound
mfFix

Calling syntax:

```
C = mfFix( A )
```

Description:
Rounds towards zero each element of the mfArray A (dense or sparse).
See also: mfCeil, mfFloor, mfRound
mfFloor
round towards minus infinity

Calling syntax:

```
C = mfFloor( A )
```

Description:
Rounds towards minus infinity each element of the mfArray A (dense or sparse).
See also: mfCeil, mfFix, mfRound

## mfMod

## modulus after division

Calling syntax:

```
C = mfMod( A, B )
```

Description:
Returns the modulus after division of each element of the mfArray A (numerical dense only).
N.B.: this is the Fortran 90 modulo function

See also: mfRem

## mfRem

## remainder after division

Calling syntax:

```
C = mfRem( A, B )
```

Description:
Returns the modulus after division of each element of the mfArray A (numerical dense only).
N.B.: this is the Fortran 90 mod function

See also: mfMod

## mfSign

Calling syntax:

```
C = mfSign( A )
```

Description:
Applies the signum function to each element of the mfArray A (numerical dense only). It returns only 1 or -1 .
$N . B .:$ It differs from the Fortran 90 sign intrinsic function.
See also: mfCSign

## mfCSign

Calling syntax:

```
C = mfCSign( A )
```

Description:
Applies the complex signum function to each element of the mfArray A (numerical dense only).
See also: mfSign

### 1.6 Specialized Math Functions

mfErf
mfErfInv
mfErfC
mfErfCInv
mfErfCScaled
mfExpInt
mfGamma
mfGammaLn
mfBesselJ
mfBesselY
mfBesselI
mfBesselK
$m f A i r y$
$m s E l l i p K E ~$
$m f I s P r i m e ~$
$m f F a c t o r ~$
$m s C l e a n P r i m e N u m b e r s ~$

Error function
inverse Error function
Complementary Error function inverse Complementary Error function Scaled Complementary Error function

Exponential Integral function

Gamma function
Logarithm of the Gamma function
Bessel functions of the first kind
Bessel functions of the second kind modified Bessel functions of the first kind modified Bessel functions of the second kind Airy function
complete elliptic integrals of first and second kinds
prime number test prime numbers factorization prime numbers storage cleaning rational approximation

See also:
Core Routines
File Input/Output
Data Analysis Functions
Operators
Elementary Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Optimization and Function Functions
Sparse Matrices

## mfErf

Error function

Calling syntax:

```
C = mfErf( A )
```

Description:
Returns the Error function (element-wise) of the mfArray A (dense only, real or complex).
See also: mfErfC, mfErfInv, mfErfCInv, mfErfCScaled

## mfErfInv

## inverse Error function

Calling syntax:

```
C = mfErfInv( A )
```

Description:
Returns the inverse Error function (element-wise) of the mfArray A (dense only).
Restriction: A must be of type real.
See also: mfErf, mfErfC, mfErfCInv, mfErfCScaled
mfErfC

## Complementary Error function

Calling syntax:

```
C = mfErfC( A )
```

Description:
Returns the Complementary Error function (element-wise) of the mfArray A (dense only, real or complex).

See also: mfErf, mfErfInv, mfErfCInv, mfErfCScaled
mfErfCInv
inverse Complementary Error function

Calling syntax:

```
C = mfErfCInv( A )
```

Description:
Returns the inverse Complementary Error function (element-wise) of the mfArray A (dense only).
Restriction: A must be of type real.
See also: mfErf, mfErfInv, mfErfC, mfErfCScaled
mfErfCScaled

## Scaled Complementary Error function

Calling syntax:

```
C = mfErfCScaled( A )
```

Description:
Returns the Scaled Complementary Error function (element-wise) of the mfArray A (dense only), defined by mfErfCScaled (x) $=\exp (x * * 2) * m f E r f C(x)$

Restriction: A must be of type real.
See also: mfErf, mfErfInv, mfErfC, mfErfCInv

## mfExpInt

## Exponential Integral function

Calling syntax:

```
C = mfExpInt( A )
```

Description:
Returns the exponential integral function (element-wise) of the mfArray A (dense only).
This function is mathematically defined as:

$$
E_{1}(x)=\int_{x}^{\infty} \frac{\exp (-t)}{t} d t
$$

For positive value of $x$, it has the following properties:

$$
E_{1}(0)=+\infty, \quad E_{1}(+\infty)=0
$$

Restriction: A must be of type real and must have a positive value. On the contrary, a warning is emitted.

## mfGamma

Calling syntax:

```
C = mfGamma( A )
```

Description:
Returns the gamma function (element-wise) of the mfArray A (dense only).
Restriction: A must be of type real.
See also: mfGammaLn

## mfGammaLn

## Logarithm of the Gamma function

Calling syntax:

```
C = mfGammaLn( A )
```

Description:
Returns the natural logarithm of the gamma function (element-wise) of the mfArray A (dense only). Restriction: A must contain positive real values.

See also: mfGamma
mfBesselJ
Bessel functions of the first kind

Calling syntax:

```
C = mfBesselJ( alpha, A )
```

Description:
Returns the Bessel functions of the first kind $J_{\alpha}(x)$ (element-wise) of the mfArray A (dense only).
The real alpha order doesn't need to be an integer: both positive and negative fractional orders are possible.

Restriction: A must be of type real but, according to the values of $\alpha$ and $x$, a complex mfArray may be returned.

Remark: Some roots of $J_{0}(x)$ and $J_{1}(x)$ are stored respectively in the arrays: MF_BESSEL_JO_ROOTS and MF_BESSEL_J1_ROOTS.

See also: mfBesselY, mfBesselI, mfBesselK, MF_BESSEL_JO_ROOTS, MF_BESSEL_J1_ROOTS
mfBesselY

## Bessel functions of the second kind

Calling syntax:

```
C = mfBesselY( alpha, A )
```

Description:
Returns the Bessel functions of the second kind $Y_{\alpha}(x)$ (element-wise) of the mfArray A (dense only).
The real alpha order doesn't need to be an integer: both positive and negative fractional orders are possible.

Restriction: A must be of type real but, according to the values of $\alpha$ and $x$, a complex mfArray may be returned.

See also: mfBesselJ, mfBesselI, mfBesselK
mfBesselI
modified Bessel functions of the first kind

Calling syntax:

```
    C = mfBesselI( alpha, A )
```

Description:
Returns the modified Bessel functions of the first kind $I_{\alpha}(x)$ (element-wise) of the mfArray A (dense only).

The real alpha order doesn't need to be an integer: both positive and negative fractional orders are possible.

Restriction: A must be of type real but, according to the values of $\alpha$ and $x$, a complex mfArray may be returned.

See also: mfBesselJ, mfBesselY, mfBesselK
mfBesselK
modified Bessel functions of the second kind

Calling syntax:

```
    C = mfBesselK( alpha, A )
```

Description:
Returns the modified Bessel functions of the second kind $K_{\alpha}(x)$ (element-wise) of the mfArray A (dense only).

The real alpha order doesn't need to be an integer: both positive and negative fractional orders are possible.

Restriction: A must be of type real but, according to the values of $\alpha$ and $x$, a complex mfArray may be returned.

See also: mfBesselJ, mfBesselY, mfBesselI

## mfAiry

Airy function

Calling syntax:

```
B = mfAiry( A )
```

Description:
Returns the Airy function (element-wise) of the mfArray A (dense only).
Restriction: A must be of type real and a real mfArray is returned. Complex function is not yet implemented.

See also: mfBesselJ

Calling syntax:

```
call msEllipKE( mfOut(K,E), A )
```

Description:
Computes the complete elliptic integrals of first and second kind $K(m)$ and $E(m)$ (element-wise) of the mfArray A (dense only), which are stored in the two mfArray K and E.

Each element of the array A is taken to be the parameter $m=k^{2}$, where $k$ is the modulus.
The parameter $m$ must be ranged in $[0,1]$ else $N a N$ values are returned for both integrals.
See also: mfOut

## mfIsPrime

Calling syntax:

```
B = mfIsPrime( A )
```

Description:
Tests if numbers in the mfArray A are prime numbers. This function returns a boolean mfArray of same shape as A. The argument A may be scalar, vector or matrix, and is supposed to contain only integer numbers.

Remark: Integers involved in this routine must be less than $2,147,000,000$.
See also: mfFactor, msCleanPrimeNumbers

## mfFactor

## prime numbers factorization

Calling syntax:

```
B = mfFactor( A )
```

Description:
Returns the prime factors of the scalar mfArray A. The argument A is supposed to contain an integer number.

Remark: Integers involved in this routine must be less than 2,147,000,000.
See also: mfIsPrime, msCleanPrimeNumbers

Example(s):

```
x = 2**4 * 3**3 * 5**2 * 7
call msDisplay( x, "x = 2**4 * 3**3 * 5**2 * 7" )
print "(/,A)", "*** test number 109 ***"
call msDisplay( mfFactor(x), "mfFactor(x)" )
```

output:

```
x = 2**4 * 3**3 * 5**2 * 7 =
    7 5 6 0 0
mfFactor(x) =
\begin{tabular}{llllllllll}
2 & 2 & 2 & 2 & 3 & 3 & 3 & 5 & 5 & 7
\end{tabular}
```

msCleanPrimeNumbers
prime numbers storage cleaning

Calling syntax:
call msCleanPrimeNumbers( )

Description:
Cleans the temporary storage of prime numbers, required for mfIsPrime and mfFactor routines. According to the magnitude of the involved integers, up to 450 MB of memory can be freed.

## msRat

## rational approximation

Calling syntax:

```
call msRat( mfOut(N,D), A [, tol] )
```


## Description:

Computes the rational approximation (element-wise) of the mfArray A (real dense only), and returns the matrix of numerators in the mfArray $N$ and the matrix of denominators in the mfArray $D$.

Default relative tolerance for the approximation is $10^{-6}$. Tolerance cannot be larger than $10^{-2}$ and cannot be smaller than $10^{-12}$.

For example, with the default tolerance, $\pi=355 / 113$. Any numerical value has a rational approximation, even $\operatorname{Inf}=1 / 0$ and $N a N=0 / 0$.

See also: mfOut

Example(s):

```
x = MF_PI
    call msDisplay( x, "x = MF_PI" )
    call msRat ( mfOut (y,z), x )
    call msDisplay( y, "numerator", z, "denominator" )
```

output:

```
x = MF_PI =
    3.1416
numerator =
    355
denominator =
    1 1 3
```

```
    x = mfHilb(4)
```

    x = mfHilb(4)
    call msDisplay( x, "x = mfHilb(4)" )
    call msDisplay( x, "x = mfHilb(4)" )
    call msRat( mfOut(y,z), x )
    call msRat( mfOut(y,z), x )
    call msDisplay( y, "numerator", z, "denominator" )
    ```
    call msDisplay( y, "numerator", z, "denominator" )
```

output:

```
x = mfHilb(4)
    1.0000 0.5000 0.3333 0.2500
    0.5000 0.3333 0.2500 0.2000
    0.3333 0.2500 0.2000 0.1667
    0.2500 0.2000 0.1667 0.1429
```

| 1 | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |
| denominator $=$ |  |  |  |
| 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 5 |
| 3 | 4 | 5 | 6 |
| 4 | 5 | 6 | 7 |

### 1.7 Elementary Matrix Manipulation Functions

| mfEye | identity matrix |
| :---: | :---: |
| mfLinSpace | linear spaced vector |
| mfLogSpace | log spaced vector |
| mfMagic | magic square |
| msMeshGrid | arrays for interpolation and 3-D plots |
| mfZeros | matrix of zeros |
| mf0nes | matrix of ones |
| mfRand | uniformly distributed random numbers |
| msRand | set the random number generator seed |
| mfRandN | normally distributed random numbers |
| mfRandPoiss | Poisson distributed random numbers |
| mfRepMat | replicate and tile an array |
| mfMerge | merge of two arrays |
| mfPack, mfUnpack | packing and unpacking arrays |
| mfCshift, mfEoshift | array shifting |
| mf/msDiag | diagonal matrices and diagonals of a matrix |
| $m f B l k D i a g ~$ | block diagonal concatenation |
| $\mathrm{mf} / \mathrm{msFind}$ | find indices of nonzero elements |
| mfNonZeros | non-zero elements |
| $\mathrm{mf} / \mathrm{msReshape}$ | shape modification |
| mfTriL | lower triangular part |
| mfTriU | upper triangular part |
| mfFlipLR | matrix flip Left-Right |
| mfFlipUD | matrix flip Up-Down |
| mfRot90 | matrix rotation |
| mfHilb | Hilbert matrix |
| mf InvHilb | inverse Hilbert matrix |
| mfVander | Vandermonde matrix |
| mfCompan | Companion matrix |
| mfHankel | Hankel matrix |
| mfToeplitz | Toeplitz matrix |
| mfPerm | permutation vector creation |
| mfRandPerm | random permutation vector creation |
| mfCheckPerm | permutation vector checking |
| isfinite, mfIsFinite | finite test |
| isinf, mfIsInf | infinity test |
| isnan, mfIsNaN | Not-a-Number test |
| mfIsDiag | diagonal pattern test |
| mfIsTril | lower triangular pattern test |
| mfIsTriu | upper triangular pattern test |

See also:
Core Routines
File Input/Output
Data Analysis Functions
Operators
Elementary Math Functions
Specialized Math Functions
Matrix Functions

Polynomial Functions
Optimization and Function Functions
Sparse Matrices

## mfEye

## identity matrix

Calling syntax:

```
A = mfEye( n1 [, n2] )
```

Description:
Returns an mfArray containing the identity matrix.
If it is called with one integer argument, the identity matrix is square, of shape ( $\mathrm{n} 1, \mathrm{n} 1$ ).
If the optional argument n 2 is present, the routine builds an identity matrix of shape ( $\mathrm{n} 1, \mathrm{n} 2$ ).
See also: mfOnes, mfZeros

## mfLinSpace

## linear spaced vector, given nb of values

First calling syntax:

```
A = mfLinSpace( start, end [, nval] )
```


## Description:

Returns an mfArray (row vector) which contains a list of reals equally spaced, from start to end, with the specified number of values nval. start and end can take any arbitrary finite values, even equal.

By default (i.e. when the integer nval is not present), the list contains nval $=100$ reals. If nval is present, it must be greater than or equal to 2 .

Second calling syntax:

```
A = mfLinSpace( [ start | end ], nval, step )
```


## Description:

In this second syntax, only one argument among start and end can be present. The additional step argument specifies the step between two consecutive values; it cannot be equal to zero.

Remarks: start, end and step must be of type real (single or double precision).
See also: mfColon, mfLogSpace

Example(s):

```
call msDisplay( mfLinspace(-MF_PI, MF_PI, 5),
    "mfLinspace(-MF_PI, MF_PI, 5)" )
```

output:

```
mfLinspace(-MF_PI, MF_PI, 5) =
    -3.1416 -1.5708 0.0000 1.5708 3.1416
call msDisplay( mfLinSpace(end=1.0d0, nval=5, step=0.1d0),
            "mfLinSpace(end=1.0d0, nval=5, step=0.1d0)" )
```

output:

```
mfLinSpace(end=1.0d0, nval=5, step=0.1d0)=
    0.6000 0.7000 0.8000 0.9000 1.0000
```


## mfLogSpace

## log spaced vector, given nb of values

First calling syntax:

```
A = mfLogSpace( start, end [, nval] )
```


## Description:

Returns an mfArray (row vector) which contains a list of reals logarithmic spaced, from $10^{\text {start }}$ to $10^{\text {end }}$, with the specified number of values nval.

By default (i.e. when the integer nval is not present), the list contains nval $=20$ reals. If nval is present, it must be greater than or equal to 2 .

Second calling syntax:

```
A = mfLogSpace( [ start | end ], nval, step )
```


## Description:

In this second syntaxe, only one argument among start and end can be present. The additional step argument specifies the step between the logarithm of two consecutive values; it cannot be equal to zero.

Remarks: start, end and step must be of type real (single or double precision).
See also: mfColon, mfLinSpace

Example(s):

```
call msDisplay( mfLogspace(0.0d0, 5.0d0, 6),
    "mfLogspace(0.0d0, 5.0d0, 6)" )
```

output:

```
mfLogspace(0.0d0, 5.0d0, 6) =
    1 10 100 1000 10000 100000
call msDisplay( mfLogSpace(start=0.0d0, nval=6, step=0.5d0), &
    "mfLogSpace(start=0.0d0, nval=6, step=0.5d0)" )
```

output:

```
mfLogSpace(start=0.0d0, nval=6, step=0.5d0) =
    1.0000 3.1623 10.0000 31.6228
```

mfMagic

Calling syntax:

```
A = mfMagic( n )
```

Description:
Returns an mfArray containing a magic square, from any positive integer $\mathrm{n} \geq 3$.

```
Example(s):
    call msDisplay( mfMagic(3), "mfMagic(3)" )
    call msDisplay( mfMagic(4), "mfMagic(4)" )
output:
mfMagic(3) =
        8 3 4
        1 5 9
        6 7 2
mfMagic(4) =
\begin{tabular}{rrrr}
1 & 12 & 8 & 13 \\
15 & 6 & 10 & 3 \\
14 & 7 & 11 & 2 \\
4 & 9 & 5 & 16
\end{tabular}
```

msMeshGrid

Calling syntax:

```
call msMeshGrid( mfOut(X,Y), v_x, v_y )
```


## Description:

Builds two mfArrays containing the $(x, y)$ coordinates of the nodes of a rectangular, structured grid, generated from the two real vectors $v_{-} x$ and $v_{-} y$.

Among the two input vectors (called generator vectors), one must be a row vector, and the other a column vector. Then the row vector is vertically replicated as many times as the size of the other to form one of the coordinate matrices, and conversely for the other.

The coordinate matrices $X$ and $Y$ are typically used to visualize a data matrix with an orientation different from that used by default. See more explanation in section "Changing Orientation/Transposition of Matrix Data" in the Muesli User Guide. They can also be used in 2D interpolation (see mfInterp2).

Remark: Note that most of the time, you don't need to compute explicitly the coordinate matrices ( $\mathrm{X}, \mathrm{Y}$ ) since you are able to pass the generator vectors directly to graphic routines (msPColor, msContour, etc.), and also to the numerical routine mfGridFun.

See also: mfOut

Example(s):

```
v_x = mfLinSpace( -1.0d0, 1.0d0, 5 )
v_y = .t. mfLinSpace( -1.5d0, 1.5d0, 7 )
call msMeshgrid( mfOut(X,Y), v_x, v_y )
call msDisplay( X, "X", Y, "Y" )
```

output:

```
X =
    -1.0000 -0.5000 0.0000 0.5000 1.0000
    -1.0000 -0.5000 0.0000 0.5000 1.0000
    -1.0000 -0.5000 0.0000 0.5000 1.0000
    -1.0000 -0.5000 0.0000 0.5000 1.0000
    -1.0000 -0.5000 0.0000 0.5000 1.0000
    -1.0000 -0.5000 0.0000 0.5000 1.0000
    -1.0000 -0.5000 0.0000 0.5000 1.0000
Y =
    -1.5000 -1.5000 -1.5000 -1.5000 -1.5000
    -1.0000 -1.0000 -1.0000 -1.0000 -1.0000
    -0.5000 -0.5000 -0.5000 -0.5000 -0.5000
        0.0000 0.0000 0.0000 0.0000 0.0000
    0.5000 0.5000 0.5000 0.5000 0.5000
    1.0000 1.0000 1.0000 1.0000 1.0000
    1.5000 1.5000 1.5000 1.5000 1.5000
```


## mfZeros

Calling syntax:

$$
\mathrm{A}=\operatorname{mfZeros}(\mathrm{n} 1[, \mathrm{n} 2])
$$

## Description:

Returns an mfArray containing zeros.
If it is called with one integer argument, the output matrix is square, of shape (n1,n1).
If the optional argument n 2 is present, the routine builds a matrix of shape ( $\mathrm{n} 1, \mathrm{n} 2$ ).
See also: mfEye, mfOnes

## mfOnes

Calling syntax:

```
A = mfOnes( n1 [, n2] )
```

Description:
Returns an mfArray containing ones.
If it is called with one integer argument, the output matrix is square, of shape ( $\mathrm{n} 1, \mathrm{n} 1$ ).
If the optional argument n 2 is present, the routine builds a matrix of shape ( $\mathrm{n} 1, \mathrm{n} 2$ ).
See also: mfEye, mfZeros

## mfRand

## uniformly distributed random numbers

Calling syntax:

```
A = mfRand( n1 [, n2] )
```


## Description:

Returns an mfArray containing pseudo-random real values, with a uniform distribution in $[0,1]$.
If it is called with one integer argument, the output matrix is square, of shape ( $\mathrm{n} 1, \mathrm{n} 1$ ).
If the optional argument n 2 is present, the routine builds a matrix of shape ( $\mathrm{n} 1, \mathrm{n} 2$ ).

Other syntax:

## $\mathrm{A}=\mathrm{mfRand}($ string $)$

When called with the string "seed", the seed (six long integers stored as double reals) of the random number generator is returned. This value can be later used to reset the seed to a previous state with the msRand routine.

Remark: Generated numbers doesn't depend on the compiler used, because the random_number Fortran 90 intrinsic routine is not use. The RngStreams package (included in the MUESLI library) is used instead.

See also: mfRandN, mfRandPoiss
msRand

## set the random number generator seed

Calling syntax:

```
call msRand( [ seed ] )
```


## Description:

Set the random number generator seed either from given values, or from the clock.
Without argument, the seed is set from the clock.
If the optional argument seed is present, then this argument must be an mfArray whose values are used to reset the seed of the RngStreams package, included in the MUESLI library. This seed must be an array of six integer numbers (stored as double precision reals) satifying the following constraints:

- the fist three values must be less than 4294967087 (and not all zero);
- the last three values must be less than 4294944443 (and not all zero).

Note that a check is done in the current routine

See also: mfRand, mfRandN, mfRandPoiss
mfRandN
normally distributed random numbers

Calling syntax:

$$
\mathrm{A}=\mathrm{mfRandN}(\mathrm{n} 1[, \mathrm{n} 2])
$$

Description:
Returns an mfArray containing pseudo-random real values, with a normal distribution (with zero mean and unit variance).

If it is called with one integer argument, the output matrix is square, of shape ( $\mathrm{n} 1, \mathrm{n} 1$ ).
If the optional argument n 2 is present, the routine builds a matrix of shape ( $\mathrm{n} 1, \mathrm{n} 2$ ).
See also: mfRand, mfRandPoiss, msRand

## mfRandPoiss

Calling syntax:

```
A = mfRandPoiss( mu, n1 [, n2] )
```

Description:
Returns an mfArray containing pseudo-random integer values, with a Poisson distribution, with mean (and variance) equal to mu.

The argument mu is a real number.
If it is called with only one size, the output matrix is square, of shape ( $\mathrm{n} 1, \mathrm{n} 1$ ).
If the optional argument n 2 is present, the routine builds a matrix of shape ( $\mathrm{n} 1, \mathrm{n} 2$ ).
See also: mfRandN, mfRand, msRand
mfRepMat
replicate and tile an array

Calling syntax:

```
C = mfRepMat( A, m, n )
```

Description:
This routine replicates the tile mfArray A, to build a bigger mfArray. The returned mfArray can be viewed as a m-by-n tiling matrix.

The behavior of this routine is similar to the spread Fortran 90 intrinsic function.

Example(s):

```
x = reshape( [ (i, i = 1, 6) ], [ 2, 3 ] )
call msDisplay( x, "x", mfRepmat(x,3,2), "mfRepmat(x,3,2)" )
```

output:
$\mathrm{x}=$
$1 \quad 3 \quad 5$
$\operatorname{mfRepmat}(x, 3,2)=$

| 1 | 3 | 5 | 1 | 3 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 4 | 6 | 2 | 4 | 6 |
| 1 | 3 | 5 | 1 | 3 | 5 |
| 2 | 4 | 6 | 2 | 4 | 6 |
| 1 | 3 | 5 | 1 | 3 | 5 |
| 2 | 4 | 6 | 2 | 4 | 6 |

## mfMerge

Calling syntax:

```
C = mfMerge( A, B, mask )
```


## Description:

The three arguments are mfArrays having the same shape. This function merge the two numeric arrays A and B under the control of the boolean mask. Its behavior is like that of the merge Fortran 90 intrinsic function.

Sparse arrays are not handled.

Example(s):

```
    A = 3.0d0*mfOnes (3,2)
    B = -1.0d0*mfOnes (3,2)
    mask = mfEye(3,2) > 0.5d0
    call msDisplay( A,"A", B,"B", mask,"mask" )
    call msDisplay( mfMerge(A,B,mask), "mfMerge(A,B,mask)" )
```

output:
$\mathrm{A}=$
33
33
3 3
$B=$
$-1 \quad-1$
$-1 \quad-1$
$-1 \quad-1$
mask $=$
T F
F T
F F
mfMerge $(A, B$, mask $)=$
$3-1$
-1 3
$-1 \quad-1$
mfPack

Calling syntax:

```
C = mfPack( A, mask )
```


## Description:

Packs the elements of the numeric mfArray A in a long column vector, under the control of the boolean mask. The behavior of this routine is similar to the pack Fortran 90 intrinsic function.

Sparse arrays are not handled.
See also: mfUnpack

Example(s):

```
    A = mfMagic(3)
    mask = A >= 5.0d0
    call msDisplay( A,"A", mask,"mask" )
    Ap = mfPack( A, mask )
    call msDisplay( Ap, "A packed" )
output:
```

A $=$
$8 \quad 3 \quad 4$
159
$\begin{array}{ll}6 & 7\end{array}$
mask =

| T | F | F |
| :--- | :--- | :--- |
| F | T | T |
| T | T | $F$ |

A packed =

```
        8 6
R = mfOnes(3)*MF_NAN
call msDisplay( mfUnpack(Ap,mask,field=R), "A unpacked" )
```

output:
A unpacked =

| 8.0000 | NaN | NaN |
| :---: | :---: | :---: |
| NaN | 5.0000 | 9.0000 |
| 6.0000 | 7.0000 | NaN |

mfUnpack
unpacking an array

Calling syntax:

```
C = mfUnpack( A, mask, field )
```


## Description:

Unpacks the elements of the numeric vector mfArray A under the control of the boolean mask. The returned mfArray is completed from values of field. A should have been previously processed by the mfPack routine.

The behavior of this routine is similar to the unpack Fortran 90 intrinsic function.
Sparse arrays are not handled.
See also: mfPack

Example(s):

```
\(\mathrm{A}=\mathrm{mfMagic}(3)\)
mask = A >= 5.0d0
call msDisplay( A,"A", mask,"mask" )
Ap = mfPack( A, mask )
call msDisplay( Ap, "A packed" )
```

output:
A $=$

| 8 | 3 | 4 |
| :--- | :--- | :--- |
| 1 | 5 | 9 |
| 6 | 7 | 2 |

mask $=$

| T | F | F |
| :--- | :--- | :--- |
| F | T | T |
| T | T | $F$ |

A packed =

| 8 | 6 | 5 | 7 | 9 |
| :--- | :--- | :--- | :--- | :--- |

```
R = mfOnes(3)*MF_NAN
call msDisplay( mfUnpack(Ap,mask,field=R), "A unpacked" )
```

output:
A unpacked =

| 8.0000 | NaN | NaN |
| :---: | :---: | :---: |
| NaN | 5.0000 | 9.0000 |
| 6.0000 | 7.0000 | NaN |

## mfCshift

Calling syntax:

```
C = mfCshift( A, shift )
```

Description:
Performs a circular shift on the vector mfArray A. shift is an integer. The behavior of this routine is similar to the cshift Fortran 90 intrinsic function.

Sparse arrays are not handled.
See also: mfEoshift

Example(s):
$A=[1,2,3,4,5,6]$
call msDisplay ( A, "A" )
call msDisplay( mfCshift(A,shift=1), "mfCshift(A,shift=1)" )
output:
$A=$
$\begin{array}{llllll}1 & 2 & 3 & 4 & 5 & 6\end{array}$
$\operatorname{mfCshift}(A$, shift $=1)=$
$\begin{array}{llllll}2 & 3 & 4 & 5 & 6 & 1\end{array}$

## mfEoshift

Calling syntax:

```
C = mfEoshift( A, shift )
```

Description:
Performs an end-off shift on the vector mfArray A. shift is an integer. The behavior of this routine is similar to the eoshift Fortran 90 intrinsic function.

Sparse arrays are not handled.
See also: mfCshift

Example(s):

```
    A = [ 1, 2, 3, 4, 5, 6 ]
    call msDisplay( A, "A" )
    b = 99
    call msDisplay( mfEoshift(A,shift=1,boundary=b),
        "mfEoshift(A,shift=1,boundary=b)" )
```

output:
$\mathrm{A}=$
$\begin{array}{llllll}1 & 2 & 3 & 4 & 5 & 6\end{array}$
mfEoshift(A,shift=1,boundary=b) =

| 2 | 3 | 4 | 5 | 6 | 99 |
| :--- | :--- | :--- | :--- | :--- | :--- |

The first form is:

```
C = mfDiag( A [, d] )
```

If A is a matrix-like mfArray (dense or sparse), this function extracts the main (or the d-) diagonal (the output is dense, whatever the storage type of A is).

If A is a vector-like mfArray (dense only), this function builds a dense square matrix which has A as main (or as d-) diagonal. For building a sparse matrix, use mfSpDiags.

The subroutine form:

```
call msDiag( A, v [, d] )
```

modifies the main (or the d-) diagonal of the mfArray A (dense or sparse) by copying the elements of the vector-like mfArray v . The data type of A may be changed because A and v can have real or complex values. The vector $v$ is not modified.

Remarks: When d is equal to zero, it points to the main diagonal; when positive, the upper part of the matrix is concerned. If the arg. $d$ is not present, the main diagonal is the default.

See also: mfBlkDiag, mfEye, mfSpDiags

## Example(s):

```
v = [ 1, 2, 3 ]
C = mfDiag(v,2)
call msDisplay( C, "C = mfDiag(v,2)" )
```

output:

```
v =
    1 2 3
C = mfDiag(v,2) =
\begin{tabular}{lllll}
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{tabular}
    v = [ 4, 5, 6 ]
    call msDisplay( v, "v" )
    call msDiag( C, v, d=-2 )
    call msDisplay( C, "C" )
```

output:

```
v =
```

    \(4 \quad 5 \quad 6\)
    C =

| 0 | 0 | 1 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 2 | 0 |
| 4 | 0 | 0 | 0 | 3 |
| 0 | 5 | 0 | 0 | 0 |
| 0 | 0 | 6 | 0 | 0 |

mfBlkDiag
block diagonal concatenation

The first calling syntax:

```
C = mfBlkDiag( A, B )
```

gives the concatenation of the two (square) blocks diagonal A and B.
The other calling syntax:

```
C = mfBlkDiag( A, n )
```

concatenates $n$ times ( n being non negative) the square matrix A on the diagonal.
Remarks: A (and B, for the first form), may be dense or sparse, and of any type accepted by the .vc. or .hc. operators.

See also: mf/msDiag

Example(s):

```
\(A=\operatorname{mfRot} 90(m f E y e(3,3))\)
call msDisplay(A,"A")
call msDisplay( mfBlkDiag(A,3), "blkdiag(A,3)" )
```

output:
$\mathrm{A}=$

| 0 | 0 | 1 |
| :--- | :--- | :--- |
| 0 | 1 | 0 |
| 1 | 0 | 0 |

blkdiag(A,3) =

| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

The first form:

```
C = mfFind( A )
```

finds non zero (resp. TRUE) elements of the numerical (resp. boolean) mfArray A and returns a long column of their indices (called also linear indices).

Usually, this routine is applied to a boolean mfArray, automatically created in comparison operators; for example, mfFind ( $\mathrm{A}>1.0 \mathrm{dO}$ ) returns indices of elements which verify the specified property.

The subroutine form:

```
call msFind( mfOut(i,j[,v]), A )
```

allows the user to retrieve the matrix-indices. The two vectors returned (i and $j$ ) contain the row-index and column-index for non-zero elements.

If the optional argument v is present, it contains the values of corresponding non-zero elements.
Warning: Contrary to MATLAB, mf/msFind always return row index vectors, instead of column vectors.
See also: mfNonZeros, mfOut

## Example(s):

```
    A = mf( [ 9, 0, 0 ] ) .vc.
            mf( [ 0, 0, 7 ] ) .vc.
                                &
                &
    mf( [ 0, 11, 0 ] )
    call msDisplay( A, "A" )
    call msFind( mfOut(i,j,v), x )
    call msDisplay( i,"row indices", j,"col indices", v,"values" )
```

output:
$A=$
$9 \quad 0 \quad 0$
$\begin{array}{lll}0 & 0 & 7\end{array}$
0110
row indices =
132
col indices =
123
values =
$\begin{array}{lll}9 & 11 & 7\end{array}$
mfNonZeros

Calling syntax:

```
C = mfNonZeros( A )
```

Description:
Returns all non-zero elements of the mfArray A in a vector.
Works as msFind, except that indices $(\mathrm{i}, \mathrm{j})$ are not returned.
See also: ms/mfFind

## mf/msReshape

## shape modification

Generic interface:

```
function mfReshape( A, m, n ) result( out )
    integer, intent(in) :: A(:)
or real(kind=MF_DOUBLE), intent(in) :: A(:)
or type(mfArray) :: A
    integer, intent(in) :: m, n
    type(mfArray) :: out
```


## Description:

This function extends the Fortran intrinsic reshape; it can be applied to a dense or sparse mfArray.
Remark: in the special call: mfReshape ( A, MF_EMPTY, $n$ ) with A being an mfArray only, the first dimension, if not specified, is deduced from the total number of elements in A (the second dimension n can also be replaced by MF_EMPTY). Same behavior using MF_NO_ARG instead of MF_EMPTY.

Another possible form is:

```
call msReshape( A, m, n )
```

which does the same job with the mfArray A, except that the shape modification is made "in place". Here, m or n cannot be replaced by MF_EMPTY or MF_NO_ARG. This second routine is more efficient than the first one, especially for sparse matrices.

See also: mfRepMat

Example(s):

```
x = [ (i, i = 1, 12) ]
call msDisplay( x, "x" )
call msDisplay( mfReshape(x, 4, 3 ), "mfReshape(x, 4, 3 )" )
call msDisplay( mfReshape(x, 2, MF_EMPTY ), "mfReshape(x, 2, MF_EMPTY )" )
```

output:

```
x =
mfReshape(x, 4, 3 ) =
    1 5 9
    2 6 10
    3
    4 8 12
mfReshape(x, 2, MF_EMPTY ) =
    1
```

    \(\begin{array}{llllllllllll}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12\end{array}\)
    mfTriL
lower triangular part

Calling syntax:

```
C = mfTriL( A [, k] )
```

Description:
This function extracts the lower triangular part of an mfArray, including the main diagonal.
If k is present, it returns the lower part of A up to the k -th diagonal (included). k must be integer; it can be negative.

See also: mfTriU
mfTriU
upper triangular part

Calling syntax:

```
C = mfTriL( A [, k] )
```

Description:
This function extracts the upper triangular part of an mfArray, including the main diagonal.
If k is present, it returns the upper part of A down to the k -th diagonal (included). k must be integer; it can be negative.

See also: mfTriL
mfFlipLR

Calling syntax:

```
C = mfFlipLR( A )
```

Description:
This function flips the mfArray A in the left-right direction.
See also: mfFlipUD, mfRot90

Example(s):
$A=\operatorname{reshape}([(i, i=1,15)],[3,5])$
call msDisplay ( A, "A" )
call msDisplay( mfFlipLR(A), "mfFlipLR(A)" )
output:
A $=$

| 1 | 4 | 7 | 10 | 13 |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 5 | 8 | 11 | 14 |
| 3 | 6 | 9 | 12 | 15 |

$\operatorname{mfFlipLR}(\mathrm{A})=$

| 13 | 10 | 7 | 4 | 1 |
| :--- | :--- | :--- | :--- | :--- |
| 14 | 11 | 8 | 5 | 2 |
| 15 | 12 | 9 | 6 | 3 |

## mfFlipUD

Calling syntax:

```
C = mfFlipUD( A )
```

Description:
This function flips the mfArray A in the up-down direction.
See also: mfFlipLR, mfRot90

Example(s):
$A=\operatorname{reshape}([(i, i=1,15)],[3,5])$
call msDisplay( A, "A" )
call msDisplay( mfFlipUD(A), "mfFlipUD(A)" )
output:
$\mathrm{A}=$

| 1 | 4 | 7 | 10 | 13 |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 5 | 8 | 11 | 14 |
| 3 | 6 | 9 | 12 | 15 |

$\operatorname{mfFlipUD}(\mathrm{A})=$

| 3 | 6 | 9 | 12 | 15 |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 5 | 8 | 11 | 14 |
| 1 | 4 | 7 | 10 | 13 |

## mfRot90

## matrix rotation

Calling syntax:

```
C = mfRot90( A [, k] )
```

Description:
This function rotates the mfArray A in the trigonometric direction, by the angle $\mathrm{k}^{*} 90$ in degrees. (i.e. for $90^{\circ}$, last column becomes first row).
k may be any integer (including zero and negative value). It is an optional argument (default value is 1).

See also: mfFlipLR, mfFlipUD

## Example(s):

```
A = reshape( [ (i,i=1,6) ], [3,2] )
call msDisplay( A, "A" )
call msDisplay( mfRot90(A,1), "mfRot90(A,1) [90` anticlockwise]" )
call msDisplay( mfRot90(A,-1), "mfRot90(A,-1) [90` clockwise]" )
```

output:
$\mathrm{A}=$

14
25
36
$\operatorname{mfRot90}(\mathrm{A}, 1) \quad\left[90^{\circ}\right.$ anticlockwise] =
$4 \quad 5 \quad 6$
123
mfRot90(A,-1) [90 clockwise] =
$3 \quad 2 \quad 1$
$6 \quad 5 \quad 4$
mfHilb

Calling syntax:

```
C = mfHilb( n )
```

Description:
Builds a square mfArray which contains an Hilbert matrix. It is a well known example of badly conditioned matrix.

See also: mfInvHilb, mfCond

Example(s):

```
    A = mfHilb(5)
    call msDisplay( A, "A = mfHilb(5)" )
    call msDisplay( mfCond(A), "cond(A)" )
```

output:

```
A = mfHilb(5) =
\begin{tabular}{lllll}
1.0000 & 0.5000 & 0.3333 & 0.2500 & 0.2000 \\
0.5000 & 0.3333 & 0.2500 & 0.2000 & 0.1667 \\
0.3333 & 0.2500 & 0.2000 & 0.1667 & 0.1429 \\
0.2500 & 0.2000 & 0.1667 & 0.1429 & 0.1250 \\
0.2000 & 0.1667 & 0.1429 & 0.1250 & 0.1111
\end{tabular}
cond(A) =
    4.7661E+05
```


## mfInvHilb

## inverse Hilbert matrix

Calling syntax:

```
C = mfInvHilb( n )
```

Description:
Returns the inverse of an Hilbert matrix; therefore the matrix product of mfHilb(n) by mfinvHilb(n) is the identity matrix.

Remarks: this inverse is not computed by some linear algebra algorithm but by an exact formula.
See also: mfHilb

Example(s):

```
\(\mathrm{A}=\mathrm{mfHilb}(5)\)
B \(=\mathrm{mfInvHilb}\) (5)
call msDisplay( B, "B = mfInvHilb(5)" )
call msDisplay ( mfNorm(mfEye(5) - mfMul(A,B)), "| I - A*B |" )
```

output:

```
B = mfInvHilb(5) =
```

| 25 | -300 | 1050 | -1400 | 630 |
| ---: | ---: | ---: | ---: | ---: |
| -300 | 4800 | -18900 | 26880 | -12600 |
| 1050 | -18900 | 79380 | -117600 | 56700 |
| -1400 | 26880 | -117600 | 179200 | -88200 |
| 630 | -12600 | 56700 | -88200 | 44100 |

I - A*B | =
$5.7926 \mathrm{E}-12$

## mfVander

## Vandermonde matrix

Calling syntax:

```
A = mfVander( v [, n ] )
```


## Description:

Returns a Vandermonde matrix. The input vector mfArray v may be either a row or a column vector.
If the optional argument n is present, the returned matrix will have $n$ columns, each one constituted by a power of the vector v ; else the returned matrix will be square, i.e. will have as much columns as the size of v .

Example(s):
$\mathrm{x}=[1,2,3,4,5]$
call msDisplay ( x, "x")
call msDisplay( mfVander (x,3), "A = mfVander (x,3)" )
output:
$\mathrm{x}=$
$\begin{array}{lllll}1 & 2 & 3 & 4 & 5\end{array}$
$\mathrm{A}=\mathrm{mf} \operatorname{Vander}(\mathrm{x}, 3)=$

| 1 | 1 | 1 |
| ---: | ---: | ---: |
| 4 | 2 | 1 |
| 9 | 3 | 1 |
| 16 | 4 | 1 |
| 25 | 5 | 1 |

## mfCompan

Calling syntax:

```
A = mfCompan( v )
```


## Description:

Returns the Companion matrix of a polynomial, represented by its coefficients provided in the input vector mfArray v (this latter array may be either a row or a column vector).

The eigenvalues of the matrix A are the roots of the input polynomial.
See also: mfRoots, mfEig

## Example(s):

```
    ! polynom: (x-1)*(x-2)*(x-3)*(x-4) = x^4 - 10*x^3 + 35*x^2 - 50*x + 24,
    ! so the roots are: {1, 2, 3, 4}
    v = [ 1.0d0, -10.0d0, 35.0d0, -50.0d0, 24.0d0 ]
    call msDisplay( v, "v" )
    A = mfCompan(v)
    call msDisplay( A, "A = mfCompan(v)" )
    call msDisplay( mfEig(A), "Eigenvalues of A" )
```

output:

```
v =
    1
```

$\mathrm{A}=\mathrm{mfCompan}(\mathrm{v})=$
$\begin{array}{llll}10 & -35 & 50 & -24\end{array}$
1000
$\begin{array}{llll}0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0\end{array}$
Eigenvalues of $\mathrm{A}=$
$4.0000+0.0000 i$
$3.0000+0.0000 i$
$2.0000+0.0000 i$
$1.0000+0.0000 i$

## mfHankel

Calling syntax:

```
A = mfHankel( C [, R ] )
```


## Description:

Returns the Hankel matrix of the input vectors mfArray $C$ and $R$ (these latter arrays may be either a row or a column vector).

A Hankel matrix has constant anti-diagonal values.
Hankel matrices are real (because only real input vectors are supported) and symmetric (if square).
See also: mfToeplitz

Example(s):
$C=[1.0 \mathrm{~d} 0,2.0 \mathrm{~d} 0,3.0 \mathrm{do}, 4.0 \mathrm{~d} 0$ ]
$R=$ [ 4.0d0, 5.0d0, 6.0d0 ]
call msDisplay( C, "C", R, "R" )
call msDisplay( mfHankel(C,R), "mfHankel(C,R)" )
output:
C $=$
$\begin{array}{llll}1 & 2 & 3 & 4\end{array}$
$R=$
$4 \quad 5 \quad 6$
mfHankel $(C, R)=$

| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 2 | 3 | 4 |
| 3 | 4 | 5 |
| 4 | 5 | 6 |

## mfToeplitz

Calling syntax:

```
A = mfToeplitz( C [, R ] )
```


## Description:

Returns the Toeplitz matrix of the input vectors mfArray $C$ and $R$ (these latter arrays may be either a row or a column vector).

A Toeplitz matrix has constant diagonal values and is symmetric (if square).
Complex input vectors are supported. In such a case, the complex matrix is hermitian if the first element of $R$ is real.

See also: mfHankel

Example(s):

```
C = [ 1.0d0, 2.0d0, 3.0d0, 4.0d0 ]
R = [ 1.0d0, 5.0d0, 6.0d0 ]
call msDisplay( mfToeplitz(C,R), "mfToeplitz(C,R)" )
```

output:
$\mathrm{C}=$
$\begin{array}{llll}1 & 2 & 3 & 4\end{array}$
$R=$
156
$\mathrm{A}=\mathrm{mfToeplitz}(\mathrm{C}, \mathrm{R})=$
$1 \quad 5 \quad 6$
215
$3 \quad 2 \quad 1$
432

## mfPerm

Calling syntax:

```
p = mfPerm( v )
```


## Description:

Returns a permutation vector mfArray from the vector of indices v .
v may be either a Fortran 90 integer vector, or a vector-like mfArray. In this latter case, the mfArray should contains integer values; on the contrary, a warning is emitted by the routine.

See also: mfIsPerm, mfRandPerm, mfCheckPerm, mfColPerm, mfRowPerm, msSaveAscii

## mfRandPerm

## random permutation vector creation

Calling syntax:

```
p = mfRandPerm( n [, k ] )
```

Description:
Returns a random permutation vector mfArray. Elements of p are unique integers in the range $[1, n]$.
If the second argument k is present, returns $k$ unique integers in the range $[1, n]$. In this latter case, p is not a true permutation vector if $k \neq n$.

See also: mfPerm, mfIsPerm, mfCheckPerm

## mfCheckPerm

Interface:

```
function mfCheckPerm( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool
```


## Description:

Returns '.true.' if A is a valid permutation vector, i.e. if the integers are consecutive numbers, not necessarily ordered and appearing only once in the list.

See also: mfPerm, mfIsPerm

## isfinite

Interface:

```
function isfinite( x ) result( out )
    double precision, intent(in) :: x
    logical :: out
```

Description:
This function returns a logical value, according to the value of x , which must be only of type double precision.

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or $N a N$.
See also: isinf, isnan, mfIsFinite
isinf

Interface:

```
function isinf( x ) result( out )
    double precision, intent(in) :: x
    logical :: out
```

Description:
This function returns a logical value, according to the value of x , which must be only of type double precision.

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.
See also: isfinite, isnan, mfIsInf
isnan

Interface:

```
function isnan( x ) result( out )
    double precision, intent(in) :: x
    logical :: out
```

Description:
This function returns a logical value, according to the value of x , which must be only of type double precision.

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.
See also: isfinite, isinf, mfIsNaN

## mfIsFinite

Calling syntax:

```
C = mfIsFinite( A )
```

Description:
This elemental function returns a boolean mfArray.
Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.
See also: isfinite, mfIsInf, mfIsNaN

## mfIsInf

infinity test

Calling syntax:

```
C = mfIsInf( A )
```

Description:
This elemental function returns a boolean mfArray.
Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.
See also: isinf, mfIsFinite, mfIsNaN

## mfIsNaN

Calling syntax:

```
C = mfIsNaN( A )
```

Description:
This elemental function returns a boolean mfArray.
Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.
See also: isnan, mfIsFinite, mfIsInf

## mfIsDiag

## diagonal pattern test

Calling syntax:

```
bool = mfIsDiag( A )
```

returns a $T R U E$ boolean value if the mfArray A (sparse or dense) is a diagonal matrix.
See also: mfIsTril, mfIsTriu

## mfIsTril

Calling syntax:

```
bool = mfIsTril( A )
```

returns a $T R U E$ boolean value if the mfArray A (sparse or dense) is a lower triangular matrix.
See also: mfIsDiag, mfIsTriu

## mfIsTriu

Calling syntax:

```
bool = mfIsTriu( A )
```

returns a TRUE boolean value if the mfArray A (sparse or dense) is a upper triangular matrix.
See also: mfIsDiag, mfIsTril

### 1.8 Matrix Functions

| mfDet | determinant |
| :---: | :---: |
| mfTrace | sum of diagonal elements |
| mfNorm | matrix or vector norm |
| mfRank | matrix rank |
| mfCond | matrix condition number |
| mfRCond | reciprocal condition number estimation |
| msLU | $L U$ factorization |
| msLDLT | $L D L^{t}$ factorization |
| $\mathrm{mf} / \mathrm{msChol}$ | Cholesky factorization |
| msCholSpSymb | Symbolic Cholesky factorization (sparse only) |
| msCholSpNum | Numerical Cholesky factorization (sparse only) |
| $\mathrm{mf} / \mathrm{msBalance}$ | scaling to improve eigenvalue accuracy |
| $\mathrm{mf} / \mathrm{msSVD}$ | singular value decomposition |
| mfInv, .i. | matrix inverse |
| mfPseudoInv | matrix pseudo-inverse |
| mf QR, msQR | orthogonal-triangular factorization |
| mfQleft, mfQright | left or right multiplication by a Q factor (sparse only) |
| $\mathrm{mf} / \mathrm{msEig}$ | eigenvalues and eigenvectors |
| $\mathrm{mf} / \mathrm{msHess}$ | Hessenberg form |
| $\mathrm{mf} / \mathrm{msSchur}$ | Schur decomposition |
| mfLDiv, .ix. | left matrix divide |
| mfRDiv, .xi. | right matrix divide |
| msRref | reduced row echelon form |
| mfNull | null space |
| mf0rth | matrix orthogonalization |
| mfExpm | matrix exponential |
| mfLogm | matrix logarithm |
| mfSqrtm | matrix square root |
| mfPowm | matrix power |
| $\mathrm{mf} / \mathrm{msFunm}$ | general matrix function |
| mf/msEigs | few eigenvalues and eigenvectors |
| $\mathrm{mf} / \mathrm{msSVDS}$ | few singular values and vectors |
| mfNormEst, mfNormEst1 | norm estimations |
| mfCondEst | matrix condition number estimation |
| mfIsSymm | matrix symmetry inquiry |
| mfTolForSymm | tolerance for symmetry |
| mfIsPosDef | definite positiveness inquiry |
| mfIsDiagDomCol | diagonally dominant by cols inquiry |
| mfIsStrictDiagDomCol | strictly diagonally dominant by cols inquiry |
| mfIsFullRank | full rank inquiry |
| MF_LAPACK_VERSION | LAPACK version |
| msGetBlasLib | Get BLAS library type |
| msGetLapackLib | Get LAPACK library type |
| msGetArpackInfo | Get ARPACK library information |
| msGetSuiteSparseLib | Get SuiteSparse library version |

See also:
Core Routines
File Input/Output
Data Analysis Functions

Operators
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Polynomial Functions
Optimization and Function Functions
Sparse Matrices
mfDet

Calling syntax:

```
C = mfDet( A )
```

Description:
Returns the determinant of the mfArray A.
A must be square, and cannot be sparse.
Remarks: using the determinant of a matrix to detect whether it is singular is not a good idea: use mf Cond or mfRCond instead.

See also: mfRank
mfTrace
sum of diagonal elements

Calling syntax:

```
C = mfTrace( A )
```

Description:
Returns the sum of all diagonal element of the mfArray A (dense or sparse, real or complex).
Remarks: the trace of a square matrix is also the sum of its eigenvalues.
See also: mfEig
mfNorm

Calling syntax:

```
C = mfNorm( A [, norm | p ] )
```


## Description:

Returns the norm of the mfArray A, which can be a vector or a matrix (sparse or dense).
The two optional arguments (norm is a character string whereas p is an integer) cannot be both present.

- For a matrix, only 1-, 2-, "inf"- or "fro"-norms are supported.

By default, returns the 2-norm (which is also the largest singular value of $A$ ). If $p$ is present, it returns the p-norm of A (therefore, p can take only the values 1 or 2 ). If norm is present, it must be inf (infinite-norm) or fro (frobenius-norm).

- For a vector, only p->0 and "inf"- norms are supported.

By default, returns the 2-norm. If p is present, it returns the p -norm of A . If norm is present, it must be inf (infinite-norm).

Note: For sparse matrices, use mfNormEst or mfNormEst1 instead, which are more efficient.

See also: mfCond, mfSVD

## mfRank

## matrix rank

Calling syntax:

```
C = mfRank( A [, tol ] )
```


## Description:

Computes the rank of a matrix, i.e. the number of linearly independent rows or columns.
If tol, of type real, is present then it returns the number of singular values greater or equal to this optional tolerance. Default tolerance is chosen as $\max (m, n)\|A\|_{2} \epsilon$, where $m$ and $n$ are respectively the number of rows and number of columns of the mfArray A.

## Remarks:

- it is internally based on the Singular Value Decomposition;
- for sparse matrices, the user is invited to use mfSVDS, and examine himself few singular values.

See also: mfSVD, mfSVDS, mfIsFullRank
mfCond
matrix condition number

Calling syntax:

```
C = mfCond( A )
```

Description:
Returns the condition number of the mfArray A. It is the ratio of the largest singular value to the smallest one.

Remarks: The condition number is always greater than 1. A value equal to infinity (resp. close to the inverse of the $\epsilon$ machine) indicates that the matrix A is singular (resp. nearly singular).

See also: mfCondEst, mfRCond, mfNorm, mfSVD, mfDet
mfRCond

## reciprocal condition number estimation

Calling syntax:

```
C = mfRCond( A )
```

Description:
Returns an estimation of the inverse of the condition number (which is called the reciprocal condition number).

Remarks: The reciprocal condition number is always between 0 and 1. A value equal to zero (resp. close to the $\epsilon$ machine) indicates a singular matrix (resp. nearly singular).

See also: mfCond, mfCondEst, mfNorm, mfSVD, mfDet
msLU

## $L U$ factorization

## Description:

Computes numerically the $L U$ decomposition of an mfArray.

First calling syntax:

```
call msLU( mfOut( L, U [, p] ), A )
```

For dense mfArrays, returns the $L$ (lower triangular, with a unit diagonal) and $U$ (upper triangular) factors, and, optionally, the row permutation vector $p$, such that:

$$
L U=A(p,:)
$$

If the optional argument p is not present, the routine actually returns a row permuted $L$, such that:

$$
L U=A
$$

In this latter case, be aware that you will not be able to solve a linear system using only the factors $L$ and $U$, because you must know the permutation $p$ to be applied to the right hand side vector. See the Muesli User's Guide to an example using the $L U$ decomposition.

Second calling syntax:

```
call msLU ( mfOut( L, U [, p, q, r] ), A )
```

For sparse mfArrays, the routine msLU behaves in a slightly different manner:
$p$ and $q$ are respectively row and column permutations vectors while $r$ is a vector containing the row scaling of A. All these sparse matrices should verify the following (theoretical) relationship:

$$
L U=P R A Q
$$

where the matrices $P, Q, R$ are associated to the vectors $p, q, r$. Practically, the RHS of the previous identity should be computed using mfRowScale, mfColPerm and mfRowPerm.

Moreover, the L factor contains a pointer to an internal structure containing all the five output matrices, so that a call to the mfLDiv routine is simplified.

Third calling syntax: In all cases (sparse or dense matrix A), the following syntax:

```
call msLU( mfOut(factors), A )
```

provides the possibility to factorize only, without copying factors in explicit mfArrays. In this case, the factors (accompanied by the permutations and the scaling) are internally stored (i.e. not available for the end-user) and can be referenced only via the mfMatFactor factors.

The mfMatFactor obtained is intended to be used only in mfLDiv.

Remark: If the matrix A is symmetric and positive definite, the current routine may emit a warning: indeed, a Cholesky factorization (via the $\mathrm{mf} / \mathrm{msChol}$ routine) should be more efficient and uses less memory.

See also: msQR, msRref, mfOut
msLDLT

## $L D L^{t}$ factorization

Calling syntax:
call msLDLT( mfOut(L,D,P), A )

Description:
Computes numerically the $L D L^{t}$ decomposition of a real and symmetric (indefinite) mfArray A, dense or sparse. Returns a lower triangular L, a matrix D (block diagonal 1x1 or 2 x 2 ) and a permutation matrix P such that: $L D L^{\prime}=P^{\prime} A P$.

Remark: if A is moreover positive definite, then the Cholesky factorization should be more appropriate !
N.B.: For sparse matrices:

- the matrix D is strictly diagonal;
- be aware that the $L D L^{t}$ factorization can fail, especially if the original matrix has some zeros on the main diagonal (the fact that A is non-singular is necessary but not sufficient); in this case, use the $L U$ factorization instead which makes pivoting.

See also: msLU, mf/msChol, mfOut
mf/msChol

## Cholesky factorization

## Description:

Computes numerically the Cholesky factorization of the mfArray A, which must be symmetric and positive definite; if this latter property doesn't hold, MUESLI emits an error and returns an empty mfArray.

First calling syntax:

```
U = mfChol( A )
```

For dense matrices, returns an upper triangular mfArray U , such that: $U^{\prime} U=A$
Second calling syntax: For sparse matrices, the calling syntax is slightly different, because a permutation is first applied to the matrix $A$ in order to reduce the fill-in:

```
call msChol( mfOut(L,p), A )
```

returns a lower triangular mfArray L and a permutation p such that: $L L^{\prime}=P^{\prime} A P$. The permutation p is always returned as a special mfArray (see mfPerm), thus the routines $\mathrm{mf} / \mathrm{msColPerm}$ and $\mathrm{mf} / \mathrm{msRowPerm}$ must be used to applied this permutation to the matrix $A$.

Third calling syntax: For sparse matrices only, there is also the possibility to factorize only, without copying the factors ( L and P ) in explicit mfArrays. In this case, the factors are internally stored (i.e. not available for the end-user) and can be referenced only via the mfMatFactor factors:

```
call msChol( mfOut(factors), A )
```

The mfMatFactor obtained is intended to be used only in mfLDiv.
See also: msLU, msLDLT, mfOut, msCholSpSymb, msCholSpNum

Description:
Computes the symbolic Cholesky factorization of the sparse mfArray A, which must be symmetric; if this latter property doesn't hold, MUESLI emits an error and returns an empty mfArray.

Interface:

```
subroutine msCholSpSymb( factor, A )
    type(mfArray), intent(in) :: A
    type(mfMatFactor), intent(out) :: factor
end subroutine
```

After using this routine, the usual step is to call (once or more) the msCholSpNum routine, before solving a linear system with mfLDiv.

See also: mf/msChol

## msCholSpNum

## Description:

After having called the symbolic factorisation (via the msCholSpSymb routine), computes the numerical Cholesky factorization of the sparse mfArray A, which must be positive definite; if this latter property doesn't hold, MUESLI emits an error and returns an empty mfArray.

Interface:

```
subroutine msCholSpNum( factor, A )
    type(mfArray), intent(in) :: A
    type(mfMatFactor), intent(in out) :: factor
end subroutine
```

After using this routine, the usual step should be to solve a linear system with mfLDiv.
See also: mf/msChol
mf/msBalance

## scaling to improve eigenvalue accuracy

The first form:

```
B = mfBalance( A )
```

returns an mfArray which is balanced, i.e. applies similarity transformations to A in order to make the rows and columns as close in norm as possible.

The second form:
call msBalance( mfOut(T,B), A )
also returns the (non singular) diagonal transformation matrix T , such that: $B=T^{-1} A T$
Remark: there is no permutation applied to the matrix A. Moreover, the matrix A must be square.
See also: mfNorm, mfEig, mfOut

Calling syntax:

```
S = mfSVD( A )
```

The function form returns only the singular values of the mfArray A (size $m$ by $n$ ), in a column vector that is sorted from the greatest to the smallest; the size of this vector is $\min (m, n)$ and its type is always real, whatever the type of A.
The subroutine form allows the user to access to the full decomposition:
call msSVD ( mfOut (U,S,V), A [, economy_size ] )
which returns three mfArrays such that $A=U S V^{\prime} . \mathrm{U}$ and V are both unitary matrices whose columns contain, respectively, the left- and right-singular vectors of $A$. In this case, $S$ is returned as a rank- 2 array.

If the logical optional argument economy_size is present and equal to $T R U E$, then U contains only the $\min (m, n)$ first columns of the matrix $U$, and V contains only the $\min (m, n)$ first columns of the matrix $V . \mathrm{S}$ is always a square matrix of order $\min (m, n)$.

Remarks: sparse matrices are not handled; please use mf/msSVDS instead.
See also: mfNorm, mfRank, mfDet, mfOut
mfInv, .i.

Calling syntax:

```
C = mfInv( A )
```

.i. A $\left(A^{-1}\right)$ is a shortcut for writing: $m f \operatorname{Inv}(A)$

## Description:

Returns the inverse of the square mfArray A , mathematically noted $A^{-1}$.
If $A$ is singular (or close to, ought to the machine $\epsilon$ ), it returns an infinite $m f$ Array of same shape.
For singular of non square matrices, the Moore-Penrose pseudo-inverse routine mfPseudoInv could also be used.

Remarks: sparse matrices are not handled; it is very unusual to have to explicitly compute the inverse of a matrix. If necessary, solving the system $A x=e_{i}$ for each basis vector $e_{i}$ gives each column of $A^{-1}$; but be aware that you will obtain a (nearly) dense matrix.

See also: mfPseudoInv, mfLDiv, mf/msSVD, mfRank
mfPseudoInv

## matrix pseudo-inverse

Calling syntax:

```
C = mfPseudoInv( A [, tol ] )
```

Description:
Returns the Moore-Penrose pseudo-inverse of the mfArray A.
A may be non square or square singular. mfPseudoInv returns a matrix, based on the SVD of A and any singular values less than the tolerance tol are treated as zero. The default tolerance is $\max (m, n)\|A\|_{2} \epsilon$, where $m$ and $n$ are respectively the number of rows and number of columns of the mfArray A.

See also: mfInv, mf/msSVD, mfRank
$m f Q R$

## orthogonal-triangular decomposition

Calling syntax:

```
R = mfQR( A )
```

performs a QR factorization of the matrix A ( $m \times n$, dense or sparse mfArray), but returns only the R factor (upper triangular matrix of size $n \times n$ ) verifying $R^{\prime} R=A^{\prime} A$. This is the $Q$-less decomposition.

Remark for sparse matrices: The use of this routine should be restricted to small or moderate size sparse matrices because no ordering is applied to prevent the fill-in. For large sparse matrices, use the msQR routine instead, which has also more options.

Note: The complex type is not yet supported for sparse matrices.
See also: msQR, msLU
msQR

## orthogonal-triangular decomposition

Calling syntax:

```
call msQR( mfOut(Q,R[,p,RANK]), A [, tol] )
```

applies the orthogonal-triangular decomposition to the mfArray A , dense or sparse, of size $m \times n$. It returns the two mf Arrays Q and R such that $A=Q R$.

Usually, $\mathbf{Q}$ is an orthogonal matrix (size $m \times m$ ) and R is a upper-triangular matrix (size $m \times n$ ).
If $m>n$, the economy size is always used: $\mathbb{Q}$ contains only the $n$ first columns of the orthogonal full matrix Q (therefore of size $m \times n$ ), and R is the square upper part ( $i . e$. the non-zero part) of the full R (therefore of size $n \times n$ ). This latter case is used for least-square problems.

Optionally, a column permutation vector $p$ may be applied to the matrix $A$, such that $Q R=A(:, p)$ :

- for the dense case, the use of this option leads to a factor $R$ having the module of its diagonal terms in decreasing order;
- for the sparse case, the permutation is chosen to obtain a better sparsity for both $Q$ and $R$ factors.

If the mfArray RANK is present, it contains on return the numerical rank of the matrix A:

- for the dense case, a different method is used in order to compute safely the rank: it is the rankrevealing QR;
- for the sparse case, an estimation of the rank is returned.

If the argument tol (real double) is present, it specifies to which accuracy the rank is computed. Default value is the machine precision, MF EPS.

Other syntax: for sparse matrices only, the additional syntax can be used:

```
call msQR( mfOut(Qhouse,R[,p,RANK]), A[, tol] )
```

which returns the Householder vectors of the $Q$ matrix in the mfMatFactor Qhouse variable, and the $R$ triangular factor in the mfArray R. Although Qhouse doesn't contain explicitly the matrix Q, the Shape routine applied to it will returned the shape of $\mathbf{Q}$, i.e. $(m, m)$. The shape of R is $(m, n)$.

This second form should be used for large size matrices, Qhouse being much more sparse than Q. The user is invited to use one of the routines mfQleft or mfQright in order to perform the multiplication of $Q$ with another vector or matrix.

The other parameters have the same meaning as in the first calling syntax.
Note: The complex type is not yet supported for sparse matrices.
See also: mfQR, msLU, mfOut
mfQleft

## left multiplication by a sparse Q factor

Calling syntax:

```
B = mfQleft( Qhouse, A )
```

returns the product $Q^{\prime} A$.
The mfMatFactor Qhouse holds a sparse $Q$ factor (obtained from a $Q R$ decomposition of a sparse matrix); the mfArray A may be a vector (dense) or a matrix (dense or sparse). B has the same structure (dense or sparse) than A.

By performing the operation: $Q=. t . m f Q 1 e f t($ Qhouse, $m f S p E y e(m, m)$ ) the standard form of the $Q$ factor can be obtained (but be aware that Qhouse is often much more sparse than Q ).

Note: The complex case is not yet implemented.
See also: msQR, mfQright
mfQright
right multiplication by a sparse $Q$ factor

Calling syntax:

```
B = mfQright( A, Qhouse )
```

returns the product $A Q$.
The mfMatFactor Qhouse holds a sparse $Q$ factor (obtained from a $Q R$ decomposition of a sparse matrix); the mfArray A may be a vector (dense) or a matrix (dense or sparse). B has the same structure (dense or sparse) than A.
By performing the operation: $\quad Q=m f Q r i g h t(m f \operatorname{SpEye}(m, m)$, Qhouse ) the standard form of the $Q$ factor can be obtained (but be aware that Qhouse is often much more sparse than Q ).

Note: The complex case is not yet implemented.
See also: msQR, mfQleft
mf/msEig

## eigenvalues and eigenvectors

The first form:

```
E = mfEig( A )
```

returns the eigenvalues of the mfArray A in the vector mfArray E.
The subroutine form:
call msEig( mfOut(V,D), A )
returns two mfArrays. D is a diagonal matrix containing the eigenvalues and V is the matrix whose columns are the corresponding eigenvectors so that: $A V=V D$ (or, equivalently, $A=V D V^{\prime}$ ).

Remarks:

- Sparse matrices are not handled; please use $\mathrm{mf} / \mathrm{msEigs}$ instead.
- Eigenvalues are returned in ascending order, according to the magnitude for complex ones.

See also: mf/msEigs, mf/msSchur, mfOut

## mf/msHess

The first form:

```
H = mfHess( A )
```

computes the Hessenberg form of the mfArray A. The returned mfArray has the same eigenvalues as A but contains zeros below the first subdiagonal.

The subroutine form:

```
call msHess ( mfOut(P,H), A )
```

returns two mfArrays. H is the Hessenberg form and P is a unitary matrix so that: $A=P H P^{\prime}$.
Remarks: sparse matrices are not (yet) handled.
See also: mfOut
$\mathrm{mf} / \mathrm{msSchur}$

## Schur decomposition

Calling syntax:

```
call msSchur ( mfOut(U,T), A [, form ] )
```

produces a upper triangular Schur matrix T and a unitary matrix U so that: $A=U T U^{\prime}$. A must be square.

For a real, dense mfArray A the Schur matrix $T$ is generally quasi-triangular (real form by default, see below).

The optional character argument form may be equal to "real" or "complex".
When processing a complex, dense mfArray A, or when the optional form string is equal to "complex", the matrix T is complex, (strictly) upper triangular.

Other syntax:

```
T = mfSchur( A [, form ] )
```

just returns the T factor.

## Remarks:

- in the real form, the matrix T contains 1 -by-1 and 2-by-2 diagonal blocks: the 2 -by- 2 blocks correspond to complex conjugate pairs of eigenvalues of A .
Each 2-by-2 block is in the form:

$$
\left(\begin{array}{ll}
a & b \\
c & a
\end{array}\right)
$$

where $b c<0$. The eigenvalues of such a block are $a \pm \sqrt{b c}$.

- the Schur decomposition is intended to be used for non-symmetric eigenvalue problems; if the mfArray $A$ is symmetric/hermitian, it is preferable to compute eigenvalues/vectors via the $\mathrm{mf} / \mathrm{msEig}$ routines (in such a case, V and D mfArrays returned by mf/msEig correspond exactly to U and T ). A warning is therefore emitted if A is symmetric or hermitian.
- sparse matrices are not (yet) handled.

See also: mfOut
mfLDiv, .ix.

## left matrix divide

Calling syntax:
$\mathrm{x}=\operatorname{mfLDiv}(\mathrm{A}, \mathrm{b})$

A .ix. b $\left(A^{-1} b\right)$ is a shortcut for writing: $\operatorname{mfLDiv}(\mathrm{A}, \mathrm{b})$

Description: solves a linear system of the form $A x=b$, i. e. returns $x=A^{-1} b$ without computing $A^{-1}$.
The mfArray A (of size $m \times n$ ) may be real or complex, sparse or dense, and doesn't need to be square; it can be even rank deficient (tests are made internally to use the most appropriate method):

- if $m>n$ the mean-square solution is returned;
- if $m<n$ the minimum 2-norm solution is returned.

This routine automatically detects if A is triangular and chooses the appropriate method.
Multiple right hand sides may be simultaneously solved, by concatenating all the RHS vectors in one rank- 2 mfArray b . This feature is not available for sparse matrices $A$ (but see below the way of solving simultaneously equations by using intermediate factors).

Note: For the sparse complex case, only square matrices $A$ are supported.
Using this routine is always more efficient than computing the inverse of the matrix A first followed by the product with b .

## Remarks:

- After doing an $L U$ factorization of the (dense) matrix A, this routine can also be called to solve the linear system $A x=P L U x=b$, by using the following syntax:

```
x = mfLDiv( L, U, P, b[, option="transp" ] )
```

The optional last argument allows the user to solve the system $A^{\prime} x=b$ (transpose in the real case, conjugate transpose in the complex case).

- After doing a Cholesky factorization of the (dense) matrix A via mfChol, this routine can also be called to solve the linear system $A x=U^{\prime} U x=b$, by using the following syntax:

$$
\mathrm{x}=\mathrm{mfLDiv}(\mathrm{U}, \mathrm{~b}, \text { form="cholesky" ) }
$$

- Similarly,

$$
\mathrm{x}=\operatorname{mfLDiv}(\mathrm{L}, \mathrm{U}, \mathrm{~b})
$$

is used for a sparse matrix. As noted in the msLU page, the mfArray $L$ factor contains a pointer to an internal structure containing all necessary data concerning the $L U$ factorization. Note that the $R H S \mathrm{~b}$ cannot have here multiple columns.

- For using with an mfMatFactor factors (from, e.g., msLU or mf/msChol), the routine must be called as:

$$
\mathrm{x}=\operatorname{mfLDiv}(\text { factors, } \mathrm{b}[\text {, option="transp" ] ) }
$$

This latter interface allows the user to simultaneously solve multiple RHS. Moreover, for this case only, the RHS b may be dense or sparse. The optional last argument allows the user to solve the system $A^{\prime} x=b$.

- When a (near) singular matrix is encountered, a Warning message may be emitted, according to the message level parameter (see msSetMsgLevel to change this level).

See also: mfRDiv
mfRDiv, .xi.

Calling syntax:
$\mathrm{x}=\mathrm{mfRDiv}(\mathrm{b}, \mathrm{A})$
b .xi. A $\left(b A^{-1}\right)$ is a shortcut for writing: $\operatorname{mfRDiv}(\mathrm{b}, \mathrm{A})$

Description: solves linear system of the form $x A=b$, i. e. returns $x=b A^{-1}$ without computing $A^{-1}$.
The mfArray A (of size $m \times n$ ) may be real or complex, sparse or dense, and doesn't need to be square; it can be even rank deficient (tests are made internally to use the most appropriate method):

- if $n>m$ the least-square solution is returned;
- if $n<m$ the minimum 2-norm solution is returned.

This routine automatically detects if A is triangular and chooses the appropriate method.
Multiple right hand sides may be simultaneously solved, by concatenating all the RHS vectors in one rank- 2 mfArray b . This feature is not available for sparse matrices $A$ (you should use mfLDiv instead, by using transposition).

Note: For the sparse complex case, only square matrices $A$ are supported.
Using this routine is always more efficient than computing the inverse of the matrix A first followed by the product with b .

## Remarks:

- After doing an $L U$ factorization of the (dense) matrix A, this routine can also be called to solve the linear system $x A=x P L U x=b$, by using the following syntax:

$$
\mathrm{x}=\operatorname{mfRDiv}(\mathrm{b}, \mathrm{~L}, \mathrm{U}, \mathrm{P})
$$

- After doing a Cholesky factorization of the (dense) matrix A via $\mathrm{mf} / \mathrm{msChol}$, this routine can also be called to solve the linear system $x A=x U^{\prime} U=b$, by using the following syntax:

$$
\mathrm{x}=\operatorname{mfRDiv}(\mathrm{b}, \mathrm{U}, \text { form="cholesky" ) }
$$

- Similarly,

$$
\mathrm{x}=\operatorname{mfRDiv}(\mathrm{L}, \mathrm{U}, \mathrm{~b})
$$

is used for a sparse matrix. As noted in the msLU page, the mfArray L factor contains a pointer to an internal structure containing all necessary data concerning the $L U$ factorization. Note that the $R H S \mathrm{~b}$ cannot have here multiple columns.

- When a (near) singular matrix is encountered, a Warning message may be emitted, according to the message level parameter (see msSetMsgLevel to change this level).

See also: mfLDiv
msRref

## reduced row echelon form

Calling syntax:

```
call msRref ( mfOut(R,jpiv), A [, tol ] )
```

computes the reduced row echelon form of the mfArray A.
The optional real tol is the tolerance (should be positive) used to compute the rank of the matrix. This tolerance cannot be too small. Minimum is the machine precision. When it is known, use the precision of the elements of the matrix A .

Note that the rank provided by this method is only an approximation which may differ from that computed with other methods (e.g., the Singular Value Decomposition msSVD). However, this approach is pedagogically interesting.

See also: mfRank, mfOrth, mfNull, msQR, mfOut
mfNull
null space

Calling syntax:

```
B = mfNull( A [, rational, tol ] )
```

returns an orthonormal basis for the null space of the mfArray A.
The optional logical rational argument specifies whether the rational basis is expected or not.
The optional real tol argument is the tolerance (should be positive) used to compute the rank of the matrix. As mfNull calls msRref, tol cannot be less than the machine $\epsilon$.

See also: mfRank, mfOrth, msRref, msQR, msSVD
mfOrth
matrix orthogonalization

Calling syntax:

```
B = mfOrth( A )
```

returns an orthonormal basis for the range of the mfArray A.
See also: mfRank, mfNull, msRref, msQR, msSVD
mfExpm

Calling syntax:

```
B = mfExpm( A )
```

computes the matrix exponential of the mfArray A (dense only).
See also: mfExp, mfLogm, mfSqrtm, mfPowm, mfFunm

## mfLogm

Calling syntax:

```
B = mfLogm( A )
```

computes the matrix logarithm of the mfArray A (dense only).
See also: mfLog, mfExpm, mfSqrtm, mfPowm, mfFunm
mfSqrtm

Calling syntax:

```
B = mfSqrtm( A )
```

computes the matrix square root of the mfArray A (dense only).
The property $B B=A$ doesn't always hold, because $A$ may not have a square root.
See also: mfSqrt, mfExpm, mfLogm, mfPowm, mfFunm

## mfPowm

Calling syntax:

```
B = mfPowm( A, e )
```

computes the matrix power of the mfArray A, i.e. $A^{e}$.
If e is integer, then A may have a sparse or a dense structure. Any positive integer is accepted.
If e is real, then A must have a dense structure. Any real value for e is accepted.
See also: mfPow10, mfExpm, mfLogm, mfSqrtm, mfFunm
mf/msFunm
general matrix function

Calling syntax:

```
F = mfFunm( A, fun )
```

applies the matrix function fun to the mfArray $A$ (dense only).
fun is either a character string giving the name of a usual function (as $\sin , \cos$, asin, ...), or the name of a user function. In this latter case, the user function must have the following interface:

```
function fun(z1) result(z2)
    complex(kind=MF_DOUBLE) :: z1, z2
end function
```

The subroutine form, which has the calling syntax:

```
call msFunm( mfOut(F,e), A, fun )
```

returns also an estimation of the error in the scalar mfArray e. Note that in the case of the exponential function (fun set to "exp"), the mfExpm is directly called, so that the error estimation will be not available (the mfArray e will be empty).

See also: mfFun, mfExpm, mfLogm, mfSqrtm, mfPowm, mfOut
mf/msEigs

## few eigenvalues and eigenvectors

The first form:

```
E = mfEigs( A, k [, which|sigma] [, tol, ncv] )
```

returns few eigenvalues of the matrix A using ARPACK in the vector mfArray E. A may be real/complex, sparse/dense.

The number of requested eignevalues k must be ranged in $[2, n-2]$.
which is a character(len=2) optional argument, can take the values "LM" or "SM", depending on whether the Largest or the Smaller Magnitude eigenvalues/vectors are requested. In addition, which can take the following values:

- for real, symmetric matrices:
"LA" = Largest Algebraic, with an algebraic selection;
"SA" = Smallest Algebraic, with an algebraic selection;
"BE" = Both Ends, with an algebraic selection (half from the largest ones and half from the smallest ones.
- for real, unsymmetric matrices:
"LR" = Largest Real part, with an algebraic selection;
"SR" = Smallest Real part, with an algebraic selection;
"LI" = Largest Imaginary part, with a magnitude ${ }^{1}$ selection;
"SI" = Smallest Imaginary part, with a magnitude ${ }^{1}$ selection;
- for complex matrices:
"LR" = Largest Real part, with an algebraic selection;
"SR" = Smallest Real part, with an algebraic selection;
"LI" = Largest Imaginary part, with a algebraic selection;
"SI" = Smallest Imaginary part, with a algebraic selection;

Eigenvalues are generally returned in a ascending order, excepted for the "LM" and "LA" cases, for which they are returned in a descending order.
sigma is a complex optional argument. If present, mfEigs tries to find the $k$ eigenvalues closest to the complex sigma.

If tol is present, it indicates the tolerance of the computed eigenvalues. By default, the machine epsilon is taken.

If ncv is present, it indicates how many Arnoldi vectors are generated (following the ARPACK documentation, the minimum value for $n c v$ is $2^{*} k+1$ and the maximum value is the order $n$ of the matrix; a warning is emitted if these conditions are not fulfilled).

If ncv is not present, the mfEigs routine tries larger and larger values of ncv, until (hopefully) the convergence of all requested eigenvalues.

In both cases (ncv present or not present), a warning is issued if not all eigenvalues converged. Use the subroutine form, explained below, to obtain a quiet behavior.

$$
\cdots / \ldots
$$

[^1]The subroutine form:

```
call msEigs( mfOut(V,E,flag), A, k [, which|sigma] [, tol, ncv] )
```

returns the requested eigenvalues in the vector mfArray E , the corresponding eigenvectors in the columns of the mfArray V , and the convergence of the iterative process. If the boolean mfArray flag is TRUE then all the eigenvalues converged; otherwise not all converged. Note that non-converged eigenvalues/eigenvectors are set to NaN .

The remaining arguments are processed as described above.
See also: mf/msEig, mf/msSVDS, mfOut

The first form:

```
S = mfSVDS( A, k [, which, tol, ncv ] )
```

returns few singular values of the matrix A using ARPACK in the vector mfArray S. A may be real/complex, sparse/dense; it doesn't need to be square.
k , the number of required singular values, cannot be larger than $\min (m, n)-2$. If so, $N a N$ values are used to fill the vector S .
which is a character (len=2) optional argument, which must be equal to "LM", "SM" or "BE". If present, mf SVDS tries to find the $k$ singular values of largest magnitude ("LM"), or smallest magnitude ("SM"). Only for real matrices, "BE" allows to compute the both-end singular values. Default value is "LM".

If tol is present, it indicates the tolerance of the computed singular values. By default, the machine epsilon is taken.

If ncv is present, it indicates how many Arnoldi vectors are generated (following the ARPACK documentation, the minimum value for $n c v$ is $2^{*} \mathrm{k}+1$ and the maximum value is $\min (m, n)$ of the matrix; a warning is emitted if these conditions are not fulfilled).

If ncv is not present, the mfSVDS routine tries larger and larger values of ncv, until (hopefully) the convergence of all requested singular values.

In both cases (ncv present or not present), a warning is issued if not all singular values converged. Use the subroutine form, explained below, to obtain a quiet behavior.

The subroutine form:

```
call msSVDS( mfOut(S,flag), A, k [, which, tol, ncv ] )
```

returns the requested singular values in the vector mfArray $S$ and the convergence of the iterative process. If the boolean mfArray flag is $T R U E$ then all the singular values converged; otherwise not all converged. The remaining arguments are processed as described above.

See also: mf/msSVD, mf/msEigs, mfOut
mfNormEst, mfNormEst1

Syntax:

```
C = mfNormEst( A [, tol ] )
```

Description:
Returns an estimation of the 2-norm of a rank-2 mfArray (dense or sparse, real or complex, square or non-square).

The optional argument allows the user to choose the tolerance used in the iterative algorithm. By default, tolerance is $10^{-6}$.

Syntax:
$C=m f N o r m E s t 1(A)$
returns an estimation of the 1-norm of a square matrix (sparse or dense).

See also: mfNorm
mfCondEst
matrix condition number estimation

Calling syntax:

```
C = mfCondEst( A )
```

Description:
Returns an estimation of the condition number of the rank- 2 mfArray A.
Remarks: The condition number is always greater than 1. A value equal to infinity (resp. close to the inverse of the $\epsilon$ machine) indicates that the matrix A is singular (resp. nearly singular).

See also: mfCond, mfRCond

## mfIsSymm

## matrix symmetry inquiry

Calling syntax:

```
bool = mfIsSymm( A [, option="pattern", tol ] )
```

returns a boolean value according to the symmetry property of the mfArray A. If the matrix is complex, this property means hermitian.

For a sparse mfArray A, if the optional argument option is present and equal to "pattern", then the routine checks only for the symmetry of the sparse structure.

If the optional tolerance tol is present, the symmetry test (for the real case) is made according to the rule:

$$
\left\|A-A^{\prime}\right\|_{1}<2 \text { tol }\|\mathrm{A}\|_{1}
$$

Default tolerance is MF EPS.
See also: mfIsPosDef, mfTolForSymm
mfTolForSymm
tolerance for symmetry

Calling syntax:

```
tol = mfTolForSymm( A )
```

returns the tolerance for which the mfArray A should be symmetric.
See also: mfIsSymm
mfIsPosDef
definite positiveness inquiry

Calling syntax:

```
bool = mfIsPosDef( A )
```

returns a boolean value according to the definite positiveness property of the mfArray A.
See also: mfIsSymm
mfIsDiagDomCol
diagonally dominant by cols inquiry

Calling syntax:

```
bool = mfIsDiagDomCol( A )
```

returns a boolean value according to the "diagonally dominant by cols" property of the mfArray A. See also: mfIsStrictDiagDomCol
mfIsStrictDiagDomCol
strictly diagonally dominant by cols inquiry

Calling syntax:

```
bool = mfIsStrictDiagDomCol( A )
```

returns a boolean value according to the "strictly diagonally dominant by cols" property of the mfArray A.

See also: mfIsDiagDomCol

## mfIsFullRank

## full rank inquiry

Calling syntax:

```
bool = mfIsFullRank( A [, tol ] )
```

returns '.true.' if the mfArray A is full rank, i.e. if its rank is equal to the minimum of its dimensions.
If tol, of type real, is present then this value is used as tolerance in the rank computation. Default tolerance is chosen as $\max (m, n)\|A\| \epsilon$.

See also: mfRank

## MF_LAPACK_VERSION

LAPACK version

Calling syntax:

```
    string = MF_LAPACK_VERSION()
```

returns the LAPACK version used during the link of the executable.
Note the parenthesis used after the name of the variable, because it is implemented as a function.
The returned string may be used by the mfIsVersion boolean function.
See also: MF_COMPILER_VERSION, MF_MUESLI_VERSION, msGetBlasLib, msGetLapackLib, msGetArpackInfo, msGetSuiteSparseLib

## msGetBlasLib

## Get BLAS library type

Calling syntax:

```
call msGetBlasLib( string )
```

returns the BLAS library type used during the link of the executable, that is, either Reference for most compilers or a specific package for others (e.g. MKL for the INTEL compiler suite; OpenBLAS or ATLAS for the GNU compiler suite).
string is a character string of length at least 128.
The current routine is able to detect the dynamic switch between BLAS implementations like OpenBLAS and ATLAS.

See also: MF_COMPILER_VERSION, MF_MUESLI_VERSION, MF_LAPACK_VERSION, msGetLapackLib, msGetArpackInfo, msGetSuiteSparseLib
msGetLapackLib
Get LAPACK library type

Calling syntax:

```
call msGetLapackLib( string )
```

returns the LAPACK library type used during the link of the executable, that is, either Reference for most compilers or a specific package for others (e.g. MKL for the INTEL compiler suite or ATLAS for the GNU compiler suite).
string is a character string of length at least 128.
The current routine is able to detect the dynamic switch between BLAS implementations like Reference and ATLAS.

See also: MF_COMPILER_VERSION, MF_MUESLI_VERSION, MF_LAPACK_VERSION, msGetBlasLib, msGetArpackInfo, msGetSuiteSparseLib
msGetArpackInfo
Get ARPACK library information

Calling syntax:

```
call msGetArpackInfo( version[, info] )
```

version is a character string of length at least 24, containing on return the version of the library. The "NG" letters indicates the new generation version, which can be found at: http://forge.scilab.org/ index.php/p/arpack-ng/.
info is an optional character string of length at least 128, containing on return additional information, especially various non official fixes applied to the library.

See also: MF_COMPILER_VERSION, MF_MUESLI_VERSION, MF_LAPACK_VERSION, msGetBlasLib, msGetLapackLib, msGetSuiteSparseLib
msGetSuiteSparseLib
Get SuiteSparse library version

Calling syntax:
call msGetSuiteSparseLib( version )
version is a character string of length at least 8 , containing on return the version of the library.
The returned string may be used by the mfIsVersion boolean function.
See also: MF_COMPILER_VERSION, MF_MUESLI_VERSION, MF_LAPACK_VERSION, msGetBlasLib, msGetLapackLib, msGetArpackInfo

### 1.9 Polynomial Functions

```
mfPolyVal
mfPolyFit,msPolyFit
mfFunFit, msFunFit
mfRoots
mfPoly
mfSpline, msSpline
mfLegendre
mfPPVal
mfPPDer
mfInterp1
mfInterp2
mfPSLG
msPrintPSLG
mfDelaunay
mfTriConnect
msBuildTriConnect
msUpdateTriConnect
msExtractTriConnect
msCheckDomainConvexity
msPrintTriConnect
msTriNodeNeighbors
mfTriSearch, mfNodeSearch
mfGridData
mfDelaunay3D
msEndDelaunay3D
mfTetraConnect
msBuildTetraConnect
msPrintTetraConnect
msDel3DNodeNeighbors
mfTetraSearch, mfNodeSearch3D
mfGridData3D
polynomial evaluation polynomial fitting nonlinear function fitting roots of a polynomial polynomial with specified roots cubic spline interpolation or smoothing Legendre polynomials piecewise polynomial evaluation piecewise polynomial derivative 1D interpolation regular 2D interpolation
Planar Straight Line Graph (derived type)
Planar Straight Line Graph display
Delaunay 2D triangulation
2D triangular connectivity (derived type)
2D triangular connectivity initialization
2D triangular connectivity update
get 2D connectivity from a mfTriConnect structure
Check convexity of a 2D triangulation
2D triangular connectivity display
2D triangulation node neighbors
2 D triangulation search
irregular 2D interpolation
Delaunay 3D tetrahedralization Internal free storage after Delaunay 3D
3D tetrahedral connectivity (derived type)
3D tetrahedral connectivity initialization
3D tetrahedral connectivity display
3D tetrahedralization node neighbors
3D tetrahedralization search
irregular 3D interpolation
```

See also:
Core Routines
File Input/Output
Data Analysis Functions
Operators
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Optimization and Function Functions
Sparse Matrices
mfPolyVal

Interface:

```
function mfPolyVal( p, x ) result( out )
    type(mfArray), intent(in) :: p, x
    type(mfArray) :: out
```

Description:
Evaluates polynomial p at points x.
The mfArray p is a vector of length $\mathrm{k}+1$ whose elements are the coefficients of the polynomial in descending powers.

$$
\text { out }=p_{1} x^{k}+p_{2} x^{k-1}+\ldots+p_{k} x+p_{k+1}
$$

See also: mfPPVal, mfPolyFit

## mfPolyFit

Interface:

```
function mfPolyFit( x, y, n ) result( out )
    type(mfArray), intent(in) :: x, y
    integer, intent(in) :: n
    type(mfArray) :: out
```

Description:
Fit polynomial to real data.
The mfArray x and y must be column vectors of same length. They are coordinates of the data. n is the degree of the polynomial to be obtained, so $n+1$ real coefficients are returned in the vector out. These coefficients are sorted for descending powers.

See also: msPolyFit, mfPolyVal, mfFunFit
msPolyFit
polynomial fitting

Calling syntax:

```
call msPolyFit( mfOut(p, normr[, r2]), x, y, n )
```

Description:
Similar to mfPolyFit, but also returns (in mfArrays) the norm- 2 of the residuals (in normr) and optionally the correlation coefficient $r_{\text {corr }}^{2}$ (in r2).

Nota: The correlation coefficient of the approximation is computed as follows:

$$
r_{\mathrm{corr}}^{2}=1-\frac{\mathrm{normr}^{2}}{M \sigma^{2}}
$$

where $\sigma^{2}$ is the standard deviation (as computed by $m f \operatorname{Var}(\mathrm{y})$ ) and $M$ is the number of data points in x and y .

See also: mfPolyVal, msFunFit, mfOut
mfFunFit
nonlinear function fitting

## Interface:

```
function mfFunFit( x, y, fun, p0, n, tol ) result( out )
type(mfArray), intent(in) :: x, y, p0
interface
        function fun( x, p, n ) result( res )
            import :: MF_DOUBLE
            real(kind=MF_DOUBLE), intent(in) :: x
            real(kind=MF_DOUBLE), intent(in) :: p(n)
            integer, intent(in) :: n
            real(kind=MF_DOUBLE) :: res
        end function fun
    end interface
integer, intent(in) :: n
real(kind=MF_DOUBLE), intent(in), optional :: tol
type(mfArray) :: out
```


## Description:

Fit arbitrary (nonlinear) function to real data.
Data is stored in the mfArrays x and y , which must be column vectors of same length. There are $m$ couples $(x, y)$.
fun is the real function which tries to model the data. This function is parameterized by n variables stored in the mfArray p.

At input, p0 contains an initial guess for the values of the n unknown parameters. Final values are stored in the mfArray out.
tol is the tolerance for the convergence. Its default value is $10^{-6}$.

## Remarks:

- the problem is formulated as a non-linear least-square problem, solved by package MINPACK;
- therefore, $m$ (the number of couples $(x, y)$ ) must be bigger than the number of parameters $n$.

See also: msFunFit, mfPolyFit, mfLsqNonLin
msFunFit
nonlinear function fitting

Calling syntax:

```
call msFunFit( mfOut(p[, r2]), x, y, fun, p0, n [, tol ] )
```

Description:
Similar to mfFunFit, but in addition it returns the correlation coefficient $r_{\text {corr }}^{2}$ (in the mfArray r2).
For this subroutine, the output of the desired $n$ variables is stored in the mfArray $p$.

Remark: Please note that the correlation coefficient $r_{\text {corr }}^{2}$ is ranged in the interval $[-1,+1]$. A value close to +1 indicates a perfect correlation while a null value indicates no correlation. A negative value means that the relationship between original data and fitted data is very bad most of time (practically, this should indicates that your function fun is not adapted to the data).

See also: mfOut
mfRoots

Calling syntax:

```
    r = mfRoots( p )
```

Description:
Returns the roots of the polynomial whose coefficients are provided in the mfArray p (row or column vector).

This polynomial writes:

$$
P(x)=p_{1} x^{n-1}+p_{2} x^{n-2}+\ldots+p_{n-1} x+p_{n}
$$

where $\left[p_{1}, p_{2}, \ldots, p_{n}\right]$ are the $n$ components of the input vector p .
Even if the roots are real, the returned mfArray is complex. The roots are not sorted.
Remarks:

- p must be of type real.
- The mfPoly function is the inverse of the mfRoots function.

See also: mfCompan, mfEig, mfFZero
mfPoly
polynomial with specified roots

Calling syntax:

```
    p = mfPoly( r )
```

Description:
Returns the coefficients of the polynomial whose roots are provided in the mfArray $r$ (row or column vector).

If $n$ is the size of p then the output polynomial reads:

$$
P(x)=p_{1} x^{n-1}+p_{2} x^{n-2}+\ldots+p_{n-1} x+p_{n}
$$

Remarks:

- The specified roots in r may be complex.
- The mfRoots function is the inverse of the mfPoly function.

See also: mfCompan, mfEig
mfSpline

## cubic spline interpolation

## Interface:

```
function mfSpline( x, y, &
    xi, &
    BC_type_1, BC_val_1, BC_type_2, BC_val_2, &
    periodic ) &
result( y_sec )
    type(mfArray), intent(in) :: x, y
    type(mfArray), intent(in), optional :: xi
    integer, intent(in), optional :: BC_type_1
    real(kind=MF_DOUBLE), intent(in), optional :: BC_val_1
    integer, intent(in), optional :: BC_type_2
    real(kind=MF_DOUBLE), intent(in), optional :: BC_val_2
    logical, intent(in), optional :: periodic
    type(mfArray) :: y_sec
```


## Description:

If $x i$ is not present, this function returns the coefficients of the spline (piecewise cubic polynomial form) which interpolates the values y at the abscissas x . The returned coefficients in y_sec (second derivative at abscissas) are intended to be used with the mfPPVal function.

If xi is present, mfSpline returns only the interpolated values at the entries contained in the mfArray xi.

By default, the spline form is natural, which means that the first and the last coefficients are null (i.e. curvature is null at both ends). Optionally, boundary conditions of various (and mixed) types may be specified at the ends of the interval:

> BC_type_1 $=1$ implies that $y^{\prime}=$ BC_val_ $^{1}$ at first end;
> BC_type_1 $=2$ implies that $y^{\prime \prime}=$ BC_val_ $^{\prime} 1$ at first end;

BC_type_2 $=1$ implies that $y^{\prime}=$ BC_val_2 at second end;
BC_type_2 $=2$ implies that $y^{\prime \prime}=$ BC_val_2 at second end;
If periodic is present and equal to $T R U E$, then the value of y , along with the first and second derivatives are equal at the two ends of the interval. Default is $F A L S E$.

## Remarks:

- x and y must be vectors having the same shape, but xi may have any shape.
- Abscissas x doesn't need to be equally spaced; x must contain at least 2 points.
- $\mathrm{x}, \mathrm{y}$ and xi may be temporary mfArrays.
- interpolated values, at abscissas different than the nodes in x may be obtained by using the mfPPVal function.
- smoothing of data may be obtained by using the msSpline routine.


## Interface:

```
call msSpline( mfOut(y_smooth, y_sec), x, y, weights
&
[, BC_type_1, BC_val_1, BC_type_2, BC_val_2] )
```


## Description:

Similar to the mfSpline function (in particular the last four optional arguments), but apply a smoothing to the data, instead of a strict interpolation. The new (smoothed) values of y are stored in the y_smooth mfArray, and the cubic spline coefficients are stored in the y_sec mfArray.

## Remarks:

- The weights mfArray contains the weight of the smoothing, to be applied to each value of the $y$ mf Array; it may be a scalar: in such a case, all weights are equal.
- When choosing infinite weights, this routine gives the same results than the mfSpline function, i.e. a strict interpolation of the data; on the other end, choosing null weights leads to the leastsquare linear regression of all the data points.
- Typically, the values of the weights vary according to the powers of ten.

See also: mfOut

## mfLegendre

Calling syntax:

```
C = mfLegendre( n, A )
```

Description:
Returns the evaluation of the n -th Legendre polynomial (degree $n$ ) of each element of the mfArray A . Restriction: A must be of type real.
mfPPVal

## Interface:

```
function mfPPVal( x, y, p, xi, extrapol ) result( yi )
    type(mfArray), intent(in) :: p, xi
    (logical|real(kind=MF_DOUBLE)), intent(in), optional :: extrapol
    type(mfArray) :: yi
```


## Description:

Computes the interpolated values at the entries xi.
The way this function treats out-of-range input values in xi is similar to that of mfInterp1 (see the remarks below).
p is the vector of Spline coefficients - of the curve ( $\mathrm{x}, \mathrm{y}$ ) - returned by mfSpline.

## Remarks:

- $\mathrm{x}, \mathrm{y}, \mathrm{p}$ and xi may be temporary mfArrays.
- xi mfArray may have any shape.
- By default, an element of yi is set to MF_NAN if the corresponding value in xi is outside the range of x . The optional argument extrapol may be used to override this behaviour: a real value allows the use of a constant for out-of-range inputs. More interestingly, a logical value can be used to compute an extrapolated value by using the interpolation law of the nearest valid interval.

See also: mfPolyVal, mfPPDer
mfPPDer

Interface:

```
function mfPPDer( x, y, p, xi, extrapol ) result( yi )
\begin{tabular}{lll} 
type(mfArray), & intent(in) & \(:: ~ p, ~ x i\) \\
(logical|real(kind=MF_DOUBLE)), & intent(in), optional & \(::\) extrapol \\
type(mfArray) & & \(::\) yi
\end{tabular}
```

Description:
Similar to mfPPVal but returns the derivative instead of the function value.
p is the vector of Spline coefficients - of the curve ( $\mathrm{x}, \mathrm{y}$ ) - returned by mfSpline.

## mfInterp1

## 1D interpolation

## Interface:

```
function mfInterp1( x, y, xi, order, extrapol ) result( yi )
    type(mfArray), intent(in) :: x, y, xi
    integer, intent(in), optional :: order
(logical|real(kind=MF_DOUBLE)), intent(in), optional :: extrapol
    type(mfArray) :: yi
```


## Description:

Interpolates data y , which are defined along a vector x , at the entry(ies) specified by xi .
x and y must be vectors with the same shape; x must have strictly monotonous values.
xi may be a scalar or a vector; yi, which contains interpolated data, will have the same shape as xi.

## Remarks:

- By default a linear interpolation (2-point stencil) is used (order=1); set the optional argument order to 2 (resp. 3) in order to use a quadratic (resp. cubic) interpolation using a 3-point (resp. 4-point) stencil. A nearest interpolation (1-point stencil) is also possible: set order to 0 .
- By default, an element of yi is set to MF_NAN if the corresponding value in xi is outside the range of x . The optional argument extrapol may be used to override this behaviour: a real value allows the use of a constant for out-of-range inputs. More interestingly, a logical value can be used to compute an extrapolated value by using the interpolation law of the nearest valid interval.

See also: mfSpline, mfInterp2, mfGridData, msMeshGrid

## Example(s):

```
    x = mfLinspace( 0.0d0, 1.0d0, 5 )
    call msDisplay( x, "x" )
    print *, "y = x**2"
    y = x**2
    xi = mfLinspace( 0.05d0, 0.95d0, 4 )
    call msDisplay( xi, "xi" )
    yi = mfInterp1( x, y, xi, order=2 )
    call msDisplay( yi, "interpolated data at many points" )
```

output:
$\mathrm{x}=$

| 0.0000 | 0.2500 | 0.5000 | 0.7500 | 1.0000 |
| :--- | :--- | :--- | :--- | :--- |

$\mathrm{y}=\mathrm{x} * * 2$
xi $=$
$\begin{array}{llll}0.0500 & 0.3500 & 0.6500 & 0.9500\end{array}$
interpolated data at many points =
$\begin{array}{llll}0.0025 & 0.1225 & 0.4225 & 0.9025\end{array}$

$$
\cdots / \ldots
$$

```
xi = mfLinspace( -0.15d0, 1.15d0, 7 )
call msDisplay( xi, "xi" )
yi = mfInterp1( x, y, xi, order=2 )
call msDisplay( yi, "interpolated data at many points (2 NaNs expected)" )
```

output:

```
xi =
    -lllllll
interpolated data at many points (2 NaNs expected) =
```

NaN
0.0044
0.0803
0.2500
0.5136
0.8711
NaN

```
yi \(=m f \operatorname{Interp} 1(\mathrm{x}, \mathrm{y}, \mathrm{xi}, \operatorname{order}=2\), extrapol=MF_INF )
call msDisplay( yi, "interpolated data at many points (2 Infs expected)" )
```

output:

```
interpolated data at many points (2 Infs expected) =
\begin{tabular}{llllll} 
Inf 0.0044 & 0.0803 & 0.2500 & 0.5136 & 0.8711
\end{tabular}
yi = mfInterp1( x, y, xi, order=2, extrapol=.true. )
call msDisplay( yi, "interpolated data at many points (using extrapolation)" )
```

output:

```
interpolated data at many points (using extrapolation) =
\begin{tabular}{lllllll}
0.0225 & 0.0044 & 0.0803 & 0.2500 & 0.5136 & 0.8711 & 1.3225
\end{tabular}
```


## mfInterp2

regular 2D interpolation

## Interface:

```
function mfInterp2( x, y, z, xi, yi, order, extrapol ) result( zi )
    type(mfArray), intent(in) :: x, y, z, xi, yi
    integer, intent(in), optional :: order
(logical|real(kind=MF_DOUBLE)), intent(in), optional :: extrapol
    type(mfArray) :: zi
```


## Description:

Interpolates data $\mathbf{z}$, which are defined on a rectangular, horizontal $\operatorname{grid}(x, y)$, at the entry(ies) specified by xi and yi.
$\mathrm{x}, \mathrm{y}$ and z must be matrices with the same shape; x , y must have strictly monotonous values. Usually, x and y are built with the routine msMesh Grid.
(xi,yi) also must have the same shape (scalars, vectors, or matrices); zi, which contains interpolated data, will have the same shape as xi and yi.

## Remarks:

- By default a bilinear interpolation (4-point stencil) is used (order=1); set the optional argument order to 2 (resp. 3) in order to use a biquadratic (resp. bicubic) interpolation using a 9 -point (resp. 16-point) stencil. A nearest interpolation (1-point stencil) is also possible: set order to 0 .
- Note that the method used is valid only for rectangular, horizontal grid. For a general, irregular grid, see mfGridData.
- By default, an element of zi is set to MF_NAN if the corresponding value in (xi,yi) is outside the range of $x$ and $y$. The optional argument extrapol may be used to override this behaviour: a real value allows the use of a constant for out-of-range inputs. More interestingly, a logical value can be used to compute an extrapolated value by using the interpolation law of the nearest valid interval.

See also: mfInterp1, msMeshGrid
mfPSLG
Planar Straight Line Graph (derived type)

## Description:

This derived type encapsulates internal arrays describing a 2D domain definition, using polygonal shapes.
Declaration is made as follows:

```
type(mfPSLG) :: PSLG_domain
```

Contrary to other Muesli derived types, all internal fields are public, letting the user to allocate/deallocate and set the data himself. The complete definition of this kind of object is:
type (mfPSLG)

```
    real(kind=MF_DOUBLE), allocatable :: n_xy(:,:), holes_xy(:,:)
    integer, allocatable :: edge_n(:,:)
```

end type
$\mathrm{n} \_\mathrm{xy}$ contains the coordinates of the nodes; must be of size $(n n, 2)$, with $n n \geq 3$.
edge $\_$n contains the definition of each edge segment, by its two endpoints; ; must be of size ( $n e, 2$ ), with $n e \geq 0$.
holes_xy contains the definition of holes; they are points having two coordinates; must be of size ( $n h, 2$ ), with $n h \geq 0$. A hole should be, ideally, located inside a closed boundary of edges or, alternatively, outside the domain but inside the convex hull (in this later case, its role is to allow concavities). The hole points must not be located exactly on any edge.

## Remarks.

- This structure can be used directly by msDelaunay to triangulate the corresponding domain.
- See examples of PSLG domains in the Muesli User's Guide.
- A variable of this type may be freed, at the end of its use, by the msRelease routine.

See also: msPrintPSLG, msPlotPSLG

Calling syntax:

```
call msPrintPSLG( PSLG_domain [, short_info ] )
```

Description:
For a small number of nodes (typically less than few dozens), this routine print on the screen the definition of a Planar Straight Line Graph, which is used to represent a domain geometry.

The input argument PSLG_domain must be of type mfPSLG.
When the boolean optional argument short_info is present and equal to $T R U E$ then the routine just print the number of items, i.e. nodes, edges and holes. Therefore, it is the only way to inspect the structure when it is big.

See also: msPlotPSLG
$m f D e l a u n a y$

First calling syntax:

```
tri = mfDelaunay( x_in, y_in )
```


## Description:

Builds a Delaunay 2D triangulation from a list of nodes whose coordinates are given in x_in and y_in (these two mfArrays must be vectors having the same length).

The returned mfArray contains indices for the triangles: each row of tri provides three integers which describes a triangle (direct orientation, when travelling from first node to the second and the third ones).

## Second calling syntax:

```
call msDelaunay( mfOut( x, y, tri ), x_in, y_in,
[ theta_min, area_max ] )
```


## Description:

Builds a constrained conforming Delaunay 2D triangulation (as previously, from a given list of nodes whose coordinates are stored in $x_{\text {_in }}$ and $y$ _in), giving a quality mesh by adding new nodes where it is required. Two different criteria are used:

- A minimal angle theta_min, to avoid triangles with too small angles. This angle, in degree, must be less than 28.6 (otherwise the algorithm may fail to converge - anyway, the routine will emit an error). The recommanded value ranges from 20 to 25 .
- A maximum area area_max, to avoid big triangles.

These two optional arguments are real. At least one is required.

Third calling syntax:

```
call msDelaunay( mfOut( x, y, tri ), PSLG_domain, &
    [ theta_min, area_max ] )
```


## Description:

Same as the above interface, but take a PSLG domain on input (see mfPSLG), instead of a list of nodes. PSLG means Planar Straight Line Graph and, very briefly, ca be viewed as a polygonal description of the boundaries of a domain; it is intended to store the description of a plane domain to be meshed.

See also: msBuildTriConnect, mfNodeSearch, mfTriSearch, mfDelaunay3D

# 2D triangular connectivity (derived type) 

## Description:

This derived type encapsulates internal arrays describing the connectivity of a 2 D triangulation. It must be initialized with the msBuildTriConnect routine.

Declaration is made as follows:

```
type(mfTriConnect) :: tri_connect
```

Note that this structure embeds not only the connectivity tables, but also all nodes coordinates. See msExtractTriConnect for more information about the content of this structure.

A variable of this type must be freed, at the end of its use, by the msRelease routine.
See also: msPrintTriConnect, msTriMesh
msBuildTriConnect

## 2D triangular connectivity initialization

## Interface:

```
subroutine msBuildTriConnect( x, y, tri, tri_connect,
&
check_tri_orient,
&
tri_renum, equil_face_orient )
type(mfArray), intent(in) :: x, y
type(mfArray), intent(in out) :: tri
type(mfTriConnect), intent(out) :: tri_connect
logical, intent(in), optional :: check_tri_orient
logical, intent(in), optional :: tri_renum, equil_face_orient
```


## Description:

This routine initializes the internal structures of the 2D triangulation connectivity tri_connect. See mfTriConnect.

Arguments are mfArrays describing (i) the coordinates of all nodes (in x and y ) and (ii) the definition of the triangles in terms of nodes (in tri).

The indices in the mfArray tri describe the 2D triangulation; they may come from the mfDelaunay routine applied to the coordinates x and y .

The optional logical argument check_tri_orient is used to check that all triangles, described in the mfArray tri, have a direct orientation; this is required if you plan to apply, later on, the msCheckDomainConvexity routine. Default is to apply such a check. As this check is costly, you should omit it only if you are sure on the constraint about triangles' orientation, for example when tri results from the use of the mfDelaunay routine. When used, this check may change some indices in the tri mfArray.

The optional logical argument tri_renum is used to apply a renumbering of the triangles, in such a way that they are, wherever possible, geometrically contiguous. In this case, the table of triangles tri can be modified (row ordering). Default is to not apply such a renumbering.

The optional logical argument equil_face_orient is used to modify the orientation of triangles' faces, in such a way that they are, wherever possible, equilibrated at each node. Default is to not modify the faces' orientation.

## Remarks:

- After use, the tri_connect object should be freed via a call to the routine msRelease.
- The nodes coordinates are embedded in the tri_connect structure. Therefore, you should NOT change the nodes coordinates after calling the current routine: indeed, while the connectivity remains the same, the convexity (which is a geometrical property embedded in the structure, see msExtractTriConnect) may be affected. Otherwise, especially if you DO modify the coordinates of the boundary nodes, you will have to compute again the connectivity.

See also: mfTriSearch, mfNodeSearch, msPrintTriConnect
msUpdateTriConnect

## 2D triangular connectivity update

Interface:
subroutine msUpdateTriConnect( $\mathrm{x}, \mathrm{y}$, tri_connect )
type(mfArray), :: x, y
type(mfTriConnect), intent(in out) :: tri_connect

Description:
Update the nodes' position in an mfTriConnect structure.
Arguments x and y are mfArrays containing the new $(x, y)$ position of the nodes in the 2 D triangulation.

See also: msBuildTriConnect
msExtractTriConnect
get 2 D connectivity from a mfTriConnect structure

## Interface:

```
subroutine msExtractTriConnect( tri_connect, tri_f, face_n,
    face_tri, convexity )
    type(mfTriConnect), intent(in) :: tri_connect
    integer, allocatable, intent(out), optional :: tri_f(:,:), face_n(:,:), &
    face_tri(:,:)
    integer, intent(out), optional :: convexity
```


## Description:

Usually, the 2D mesh connectivity is embedded (with hidden components) in a mfTriConnect structure. If the user want to know/use the links between nodes, triangles and faces, he has to extract himself the corresponding connectivity tables.

The input 2D connectivity tri_connect must have been initialized by the msBuildTriConnect routine.
According to the optional argument presence, this routine returns the following integer tables, containing positive integers (when nothing else is specified):

- tri_f, of dimension ( $\mathrm{nt}, 3$ ), which contains the three faces of a given triangle; nt is the total number of triangles.
- face_n, of dimension ( $\mathrm{nf}, 2$ ), which contains the two nodes which define a given face; nf is the total number of faces.
- face_tri, of dimension ( $\mathrm{nf}, 2$ ), which contains the two triangles separated by a given face; nf is the total number of faces. In the case where the given face is on the boundary domain, only the first triangle number is valid; the second one is set to a value which is negative or zero (see below)
- convexity, an integer variable whose value may be $-1(U N K N O W N), 0(F A L S E)$ or 1 (TRUE). It is recommended to ask for its value when interpreting the value of the second column of the face_tri table. When the domain convexity is $U N K N O W N$ or TRUE, the second element of the face_tri table is always zero for a boundary face. When the domain convexity is $F A L S E$, the value zero for a boundary face indicates the convex exterior part, the value -1 indicates the non-convex part of the exterior boundary, whereas other negative values indicates an interior bondary (i.e. a hole; add one to this number to obtain the numbering of the hole).
Note that the order of the interior boundaries' numbering is determined by the smallest value of the nodes for each of these internal boundaries.
Note also that the convexity can be explicitely determined by use of the msCheckDomainConvexity routine.


## Remarks:

- the connectivity doesn't depend on the exact location of the nodes, but the convexity does.
- the number of faces (nf) can be retrieve by asking the shape of the face_n array.
- the allocatable arrays tri_f, face_n and face_tri are allocated by the routine itself; in the case where one of them is already allocated, it will be deallocated and reallocated if needed.

See also: mfTriConnect, msBuildTriConnect

## msCheckDomainConvexity

## Check convexity of a 2D triangulation

Calling syntax:
call msCheckDomainConvexity( tri_connect )

Description:
Computes the convexity of a 2 D triangulation, given as argument in the tri_connect connectivity.
After use of this routine, the convexity component of the connectivity may be only TRUE or FALSE, not UNKNOWN (see msExtractTriConnect).

See also: mfTriConnect, msBuildTriConnect, msPrintTriConnect

Calling syntax:

```
call msPrintTriConnect( tri_connect [, short_info ] )
```

Description:
For a small number of nodes (typically less than few dozens), this routine print on the screen the connectivity tables of a triangular mesh (the connectivity could be computed by the msBuildTriConnect routine, or by other ways).

The input argument tri_connect must be of type mfTriConnect.
When the boolean optional argument short_info is present and equal to TRUE then the routine just print the number of items, i.e. nodes, triangles and faces. Therefore, it is a simple way to inspect the structure when it is big. Another way is to extract some components of tri_connect by using the msExtractTriConnect routine.

See also: msTriMesh

## Interface:

```
subroutine msTriNodeNeighbors( tri_connect [, connected_nodes | connected_faces ] )
type(mfTriConnect), intent(in) :: tri_connect
type(mf_Int_List), intent(out), optional :: connected_nodes(:)
type(mf_Int_List), intent(out), optional :: connected_faces(:)
```


## Description:

This routine computes the list of the neighbors (either nodes, stored in connected_nodes, or faces, stored in connected_faces) for each node belonging to the triangulation given in tri_connect.

The ouput lists, connected_nodes and connected_faces are both vectors of type mf_Int_List, a special structure which can contain a variable number of elements. Be aware that the allocation is done by the current routine, not by the user. The length of these vectors is, of course, equal to the number of nodes of the triangulation.

The input mesh connectivity tri_connect must have been built before via a call to msBuildTriConnect.

## mfTriSearch

## 2D triangulation search (for triangle)

## Generic Interface:

```
function mfTriSearch( tri_connect, x, y, strict ) result( num )
    type(mfTriConnect), intent(in) :: tri_connect
    type(mfArray), intent(in) :: x, y
or real(kind=MF_DOUBLE), intent(in) :: x, y
logical, intent(in), optional :: strict
integer :: num
```


## Description:

Searches in a Delaunay triangulation the triangle which encloses the point ( $\mathrm{x}, \mathrm{y}$ ).
The input mesh connectivity tri_connect must have been built before via a call to msBuildTriConnect.
For a general search (i.e. when the optional argument strict is $F A L S E$ ) the routine always returns a valid triangle index which is the enclosing triangle to the targeted point, or the nearest triangle if the targeted point is outside the meshed domain.

On the contrary, for a strict search (i.e. when the optional argument strict is $T R U E$, which is the default), if the search successes, then num contains the (non zero) index of the triangle; else it contains a negative or null value, which is related to concavities in the meshed domain: 0 means a targeted point outside the domain, whereas $-k$ means a point located in the hole number $k$ (a warning is emitted).

Remark: The belonging of a point to a triangle is not strict: this means that the same point can belong to several adjacent triangles. In particular, a node of the triangulation belongs to all triangles who share it.

See also: mfNodeSearch
mfNodeSearch

## 2D triangulation search (for node)

## Interface:

```
function mfNodeSearch( tri_connect, x, y, strict ) result( num )
    type(mfTriConnect), intent(in) :: tri_connect
    type(mfArray), intent(in) :: x, y
or real(kind=MF_DOUBLE), intent(in) :: x, y
    logical, intent(in), optional :: strict
    integer :: num
```


## Description:

Searches in a Delaunay triangulation the nearest node to the point ( $\mathrm{x}, \mathrm{y}$ ).
The input mesh connectivity tri_connect must have been built before via a call to msBuildTriConnect.
For a general search (i.e. when the optional argument strict is $F A L S E$ ) the routine always returns a valid node number which is the nearest node to the targeted point, even if this latter point is outside the meshed domain.

On the contrary, for a strict search (i.e. when the optional argument strict is $T R U E$, which is the default), if the search successes, then num contains the (non zero) index of the node; else it contains a negative or null value, which is related to concavities in the meshed domain: 0 means a targeted point outside the domain, whereas $-k$ means a point located in the hole number $k$ (a warning is emitted).

See also: mfTriSearch

## mfGridData

Interface:

```
function mfGridData( x, y, f, xi, yi ) result( fi )
    type(mfArray), intent(in) :: x, y, f, xi, yi
    type(mfArray) :: fi
```


## Description:

Interpolates data $f$, which are defined on an irregular grid ( $x, y$ ), at the entries specified by xi and yi. $x, y$ and $f$ must be vectors with the same shape.
(xi,yi) also must have the same shape (scalars, vectors, or matrices); fi, which contains interpolated data, will have the same shape as them.
fi is set to MF_NAN if (xi,yi) is outside the convex hull of $x$ and $y$.
Remarks: a linear interpolation is used.
See also: mfInterp2, mfGridData3D

```
mfDelaunay3D
```


## Delaunay 3D tetrahedralization

## Interface:

```
    function mfDelaunay3D( x, y, z ) result( tetra )
        type(mfArray), intent(in) :: x, y, z
        type(mfArray) :: tetra
    function mfDelaunay3D( coords ) result( tetra )
        type(mfArray), intent(in) :: coords
        type(mfArray) :: tetra
```

or

## Description:

Builds a Delaunay 3D tetrahedralization from a list of nodes whose coordinates are given in $x, y$ and $z$ (these three mfArrays must be vectors having the same length). In the second possible interface, coords must have three columns.

The returned mfArray contains indices for the tetrahedra: each row of tetra provides four integers which describes a tetrahedron.

All tetrahedra have a direct orientation: nodes 1,2 and 3 is a direct triangle if node 4 is above them.
See also: msBuildTetraConnect, mfDelaunay
msEndDelaunay3D
Internal free storage after Delaunay 3D

Calling syntax:

```
call msEndDelaunay3D( )
```

Description:
After using the mfDelaunay3D routine, it is recommended to call msEndDelaunay3D in order to clean the auxiliary memory used to build the tetrahedralization.

## mfTetraConnect

## Description:

This derived type encapsulates internal arrays describing the connectivity of a 3D tetrahedralization. It must be initialized with the msBuildTetraConnect routine.

Declaration is made as follows:

```
type(mfTetraConnect) :: tetra_connect
```

A variable of this type must be freed, at the end of its use, by the msRelease routine.
See also: mfDelaunay3D
msBuildTetraConnect

## 3D tetrahedral connectivity initialization

## Interface:

```
    subroutine msBuildTetraConnect( x, y, z, tetra, tetra_connect )
            type(mfArray), intent(in) :: x, y, z, tetra
            type(mfTetraConnect), intent(out) :: tetra_connect
        subroutine msBuildTetraConnect( coords, tetra, tetra_connect )
            type(mfArray), intent(in) :: coords, tetra
            type(mfTetraConnect), intent(out) :: tetra_connect
```

or

## Description:

This routine initializes the internal structures of a 3D tetrahedralization connectivity.
The indices in the mfArray tetra describe the 3D tetrahedralization; they may come from the mfDelaunay3D routine applied to the coordinates $\mathrm{x}, \mathrm{y}$ and z .

Arguments are mfArrays describing (i) the coordinates of all nodes (in $x, y$ and $z$ ) and (ii) the definition of the triangles in terms of nodes (in tetra).

The second possible interface allows the user to group all the columns vector coordinates in one matrix having a coherent shape.

After use, the tetra_connect object should be freed via a call to the routine msRelease.
See also: mfTetraSearch, mfNodeSearch3D
msPrintTetraConnect

## Display of a 3D tetrahedral connectivity

Calling syntax:

```
call msPrintTetraConnect( tetra_connect )
```

Description:
For a few numbers of nodes (typically less than few dozens), this routine print on the screen the connectivity tables computed by the msBuildTetraConnect routine.

## msDel3DNodeNeighbors

## 3D tetrahedralization node neighbors

## Interface:

```
subroutine msDel3DNodeNeighbors( tetra, node_neighbors )
type(mfArray), intent(in) :: tetra
type(mf_Int_List), intent(out) :: node_neighbors(:)
```


## Description:

This routine computes the list of the neighbors of all nodes in the allocatable array node_neighbors (of type mf_Int_List). Be aware that the allocation is done by the current routine, not by the user.

The indices in the mfArray tetra describe the 3 D tetrahedralization; they may come from the mfDelaunay3D routine applied to the coordinates $\mathrm{x}, \mathrm{y}$ and z .

See also: msBuildTetraConnect

## mfTetraSearch

## 3D tetrahedralization search (for tetrahedron)

Generic Interface:

```
function mfTetraSearch( tetra_connect, x, y, z ) result( num )
    type(mfTetraConnect), intent(in) :: tetra_connect
    type(mfArray), intent(in) :: x, y, z
or real(kind=MF_DOUBLE), intent(in) :: x, y, z
    integer :: num
```


## Description:

Searches in a Delaunay tetrahedralization the tetrahedron which encloses the point ( $x, y, z$ ).
tetra_connect must be built before via a call to msBuildTetraConnect.
If the search successes, then num contains the (non zero) index of the tetrahedron, else it contains 0 .
Remark: The belonging of a point to a tetrahedron is not strict: this means that the same point can belong to several adjacent tetrahedra. In particular, a node belongs to all tetrahedra who share it.

See also: mfNodeSearch3D, msBuildTetraConnect
mfNodeSearch3D

## 3D tetrahedralization search (for node)

## Interface

```
function mfNodeSearch3D( tetra_connect, x, y, z ) result( num )
    type(mfTetraConnect), intent(in) :: tetra_connect
        type(mfArray), intent(in) :: x, y, z
or real(kind=MF_DOUBLE), intent(in) :: x, y, z
        integer :: num
```


## Description:

Searches in a Delaunay tetrahedralization the nearest node to the point ( $x, y, z$ ).
tetra_connect must be built before via a call to msBuildTetraConnect.
If the search successes, then num contains the (non zero) index of the node, else it contains 0 . If num is 0 , this means that the point is outside the convex hull (a Warning is emitted).

See also: mfTetraSearch, msBuildTetraConnect

## mfGridData3D

Interface:

```
function mfGridData3D( x, y, z, f, xi, yi, zi ) result( fi )
    type(mfArray), intent(in) :: x, y, z, f, xi, yi, zi
    type(mfArray) :: fi
```


## Description:

Interpolates data $f$, which are defined on an irregular $3 \mathrm{D} \operatorname{grid}(\mathrm{x}, \mathrm{y}, \mathrm{z})$, at the entries specified by xi , yi and zi.
$\mathrm{x}, \mathrm{y}, \mathrm{z}$ and f must be vectors with the same shape.
(xi,yi,zi) also must have the same shape (scalars, vectors, or matrices); fi, which contains interpolated data, will have the same shape as them.
fi is set to MF_NAN if (xi,yi,zi) is outside the convex hull of $x, y$ and $z$.
Remarks: a linear interpolation is used.
See also: mfGridData

### 1.10 Optimization and Function Functions

mf/msFZero
mf/msFSolve
mf/msLsqNonLin
mfTrapz
mfSimpson
mfQuad, msQuad
mfDblQuad, msDblQuad
mfOdeSolve, msOdeSolve
mfDaeSolve, msDaeSolve
mf_DE_Options
mf_NL_Options
single-variable non-linear zero finding multiple-variable non-linear zero finding non-linear least-square minimization numerical evaluation of integrals by trapezoidal method numerical evaluation of integrals by Simpson method numerical evaluation of integrals (adaptive method) numerical evaluation of double integrals (adaptive method)
integrator of ODE (explicit) systems integrator of DAE (implicit) systems
options for differential solvers (derived type) options for non-linear solvers (derived type)

See also:

## Core Routines

File Input/Output
Data Analysis Functions
Operators
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Sparse Matrices
$\mathrm{mf} / \mathrm{msFZero}$

## single-variable non-linear zero finding

## Interface:

```
function mfFZero( fun, x0, tol ) result( out )
    interface
    real(kind=MF_DOUBLE) function fun( x )
            real(kind=MF_DOUBLE), intent(in) :: x
        end function
end interface
type(mfArray), intent(in) :: x0
real(kind=MF_DOUBLE), optional :: tol
type(mfArray) :: out
```


## Description:

Search for a solution of the non-linear equation fun $(x)=0$, via the Dekker algorithm (combination of bisection, linear and quadratic interpolation).
fun is a user-supplied function which has the prescribed interface above.
x 0 must be a real vector mfArray containing exactly two values; it defines the interval for searching $x$. These two values must be set such that the sign of fun $(x)$ differs.

The optional argument tol specifies the relative tolerance for the stopping criterium. It should be a non negative and not too small real value; default value is $2 \epsilon$.

## Remarks:

- the convergence is usually fast and the Dekker method theoretically never fail, providing the initial interval given in x 0 is correct;
- fun has an imposed list of arguments; the user may use module's data in order to exchange other information between his program and this function. Therefore, except for very simple cases, this user's routine must be located inside a module, USEd also by the user's program.

The subroutine form:

```
call msFZero( mfOut(x,fval[,status]), fun, x0 [, tol ] )
```

allows the user to access to other information: $f v a l$ is the function value associated to the zero x , and a status (optional) which has the following values:

| status | explanation |
| :---: | :--- |
| 0 | Normal termination of the algorithm. |
| -1 | Bad initialization: fun, applied to the two elements of $x 0$, must have opposite <br> sign. |
| -2 | Bad argument value: x 0 has not enough components. |

Remark: Adding the optional argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). On the contrary (i.e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfRoots, mfFSolve, mfOut
mfFSolve
multiple-variable non-linear zero finding

## Interface:

```
function mfFSolve( fcn, n, x0, options, jac, sparse ) result( out )
    interface
        subroutine fcn( n, x, fvec, flag )
            integer, intent(in) :: n
            real(kind=MF_DOUBLE), intent(in) :: x(n)
            real(kind=MF_DOUBLE) :: fvec(n)
            integer :: flag
        end subroutine
end interface
type(mfArray), intent(in) :: x0
type(mf_NL_Options), intent(in), optional :: options
external, optional :: jac
logical, intent(in), optional :: sparse
type(mfArray) :: out
```


## Description:

Search for a solution of n non-linear equations with n unknowns:

$$
F_{i}\left(x_{1}, x_{2}, \ldots, x_{n}\right)=0, \quad i=1, \ldots, \mathrm{n}
$$

described in the subroutine fcn . This user-supplied subroutine has the interface prescribed above and computes $\mathrm{fvec}(\mathrm{i})=F_{i}\left(x_{1}, x_{2}, \ldots, x_{n}\right), i=1, \ldots, \mathrm{n}$.
flag is actually an in-out argument of fcn : on entry, its value is always zero. The user can stop the iteration process, by setting it to any negative integer.
x 0 is an initial guess for the solution $x$.
The mfArray options $\%$ tol contains the tolerance(s) for the convergence:

- when it is a scalar, both the function tolerance FTOL and the parameter tolerance $X T O L$ are set to the corresponding value;
- when it is a vector of two elements, the values are used to set the two tolerances: FTOL and $X T O L$, respectively.
When the mfArray options\%tol is empty (i.e. not set by the user), FTOL and XTOL are both set to $10^{-9}$.

See mf_NL_Options for other options.
Optionally, the user may provide the computation of the jacobian via the subroutine jac. Its interface must be as follows:

```
interface
    subroutine jac( n, x, jacobian )
        integer, intent(in) :: n
        real(kind=MF_DOUBLE), intent(in) :: x(n)
        real(kind=MF_DOUBLE) :: jacobian(n,n)
    end subroutine jac
end interface
```

The jacobian matrix is defined as $J=\frac{\partial F}{\partial x}$, therefore, the rank-2 array jacobian must be filled as follows: jacobian $(\mathrm{i}, \mathrm{j})=J_{i, j}=\frac{\partial F_{i}}{\partial x_{j}}$. Only non-zero values of $J$ need to be defined.

For the sparse case, the optional argument sparse must be set to .true. and the jac routine must be defined as:

```
interface
    subroutine jac( n, x, job, pd, ipd, jpd, nnz )
        integer, intent(in) :: n, job
            real(kind=MF_DOUBLE), intent(in) :: x(n)
            real(kind=MF_DOUBLE) :: pd(*)
            integer, :: ipd(*), jpd(*), nnz
    end subroutine
end interface
```

The pd,ipd,jpd f90 arrays describe the CSC (Compact Sparse Column) representation of the sparse jacobian matrix, as follows:

- pd(1:nnz) contains the non-zero matrix entries;
- ipd(1:nnz) contains the row indices;
$-\operatorname{jpd}(1: n c o l+1)$ is the pointer to the beginning of the columns, in arrays (pd,ipd).
pd,ipd must contain all diagonal terms, even if they are null. Moreover, row indices must be sorted in ascending order. Lastly, this routine must include a mechanism such that only the value of nnz is computed when $j o b=0$. You must also use the convention: $j p d(n c o l+1)=n n z+1$.


## Remarks.

- the problem is solved by the package MINPACK;
- the algorithm may fail (it is not unusual to have no solution for the non-linear problem - the returned value will be therefore a vector of $N a N s$ ); in such a case, using the msFSolve subroutine helps in giving some information about the reason of the failure.
- fcn and jac have imposed lists of arguments; as for mfLsqNonLin, mfOdeSolve and mfDaeSolve the user may use module's data in order to exchange other information between his program and these routines (especially for checking the number of equations).Therefore, except for very simple cases, these two user's routines must be located inside a module, USEd also by the user's program.

See also: mfFZero, mfLsqNonLin
msFSolve
multiple-variable non-linear zero finding

Calling syntax:

```
call msFSolve( mfOut( x[, fvec, status ] ), fcn, n, x0 [, options, jac] )
```


## Description:

Similar to mfFSolve, but also returns the vector function (in the mfArray fvec) and, optionally, a status which has the following values:

| status | explanation |
| :---: | :--- |
| 1 | Normal termination of the algorithm. The stopping criterion about xtol has <br> been reached first. |
| 2 | Normal termination of the algorithm. The stopping criterion about ftol has <br> been reached first. |
| -1 | Bad initialization for x0 (size or type). |
| -2 | Bad argument value: tol is not strictly positive. |
| -3 | Bad termination: number of calls of fcn has reached or exceeded <br> max_iter*( $\mathrm{n}+1$ ) (if jac is not present) or max_iter (if jac is present). |
| -4 | Bad termination: tol is too small. No further improvement in the approximate <br> solution x is possible. |
| -5 | Bad termination: iteration is not making good progress. <br> -10The user has stopped the algorithm via the argument flag of the fcn subrou- <br> tine. |
| -14 | The user-supplied fcn routine returned invalid entries (NaN value). |

For this subroutine, the solution is stored in the mfArray x .
If the status value is not positive (i.e. when algorithm fails), both x and fvec are vectors of $N a N \mathrm{~s}$.
Remark: Adding the optional argument status allows the user to launch his program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). On the contrary (i.e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfOut

## mfLsqNonLin

## Interface:

```
function mfLsqNonLin( m, fcn, p0, n, options, jac ) result( out )
    integer, intent(in) :: m, n
type(mfArray), intent(in) :: p0
    interface
        subroutine fcn( m, n, p, fvec, flag )
            integer, intent(in) :: m, n
            real(kind=MF_DOUBLE), intent(in) :: p(n)
            real(kind=MF_DOUBLE) :: fvec(m)
            integer :: flag
        end subroutine fcn
end interface
type(mf_NL_Options), optional :: options
external, optional :: jac
type(mfArray) :: out
```


## Description:

Minimize the sum of the squares of m nonlinear functions $F_{i}$ depending on n parameters.
fcn is the user-supplied subroutine which calculates the functions; it has the interface prescribed above and computes $\mathrm{fvec}(\mathrm{i})=F_{i}(p), i=1, \ldots, \mathrm{~m}$, where $p$ represents the vector of the n parameters.
flag is actually an in-out argument of fcn : on entry, its value is always zero. The user can stop the iteration process, by setting it to any negative integer. See just below for constrained minimization.

Constraints of type $B O X$ are possible, for each parameter(see mf_NL_Options).
At input, p0 contains an initial guess for the values of the n unknown parameters. Final values are stored in the mfArray out.

All the following options are stored in the structure options (see mf_NL_Options).
The mfArray options\%tol contains the tolerance(s) for the convergence:

- when it is a scalar, both the function tolerance FTOL and the parameter tolerance $X T O L$ are set to the corresponding value, while the orthogonality tolerance GTOL (it measures the orthogonality between the function vector and the columns of the jacobian) is set to zero;
- when it is a vector of three elements, the values are used to set the three tolerances: FTOL, XTOL and $G T O L$, respectively.

When the mfArray options\%tol is empty (i.e. not set by the user), FTOL and XTOL are both set to $10^{-6}$ while $G T O L$ is set to zero.
options\%max_iter is the maximum number of iterations. Its default value is 50 .

$$
\cdots / \ldots
$$

options\%epsfen is the approximation error of the functions $F_{i}$. Its default value is zero. It is used only to compute the step for the finite differences computation of the Jacobian; this argument will not be referenced if jac is present. In the case of an approximate computation in the fcn routine (e.g. an integration or whatelse), the value of epsfen may be critical to obtain good convergence; it must be chosen carefully; see mf_NL_Options for additional information.

The logical options\%print can be used to print some information during the iterative process. Default is FALSE .

Optionally, the user may provide the computation of the jacobian via the subroutine jac. Its interface must be as follows:

```
interface
    subroutine jac( m, n, p, jacobian )
        integer, intent(in) :: m, n
        real(kind=MF_DOUBLE), intent(in) :: p(n)
        real(kind=MF_DOUBLE) :: jacobian(m,n)
    end subroutine jac
end interface
```

The jacobian matrix is defined as $J=\frac{\partial F}{\partial p}$, and the rank-2 array jacobian must be filled as follows: $\operatorname{jacobian}(i, j)=J_{i, j}=\frac{\partial F_{i}}{\partial p_{j}}$. Only non-zero values of $J$ need to be defined.
The integer options\%check_jac allows the user to numerically check (via finite differences) the components of the jacobian each time it is called. A quick global check is done by setting this argument to 1 while a complete (but more expensive) check is done by setting it to 2 . Default is 0 , i.e. no check.

Lastly, the integer print_check_jac allows the printing (either on the screen if the value is 1 , or in files if the value is 2 ) of the results of the previous jacobian check. Default is $0, i . e$. no check.

## Remarks:

- the problem is solved by the LMA method (Levenberg-Marquardt Algorithm) of the MINPACK package;
- $m$ must be bigger than the number of parameters $n$.
- fcn and jac have imposed lists of arguments; as for mfFSolve, mfOdeSolve and mfDaeSolve the user may use module's data in order to exchange other information between his program and these routines (especially for checking the number of equations). Therefore, except for very simple cases, these two user's routines must be located inside a module, USEd also by the user's program.
- the initial step bound for the parameters' vector is based on a factor whose default value is 100 . Sometimes, we may want to change this factor to a smaller value, especially if you use the $\log ()$ function to ensure the positiveness of your unknowns; in such a case, factor $=\log (100)$ is more appropriate, by setting options\%init_step_bound_factor to this value.

See also: mfFunFit, msLsqNonLin

Calling syntax:

```
call msLsqNonLin( mfOut( p, resnorm[, status, res_log, p_log, ident ] ),
    &
    m, fcn, p0, n[, options, jac ] )
```


## Description:

Similar to mfLsqNonLin, but also returns the 2-norm residue (in the mfArray resnorm) and, optionally, a status which has the following values:

| status | explanation |
| :---: | :--- |
| 1 | Normal termination of the algorithm. Both actual and predicted relative re- <br> ductions in the sum of squares of the functions fvec are at most ftol. |
| 2 | Normal termination of the algorithm. Relative error between two consecutive <br> iterates of the parameters' vector is at most xtol. |
| 3 | Normal termination of the algorithm. Conditions for status $=1$ and status $=$ <br> 2 both hold. |
| -1 | Improper input parameters. |
| -4 | fvec is orthogonal to the columns of the jacobian. This means that the gra- <br> dient is approximatively null (with respect to gtol): it may correspond to a <br> minimum, or a maximum, or a saddle point, so the algorithm cannot continue. |
| -5 | number of calls of fcn has reached max_iter*(n+1) (if jac is not present) or <br> max_iter (if jac is present). |
| -7 | tol is too small. No further reduction in the sum of squares is possible. <br> -8 <br> tol is too small. No further improvement in the approximate solution p is <br> possible. |
| fvec is orthogonal to the columns of the jacobian to machine precision. This <br> means that the gradient is approximatively null: it may correspond to a min- <br> imum, or a maximum, or a saddle point, so the algorithm cannot continue. |  |
| Incidently, gtol is too small. |  |

For this subroutine, the output of the desired $n$ variables is stored in the mfArray $p$.
res_log and p_log are mfArrays which contain the history of the norm of the residue and the parameters, along the iterations. Each row of these matrices corresponds to an iteration.
ident is an mfArray which contains the identifiability of each parameter. This vector has values ranged between 0 and 1 , and computed internally by the ratio $\left|R_{i}\right| /\left|R_{1}\right|$, where $R_{i}, i=1 \ldots \mathrm{n}$, are the diagonal values of the $R$ matrix obtained after a $Q R$ decomposition of the jacobian (sensitivity matrix). The elements of ident have been reordered to match the original order of the parameters. A value equal or close to 0 means that the corresponding parameter is not identifiable. Actually, the user should compare the values of ident to the numerical precision of the fvec functions.

$$
\cdots / \ldots
$$

Remark: Adding the optional argument status allows the user to launch his program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). On the contrary (i.e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfOut
mfTrapz

## integration by trapezoidal method

First calling syntax:

```
res = mfTrapz( y )
```


## Description:

Performs the numerical integration of the data $y$, assuming constant unit spacing for the abscissas, using the trapezoidal rule. If the spacing is different from unity, the result res must be multiplied by the effective spacing.

Second calling syntax:

```
res = mfTrapz( x, y )
```

Description:
Performs the numerical integration of the data y , spread against the abscissas x .
Remark: x and y must be real vector mfArrays.
See also: mfSimpson, mfQuad
mfSimpson
integration by Simpson method

Calling syntax:

```
    res = mfSimpson( y )
```


## Description:

Performs the numerical integration of the data $y$, assuming constant unit spacing for the abscissas, using the Simpson rule. If the spacing is different from unity, the result res must be multiplied by the effective spacing.

Second calling syntax:

```
res = mfSimpson( x, y )
```

Description:
Performs the numerical integration of the data y , spread against the abscissas x .
Remarks:

- x and y must be real vector mfArrays.
- the number of points must be odd.

See also: mfTrapz, mfQuad
mfQuad

## Interface

```
function mfQuad( fun, a, b, abs_tol, rel_tol, sing ) result( out )
    interface
        real(kind=MF_DOUBLE) function fun( x )
            real(kind=MF_DOUBLE), intent(in) :: x
        end function
    end interface
real(kind=MF_DOUBLE), intent(in) :: a, b
real(kind=MF_DOUBLE), intent(in), optional :: abs_tol, rel_tol
logical, intent(in), optional :: sing
type(mfArray) :: out
```


## Description:

Performs the numerical computation of the integral of the function $\mathrm{fun}(x)$ over the interval $[\mathrm{a}, \mathrm{b}]$.
The optional arguments abs_tol and rel_tol allows the user to give the absolute and/or the relative precision of the result. Default tolerance is $1 \times 10^{-12}$.

The second optional argument sing should be used when the function fun $(x)$ has a singularity at $x=\mathrm{a}$ or/and at $x=\mathrm{b}$. 'sing=. true.' leads to use another quadrature method, which usually involves a fewer number of function evaluation.

Remarks: out is set to MF_NAN if required precision cannot be achieved; an error is emitted by the library. Call the subroutine version if you don't want such a behavior.

See also: msQuad, mfTrapz, mfDblQuad, msDblQuad
msQuad

## Calling syntax:

```
call msQuad( mfOut( q, abserr[, status, neval] ),
    fun, a, b [, abs_tol, rel_tol, sing] )
```


## Description:

The behavior of this routine is similar to those of mfQuad.
The subroutine call allows to get more information returned by the integrator:

- q contains the numerical result
- abserr contains an estimation of the absolute error
- status (optional) contains the returned error code:
status $=0$ : normal and reliable termination of the routine. It is assumed that the requested accuracy has been achieved.
status < 0: abnormal termination of the routine. The estimates for $q$ and abserr are less reliable. It is assumed that the requested accuracy has not been achieved. Only a warning is emitted by the library. (Exact meaning of the error can be found below)
- neval (optional) contains the effective number of function evaluation

All these variable must be of type (mfArray).
Remark: Adding the optional argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). On the contrary (i.e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mf Quad, mf Out, $\mathrm{mfDblQuad}$,

| status | explanation |
| :--- | :--- |
| -1 | Maximum number of subdivisions allowed has been achieved. |
| -2 | The occurrence of roundoff errors is detected, which prevents the requested <br> tolerance from being achieved. The error may be under-estimated. |
| -3 | Extremely bad integrand behaviour occurs at some points of the integration <br> interval. |
| -4 | The algorithm does not converge. Roundoff error is detected in the extrapo- <br> lation table. It is presumed that the requested tolerance cannot be achieved, <br> and that the returned result is the best which can be obtained. |
| -5 | The integral is probably divergent, or slowly convergent. It must be noted that <br> divergence can occur with any other value of status. |
| -6 | Invalid input (this error should have been trapped by the routine msQuad itself). |

mfDblQuad
numerical evaluation of double integrals

## Interface:

```
function mfDblQuad( fun, xa, xb, ya, yb, abs_tol, rel_tol ) result( out )
        interface
            real(kind=MF_DOUBLE) function fun( x, y )
                real(kind=MF_DOUBLE), intent(in) :: x, y
        end function
end interface
real(kind=MF_DOUBLE), intent(in) :: xa, xb, ya, yb
real(kind=MF_DOUBLE), intent(in), optional :: abs_tol, rel_tol
type(mfArray) :: out
```

Other calling syntaxes:

```
out = mfDblQuad( fun, fun_xa, fun_xb, ya, yb, abs_tol, rel_tol )
```

or

```
out = mfDblQuad( fun, xa, xb, fun_ya, fun_yb, fun_y=.true., abs_tol, rel_tol )
```

where fun xa, fun_xb, fun_ya and fun_ya are all double precision real function of one real argument. By using these latter syntaxes, the user is able to integrate over non rectangular domains. Note that in the second case, the additional argument fun_y is mandatory (no other way to disambiguate a unique interface for differente routines).

## Description:

Performs the numerical computation of the double integral of the function $f u n(x, y)$ over the interval $[\mathrm{xa}, \mathrm{xb}] \times[\mathrm{ya}, \mathrm{yb}]$.

The optional arguments abs_tol and rel_tol allows the user to give the absolute and/or the relative precision of the result. Default tolerance is $1 \times 10^{-12}$.

Remarks: out is set to MF_NAN if required precision cannot be achieved; an error is emitted by the library. Call the subroutine version if you don't want such a behavior.

See also: mfQuad, msQuad

## Calling syntax:

```
call msDblQuad( mfOut( q, abserr[, status, neval] ),
    fun, xa, xb, ya, yb [, abs_tol, rel_tol] )
```

\&

## Description:

The behavior of this routine is similar to those of mfDblQuad (including the fact to do integration on non rectangular domains).

The subroutine call allows to get more information returned by the integrator:

- q contains the numerical result
- abserr contains an estimation of the absolute error
- status (optional) contains the returned error code:
status $=0$ : normal and reliable termination of the routine. It is assumed that the requested accuracy has been achieved.
status < 0: abnormal termination of the routine. The estimates for $q$ and abserr are less reliable. It is assumed that the requested accuracy has not been achieved. Only a warning is emitted by the library. (Exact meaning of the error can be found below)
- neval (optional) contains the effective number of function evaluation

All these variable must be of type(mfArray).
Remark: Adding the optional argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). On the contrary (i.e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfDblQuad, mfOut

| status | explanation |
| :--- | :--- |
| -1 | Maximum number of subdivisions allowed has been achieved. |
| -2 | The occurrence of roundoff errors is detected, which prevents the requested <br> tolerance from being achieved. The error may be under-estimated. |
| -3 | Extremely bad integrand behaviour occurs at some points of the integration <br> interval. |
| -4 | The algorithm does not converge. Roundoff error is detected in the extrapo- <br> lation table. It is presumed that the requested tolerance cannot be achieved, <br> and that the returned result is the best which can be obtained. |
| -5 | The integral is probably divergent, or slowly convergent. It must be noted that <br> divergence can occur with any other value of status. |
| -6 | Invalid input (this error should have been trapped by the routine msDblQuad <br> itself). |

mfOdeSolve

## integrator of ODE (explicit) systems

Main interface:

```
function mfOdeSolve( deriv, t_span, y_0, options, jac, sparse ) result( y )
    interface
        subroutine deriv( t, y, yprime, flag )
            real(kind=MF_DOUBLE), intent(in) :: t, y(*)
            real(kind=MF_DOUBLE), intent(out) :: yprime(*)
            integer, intent(in out) :: flag
        end subroutine
    end interface
    type(mfArray), intent(in) :: t_span, y_0
    type(mf_DE_Options), intent(in), optional :: options
    external, optional :: jac
    logical, intent(in), optional :: sparse
    type(mfArray) :: y
```


## Description:

Performs the numerical integration of a system of (explicit ${ }^{1}$ ) ordinary differential equations (ODEs) of first order, which is written under the form:

$$
y^{\prime}=F(t, y)
$$

described via the subroutine deriv (the variables $t$, y and yprime represent respectively $t, y$ and $y^{\prime}$ ). Usually, the integration is done between time $t_{0}$ and $t_{\text {end }}$ (extremal values of $t_{\_}$span, which is a column vector mfArray of at least two components) or, in the continuation mode ${ }^{2}$, up to the new values specified in t_span (in this case, $t_{-}$span may contain only one value).

The output of mfOdeSolve is the row vector y which represents the solution $y$ at time $t_{\text {end }}$. If the vector t_span contains more than two values (or more than one value in the continuation mode), the routine returns in y the intermediate values of $y$, as specified by all time values present in $\mathrm{t}_{-}$span; in this case, y is a rank- 2 mf Array with as many rows as components in $\mathrm{t}_{-}$span.

The integration uses the prescribed initial condition stored in the row vector y_0. Note that this mfArray is not referenced in the continuation mode.

The last argument flag of deriv is always set to zero (internally by the solver) on input, and should be altered (inside the routine deriv) only in some special situations:

- set flag to -1 for an Illegal Condition; the value of $y^{\prime}$ at this time may not be available. The routine will try to continue the integration (reducing the time step) as long as possible without getting the illegal condition. [You can put this assignment anywhere in the deriv routine.]
- set flag to -2 for an Emergency Exit; the value of $y^{\prime}$ at this time may not be available. The routine will return control immediately to the calling program. [You can put this assignment anywhere in the deriv routine.]
- set flag to 3 for an End Condition; the value of $y^{\prime}$ at this time must be always available. The routine will adapt its time step to finish exactly at the current time given by the argument $t$ of deriv. [WARNING: you must put this assignment at the end of the deriv routine, to insure that all the derivatives have been computed.]

[^2]The previous three situations are called stopping conditions. Note that when a stopping condition is found in the user subroutine resid, then the vector $\mathrm{t}_{-}$span (which contains the value of $t_{\text {end }}$ ) is modified. Note also that the three stopping conditions doesn't have the same numerical cost; their cost is mentioned in the following table:

| Flag to used in resid | Stopping condition | Numerical cost |
| :---: | :--- | :--- |
| -1 | Illegal Condition | expensive |
| -2 | Emergency Exit | very cheap |
| 3 | End Condition | cheap |

All following options are stored in the structure options (see mf DE_Options).
The boolean mfArray options\%continuation allows the user to continue the integration to a new value of $t_{\text {end }}$, without specify a new $t_{0}$. Thanks to specific calls to $\mathrm{mf} / \mathrm{ms}$.deSolve, the user is even able to save internal data of the integrator, to stop the program, and to launch again the integrator after the reload of saved data. See the next section Other calling syntaxes.

The mfArray options\%tol allows the user to give the precision of the result, and must contain strictly positive values. Default values are $10^{-3}$ for the relative tolerance and $10^{-6}$ for the absolute tolerance; this mfArray may be of rank 0,1 or 2 :

- if it is a scalar, it specifies the relative tolerance for all the components of the solution; the absolute tolerance is fixed to $10^{-6}$.
- if it is a vector, it must have two elements which specifies the relative and the absolute tolerance, respectively.
- if it is a matrix, it must have two columns (corresponding to the relative and absolute tolerance) and a number of rows equal to the number of equations.

The mfArray options\%y_ind_out allows the user to get only a subpart of the mfArray y; if present, it must contain integer indices of the wanted components of the $y$ vector.

The string options\%method specifies the method to be used. Possible choices are "RKF" (Runge-Kutta Fehlberg), "ABM" (Adams-Bashforth-Moulton), or "BDF" (Backward Differentiation Formula). By default, the Runge-Kutta Fehlberg method is selected.

When the "BDF" method is used, the user may provide the jac routine to compute the jacobian matrix (otherwise, it will be numerically evaluated by finite differences). Moreover, this jacobian matrix may be defined in dense, band or sparse format. For the first two formats, the interface of the jac routine is:

```
interface subroutine jac( t, y, jacobian, ldjac )
    real(kind=MF_DOUBLE), intent(in) :: t, y(*) integer,
    real(kind=MF_DOUBLE), intent(out) :: jacobian(ldjac,*)
    end subroutine end interface
```

The jacobian matrix is defined as $J=\frac{\partial F}{\partial y}$, therefore, the rank-2 array jacobian must be filled as follows: $\operatorname{jacobian}(i, j)=J_{i, j}=\frac{\partial F_{i}}{\partial y_{j}}$.
Please note that, for the dense format, only non-zero values need to be defined.
If the jacobian is symmetric and positive definite (abbreviated as SPD), you should set the jac_symm_pos_def option (see mf_DE_Options) in order to economize storage and obtain a better performance for its factorization; in such a case, you must give only the upper part of the jacobian.

$$
\cdots / \ldots
$$

Furthermore, when the jacobian $J$ is banded, the mfArray options\%band may be used to economize both memory and CPU time. It must be a vector of exactly two elements: $M L$ and $M U$ which are the widths of the lower and upper parts of the band, respectively, with the main diagonal being excluded (e.g. for a tridiagonal system, both elements are equal to 1 ).

In this case, routine jac must store the elements $J_{i, j}$ in the matrix jacobian as follows:

$$
\text { irow = i - j + ML + MU + } 1
$$

Actually, ldjac is equal to $2 * M L+M U+1$ : indeed, an extra storage is needed for the matrix factorisation. For an SPD matrix, simply take ML equal to zero in the previous formulae.

For the sparse case, the optional argument sparse must be set to .true. and the jac routine must be defined as:

```
interface
    subroutine jac( t, y, nrow, job, pd, ipd, jpd, nnz )
            real(kind=MF_DOUBLE), intent(in) :: t, y(*)
            integer, intent(in) :: nrow, job
            real(kind=MF_DOUBLE), intent(out) :: pd(*)
            integer, intent(out) :: ipd(*), jpd(*)
            integer, intent(in out) :: nnz
    end subroutine
end interface
```

The pd,ipd,jpd f90 arrays describe the CSC (Compact Sparse Column) representation of the sparse jacobian matrix, as follows:

- pd(1:nnz) contains the non-zero matrix entries;
- ipd(1:nnz) contains the row indices;
$-j p d(1: n c o l+1)$ is the pointer to the beginning of the columns, in arrays (pd,ipd).
pd,ipd must contain all diagonal terms, even if they are null. Moreover, row indices must be sorted in ascending order. Lastly, this routine must include a mechanism such that only the value of nnz is computed when $j o b=0$. You must also use the convention: $j p d(n c o l+1)=n n z+1$.

Be aware that the sparse structure (that is to say, the indices' vectors ipd and jpd) must remain the same during all the integration process. On the contrary, use the option spjac_const_struct of the $m f$ _DE_Options derived type. Moreover, you can even force to reuse the same sparse structure if you make many calls to $\mathrm{mf} / \mathrm{msOdeSolve}$ during, e. g., an iterative procedure: see the option reuse_spjac_struct of the mf_DE_Options derived type.

## Remarks:

- on output, y may contain $N a N$ values for some reasons. In such a case, use the msOdeSolve routine to further investigate the problem.
- deriv and jac have imposed lists of arguments; the user may use module's data in order to exchange other information between his program and these routines (especially for checking the number of equations). Therefore, except for very simple cases, these two user's routines must be located inside a module, USEd also by the user's program.
- for debugging purposes, the user may inspect the values of $y$ and $y^{\prime}$ during the integration process. See explanations about the fields monitor_y_ind, monitor_yp_ind and monitor_pause of the mf_DE_Options derived type.
- the stopping conditions are not (yet) available for the "ABM" method.

Other calling syntaxes

```
call msOdeSolve( A, "save" [, jac_sparse] )
```

By using this call, the user is able to save all internal data of the integrator in the mfArray A, and then continue the integration later on. The optional argument jac_sparse, which is the name of the routine computing the sparse Jacobian matrix, is required when the Jacobian has a sparse structure.

```
call msOdeSolve( A, "restore" [, jac_sparse] )
```

This call is required to restore all internal data of the integrator, before a new integration sequence. The optional argument jac_sparse, which is the name of the routine computing the sparse Jacobian matrix, is required when the Jacobian has a sparse structure.

```
call msOdeSolve( "finalyze" )
```

This call is useful to avoid memory leaks at the end of the integration process.

See also: mfDaeSolve

# integrator of ODE (explicit) systems 

## Calling syntax:

```
call msOdeSolve( mfOut( y, status[, tolout, yp, solve_log, t_log, order_log] ), &
    deriv, t_span, y_0[, options, jac, sparse] )
```


## Description:

The behavior of this routine is similar to those of mfOdeSolve, but it allows to get more information returned by the integrator (via mfArrays which must be enclosed to the mfOut function):

- y contains the numerical result.
- status contains a returned error code:
status $>0$ : normal and reliable termination of the routine. It is assumed that the requested accuracy has been achieved. (Exact meaning of the status can be found below)
status < 0: abnormal termination of the routine. The estimates for y is less reliable. It is assumed that the requested accuracy has not been achieved. Moreover, in some circumstances, a message is printed; in such cases, the user cannot use numerical values in y and yp and therefore an appropriate decision must be taken. (Exact meaning of the error can be found below)
- tolout is different from tol only if status $=-2$ : in such a case, it contains an appropriate value for continuing the integration.
- yp contains an estimation of the derivative at time $t_{\text {end }}$ (or at all intermediate values defined in t_span); moreover, the role of the option options\%y_ind_out applies also to yp.
- solve_log contains additional information about the integration process. It is a vector of 4 or 6 elements, according to the method used. These elements are:
nb_step is the total number of steps used
dt_min the smallest step used
dt_max the largest step used
nb_deriv is the number of calls of the deriv routine
nb_jac is the number of evaluation of the jacobian matrix (only for the "BDF" method)
nb_solve is the number of times the linear system is solved using the jacobian matrix (only for the "BDF" method)
- t_log contains the whole history of the times used during the integration. This can be useful to detect where a critical behavior is located.
- order_log contains the whole history of the order used during the integration. The first value, not defined, is set to NaN (Not-a-Number special IEEE floating-point value); as a consequence, this mfArray has the same size as t_log.

Among these returned variables, only the first two are required, while the others are optional. Moreover, it is forbidden to use the same mfArray in place of two or more actual arguments, even empty.

## Remarks:

- The presence of the argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (it is the responsability of the programmer to test the value of the status variable before using the result). If the mfOdeSolve routine was used instead, an error would stop the program and, in debug mode, a traceback of this error would be displayed.
- The special calling syntaxes used to save/restore the internal data (in case of continuation in overlay situations), or to finalize the integrator, have been presented at the end of the previous entry.

| status | explanation |
| :---: | :--- |
| 0 | Normal termination of the solver. |
| 4 | Normal termination of the solver, but a large amount of work has been ex- <br> pended. |
| 5 | Normal termination of the solver, but the $R K F$ integrator has been used very <br> inefficiently because the natural step size has been restricted by too frequent <br> output. |
| 12 | Normal termination of the solver, but an End Condition was found (flag, last <br> argument in deriv, was equal to 3 and the solver has been called to finish at <br> the new current point). |
| -2 | The error tolerances are too stringent. |
| -3 | The local error test cannot be satisfied because you specified a zero value for <br> the absolute error in tol and the corresponding computed solution component <br> is zero. Thus, a pure relative error test is impossible for this component. |
| -4 | The problem appears to be stiff (RKF and $A B M$ integrators only). |
| -6 | The BDF integrator had repeated convergence test failures on the last at- <br> tempted step. |
| -7 | The BDF integrator had repeated error test failures on the last attempted step. |
| -10 | Abnormal termination of the solver, due to an Illegal Condition (flas, last <br> argument in deriv, was equal to -1 and the solver did its best to not obtain <br> -1). |
| -11 | Abnormal termination of the solver, due to an Emergency exit (flag, last <br> argument in deriv, was equal to -2 and control has been returned to the <br> calling program). |
| -14 | The user-supplied deriv routine returned invalid entries (NaN value). |
| -20 | Allocation error because the maximum of available memory has been reached. <br> An explanation is printed. |
| -33 | Invalid argument. |
| -34 | Invalid implementation of the user-defined sparse jacobian. |
| -40 | On calling, the number of output argument(s) is incorrect. |
| -100 | The routine wasn't able to start, nor choose an initial timestep (flag, last <br> argument in deriv, was different from 0). |

See also: mfOdeSolve, msDaeSolve, mfOut, mf DE_Options
mfDaeSolve

integrator of DAE (implicit) systems

## Interface:

```
function mfDaeSolve( resid, t_span, y_0, yp_0, options, jac, sparse ) result( y )
    interface
        subroutine resid( t, y, yprime, delta, flag )
            real(kind=MF_DOUBLE), intent(in) :: t, y(*), yprime(*)
            real(kind=MF_DOUBLE), intent(out) :: delta(*)
            integer, intent(in out) :: flag
        end subroutine
    end interface
    type(mfArray), intent(in out) :: t_span
    type(mfArray), intent(in) :: y_0, yp_0
    type(mf_DE_options), intent(in), optional :: options
    external,
    logical, intent(in), optional :: sparse
    type(mfArray) :: y
```


## Description:

Performs the numerical integration of a system of (fully implicit ${ }^{3}$ ) differential algebraic equations (DAEs) of first order (and differential index 1 only ${ }^{4}$ ), which can be written in the general form:

$$
R\left(t, y, y^{\prime}\right)=0
$$

described via the user subroutine resid, which must define the components $R_{i}\left(t, y, y^{\prime}\right)$ and store the values in the delta array (the variables t , y and yprime represent respectively $t, y$ and $y^{\prime}$ ). Usually, the integration is done between time $t_{0}$ and $t_{\text {end }}$ (extremal values of $\mathrm{t}_{-}$span, which is a column vector mfArray of at least two components) or, in the continuation mode, up to the new values specified in $t_{\text {_span }}$ (in this case, $\mathrm{t}_{-}$span may contain only one value).

The output of mfDaeSolve is the row vector y which represents the solution $y$ at time $t_{\text {end }}$. If the vector $t_{\text {_span }}$ contains more than two values, the routine returns in $y$ the intermediate values of $y$, as specified by all time values present in $\mathrm{t}_{-}$span; in this case, y is a rank- 2 mfArray with as many rows as components in t_span.

The integration uses the prescribed initial conditions stored in the row vectors y_0 and yp_0. By default, it is assumed that the data y_0 and yp_0 are consistent (note that a consistency test is performed; if it fails, an error is returned), i.e. verify the equation

$$
R\left(t_{0}, y_{0}, y_{0}^{\prime}\right)=0
$$

otherwise, the user must set the options\%IC_known argument to .false., so that the initial conditions will be computed by the solver, taking y_0 and yp_0 as initial guesses; yp_0 may be empty but y_0 must always be allocated and must have its size equal to the number of equations (see mf_DE_Options for more details; in particular, the user must indicate which components of these two vectors should be held fixed).

$$
\cdots / \ldots
$$

[^3]The last argument flag of resid is always set to zero (internally by the solver) on input, and should be altered (inside the routine resid) only in some special situations:

- set flag to -1 for an Illegal Condition; the value of $y^{\prime}$ at this time may not be available. The routine will try to continue the integration (reducing the time step) as long as possible without getting the illegal condition. [You can put this assignment anywhere in the resid routine.]
- set flag to -2 for an Emergency Exit; the value of $y^{\prime}$ at this time may not be available. The routine will return control immediately to the calling program. [You can put this assignment anywhere in the resid routine.]
- set flag to 3 for an End Condition; the value of $y^{\prime}$ at this time must be always available. The routine will adapt its time step to finish exactly at the current time given by the argument $t$ of resid. [WARNING: you must put this assignment at the end of the resid routine, to insure that all the residues delta have been computed.]

The previous three situations are called stopping conditions. Note that when a stopping condition is found in the user subroutine resid, then the vector $\mathrm{t}_{-}$span (which contains the value of $t_{e n d}$ ) is modified. Note also that the three stopping conditions doesn't have the same numerical cost; their cost is mentioned in the following table:

| Flag to used in resid | Stopping condition | Numerical cost |
| :---: | :--- | :--- |
| -1 | Illegal Condition | expensive |
| -2 | Emergency Exit | very cheap |
| 3 | End Condition | cheap |

All following options are stored in the structure options (see mf_DE_Options).
The boolean mfArray options\%continuation allows the user to continue the integration to a new value of $t_{\text {end }}$, without specify a new $t_{0}$. Thanks to specific calls to $\mathrm{mf} / \mathrm{msDaeSolve}$, the user is even able to save internal data of the integrator, to stop the program, and to launch again the integrator after the reload of saved data. See the next section Other calling syntaxes.

The mfArray options\%tol allows the user to give the precision of the result, and must contain non negative values. Default values are $10^{-3}$ for the relative tolerance and $10^{-6}$ for the absolute tolerance; this mfArray may be of rank 0,1 or 2 :

- if it is a scalar, it specifies the relative tolerance for all the component of the solution; the absolute tolerance is fixed to $10^{-6}$.
- if it is a vector, it must have two elements which specifies the relative and the absolute tolerance, respectively.
- if it is a matrix, it must have two columns (corresponding to the relative and absolute tolerance) and a number of rows equal to the number of equations.

The mfArray options\%non_neg allows the user to apply non-negativity constraints to a subset (or all the set) of the components of the solution y. It must be a vector describing the indices of such constraints.

The mfArray options\%y_ind_out allows the user to get only a subpart of the mfArray y; if present, it must contain integer indices of the wanted components of the $y$ vector.

The user may provide the jac routine to compute the jacobian matrix (otherwise, it will be numerically evaluated by finite differences). This matrix may be defined in dense, band or sparse format. For the first two formats, the interface of the jac routine is:

```
interface subroutine jac( t, y, yprime, jacobian, cj, ldjac )
    real(kind=MF_DOUBLE), intent(in) :: t, y(*), yprime(*), cj
    integer, intent(in) :: ldjac real(kind=MF_DOUBLE), intent(out)
    end subroutine
    end interface
```

The generalized jacobian ${ }^{5} \widetilde{J}$ must be defined as:

$$
\widetilde{J}=\frac{\partial R}{\partial y}+c_{J} \frac{\partial R}{\partial y^{\prime}}
$$

where $c_{J}$ is a scalar computed by the solver; therefore the numerical values of $\widetilde{J}$ must be stored in the jacobian rank-2 array as follows: $\operatorname{jacobian}(\mathbf{i}, \mathrm{j})=\widetilde{J}_{i, j}=\frac{\partial R_{i}}{\partial y_{j}}+c_{J} \frac{\partial R_{i}}{\partial y_{j}^{\prime}}$.

Please note that, for the dense format, only non-zero values of $\widetilde{J}$ need to be defined.
If the jacobian is symmetric and positive definite (abbreviated as SPD), you should set the jac_symm_pos_def option (see mf_DE_Options) in order to economize storage and obtain a better performance for its factorization; in such a case, you must give only the upper part of the jacobian.
Furthermore, the jacobian $\widetilde{J}$ may be banded, in order to economize both memory and CPU time. In such a case, the mfArray options\%band must be set; it must contain a vector of exactly two elements: $M L$ and $M U$ which are the widths of the lower and upper parts of the band, respectively, with the main diagonal being excluded (e.g. for a tridiagonal system, both elements are equal to 1 ). In this case, routine jac must store the elements $\widetilde{J}_{i, j}$ in the matrix jacobian as follows:

```
irow = i - j + ML + MU + 1
```

Actually, ldjac is equal to $2 * M L+M U+1$ : indeed, an extra storage is needed for the matrix factorisation. For an SPD matrix, simply take ML equal to zero in the previous formulae.

For the sparse case, the optional argument sparse must be set to .true. and the jac routine must be defined as:

```
interface
    subroutine jac( t, y, yprime, cj, nrow, job, pd, ipd, jpd, nnz )
        real(kind=MF_DOUBLE), intent(in) :: t, y(*), yprime(*), cj
        integer, intent(in) :: nrow, job
        real(kind=MF_DOUBLE), intent(out) :: pd(*)
        integer, intent(out) :: ipd(*), jpd(*)
        integer, intent(in out) :: nnz
    end subroutine
end interface
```

$c_{J}$ having the same meaning as before.

[^4]The pd,ipd,jpd f90 arrays describe the CSC (Compact Sparse Column) representation of the sparse jacobian matrix, as follows:

- pd(1:nnz) contains the non-zero matrix entries;
- ipd(1:nnz) contains the row indices;
- $\operatorname{jpd}(1: n c o l+1)$ is the pointer to the beginning of the columns, in arrays (pd,ipd) (actually, a jacobian matrix is always square, hence ncol=nrow).

Row indices must be sorted in ascending order. Moreover, this routine must include a mechanism such that only the value of $n n z$ is computed when $j o b=0$; therefore the argument $n n z$ has generally an intent ( $y$ ) for the first call and an intent (in) for subsequent calls.

Be aware that the sparse structure (that is to say, the indices' vectors ipd and jpd) must remain the same during all the integration process. On the contrary, use the option spjac_const_struct of the $m f$ _DE_Options derived type. Moreover, you can even force to reuse the same sparse structure if you make many calls to $\mathrm{mf} / \mathrm{msOdeSolve}$ during, e. g., an iterative procedure: see the option reuse_spjac_struct of the mf_DE_Options derived type.

## Remarks:

- on output, y may contain $N a N$ values for some reasons. In such a case, use the msDaeSolve routine to further investigate the problem;
- resid and jac have imposed lists of arguments; the user may use module's data in order to exchange other information between his program and these routines (especially for checking the number of equations). Therefore, except for very simple cases, these two user's routines must be located inside a module, USEd also by the user's program.
- for debugging purposes, the user may inspect the values of $y$ and $y^{\prime}$ during the integration process. See explanations about the fields monitor_y_ind, monitor_yp_ind and monitor_pause of the mf DE_Options derived type.


## Other calling syntaxes

```
call msDaeSolve( A, "save" [, jac_sparse] )
```

By using this call, the user is able to save all internal data of the integrator in the mfArray A, and then continue the integration later on. The optional argument jac_sparse, which is the name of the routine computing the sparse Jacobian matrix, is required when the Jacobian has a sparse structure.

```
call msDaeSolve( A, "restore" [, jac_sparse] )
```

This call is required to restore all internal data of the integrator, before a new integration sequence. The optional argument jac_sparse, which is the name of the routine computing the sparse Jacobian matrix, is required when the Jacobian has a sparse structure.

```
call msDaeSolve( "finalyze" )
```

This call is useful to avoid memory leaks at the end of the integration process.

See also: mfOdeSolve
msDaeSolve

## integrator of DAE (implicit) systems

## Calling syntax:

```
call msDaeSolve( mfOut( y, status\&
[, tolout, yp, solve_log, init_log, t_log, order_log] ), \& resid, t_span, y_0, yp_0[, options, jac, sparse] )
```


## Description:

The behavior of this routine is similar to those of mfDaeSolve, but it allows to get more information returned by the integrator (via mfArrays which must be enclosed by the mfOut function):

- y contains the numerical result
- status contains a returned error code:
status $>0$ : normal and reliable termination of the routine. It is assumed that the requested accuracy has been achieved. (Exact meaning of the status can be found below)
status < 0: abnormal termination of the routine. The estimates for y is less reliable. It is assumed that the requested accuracy has not been achieved. Moreover, in some circumstances, a message is printed; in such cases, the user cannot use numerical values in y and yp and therefore an appropriate decision must be taken. (Exact meaning of the error can be found below)
- tolout is different from tol in some few cases. First, if status $=-2$, it contains an appropriate value for continuing the integration. Second, if status $=-101$, it contains the absolute value of the residual vector $R\left(t_{0}, y_{0}, y_{0}^{\prime}\right)$ (initial data consistency).
- yp contains an estimation of the derivative at time $t_{\text {end }}$ (or at all intermediate values defined in t_span); moreover, the role of the optional argument y_ind_out applies to yp.
- solve_log contains additional information about the integration process. It is a vector of 6 elements, which are:
nb_step is the total number of steps used;
dt_min the smallest step used;
dt_max the largest step used;
nb_resid is the number of calls of the resid routine;
$n b_{-} \mathrm{jac}$ is the number of evaluation of the jacobian matrix;
nb_solve is the number of solving the linear system using the jacobian matrix.
- init_log contains specific information about the computation of consistent initial conditions (when this has been required). It is a vector of 4 elements, which are:
nb_resid_0 is the number of calls of the resid routine;
nb_jac_0 is the number of evaluation of the jacobian matrix;
nb_solve_0 is the number of solving the linear system using the jacobian matrix;
cpu_time_0 is the CPU time spent in this initialization part.
- t_log contains the whole history of the times used during the integration. This can be useful to detect where a critical behavior is located.
- order_log contains the whole history of the order used during the integration. The first value, not defined, is set to NaN (Not-a-Number special IEEE floating-point value); as a consequence, this mfArray has the same size as t_log.

$$
\cdots / \ldots
$$

Among these returned variables, only the first two are required, while the others are optional. Moreover, it is forbidden to use the same mfArray in place of two or more actual arguments, even empty.

Remark: The presence of the argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (it is the responsability of the programmer to test the value of the status variable before using the result). If the mfDaeSolve routine was used instead, an error would stop the program and, in debug mode, a traceback of this error would be displayed.

| status | explanation |
| :---: | :---: |
| 0 | Normal termination of the solver. |
| 4 | Normal termination of the solver, but a large amount of work has been expended. |
| 12 | Normal termination of the solver, but an End Condition was found (flag, last argument in resid, was equal to 3 and the solver has been called to finish at the new current point). |
| -2 | The error tolerances are too stringent. |
| -3 | The local error test cannot be satisfied because you specified a zero value for the absolute error in tol and the corresponding computed solution component is zero. Thus, a pure relative error test is impossible for this component. |
| -6 | The method used had repeated error test failures on the last attempted step. |
| -7 | The corrector could not converge. |
| -8 | The matrix of partial derivatives is singular. |
| -9 | The corrector could not converge: there were repeated error test failures in this step. |
| -10 | Abnormal termination of the solver, due to an Illegal Condition (flag, last argument in resid, was equal to -1 and the solver did its best to not obtain $-1)$. |
| -11 | Abnormal termination of the solver, due to an Emergency Exit (flag, last argument in resid, was equal to -2 and control has been returned to the calling program). |
| -13 | Some indices for the non-negativity constraints stored in the non_neg array of the mf DE_Options are not in the range [1,Neq] (where Neq is the number of equations). |
| -14 | The user-supplied resid routine returned invalid entries ( $N a N$ value). |
| -15 | The user-supplied jac routine (either dense or sparse) returned invalid entries ( NaN value). |
| -20 | Allocation error because the maximum of available memory has been reached. An explanation is printed. |
| -33 | Invalid argument. |
| -34 | Invalid implementation of the user-defined sparse jacobian. |
| -40 | On calling, the number of output argument(s) is incorrect. |
| -100 | The routine wasn't able to start, neither choose an initial timestep, nor check initial conditions for consistency (flag, last argument in resid, was different from 0). |
| -101 | Initial conditions y_0 and yp_0 are not consistent. |
| -102 | msDaeSolve failed to compute the initial vector yp_0. |

See also: msOdeSolve, mf_DE_Options
mf_NL_Options

## options for non-linear solvers (derived type)

## Description:

This derived type gathers options used by the two routines mfFSolve and mfLsqNonLin, and is defined as follows:

```
type, public :: mf_NL_Options
    type(mfArray) :: tol
    integer :: max_iter = 50
    type(mfArray) :: epsfcn
    real(kind=MF_DOUBLE) :: init_step_bound_factor = 1.0d2
    logical :: print = .false., &
        print_using_transf = .false., &
            reuse_spjac_struct = .false.
    type(func_ptr), allocatable :: f_inv_transf(:)
    integer :: check_jac = 0,
        print_check_jac = 0
    logical :: box_constrained = .false
    type(mfArray) :: lower_bounds,
        upper_bounds
    real(kind=MF_DOUBLE) :: sing_jac_tol = epsilon(1.0d0)
    logical :: print_sing_val = .false.
end type mf_NL_Options
```

The following fields may be used in both two routines:

- tol specifies the tolerance to be used. See the appropriate routine to know the usage of this option.
- max_iter specifies the approximated maximum number of iterations.
- epsfcn specifies the optimum small increment (in a relative sense) used for computing the derivatives (the elements of the jacobian matrix) by finite differences. Therefore, its value is not used when the jacobian matrix is provided by the user. It may be a scalar or a vector; in the latter case (but only for LsqNonLin), each components of epsfcn corresponds to an unknown.
The value(s) of epsfon must be carefully chosen; there doesn't exist a universal constant because the optimum value depends on the algorithm in the user-supplied fcn routine. Its role is basically to equalize the numerical error which comes from the scheme and the roundoff error inherent to the floating-point model. If $f \mathrm{cn}$ is coded as a mathematical function, it can be set to zero (because internally, the finite-difference routine will choose $\sqrt{\epsilon}$, where $\epsilon$ is the machine precision; this is the default). If fcn uses an interpolation over a given interval $\Delta x$ (e.g. from a table lookup) then the epsfon should be related to some power of this $\Delta x$ value, according the accuracy of the interpolation scheme.
- init_step_bound_factor specifies the initial step bound factor. The default value of 100 is recommended in most cases. However it can be changed for another value as low as 0.1. For example, if you use the $\log ()$ function to ensure the positiveness of your unknowns, you should choose $\log (100)$ instead.
- print is used to print on screen the following information during the iterative process: the iteration number, the value of each unknown parameter and the norm of the residue. If some transformation is used between the physical value of the parameters and the actual value in the algorithm, it may be convenient to print the physical value of each parameter; to do that, the user must set the print_using_transf field to .true. and associate the function pointers stored in the field f_inv_transf to appropriate user-defined inverse functions. The last pointer array must be allocated by the user. See the Muesli User's Guide for more details.

$$
\cdots / \ldots
$$

- reuse_spjac_struct must be set to .true. only when you are sure that the sparse structure of the jacobian is not modified during many different calls of the FSolve solver: this can occur inside a loop when using the same set of equations. Be aware that this option can only set to .true. after the first call of the solver; on the contrary, an error will arise. Note that this option is used only for the sparse case.
- for debugging purposes (but only when MF_NUMERICAL_CHECK is TRUE), check_jac can be used to make a check of the user jacobian (dense only):

1. when check_jac is set to 1 , a global and quick check is done;
2. when check_jac is set to 2 , a complete (but more expensive) check is done.

In both cases, a message is displayed according to the errors obtained concerning the accuracy of the jacobian. Note that this is an estimation because (i) discrepancies found may be strongly affected by both floating-point roundoff errors and badly scaled variables or equations, and (ii) relative errors are based on a sort of norm of the jacobian; when this latter is large, some terms in the error matrix may remain small.
print_check_jac can be used to print the results of the jacobian check:

1. when print_check_jac is set to 1 , information is displayed on the screen;
2. when print_check_jac is set to 2 , information is stored in files (one different file for each jacobian call).

In both cases, (i) the given information depends on the level of the check and (ii) the output is done at each call of the jac routine, regardless of the severity of the discrepancies. See the Muesli User's Guide where a detailed example of use can be found.

- (for LsqNonLin only) box_constrained can be used to constrain the parameters to be inside a box. Set it to .true. and specify the box bounds as follows:

1. lower_bounds is a vector containing the lower bound for each parameter. The length of this mfArray vector must be equal to the number of unknown parameters; use -MF_INF to release the constraint.
2. upper_bounds is a vector containing the upper bound for each parameter. The length of this mfArray vector must be equal to the number of unknown parameters; use +MF_INF to release the constraint.

These constraints of type box are dynamic, which means that during the iterations, the user can change the value of the bounds (e.g. in the fon routine).

- Only when MF_NUMERICAL_CHECK is TRUE, sing_jac_tol allows the user to know whether his system of equations is singular or not. It concerns only the LsqNonLin routine. Normally the jacobian is not singular, which means that all the parameters are independant; on the contrary, the routine displays the linear relation between the parameters, by showing the vector of the basis of the nullspace. Note that the nullspace basis is always computed from a local point-of-view (linearized equations); if this basis changes along the iteration, this means that the relationship between the parameters is nonlinear. The default value of this tolerance is the machine $\epsilon$. Set sing_jac_tol to zero to avoid the computation of the nullspace.
- Only when MF_NUMERICAL_CHECK is TRUE, print_sing_val allows the user to print the singular values of the jacobian matrix. Default is FALSE.

All fields of this derived type are public, so the user can access each component via the $\% \%$ Fortran 90 selector.

Lastly, the user can use the msRelease routine to free all fields of this structure (and to re-initialize the components between two different calls to any of these integrators).

See also: mf_DE_Options
mf_DE_Options

## options for differential solvers (derived type)

## Description:

This derived type gathers options used by the two integrators mfOdeSolve and mfDaeSolve, and is defined as follows:

```
type, public :: mf_DE_Options
    logical :: continuation = .false.
    character(len=3) :: method = ""
    type(mfArray) :: tol, y_ind_out, band
    type(mfArray) :: non_neg, y0_ind, yp0_ind
    logical :: IC_known = .true., &
        print_progress = .false., &
        disp_times = .false., &
        jac_symm_pos_def = .false., &
        spjac_const_struct = .true., &
        reuse_spjac_struct = .false., &
        jac_investig = .false., &
        rational_null_basis = .false., &
        pseudo_inverse = .false.
    real(kind=MF_DOUBLE) :: jac_rcond_min = MF_EPS
    logical :: save_sing_jac = .false.
    type(mfArray) :: monitor_y_ind, &
        monitor_yp_ind
    logical :: monitor_pause = .false.
    integer :: check_jac = 0, &
        print_check_jac = 0
    type(mf_DE_Named_Group), allocatable :: named_eqn(:),
&
    named_var(:)
```

end type

The following fields may be used in both integrators:

- continuation specifies that the integrator is not restarted. Using such a feature to continue the integration leads to much better performance than restarting the integrator.
- tol specifies the tolerance in many ways (relative or absolute, as a whole or by component).
- y_ind_out allows the user to get only a subpart of the result.
- band specifies that the jacobian is stored in a band format.
- print_progress specifies that the percentage of work already done is printed in the terminal. This options should not be used when using the main program in a batch mode, redirecting output in a file.
- disp_times specifies that time values will also be displayed - wall clock time is used, not CPU time.
(See msPrepProgress and msPrintProgress for other information concerning the last two options.)
- jac_symm_pos_def must be set to .true. only when you are sure that the jacobian is symmetric and positive definite; in such a case, you must only provide the upper part of the jacobian: this will economize some storage. A Cholesky factorization will be used (instead of a general LU one) and, especially for sparse, large matrices, this will be more efficient.
- spjac_const_struct must be set to .false. only when you are sure that the sparse structure of the jacobian is modified along all the time integration (this is not common!); note that this option is used only for the sparse case.
- reuse_spjac_struct must be set to .true. only when you are sure that the sparse structure of the jacobian is not modified during many different calls of the DAE or ODE solvers: this can occur during an iterative procedure using the same mesh. Be aware that this option can only set to .true. after the first call of the solver; on the contrary, an error will arise. Note that this option is used only for the sparse case.
- for debugging the DAE integration (but only if jac_investig is TRUE), the user may set a threshold value (not greater than unity) in jac_rcond_min in order to detect whether the jacobian is singular or not ${ }^{6}$. When (an estimation of) the reciprocal of condition number is less than or equal to the value of jac_rcond_min, then the jacobian is considered as singular: a warning message is then emitted. Moreover, for the dense case only:

1. additional information is provided to the user, about the null space of the transpose of the jacobian matrix: this may help the user in understanding why the jacobian is singular.
2. the jacobian and the nullspace of its transpose can be saved on disk according to the value of the boolean save_sing_jac (the filenames are respectively 'jacobian.dat' and 'nullspace.dat').

The default value of jac_rcond_min is the machine epsilon and, if set by the user, it cannot be smaller than this default value. The nullspace basis may be returned in the form of a rational basis (set rational_null_basis to .true.) but this should be useful for small problem only (few number of equations) or for pedagogical reasons. The pseudo_inverse component is reserved for a future usage.

- for debugging purposes again, the user can tell the appropriate integrator to store all intermediate values (note: at the internal time steps!), by specifying a number of indices in monitor_y_ind, and/or indices in monitor_yp_ind. In such a case, the selected values of the vector $y$ (resp. $y^{\prime}$ ) are copied in the file 'odesolve_y.out' or 'daesolve_y.out' (resp. 'odesolve_yp.out' or 'daesolve_yp.out'); each line contains the current time (independant variable) in the first column, then the selected values of the dependant variables. The boolean monitor_pause allows the user to pause after each successful internal step of the ODE/DAE solver (the default behavior is not to make such an interruption). Moreover, the data files are flushed continuously, so you can use an external graphic tool to plot the data which are monitored.
- recall that the use of MF_NUMERICAL_CHECK is recommended, because it allows internal routines to check the validity of the equations or of the jacobian elements, especially non finite values (Infinities, NaNs ).

[^5]- for debugging purposes (but only when MF_NUMERICAL_CHECK is TRUE), check_jac can be used to make a check of the user jacobian (dense only):

1. when check_jac is set to 1 , a global and quick check is done;
2. when check_jac is set to 2 , a complete (but more expensive) check is done.

In both cases, a message is displayed according to the errors obtained concerning the accuracy of the jacobian. Note that this is an estimation because (i) discrepancies found may be strongly affected by both floating-point roundoff errors and badly scaled variables or equations, and (ii) relative errors are based on a sort of norm of the jacobian; when this latter is large, some terms in the error matrix may remain small.
print_check_jac can be used to print the results of the jacobian check:

1. when print_check_jac is set to 1 , information is displayed on the screen;
2. when print_check_jac is set to 2 , information is stored in an overwritten file (a unique file along all the integration process). The library makes as many pauses as needed to let the user to inspect this file.

In both cases, (i) the given information depends on the level of the check and (ii) the output is displayed at each call of the jac routine, regardless of the severity of the discrepancies. See the Muesli User's Guide where a detailed example of use can be found.

- the named_eqn and named_var fields allow the user to name his equations and variables by grouping them under useful, physical names. The derived type mf_DE_named_group is defined as follows:

```
type, public :: mf_DE_Named_Group
    character(len=132) :: name
    integer :: begin, last
end type
```

The user must allocate himself the arrays named_eqn(:) and named_var(:), then correctly initializes the three internal fields: name (the name of the group), begin and last (the indices of equation in deriv or resid which define the beginning and the end of the each named group). See the Muesli User's Guide where an example of use can be found.

The field method is specific to the mfOdeSolve integrator. It may contains "RKF" (Runge-Kutta Fehlberg, which is the default method), "ABM" (Adams-Bashforth-Moulton), or "BDF" (Backward Differentiation Formula).

The remaining are specific to the mfDaeSolve integrator:

- non_neg allows the user to constrain some components of the solution to remain non negative.
- IC_known indicates whether initial conditions are known (and then supposed to be consistent) or not (see also just below).
- in the case where initial conditions have to be computed (i.e. when IC_known is equal to .false.) and only when the jacobian matrix is provided, the user may indicate which components of the vectors y_0 and yp_0 are prescribed in, respectively, y0_ind and yp0_ind (at most $N$ components may be prescribed, where $N$ is the number of the system's equations). If used, these two vectors must have a length equal to $N$; free components are selected by putting a zero at appropriate locations, whereas prescribed components may be selected by putting any non-zero value. The user may also leave the fields empty, which is equivalent to specify that no component at all are prescribed. Values present in the two mfArrays y_0 and yp_0 are then considered as initial guesses. It is worth mention that initialization may fail in the case where the user prescribed too many components. A very common error is to prescribed the $N$ components of y_0 by setting all components of y0_ind to one, for a DAE systems which have $N_{a}$ algebraic equations; in such a case, all components referencing to the algebraic equations should be left free, and the user can only prescribed the $N-N_{a}$ components of y_0 corresponding to the differential equations.

All fields of this derived type are public, so the user can access each component via the $\%$ Fortran 90 selector.

Lastly, the user can use the msRelease routine to free all fields of this structure (and to re-initialize the components between two different calls to any of these integrators).

Remark: The jacobian matrix of the ODE solver may be singular, because the iteration matrix $M$ writes:

$$
M=I+\Delta t J=I+\Delta t \frac{\partial F}{\partial y}
$$

It can be seen easily that for a sufficiently small time-step $\Delta t, M$ is not singular so there is no constraint over $J$ : it could be even the null matrix! On the contrary, the iteration matrix $M$ used by the DAE solver is close to the following generalized jacobian:

$$
M \approx \widetilde{J}=\frac{\partial R}{\partial y}+c_{J} \frac{\partial R}{\partial y^{\prime}}
$$

where $c_{J}$ is related to the inverse of the time step $\Delta t$. In this latter case, $\widetilde{J}$ cannot be singular. (The two functions $F(t, y)$ and $R\left(t, y, y^{\prime}\right)$ have been defined respectively at the mfOdeSolve and mfDaeSolve entries.)

See also: mf_NL_Options

### 1.11 Sparse Matrices

```
mfSparse sparse matrix conversion
mfFull
mfSpAlloc, msSpReAlloc
mfNnz
mfNzMax
mfNcolMax
mfSpEye
mfSpOnes
mfSpRand
mfSpRandN
mfSpDiags
mfSpCut
mfSpImport, msSpExport
mfIsRowSorted row sorted inquiry
msRowSort
msGetAutoRowSorted
msSetAutoRowSorted
mfMatFactor
msFreeMatFactor
sparse matrix conversion
dense matrix conversion
sparse matrix space allocation number of nonzero matrix elements sparse matrix internal size sparse matrix internal size sparse identity matrix replace nonzero elements with ones sparse uniformly distributed random matrix sparse normally distributed random matrix sparse matrix formed from diagonals cut small elements sparse matrix conversion from/to f90 arrays
row sorted inquiry
sparse matrix row sort
row sorting policy
row sorting policy
handle to internal matrix factors (derived type) deallocation of internal matrix factors
```

See also:
Core Routines
File Input/Output
Data Analysis Functions
Operators
Elementary Math Functions
Specialized Math Functions
Elementary Matrix Manipulation Functions
Matrix Functions
Polynomial Functions
Optimization and Function Functions
mfSparse

Calling syntax:

```
B = mfSparse( A )
```

converts the mfArray A (which should be dense) into a sparse mfArray.
To create a sparse matrix, use the mfSpAlloc routine instead.
See also: mfFull, mfSpImport, msSpExport
mfFull
dense matrix conversion

Calling syntax:

```
B = mfFull( A )
```

converts the mfArray A (which should be sparse) into a dense mfArray.
See also: mfSparse

Interface:

```
function mfSpAlloc( m, n, nzmax, ncolmax, kind ) result( out )
    integer, intent(in) :: m, n
    integer, intent(in), optional :: nzmax, ncolmax
    character(len=*), intent(in), optional :: kind
    type(mfArray) :: out
```

Description:
This routine is used to create new space for a sparse mfArray of size m by (logical dimensions) which will contain at most nzmax (if present) non-zero values. If nzmax is not present, it is set to zero.

If ncolmax is present, it reserves space for a greater number of columns. The value of ncolmax cannot be less than n ; its default value is equal to n . This argument is usually employed with the routine msHorizConcat.
kind may be equal to "real" or "complex"; it defaults to "real".
See also: msSpReAlloc, msHorizConcat
msSpReAlloc

## Interface:

```
subroutine msSpReAlloc( A, nzmax, ncolmax )
type(mfArray), intent(in out) :: A
integer, intent(in), optional :: nzmax
integer, intent(in), optional :: ncolmax
```


## Description:

This routine changes the space for elements in the sparse mfArray A. At least one argument, among the two optional ones nzmax and ncolmax, must be present.

The argument nzmax corresponds to the maximum number of non zero entries in the sparse structure. It can be increased to any value, but you will get an error if the value of nzmax is too small.

The argument ncolmax corresponds to the maximum number of the columns. Here again, you will get an error if the value of ncolmax is too small. Be careful when changing its value, because the logical size of the matrix is determine by ncolmax.

Usually the two arguments are used to increase the corresponding value. It may be dangerous to try to decrease their value without knowledge about the internal sparse structure. See below to decrease as much as possible the internal size of the structure automatically, without loss of data.

## Other calling syntax:

To compact an mfArray to its minimal room space (both in terms of the maximum number of non zero entries nzmax), and maximum number of columns ncolmax), use the following special calling sequence:

```
call msSpReAlloc( A, "minimal" )
```

Note that this will remove also any (unwanted) zeros actually stored.

See also: mfSpAlloc
$m f N n z$
number of nonzero matrix elements

Interface:

```
function mfNnz( A ) result( out )
    type(mfArray), intent(in) :: A
    integer :: out
```

Description:
Returns the number of non-zero element of the sparse mfArray A.
See also: mfNzMax
mfNzMax
sparse matrix internal size

Interface:

```
function mfNzMax( A ) result( out )
    type(mfArray), intent(in) :: A
    integer :: out
```


## Description:

Returns the maximum number of non-zero element of the sparse mfArray A.
See also: mfNnz, mfNcolMax
mfNcolMax

Interface:

```
function mfNcolMax( A ) result( out )
    type(mfArray), intent(in) :: A
    integer :: out
```

Description:
Returns the maximum number of columns of the sparse mfArray A.
See also: mfNnz, mfNzMax
mfSpEye
sparse identity matrix

Interface:

```
function mfSpEye( m, n, kind ) result( out )
    integer, intent(in) :: m
    integer, intent(in), optional :: n
    character(len=*), intent(in), optional :: kind
    type(mfArray) :: out
```

Description:
This routine creates a sparse mfArray of size m by $n$.
The optional argument kind allows the user to choose a complex array (if kind is equal to "complex"); by default, a real matrix is returned.

See also: mfSpDiags, mfSpOnes
mfSpOnes
replace nonzero elements with ones

Calling syntax:

```
B = mfSpOnes( A )
```

Description:
This routine creates a sparse matrix by replacing all non-zero values of the mfArray A by one. See also: mfSpEye
mfSpRand
sparse uniformly distributed random matrix

First calling syntax:

```
B = mfSpRand( A )
```


## Description:

This routine creates a sparse matrix by replacing all non-zero values of the mfArray A (which may be dense or sparse) by a uniformly distributed random value. Therefore, the mfArray B has the same sparsity than A.

Second calling syntax:

```
B = mfSpRand( m, n, density )
```

Description:
creates a random, m-by-n, sparse matrix with approximately density*m*n uniformly distributed nonzero entries (the real density must be in $[0,1]$ ).

See also: mfSpEye, mfSpOnes, mfSpRandN
mfSpRandN
sparse normally distributed random matrix

First calling syntax:

```
B = mfSpRandN( A )
```


## Description:

This routine creates a sparse matrix by replacing all non-zero values of the mfArray A (which may be dense or sparse) by a normally distributed random value. Therefore, the mfArray B has the same sparsity than A.

Second calling syntax:

```
B = mfSpRandN( m, n, density )
```

Description:
creates a random, m-by-n, sparse matrix with approximately density*m*n normally distributed nonzero entries (the real density must be in $[0,1]$ ).

See also: mfSpEye, mfSpOnes, mfSpRand
mfSpDiags
sparse matrix formed from diagonals

## Interface:

```
function mfSpDiags( m, n, v, d ) result( out )
    integer, intent(in) :: m, n
    type(mfArray) :: v
    integer, intent(in), optional :: d
    type(mfArray) :: out
```


## Description:

Creates a sparse matrix (of size $m$ by $n$ ) whose the d-diagonal is the input vector $v$ (dense).
The vector v may have a length greater than those of the d-diagonal (in such a case, subsequent elements will not be referenced); it can be also a scalar (in such a case, the scalar value is spread over the diagonal).

Remarks: When d is equal to zero, it points to the main diagonal; when positive, the upper part of the matrix is concerned. If the arg. $d$ is not present, the main diagonal is the default.

## Other interface:

```
function mfSpDiags( m, n, A, d ) result( out )
    integer, intent(in) :: m, n
    type(mfArray) :: A
    integer, intent(in) :: d(:)
    type(mfArray) :: out
```


## Description:

In this second interface the input mfArray A may be a rank-2 array. So, each column of A is used to fill a diagonal of the returned sparse matrix, whose index is specified in the vector d .

If A is a row vector then each constructed diagonal will have the same elements.

See also: mfSpEye, mf/msDiag
mfSpCut
cut small elements

Calling syntax:

```
B = mfSpCut( A, threshold )
```


## Description:

This routine removes all elements of the mfArray A (sparse, real or complex) whose magnitude is less or equal than the prescribed threshold (positive real arg.).

## Calling syntax:

```
A = mfSpImport( i, j, d[, m, n, nzmax, format, duplicated_entries ] )
```

creates the sparse mfArray A from a sparse structure stored in classical f90 arrays; i and $j$ are integer f90 vectors whereas d is a real or a complex f90 vector.

The vector d may have a null size, in which case the allocation of data space is made by this routine but filled with $N a N$ value (initialization may occur later on).
If present, the character string format specifies the sparse format; it may be equal to "COO" (COOrdinates), "CSC" (Compact Sparse Columns) or "CSR" (Compact Sparse Rows). By default, format="COO" is assumed.

If present, $m$ and $n$ define the (logical) shape of the sparse mfArray; these shape must be large enough to contain all the input indices specified in $i$ and $j$. By default, the logical shape is deduced as follows:

- if the format is "COO", then the matrix shape is deduced from the maximum value of arrays i and j. It follows than all these arrays are inspected.
- if the format is "CSC" (resp. "CSR"), the number of columns (resp. rows) comes from the size of the $j$ array (resp. of the i array). Then the other dimension is deduced by inspected the appropriate part of the other array; this is because the long index array may be overdimensioned.

If present, nzmax allows to reserve more place than the actual size.
After importing f90 arrays:

- eventual duplicated entries are treated according to the optional character argument duplicated_entries. When this argument is present and equal to "ignored", duplicated entries are ignored; when it is equal to "added" (default value), duplicated entries are added; when it is equal to "replaced", last entries found overwrite previous ones.
- entries containing a null value are removed.

See also: msSpExport, mfSparse, mfLoadSparse, msSaveSparse
msSpExport
sparse matrix conversion to f 90 arrays

Calling syntax:

```
call msSpExport( A, i, j, d [, format ] )
```

copies the sparse mfArray A to a sparse structure using classical f90 arrays.
$i$ and $j$ are integer f90 vectors. d is a real or a complex f90 vector.
If present, the character string format specifies the sparse format; it may be equal to "COO" (COOrdinates), "CSC" (Compact Sparse Columns) or "CSR" (Compact Sparse Rows). By default, format="COO" is assumed.

See also: mfSpImport, mfSparse, mfLoadSparse, msSaveSparse

## mfIsRowSorted

## row sorted inquiry

Interface:

```
function mfIsRowSorted( A ) result( bool )
    type(mfArray), intent(in out) :: A
    logical :: bool
```

Description:
Returns '.true.' if the columns of the sparse mfArray A are "row sorted" and contain no duplicated entries.

See also: msRowSort, msGetAutoRowSorted, msSetAutoRowSorted

## msRowSort

Calling syntax:

```
call msRowSort( A [, struct_only ] )
```

Description:
Sorts all columns of the sparse mfArray A in increasing order, and remove all duplicated entries.
If the logical optional argument struct_only is present and equal to '.true.' then the data array is discarded.

See also: mfIsRowSorted, msGetAutoRowSorted, msSetAutoRowSorted
msGetAutoRowSorted

Interface:

```
subroutine msGetAutoRowSorted( auto_row_sorted )
    logical, intent(out) :: auto_row_sorted
```

Description:
If the returned logical auto_row_sorted is $T R U E$, then all sparse matrices are row sorted when needed.
See also: msRowSort, mfIsRowSorted, msSetAutoRowSorted

## msSetAutoRowSorted

## Interface:

```
subroutine msSetAutoRowSorted( auto_row_sorted )
    logical, intent(in) :: auto_row_sorted
```

Description:
Only if auto_row_sorted is TRUE, then all sparse matrices will be row sorted when needed. Therefore, this routine can change this row sorting policy.

By default, auto_row_sorted is TRUE.
See also: msRowSort, mfIsRowSorted, msGetAutoRowSorted

## mfMatFactor

## handle to internal matrix factors (derived type)

## Description:

This derived type is a handle to (sparse or dense) factors computed by some factorization routine (e.g. $\mathrm{msLU}, \mathrm{msQR}$ or msChol). Declaration is made as follows:
type(mfMatFactor) :: factor
The user cannot manipulates the contents of this derived type: it is indeed intended to be used only by other MUESLI routines.

The user may release this structure by use of the msFreeMatFactor routine (but use of msRelease is equivalent).
msFreeMatFactor
deallocation of internal matrix factors

Calling syntax:
call msFreeMatFactor ( factor )

Description:
Deallocate the mfMatFactor factor.

Remark: The routine msRelease may also be used in an equivalent way.

## 2 FGL: Graphical Library

FGL contains all routines to make figures and plots (only in 2D).
Routines beginning with ' ms ' are subroutines, whereas those beginning with ' mf ' are functions. Usually, the two forms of the same routine do the same job, but the latter returns an identification (actually, a handle to the graphic object).

The available routines have been grouped into sub-parts:
Global graphic settings
Window's and figure's management
Figure properties
Figure annotation - Low level graphic object's manipulation
High level plotting routines
Interactive routines

### 2.1 Global graphic settings

```
msSetBackgroundColor
msSetColorOverflowPolicy
mfGetColorOverflowPolicy
msExitFgl
mfGetX11Device, msSetX11Device
mfGetX11ColorDepth
mfGetDefaultCapStyle, msSetDefaultCapStyle
mfGetDefaultJoinStyle, msSetDefaultJoinStyle
mfGetCharEncoding, msSetCharEncoding
background color setting set color overflow policy get color overflow policy FGL closing
X11 device status get X11 screen color depth default cap style for lines default join style for lines character encoding
```

See also:
Window's and figure's management
Figure properties
Figure annotation - Low level graphic object's manipulation
High level plotting routines
Interactive routines

## msSetBackgroundColor

## background color setting

## Interface:

```
subroutine msSetBackgroundColor( color_string )
    character(len=*), intent(in) :: color_string
```

Description:
Set the background color. The only two possibilities are "white" (the default) and "black".
This routine must be used at the beginning of the graphic part. The background color must be the same for all the opened windows; it cannot be changed during the execution, until $F G L$ graphic part is closed (msExitFgl) and automatically reinitialized by opening any figure.

Remark: When the background has been selected as black, printing leads to different behavior for EPS and PDF formats (see msPrint).

## mfGetColorOverflowPolicy

## get color overflow policy

Interface:

```
function mfGetColorOverflowPolicy() result( policy )
    character(len=10) :: policy(2)
```

Description:
Returns two strings. The first one (resp. second one) refers to color overflow in the low part (resp. high part) of the colormap.

The color overflow policy may be "signaled" or "truncated". When it is "signaled", a contrasted color is used to signal the colormap overflow. When it is "truncated", the lowest color (or highest color) is used, giving a smooth aspect to the colors.

Default behavior is "signaled" for both ends.
See also: msSetColorDverflowPolicy

## msSetColorOverflowPolicy

## set color overflow policy

## Interface:

```
subroutine msSetColorOverflowPolicy( low, high )
    character(len=*), optional :: low, high
```

Description:
This routine is used to monitor the colormap overflow. low and high are two strings which must contain "signaled" or "truncated".

When it is "signaled", a contrasted color is used to signal the colormap overflow. When it is "truncated", the lowest color (or highest color) is used, giving a smooth aspect to the colors.

Default behavior is "signaled" for both ends.
Remark: if none of the two arguments is present, then nothing is changed.
See also: mfGetColorOverflowPolicy

Interface:

```
status = mfGetX11Device( )
```

Description:
Returns the status of the X11 device, which can be "on" (the default) or "off".
See also: msSetX11Device

## msSetX11Device

## Interface:

```
subroutine msSetX11Device( flag )
    character(len=*), intent(in) :: flag
```

Description:
During execution, as MUESLI is able to produce printed images (EPS or PDF) in an independant way, the X11 device can be de-activated, by setting the flag parameter to "off". It can be useful for batch executions.

Remarks:

- the X11 device is "on" by default;
- it can be enabled/disabled by the MFPLOT_X11_DEVICE environment variable, which can take the value 0 or 1 .

See also: mfGetX11Device

## mfGetX11ColorDepth

## get color depth

Interface:

```
function mfGetX11ColorDepth() result( depth )
```

    integer :: depth
    Description:
Returns the color depth currently used by the X11 server. A figure must have been previously opened
See also: mf/msColormapSize, mf/msColormap, msColorbar

## msExitFgl

## FGL closing

Calling syntax:

```
call msExitFgl()
```

This routine is intended to be used at the end of your graphical program. It leaves a prompt to the user and waits for an answer (' $y$ ' or ' $Y$ '), after warning the user by a text like:

```
***
End of MUESLI plotting:
- all figures are going to be closed.
- graphic memory will be properly cleaned.
Do you really close graphics ? [y|Y]
```

It is a useful command to avoid that all graphic windows disappear; as mentioned, it also cleans the FGL internal memory (all graphic objects); moreover, X11 windows position and size are saved in a small database.

Remarks: If you want that your program terminates without a user action, you can redirect the standard input (via ' $<$ ' in a shell command) and read a file containing the ' $y$ ' symbol. Another possibility is to use the 'yes' unix command which outputs repeatedly the character 'y' to your program via a unix pipe, e. g.:
\$ yes | a.out

See also: msRelease
mfGetDefaultCapStyle

## Get default cap style for lines

Calling syntax:

```
cap_style = mfGetDefaultCapStyle( )
```

Description:
Returns the current default cap style, which may be 0 ("CapButt"), 1 ("CapRound") or 2 ("CapProjecting").

The signification of these three attributes may be found easily on the web.
See also: msSetGrObj, msSetDefaultCapStyle, msSetDefaultJoinStyle

## msSetDefaultCapStyle

## Set default cap style for lines

## Interface:

```
subroutine msSetDefaultCapStyle( cap_style )
    character(len=*), intent(in) :: cap_style
```

Description:
Change the global value for the default cap style. The argument cap_style may be set to "CapButt", "CapRound" or "CapProjecting".

The signification of these three attributes may be found easily on the web.
Note also that this graphic attribute may be change for a particular object, via the msSetGrObj routine.
See also: mfGetDefaultCapStyle, mfGetDefaultJoinStyle
mfGetDefaultJoinStyle
Get default join style for lines

Calling syntax:

```
join_style = mfGetDefaultJoinStyle( )
```

Description:
Returns the current default join style, which may be 0 ("JoinMiter"), 1 ("JoinRound") or 2 ("JoinBevel").

The signification of these three attributes may be found easily on the web.
See also: msSetGrObj, msSetDefaultJoinStyle, msSetDefaultCapStyle

## msSetDefaultJoinStyle

## Set default join style for lines

## Interface:

```
subroutine msSetDefaultJoinStyle( join_style )
    character(len=*), intent(in) :: join_style
```

Description:
Change the global value for the default join style. The argument join_style may be set to "JoinMiter", "JoinRound" or "JoinBevel".

The signification of these three attributes may be found easily on the web.
Note also that this graphic attribute may be change for a particular object, via the msSetGrObj routine.
See also: mfGetDefaultJoinStyle, mfGetDefaultCapStyle
mfGetCharEncoding
Get character encoding

Interface:

```
encoding = mfGetCharEncoding( )
```

Description:
Returns the current character encoding, which is "Latin" (the default) or "UTF-8".
This is used in the following graphic routines, which display character strings in a figure: $\mathrm{mf} / \mathrm{msText}$, msTitle, msXLabel, msYLabel, mf/msFigure and msLegend.

See also: msSetCharEncoding

## msSetCharEncoding

Interface:

```
subroutine msSetCharEncoding( encoding )
    character(len=*), intent(in) :: encoding
```

Description:
When processing character's strings (e.g. in the msTitle routine), MUESLI accepts two different character encodings. The charset may be "Latin" (default one) or "UTF-8".

So, the encoding argument can take only the following values:

- "Latin" (or "ISO-8859-1" which is an alias)
- "UTF-8"

Warning: Not all the characters defined in the "UTF-8" encoding are valid. Actually, only a subpart of the 256 first characters (i.e. mostly of the "Latin" encoded characters, but not all) are correctly processed.

See also: mfGetCharEncoding

### 2.2 Window's and figure's management

| mf/msFigure | figure creation |
| :--- | :--- |
| msResizeWindow | window resizing |
| mfGetWinId | current figure identification |
| msClose | figure deletion |
| mfGetColorScheme, msSetColorScheme | color scheme |
|  |  |
| msPrint | file printing (EPS and PDF) |
| msSetPdfOC | PDF Optional Contents management |

See also:
Global graphic settings
Figure properties
Figure annotation - Low level graphic object's manipulation
High level plotting routines
Interactive routines

## Calling syntax:

```
call msFigure( [win_id] [, position] [, size] [, title] )
```

where arguments are all optional.
This routine opens a new window or selects (and therefore makes active) a previously opened one.
The win_id argument (integer) indicates the identification of the window; if not present, a sequential integer from 1 is used.

The position argument is used to impose the position of the window on the screen. It must be a Fortran array of two integer values specifying the position of the newly created window; the position in pixels of the top-left corner of the window is taken from the top-left corner of the whole screen (virtual screen in case of many monitors), counting positively downward for the $y$ value. Default position is approximately $(0,0)$ for the first window, and shifted both in $x$ - and $y$-position, in order to be not completely overlapped (for the windows below others, only their title is visible).

The size argument is used to impose the size of the window on the screen. It must be a Fortran array of two integer values specifying the size of the newly created window; the two values indicates the width and height in pixels of the window. Default size is 800 by 600 .

The title argument (character string, of maximum length 80) specifies the title of the figure. This title is visible in the X11 window title; it applies also for the printed EPS and PDF files.

## Remarks:

- the default size of X11 windows can be modified via the environment variables MFPLOT_X11_WIDTH and MFPLOT_X11_HEIGHT.
- the window title always contains win_id and the name of the executable program.
- ordinarily, if you have no figure opened, most of graphical routines automatically open a new one.
- maximum number of concurrent figures is 64 . However, win_id values don't need to be contiguous, and may be even greater than 64 .

The other form:

```
win_id = mfFigure()
```

creates a new figure and returns its (integer) window identification.
Remarks: use mfGetWinId to get the window identification of the current active figure.
See also: msClose, msClf

Calling syntax:

```
call msResizeWindow( [width, height] )
```

Description:
If the optional arguments width and height are both present, then the window is resized by the library.
On the contrary, i.e. when the optional arguments are not present, this routine becomes interactive, prints a prompt on the terminal, and wait the resizing of the window by the mouse.

See also: msFigure, msClf

## mfGetWinId

Calling syntax:

```
win_id = mfGetWinId( )
```

Description:
Returns the window identification (integer) of the current active figure.
See also: msFigure

## msClose

Calling syntax:

```
call msClose( [win_id] )
```


## Description:

Closes the specified window, or the current window if the optional argument win_id is not present.
Note that after closing a window, the current selected window is undefined, even if other figures yet exist. It is required to select another existing window if you wish to plot inside, else a new figure will be automatically created.

See also: msFigure, msClf

## mfGetColorScheme

Interface:

```
color_scheme = mfGetColorScheme( )
```

Description:
Returns the color scheme attached to the selected window, which is an integer ranged in [1-4]. Default is 3 .

There are four different color schemes, used for recycling colors when you plot several lines on the same figure. They are described in the Muesli User's Guide (cf. § 5.2 Color management).

See also: msSetColorScheme

## msSetColorScheme

## Interface:

```
subroutine msSetColorScheme( color_scheme )
    integer, intent(in) :: color_scheme
```

Description:
Set the color scheme for the selected window, via an integer argument ranged in [1-4].
There are four different color schemes, used for recycling colors when you plot several lines on the same figure. They are described in the Muesli User's Guide (cf. § 5.2 Color management).

The default color scheme is the third one, i.e. the new colors in Matlab (from release R2014b).
See also: mfGetColorScheme
msPrint
file printing

Calling syntax:
call msPrint( filename [, file_format ] )

## Description:

Prints the current figure in a file named filename.
Currently, file_format may be only "EPS" or "PDF". If this optional argument is not present, MUESLI tries to deduce the device type from the extension of the provided filename. By default, an EPS file is created.

For EPS (resp. PDF) files, and for debugging purpose, comments can be inserted in the generated file by setting the environment variable MFPLOT_EPS_COMMENTS (resp. MFPLOT_PDF_COMMENTS) to 1 (default is no comments). Be aware that this will increase the size of the file.

Note, if you have selected a black background (see msSetBackgroundColor), that the EPS only will have also a black background. For the PDF files have always a white background.

## Remarks:

- EPS and PDF files intensively make use of language-based shading. This means that all coloured gradients (from msPatch or msPColor for example) are treated by using appropriate PostScript or PDF commands, and leads to smaller files.
- Of course, printed files may be generated without any X11 capability. This is useful under Windows (where there is no X11 access) or for batch processes.
- For PDF files only, optional contents (also known as layers) may be inserted. See msSetPdfOC.
msSetPdf0C
PDF Optional Contents management

The first interface is:

```
subroutine msSetPdfOC( handle,
    &
name, mutex, persistent, super_group )
```

```
integer, intent(in) :: handle
```

integer, intent(in) :: handle
character(len=*), intent(in), optional :: name, mutex, super_group
character(len=*), intent(in), optional :: name, mutex, super_group
logical, optional :: persistent

```
logical, optional :: persistent
```

It is used to tag a graphic object of the current window as Optional Content in the PDF. Doing so, and once the Layers of your PDF viewer is opened, you should be able to choose to display or not this graphic object (which appears under the name) on the screen.

The graphic object, referenced by its handle, may be visible or not. The initial state of visibility in the PDF layers will be consistent with the visibility in the X11 window at the time of printing, except when the persistent argument is used: in such a case, only the graphic object tagged as persistent will be initially visible in the mutex group.

If name is not present, then it will be assigned the corresponding graphic object number (as "handle $i$ ").
The mutex optional argument (MUTually EXclusive) is used to group some optional contents in a RadioButton group: at most one of these optional contents may be visible in the PDF (but that does not prevent the existence of a state where none of these are visible - see below the section concerning persistent if you want that exactly one content must be visible at a time). The mutex name is the name which will appear in the "layers" panel of the PDF viewer, which enclosed all different names. Same thing for the super_group optional argument, except that the groups inside are not in mutually exclusion.

When using the mutex feature, replacing 'name=name' by 'persistent=. true.' allows that one object is always visible, and it is the concerned object by default. A typical application is to group text annotations in the figure in mutually exclusive groups, one for a language; each group may have its own name, for example, equal to Français, Deutsch, etc. The mutex name could be then, Other languages (default: English). See the Labels_and_Languages_test.f90 program under src/test/fgl.

The other interface:

```
subroutine msSetPdfOC( handle,
    &
    merge, name, mutex, persistent, super_group )
integer, intent(in) :: handle(:)
logical, optional :: merge, persistent
character(len=*), intent(in), optional :: name, mutex, super_group
```

concerns a vector of handle, and can be used to the following aims:

- when the two arguments merge and name are both present, it merges a group of graphic objects under the same name as optional contents; as an example of use, see the PlotCubicBezier_test program under src/test/fgl.
- when the two arguments merge and name are both not present, it gets a different name for each graphic objects of the group, from the legend of the figure. Of course, such a legend should exist; on the contrary, a generic name (as "handle $i$ ") will be used instead. As an example of use, see the figure 4 of the Legend_test program under src/test/fgl.

The mutex optional argument has the same meaning as in the first interface. Its use implies the merge of all optional contents specified by the array of handles. Same thing for super_group.

Constraints about the presence of optional arguments:

- mutex and super_group arguments cannot be both present.
- persistent and name arguments cannot be both present.
- in a given group of optional contents, the persistent argument can be used only one time.
- currently (but this may change in a future version), there are only one Radio-Button group and only one Super-group for each figure.


## Remarks:

- this routine must be called before creating the PDF file via msPrint.
- you are allowed to first call the second interface, using an array of handle, getting the names from the legend and then, call the first interface to modify the name of one specific graphic object.
- the PDF Optional Contents will be ignored if you print in the EPS format.


### 2.3 Figure properties



See also:
Global graphic settings
Window's and figure's management
Figure annotation - Low level graphic object's manipulation
High level plotting routines
Interactive routines
mf/msAxis

## The function form

```
function mfAxis( [ axis ] ) result( mf_range )
    type(mfArray) :: mf_range
    character(len=*), optional :: axis
```

returns the current axis values of the selected figure, always in the order: left, right, bottom, top, whatever the mode ("xy" or "ij" - see below) is. This array is a vector of length 4 or 2 , containing extremal values of the $x$ - and/or $y$-axis, according to the presence of the optional argument axis which must be equal to the single character "x" or "y".

The subroutine form has the following calling syntax:

```
call msAxis( mode | dp_range | mf_range [, axis ] )
```

If the character string mode is present, it may have the following values:

- "on" / "off": axes are drawn (default) or hidden (and so for X label, Y label and Figure title).
- "auto": axes' ranges are computed from the actual numerical values of data to be plotted, giving smart extremal values; this is the default. Note that graphic data involved do not include arrows or texts, which are considered as annotations.
- "tight": axes' ranges are computed from actual values of data (see just above what is considered as "data"), strictly keeping their extremal values.
- "manual": axes' ranges are not automatic; the current axis values are frozen, whatever the following plots. This mode is also set when a range is explicitly used (see below). Only another call using the "auto" or "tight" argument can unlock it.
- "equal": $x$ - and $y$ - axis are drawn with the same scale; the default behaviour is "unequal". Note that the equal mode is possible only when axes have the same scaling (both linear or both logarithmic).
- "xy" / "ij": the origin of the $x$ - and $y$-axis is at the bottom left corner of the figure (default location for most plotting commands) or at the top left corner (when the data is interpreted as a matrix, which is the case for msSpy, msImage and msPColor in the absence of (X,Y) coordinates). In the "xy" mode, the $x$-axis labels are written at the bottom side of the box, whereas they are written at the top one in the other mode.
- "inverted": in this case, the second optional argument axis must be present and equal to "x" or "y". Corresponding extremal values of this axis are then swapped together. This doesn't imply that the axis becomes "manual". There is no "normal" state, i.e. two successive calls with the "inverted" argument is equivalent to do nothing; in other words, "inverted" is not a property but an action to realize.
- "linlin" / "loglog": $x$ - and $y$-axis are both linear (default) or logarithmic (base 10).
- "loglin" / "linlog": as specified, one of the two axes is linear whereas the other is logarithmic (base 10); Note that if the equal mode is set, it will be disabled.

If the real dp_range, or mfArray mf_range, is present (containing 4 values), it is used to initialize the axes' range; the manual mode is automatically set. The additional optional argument axis can be used to set only the $x$ - or the $y$-axis: in this case, the length of the range vector must be equal to 2 , instead of 4 , and the axis argument set to the single character " $x$ " or " $y$ ".

The axis optional argument can also be used when the following modes are set: auto, tight and manual.
Axes may be inverted (e.g., for the $x$-axis, the maximum value on the left and the minimum value on the right); to do this, simply give a range in decreasing order.
See also: msAxisFontSize, msAxisLineWidth, msLabelFontSize, msGrid
msAxisFontSize

## Interface:

```
subroutine msAxisFontSize( size_factor )
    real(kind=MF_DOUBLE), intent(in) :: size_factor
```

Description:
This routine is used to change the font size of the axis numbering.
size_factor must be ranged in $[0.4,2.5]$. Default size factor is unity. Actual font size also depends on the char_height_pixel mode (see the msCharInPixels routine).

See also: msAxis, msTitleFontSize, msLabelFontSize, msAxisLineWidth

## Interface:

```
subroutine msAxisLineWidth( width )
    real(kind=MF_DOUBLE), intent(in) :: width
```

Description:
This routine is used to change the line width of the axis and the associated numbering.
Default width is unity.
Remark: If you increase the value of width, you will remark that the effective line width in pixels is approximately half of this latter value. Actually, the pixel width is a discontinuous function of width, especially at small values. The corresponding behaviour is however continuous for the drawing generated in EPS and PDF files.

See also: msAxis, msAxisFontSize, msLabelFontSize
msCharInPixels

Interface:

```
subroutine msCharInPixels( mode )
    character(len=*), intent(in) :: mode
```

Description:
If mode is equal to "off", the character size is proportional to the window size (default behavior).
If mode is equal to "on", the character size is absolute. Unity is about 12 pixels.
See also: msAxisFontSize, msText
mf/msCAxis

## get or set color axis

The first form:

```
function mfCAxis() result ( out )
```

    type(mfArray) : : out
    returns the color axis of the current figure. It is a 2 -value vector which contains the min and max values used in the colorbar. The default value (if not set by the user) is the range [0,1].

The subroutine form:

```
call msCAxis( col_range | mf_range | string )
```

is used to set the color axis. One argument is required, it is either a real 2 -value vector, an mfArray or a character string.

When the argument contains numerical values, it specifies the color range. Be aware that the two values must not be too close together: in such a case, the routine enlarges slightly the given interval and emits a warning. Once the color axis has been defined, it will remain the same until the next call.

When the argument is a string, it must be equal to "reset": the color axis takes the initial $[0,1]$ range, whereas it becomes extensible again, when used repeatedly (by successive calls of msContour, for example).

Remark: The two input numerical values defining the color axis must be ordered: first is the minimum, second is the maximum.

See also: msColorbar, msColormap, msColormapSize, msPColor, msContour, msPatch
msClf

Calling syntax:
call msClf()

Description:
Clears the current figure, but don't close the corresponding window.
See also: msFigure, msClose, msCla

## msColorbar

Calling syntax:

```
call msColorbar( mode [, position ] [, label ] )
```


## Description:

If mode is equal to "on", adds a colorbar to the current figure.
Its position can be specified by an optional argument, position, which can be "vert" (i.e. right side) or "horiz" (i.e. bottom side); by default the position is automatically chosen.

The optional argument label can be used for the labelling of the color bar.
The three arguments are of type character (len=*).

## Remarks:

- In some situations, in particular when axis scaling are equal (see msAxis) and one figure side is much smaller than the other, it is recommended to left the routine choose itself the position; otherwise the colorbar may have a too short length.
- It is recommended to call this routine after drawing all objects.

See also: msCAxis, msColormap, msColormapSize

The function form:

```
colormap \(=\) mfColormap()
```

returns the current colormap in an mfArray (rank-2 array of 3 columns) which contains the Red-GreenBlue components of each color. These components are reals, ranged from 0.0 to 1.0.

The other function form:

```
bool = mfColormap( "init_status" )
```

returns a logical value according to the initialization status of the colormap.
The subroutine form:

```
call msColormap( name [, "inverted"] )
```

```
call msColormap( colormap [, int_max] )
```

```
call msColormap( colormap [, int_max] )
```

or
is used to set the colormap.
The user can choose a predefined colormap, via the character name argument: "rainbow" (or its alias "jet"), "parula", "hot", "bluered", "fusion", "flag" or "grey" (a grey scale). Number of colors is 256 , but it can be changed via msColormapSize. You can see their color representation in the Muesli User's Guide. If the optional argument "inverted" is added, then the colormap is inverted.

User-defined colormap can also be registered, via the mfArray colormap. This array must have 3 columns and a number of rows up to 4096.

By default, colormap contains real color components, ranged from 0.0 to 1.0. If the optional argument int_max is present, it indicates that entries are integers whose max value is provided (e.g. 255 for 256 values for each R,G,B component, giving a subset among 16 millions of colors. Integer color components always start from 0 ).

## Remarks:

- Before using this routine, a window must be opened via msFigure.
- There is no default colormap. The user must select one of them before using colors.

See also: msCAxis, msColorbar
mf/msColormapSize

The function form:
size $=$ mfColormapSize()
returns the size (integer) of the current colormap.
The subroutine forms:
call msColormapSize( size )
is used to set the size of the colormap. A maximum of 4096 colors is allowed; note however that in most of case, 256 colors are sufficient to eliminate color artifacts on the screen.
call msColormapSize( "auto" )
sets the size of the colormap to its default size, i.e. 256 .

See also: mf/msColormap, msColorbar

Calling syntax:

```
call msDrawBox()
```

Redraws the framed box in the current figure.
This routine is useful after drawing some plots which partially erase the ticks and marks of the box. It doesn't redraw the numerical label neither the optional grid (which should remain in the background).

See also: msRedrawFigure, msDrawGrid

## msDrawGrid

## if needed, redraw the grid in the current figure

Calling syntax:

```
call msDrawGrid()
```

Redraws the grid in the current figure, if needed.
This routine is useful during animation: if the grid has been set with the msGrid routine, the current routine must be called in the iteration loop, before any plotting with FGL.

See the Muesli User's Guide about a typical sequence of Muesli routines to include in an animation loop.
See also: msRedrawFigure, msDrawBox
msRedrawFigure

## redraw the current figure

Calling syntax:

```
call msRedrawFigure( )
```

redraws the current figure.
Typically, this routine should be used after any graphic object change, modified by the routine msSetGrObj, in order to make visible the modification on the screen.

See also: msDrawBox, msDrawGrid, msResizeFigure

## msGetX11Pixmap

## X11 pixmap saving

Calling syntax:

```
call msGetX11Pixmap( A1, A2, A3 )
```

Description:
This routine returns in three mfArrays, the color components of the X11 pixmap drawn on the screen.
If the background color is black, then it gets the R,G,B planes. If the background color is white, then it gets the $\mathrm{C}, \mathrm{M}, \mathrm{Y}$ planes.

Remark: the mfArray arguments should not be pointed by another ordinary Fortran pointer (see msPointer); on the contrary, a Warning is emitted.

See also: msSetBackgroundColor
msGrid
grid setting

Calling syntax:
call msGrid ( mode [, minor ] )
The string mode may have the following values:

- "on": a background grey dashed grid is drawn for major axis ticks; (default)
- "off": grid is hidden;

The boolean optional minor can be set to .true. to get minor lines (aligned on minor ticks). By default, these minor dotted lines are not drawn.

See also: msAxis
msHold
plot command superposition

Calling syntax:

```
call msHold( "on" | "off" )
```

Description:
Usually, each new plot command erase the previous one. If you want to keep the previous graphic objects drawn in the figure, you must call this routine with "on".

Use "off" to come back to the default behavior.

Remark: to avoid unwanted behavior, place this command after the first plot (msPlot or whatever).
msRemoveClipBox
remove a clipping box inside axes

Interface:

```
subroutine msRemoveClipBox( )
```

Description:
Remove the clipping box set by the msSetClipBox routine.
See also: msSetClipBox
msSetClipBox
define a clipping box inside axes

Interface:

```
subroutine msSetClipBox( dp_range )
    real(kind=MF_DOUBLE), intent(in) :: dp_range(:)
```

Description:
Define a clipping box (inside current axes) for subsequent plotting commands.
See also: msRemoveClipBox
msShading
color shading mode

Calling syntax:
call msShading ( mode )
where mode is a character string which can be "flat" or "interp".

## Description:

Sets the shading mode used when drawing patches:

- "flat" implies that only one color is used;
- "interp" leads to interpolated colors between the vertices of the polygonal shape.
"flat" is the default mode.
Note that the "interp" mode doesn't lead to bigger files when printing in PDF. Only for EPS, the use of transparency leads to the inclusion of bitmap images.

The shading has no effect on the display of the colorbar. Indeed, the colormap may have sharp gradients or even jumps in colors, or may use of a very small number of colors (e.g. 8 or 16): that must appear clearly in the colorbar.

See also: mf/msPatch, mf/msPColor, msColorbar

## mf/msTitle

## Interface:

```
subroutine msTitle( title )
    character(len=*), intent(in) :: title
```

Description:
Adds a title to the current figure. Not drawn if axis is off.
Maximum length of the title is 128 characters.
This routine should be called after the other data plotting routines.
Escaped sequences may be use in the string (see msText) to change the font, the style, the position (subor superscript), etc. Note also that the global character size may be changed by using msTitleFontSize.

Remark: the function mfTitle has the same argument list; it returns the (integer) handle of the created graphic objet; this is useful to create many strings in different languages, and set them as optional content for PDF creation (see msSetPdfOC).
See also: msXLabel, msYLabel, msAxis

## msTitleFontSize

title font size setting

Interface:

```
subroutine msTitleFontSize( size_factor )
    real(kind=MF_DOUBLE), intent(in) :: size_factor
```

Description:
This routine is used to change the font size of the title.
size_factor must be ranged in $[1.0,3.5]$. Default size factor is 2 . Actual font size also depends on the char_height_pixel mode (see the msCharInPixels routine).

See also: msAxis, msAxisFontSize, msTitle, msLabelFontSize

## mf/msXLabel

## Interface:

```
subroutine msXLabel( xlabel )
    character(len=*), intent(in) :: xlabel
```

Description:
Adds a label to the x-axis of the current figure. Not drawn if axis is off.
Maximum length of this label is 96 characters.
This routine should be called after the other data plotting routines.
Escaped sequences may be use in the string (see msText) to change the font, the style, the position (subor superscript), etc. Note also that the global character size may be changed by using msLabelFontSize.

Remark: the function mf XLabel has the same argument list; it returns the (integer) handle of the created graphic objet; this is useful to create many strings in different languages, and set them as optional content for PDF creation (see msSetPdfOC).

See also: msTitle, msYLabel, msAxis

## mf/msYLabel

## Interface:

```
subroutine msYLabel( ylabel )
    character(len=*), intent(in) :: ylabel
```

Description:
Adds a label to the y-axis of the current figure. Not drawn if axis is off.
Maximum length of this label is 96 characters.
This routine should be called after the other data plotting routines.
Escaped sequences may be use in the string (see msText) to change the font, the style, the position (subor superscript), etc. Note also that the global character size may be changed by using msLabelFontSize.

Remark: the function mf YLabel has the same argument list; it returns the (integer) handle of the created graphic objet; this is useful to create many strings in different languages, and set them as optional content for PDF creation (see msSetPdfOC).

See also: msTitle, msXLabel, msAxis
msLabelFontSize
descriptive labels font size setting

## Interface:

```
subroutine msLabelFontSize( size_factor )
    real(kind=MF_DOUBLE), intent(in) :: size_factor
```

Description:
This routine is used to change the font size of the axis labels.
size_factor must be ranged in $[0.4,2.5]$. Default size factor is unity. Actual font size also depends on the char_height_pixel mode (see the msCharInPixels routine).

See also: msAxis, msAxisFontSize, msTitleFontSize, msXLabel, msYLabel

## msAxisLabelFormat

## Interface:

```
subroutine msAxisLabelFormat( x_axis_mode, y_axis_mode )
    character(len=*), intent(in), optional :: x_axis_mode, y_axis_mode
```

Description:
This routine is used to set the format of labelling axis.
Default mode is "std", i.e. numeric numbers without unit. If set to "time" then numbers are supposed to be seconds and are written under the format (DD) HH MM SS.S.

See also: msAxisFontSize

## msSetWinProp

window properties setting

Interface:

```
call msSetWinProp( property, data )
```

Description: Used to modify a specified property of the current window.
property may take the following values:

- "axis_font_size", "axis_line_width", "label_font_size" or "title_font_size":
$\rightarrow$ data of type real
- "xlabel", "ylabel" or "title":
$\rightarrow$ data of type character (len=*)

Remark: Contrary to the specific following routines: msAxisFontSize, msAxisLineWidth, msLabelFontSize, msTitleFontSize, msXLabel, msYLabel, msTitle, the redraw of the figure is not done; therefore, the user must call the msRedrawFigure to see the change effects.

First interface:

```
subroutine msLegend( legend_1 [, legend_2, legend_3, ... ]
    [, location | position ] )
character(len=*), intent(in) :: legend_1
character(len=*), intent(in), optional :: legend_2, ..., legend_13
character(len=2), intent(in), optional :: location
type(mfArray), intent(in), optional :: position
```

Second interface:

```
subroutine msLegend( legend_1, handle_1 [, legend_2, handle_2, ... ] &
    [, location | position ] )
    character(len=*), intent(in) :: legend_1
    integer, intent(in) :: handle_1
    character(len=*), intent(in), optional :: legend_2, ..., legend_13
    integer, intent(in), optional :: handle_2, ..., handle_13
    character(len=2), intent(in), optional :: location
    type(mfArray), intent(in), optional :: position
```


## Description:

Adds legends to the curves plotted in the figure. Up to 13 legends (noted above as legend_1, legend_2, ...) in one frame are supported, each containing a string of at most 80 characters.

The handles of the curves may be provided (see the second interface), making possible to legend only a subset of the curves. Of course, all handled must be valid and must be integers returned by Muesli graphic functions. Note that the two calling syntaxes above cannot be mixed.

Numbered legends and handles must be used in increasing order, otherwise some legends may not appear.
The optional argument location (not to be used in conjonction with position) can be used to specify in which corner of the figure the legend is displayed. Possible values are "TL" (top-left), "TR" (top-right), "BL" (bottom-left), "BR" (bottom-right) or "outside" (see below). Default is top-left corner.

The optional argument position (not to be used in conjonction with location) specify the position of the legend's frame, i.e. the coordinates of the top-left angle of the frame. The user is even allowed to locate the legend "outside" of the axes, by specifying any value of the coordinates. The legend frame may or may not be entirely visible in the X11 window, but it should appear in the EPS and PDF files. This is useful in case of a very high legend. A better way, however, is to use location="outside"; in such a case, the legend will be drawn in a small, additional window (but the legend will appear in the same EPS or PDF after printing).

Legends are grouped in a unique frame which can be moved inside or outside axes via the msMoveLegend routine. See mfLegend to create many frames. A legend frame cannot be removed without knowing its handle, i.e. via mfLegend.

## Remarks:

- Usually, the number of string arguments (or pairs of string and associated handle) specifying the legends is equal to the current number of curves. However, it is possible to specify less legends than the actual number of curves: a simple warning will be emitted during the call. Similarly, specifying a number of legends larger than the number of current curves leads to a simple warning, and additional legends will be discarded.

$$
\cdots / \ldots
$$

- For bar plot, the different items in the legend correspond to the data series plotted; this works only for grouped or stacked bars (see mf/msBar). Currently, the second interface above is not compatible with bar plots.


## Third interface:

```
subroutine msLegend( legend_array [, handle_array ]
                                    &
            [, location | position ] )
    character(len=*), intent(in) :: legend_array(:)
    integer, intent(in), optional :: handle_array(:)
    character(len=2), intent(in), optional :: location
    type(mfArray), intent(in), optional :: position
```


## Description:

Allows the user to group all the legends (and, optionally, the associated handles) in one array of character strings.

## mfLegend

## add legends to curves, possibly in many frames

## Description:

As opposed to most of graphic routines, the function version of the Legend family behaves differently.
It has the same interfaces as msLegend and, as usual, returns the (integer) handle of the graphic objet; moreover, it can be called many times for the same figure, making it useful to create legends in different languages, and to set them as optional content for PDF creation (see msSetPdfOC).

### 2.4 Figure annotation - Low level graphic object's manipulation

| $\mathrm{mf} / \mathrm{msArrow}$ |  |
| :--- | :--- |
| $\mathrm{mf} / \mathrm{msText}$ | arrow drawing <br> text display |
| mfGetAllGrObj | get all graphic objects |
| mfGetTypeGrObj | get the type of a graphic object |
| mfSelectTypeGrObj | select graphic objects by type <br> msSetGrObj <br> gsRemoveGrObj |
| graphic object setting <br> graphic object deletion |  |

See also:
Global graphic settings
Window's and figure's management
Figure properties
High level plotting routines
Interactive routines

The first form:

```
call msArrow( x_start, y_start, x_end, y_end &
    [, color] [, linewidth] [, headsize] [, clipping ] )
```

draws an arrow from the position (x_start, y_start) to the position (x_end, y_end). These coordinates must be of type real.

The optional argument color have the same meaning as in the routine $\mathrm{mf} / \mathrm{msPlot}$.
Optional arguments linewidth and headsize (real numbers) can be used to specify the line width and the head size of the arrow. Default values are unity.

If the optional boolean argument clipping is set to $F A L S E$, then the arrow can be displayed out of the viewport (default is TRUE, i.e. the arrow is clipped at the viewport; this default behavior is recommended when it is an annotation at some location inside the viewport and you might want to pan or zoom inside the axes).

The second form:

```
call msArrow ( x, y, angle
\&
    [, color] [, headsize] )
```

draws only an arrow head, given a position $(x, y)$ and an angle in radian (all are real variables).
The remaining arguments have the same meanings as above.

## Remarks:

- the function mfArrow has the same argument list; it returns the (integer) handle of the created graphic object.
- in future, the four first arguments will be able to take also the type mfArray, in order to display many arrow at a time.

See also: mf/msText
$\mathrm{mf} / \mathrm{msText}$

## text display

## Interface:

```
call msText( x, y, text &
    [, angle] [, just] [, just_vert] [, pix_voffset] &
    [, color] [, bg] [, height] [, clipping] [, xbox, ybox] )
```


## Description:

Writes the string text at the position ( $\mathrm{x}, \mathrm{y}$ ). These coordinates may be of type real or mfArray. Many escape codes allow the use of different kinds of fonts, the use of subscript or superscript, greek letters, etc. See the Muesli User's Guide at section 5.5 for more information.

If the real argument angle is present then text is written with this angle (in degrees).
The real argument just specifies the justification of the string in comparison with the position ( -1 for left (default), 0 for centered, +1 for right - other real values ranged from -1 to +1 are permitted, giving intermediate justifications). Similarly, the real just_vert argument concerns the vertical justification (default is -1 , i.e. the baseline of the glyph located at y coordinate, and 0 to obtain the topline of the glyph at $y$; use -0.5 for a vertically centered glyph). Concerning only the vertical justification, the optional boolean argument pix_voffset can be used (under X11) to move vertically the string box by a 2 pixel shift - this is especially useful when the bottom or the top of the string box is against a drawn line (default is pix_voffset=.false.).
If the optional argument color is present, it specifies the color of the text displayed (see msSetGrObj). Default color is black when the figure background is white (see msSetBackgroundColor).

If the optional argument bg is present, it specifies the color of the background (see msSetGrObj). Default is transparent.

The optional real argument height specifies the height of the characters drawn (default is 1 ). This height may be relative (the default, character size is proportional to the window size) or absolute (character height is in pixels, unity is about 12 pixels), according to the mode defined in the routine msCharInPixels.
If the optional boolean argument clipping is set to $F A L S E$, then text can be displayed out of the viewport (default is TRUE, i.e. the text is clipped at the viewport; this default behavior is recommended when text is an annotation at some location inside the viewport and you might want to pan or zoom inside the axes).

The optional output arguments xbox and ybox allows the user to retrieve the value of the rectangle frame of the string displayed (4 values for both $x$ and $y$, because the string may be inclined). These arguments must be real arrays of length 4 .

## Remarks:

- usually, the current routine is used to annotate a graphic, so it should be employed after any use of $\mathrm{mf} / \mathrm{msPlot}$; on the contrary, the msHold routine must be added before the plot command, else all texts from $\mathrm{mf} / \mathrm{msText}$ will be erased.
- the function mfText has the same argument list; it returns the (integer) handle of the created graphic object.

See also: mf/msArrow, msSetCharEncoding

## mfGetAllGrObj

## get all graphic objects

Calling syntaxe:

```
hdle_vec = mfGetAllGrObj( )
```

Description: Returns a vector of handle for all graphic objects in the current window.
The hdle_vec variable, declared by the user, must be an integer vector. For an easy programming use, it should be allocatable: due to a feature of modern Fortran, it will be allocate with the appropriate size during the assignment; moreover, there is no need to deallocate it between calls.

See also: mfGetTypeGrObj, msSetGrObj

## mfGetTypeGrObj

Calling syntaxe:

```
type = mfGetTypeGrObj( handle )
```

Description: Returns the type of a graphic object, identified by its handle, as a character string.
The type may be one of the followings:

- "line", "point" or "line+point", typically created by mf/msPlot;
- "image", created by mf/msImage;
- "text", created by mf/msText;
- "polygon", created by mf/msPatch;
- "pcolor", created by mf/msPColor;
- "contour", created by mf/msContour;
- "quiver", created by mf/msQuiver;
- "streamline", created by mf/msStreamline or mf/msTriStreamline;
- "arrow" or "arrow_head_only", created by mf/msArrow;
- "quadr_bezier", created by mf/msPlotQuadrBezier;
- "cubic_bezier", created by mf/msPlotCubicBezier or mf/msPlotCubicSpline;
- "histogram", created by mf/msPlotHist;
- "errorbar_x_line", "errorbar_y_line", "errorbar_xy_line", "errorbar_x_pt", "errorbar_y_pt", "errorbar_xy_pt", "errorbar_x_line+pt", "errorbar_y_line+pt" or "errorbar_xy_line+pt", created by mf/msErrorBar;
- "pcolor_spy", "pcolor_spy_sparse", "pcolor_spy_sparse_2", "plot_spy_sparse", created by mf/msSpy;
- "legend", created by mf/msLegend;
- "tri_fill", created by mf/msTriFill;
- "trimesh_by_fac", "trimesh_by_tri" or "mesh_boundary_unstruct", created by mf/msTriMesh;
- "tri_pcolor", created by mf/msTriPColor;
- "tri_quiver", created by mf/msTriQuiver;
- "tri_contour", created by mf/msTriContour;
- "xlabel", created by mf/msXLabel;
- "ylabel", created by mf/msYLabel;
- "title", created by mf/msTitle;
- "set_clip_box", created by msSetClipBox;
- "remove_clip_box", created by msRemoveClipBox.

See also: mfGetAllGrObj, msSetGrObj

Calling syntaxe:

```
hdle_vec = mfSelectTypeGrObj( handles, grobj_type )
```

Description: Returns a vector of handles for the graphic objects which match the type specified as argument, among a list of input handles.
handles must be a integer vector, containing a set of valid graphic object handles.
grobj_type is a character string, containing the type of the graphic object, as returned by mfGetTypeGrObj.
The hdle_vec variable, declared by the user, must be an integer vector. For an easy programming use, it should be allocatable: due to a feature of modern Fortran, it will be allocate with the appropriate size during the assignment; moreover, there is no need to deallocate it between calls.

See also: mfGetAllGrObj, msSetGrObj

First interface:

```
call msSetGrObj( handle, property, data )
```

Description: Changes one property of the graphic object identified by its handle. This handle may be a scalar or vector integer, or even a vector of handles stored in an mfArray.

The type of data depends on the target property, as detailed below.
property may take the following values:

- "visible": data may be any string, but the object will be displayed only if data is "on"; an exception occurs if the object is tagged as Optional Content for the PDF driver (see msSetPdfOC).
- "linestyle": data must be a string among "-", "--", "-." (or ".-" as an alias) or ": " (or ".." as an alias), for continuous, dashed, dashed-dotted, and dotted, respectively.
- "linewidth": data must be of type real, positive (default value is 1).
- "cap_style": data must be a string among "CapButt", "CapRound" or "CapProjecting" (but not case sensitive). Signification of these three keywords may be found easily on the web and concerns all devices ( $X 11, E P S$ and $P D F)$. Default is "CapRound".
- "join_style": data must be a string among "JoinMiter", "JoinRound" or "JoinBevel" (but not case sensitive). Signification of these three keywords may be found easily on the web and concerns all devices ( $X 11, E P S$ and $P D F$ ). Default is "JoinRound".
- "marker": data may be a single character among ".", "+", "*", "o", "x", "s", "~" or "d" (for Dot, Plus, Asterisk, Circle, X, Square, TriangleUp and Diamond, respectively), or an escaped sequence as "\M01" to "\M26" for using the complete set of markers, described in Fig. 27 of the Muesli User's Guide. Negative codes " $\backslash \mathrm{M}-14$ " to " $\backslash \mathrm{M}-21$ " tell Muesli to draw the 8 filled markers with a small white border around them. For a more comprehensive reference, the markers may be also specified by an escaped text sequence, as "\CircleFilled" (the character case doesn't matter; it can be adapted for a better reading).
- "markersize": data must be of type real, positive (default value is 1 ).
- "color": data may be a single character, among "w", "k", "r", "g", "b", "c", "m" or "y" (for white, black, red, green, blue, cyan, magenta and yellow, respectively), or an escaped sequence of the form "\Cnn" (e.g. "\C01", "\C02", ...) for using the corresponding color of the current color scheme (see msSetColorScheme).
- "col_name": data may be any string which represents a valid colorname from the X11 RGB database.
- "col_rgb": data must be a vector of type real and size 3.
- "text": data may be any string.
- "position": data must be a string among "first", "last", "up" or "down". This affects the position in the stack, i.e. the drawing order.
- "opacity": data must be of type real, ranged from 0.0 (full transparency) to 1.0 (full opacity).
- "coordinates": (for text string) data must be of type real and size 2.
- "relat_coords": (for a legend frame created by mfLegend) data must be of type real and size 2 , and is understood as relative coordinates (left and bottom sides correspond to 0 , right and top sides correspond to 1 ).

$$
\cdots / \ldots
$$

Changing the coordinates of the points in a polyline needs to change both $x$ and $y$ coordinates together by using the Other interface:

```
call msSetGrObj( handle, "x", x, "y", y )
```


## Remarks:

- the user must call the msRedrawFigure to see the change effects.
- properties are not all editable for a given graphic object. The following table describes what can be modified for all types of graphic objects.

See also: msSetWinProp, msRemoveGrObj

## msRemoveGrObj

## graphic object deletion

Calling syntax:

```
call msRemoveGrObj( handle [, redraw ] )
```

Description:
Removes the graphic object in the current figure, specified by its handle.
handle may be a scalar integer or a vector of handle; in this latter case, handle must be an mfArray.
For performance reason, the figure is not redrawn after the removing, except if you specify the value $T R U E$ for the optional argument redraw (default value is $F A L S E$ ). Therefore, there is no need to make yourself a redraw of the figure (routine msRedrawFigure), since it is sufficient to specify redraw $=T R U E$ on the last call.

Remark: After the grobj removing, the handle is set to zero. In the case of an mfArray, all elements are set to zero but the mfArray itself is not released.

See also: msSetWinProp, msSetGrObj

### 2.5 High level plotting routines - 2D

| mf/msPlot <br> mf/msErrorBar | data plot (using straight segments) data plot with error bars |
| :---: | :---: |
| mf/msPlotQuadrBezier | data plot (using quadratic curved segments) |
| mf/msPlotCubicBezier | data plot (using cubic curved segments) |
| mf/msPlotCubicSpline | data plot (using cubic curved segments) |
| mf/msBar | bar data plot |
| $\mathrm{mf} / \mathrm{msPColor}$ | pseudo-color plot |
| $\mathrm{mf} / \mathrm{msContour}$ | data contouring |
| $\mathrm{mf} / \mathrm{msContourF}$ | data contouring with filled regions |
| $\mathrm{mf} / \mathrm{msQuiver}$ | vector field plot |
| $\mathrm{mf} / \mathrm{msStreamline}$ | vector field streamline |
| mf/msPatch | graphic patch |
| $\mathrm{mf} / \mathrm{msPlotHist}$ | data histogram |
| msCumulHist | cumulative data histogram |
| msImRead, msImWrite | image read and write |
| mf/msImage | image display |
| $\mathrm{mf} / \mathrm{msPlotPSLG}$ | plot a PSLG domain |
| mf/msTriMesh | plot a triangulation |
| mf/msTriPColor | pseudo-color on a triangulation |
| mf/msTriFill | coloring on a triangulation |
| $\mathrm{mf} / \mathrm{msTriContour}$ | data contouring on triangles |
| $\mathrm{mf} / \mathrm{msTriContourF}$ | data contouring on triangles with filled regions |
| mf/msTriQuiver | vector field plot on a triangulation |
| $\mathrm{mf} / \mathrm{msTriStreamline}$ | vector field streamline on a triangulation |
| mf/msSpy | sparsity pattern visualization |

See also:
Global graphic settings
Window's and figure's management
Figure properties
Figure annotation - Low level graphic object's manipulation
Interactive routines

## mf/msPlot

## data plot (using straight segments)

## Calling syntax:

```
call msPlot( x [, y] &
    [, linespec] [, color] [, linewidth] [, markersize] &
    [, dashes_inverted] )
```


## Description:

Plots the vector mfArray y versus the vector mfArray x .
If y is not present then x is plotted against its integer indices. One exception is when x is complex: in such a case, $\operatorname{msPlot}(x)$ is equivalent to $\operatorname{msPlot}(r e a l(x), \operatorname{imag}(x)$ ), so the drawing is done in the complex plane.

The optional argument linespec is a string which contains a multiple attribute vector (color, linestyle, marker) as in MATLAB. See msSetGrObj for further information.

The optional argument color may be a character string (one-letter color code, escaped sequence, colorname from the RGB database) or a vector of type real and length 3 containing the RGB components of a color.

The optional argument linewidth must be a positive number of type real (default is 1). Under X11 the unit is pixel, therefore specifying a value less than 1 makes no difference; however, in $E P S$ and $P D F$, this will lead to very thin lines.

The optional argument markersize must be a positive number of type real; it is used only for marker symbols. If this argument is not present, then the marker size is automatically chosen to be coherent with the line width value.

The optional argument dashes_inverted must be a boolean; it is used only for dashed lines: it shifts the dashes in such a way that two identical dashed lines using different colors are both visible, when this argument is used for the second curve.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

Log representation of the data is made by using the msAxis routine.

## Remarks:

- the drawing is always clipped at the viewport.
- the function mfPlot has the same argument list; it returns the (integer) handle of the created graphic object.
- when calling many time this routine in the same figure, without providing any color information, there is an automatic color selection, cycling the colors from the current colortable (for more information about colortables - or color schemes - see the Muesli User's Guide).
- by default, the axis range presents its minimum on the left and its maximum on the right. To invert the axis, use the msAxis routine.

See also: mf/msBar, mf/msPlotQuadrBezier, mf/msPlotCubicBezier, mf/msPlotCubicSpline

## mf/msErrorBar

## data plot with error bars

## Calling syntax:

```
call msErrorBar( [ x, ] y [, x_err] [, y_err]
            [, linespec] [, color] [, linewidth] [, markersize] )
```


## Description:

Plots the mfArray y versus the mfArray $x$, using line specification given in linespec and adding error bars at each point.

The argument x is optional (the $x$-axis will use a vector of sequential integers from 1 ). x and y must be vectors (row or col) of same length, not matrices.

The optional argument linespec (character string)) has exactly the same meaning as in the msPlot routines. In particular, when not specified, colors will be recycled from the current colortable (for more information about colortables - or color schemes - see the Muesli User's Guide).

The optional arguments linewidth, color and markersize have exactly the same meaning as in the msPlot routines.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remark:

- the drawing is always clipped at the viewport.
- the function mfErrorBar has the same argument list; it returns the (integer) handle of the created graphic object.
- currently, axis must be both linear.
- due to the simple implementation and the number of optional arguments, it is required, most of times, to use the name of each argument as keywords (on the contrary, errors will occur at compilation and even at run-time). The examples below show typical calls.

See also: mf/msPlot, msAxis

## Example(s):

For adding error bars only along the $y$-axis, the first call:

```
call msErrorBar( mf([2.00d0,1.50d0,3.00d0,2.50d0]),
    mf([0.10d0,0.15d0,0.20d0,0.25d0]) )
```

will give at run-time:
(MUESLI msErrorBar:) ERROR: bad optional arg combinaison (see doc) !
because the first actual argument is associated to x and the second one to y .
The second one:

```
call msErrorBar( mf([2.00d0,1.50d0,3.00d0,2.50d0]),
    y_err=mf([0.10d0,0.15d0,0.20d0,0.25d0]) )
```

will not compile:
Compilation Error: Missing actual argument for argument 'y'
Only this third call is correct:

```
call msErrorBar( y=mf([2.00d0,1.50d0,3.00d0,2.50d0]),
    y_err=mf([0.10d0,0.15d0,0.20d0,0.25d0]) )
```

```
mf/msPlotQuadrBezier
```


## data plot (using quadratic curved segments)

## Interface:

```
call msPlotQuadrBezier( x, y &
[, linespec] [, color] [, linewidth] &
    [, dashes_inverted] )
```

Description:
Draws a quadratic Bézier curve from the control points whose coordinates are given in the mfArrays x and y. The drawn object is a quadratic piecewise polynomial parametric curve. The curve is continuous (but doesn't have necessarily continuous derivatives); therefore, the total number of points in the x and y mfArrays must be of the form: $2 k+1$, where $k$ is the number of segments.

Optional arguments linespec, color, linewidth and dashes_inverted have the same meaning as in the routine msPlot.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- the drawing is always clipped at the viewport.
- the marker choice, when specified, is discarded.
- the function mfPlotQuadrBezier has the same argument list; it returns the (integer) handle of the created graphic object.
- when not specified, colors will be recycled from the current colortable (for more information about colortables - or color schemes - see the Muesli User's Guide).

See also: mf/msPlotCubicBezier, mf/msPlotCubicSpline

# data plot (using cubic curved segments) 

## Interface:

```
call msPlotCubicBezier( x, y &
[, linespec] [, color] [, linewidth] &
    [, dashes_inverted] )
```


## Description:

Draws a cubic Bézier curve from the control points whose coordinates are given in the mfArrays x and y. The drawn object is a cubic piecewise polynomial parametric curve. The curve is continuous (but doesn't have necessarily continuous derivatives); therefore, the total number of points in the x and y mf Arrays must be of the form: $3 k+1$, where $k$ is the number of segments.

Optional arguments linespec, color, linewidth and dashes_inverted have the same meaning as in the routine msPlot.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- the drawing is always clipped at the viewport.
- the marker choice, when specified, is discarded.
- the function mfPlotCubicBezier has the same argument list; it returns the (integer) handle of the created graphic object.
- when not specified, colors will be recycled from the current colortable (for more information about colortables - or color schemes - see the Muesli User's Guide).

See also: mf/msPlotQuadrBezier, mf/msPlotCubicSpline

# data plot (using cubic curved segments) 

## Interface:

```
call msPlotCubicSpline( abs_curv, x, y, wx, wy,
&
[, linespec] [, color] [, linewidth]
&
    [, dashes_inverted] )
```

Description:
Draws a cubic Spline curve from given points whose coordinates are given in the mfArrays x and y . The drawn object is a cubic piecewise polynomial curve parametrized by the vector abs_curv. It is $C^{2}$ continuous at segment boundaries (i.e. both the tangent vector and the curvature are continuous). The two vectors wx and wy must have been computed before the call by use of the routine mfSpline .

Optional arguments linespec, color, linewidth and dashes_inverted have the same meaning as in the routine msPlot.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- the drawing is always clipped at the viewport.
- the marker choice, when specified, is discarded.
- the function mfPlotCubicSpline has the same argument list; it returns the (integer) handle of the created graphic object.
- when not specified, colors will be recycled from the current colortable (for more information about colortables - or color schemes - see the Muesli User's Guide).
See also: mf/msPlotQuadrBezier, mf/msPlotCubicBezier


## Calling syntax:

```
call msBar( [ x, ] y
    [, color] [, width] [, baseline] [, style] )
```


## Description:

Plots the mfArray y versus the mfArray x , using vertical bars. The argument x is optional (the $x$-axis will use a vector of sequential integers from 1). When y is a vector (row or col) then x must have the same shape as y ; however, when y is a matrix (processed by columns), then x must be a column vector of same length as that of y columns.

The optional argument color specifies the color used to fill the bars; the color convention is the same as in the msPlot routine. See also msSetGrObj for further information. It may have many types and shapes:

- it may be a single character string, containing a one-letter color code or a color name from the RGB database; it may be also a vector of $N$ character strings, $N$ being the number of columns of y.
- it may be a vector of type real with 3 RGB components (describing one color); or a matrix of dimension $(3, N), N$ being the number of columns of y .

The optional argument "width" must be a positive number of type real ranged in $[0,1]$. It is the relative width of the bars (default is 0.8 ).

The optional argument "baseline" must be a positive number of type real. Its role depends on whether the Y-axis is linear or logarithmic:

- in the case of a linear Y-axis, baseline is usually equal to zero, in order to separate positive from negative data values. The routine plots a thin line to visualize the baseline.
- in the case of a logarithmic Y-axis, baseline is the minimum value for data (default value is 0.01 ). Negative values, or those which are less than baseline are discarded.

The optional argument style works only for matrix data. When it is equal to "grouped" (default value), grouped bars are used; when it is equal to "stacked", stacked bars are used.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- the function mfBar has the same argument list; it returns the (integer) handle of the created graphic object.
- only the Y-axis can be of logarithmic type; in this case, of course, all values in y must be positive.
- when not specified, colors will be recycled from the current colortable (for more information about colortables - or color schemes - see the Muesli User's Guide).
See also: mf/msPlot, msAxis

Calling syntax:

```
call msPColor( [ X, Y, ] Z [, data_centering, view]
    [, grid, grid_color, grid_step] [, lighting] )
```

Draws a pseudo color plot of the data contained in the $\mathrm{Z} \mathrm{mfArray}$.
The $(i, j)$ indices are mapped to the axis $(-Y, X)$, so data visualisation is coherent with the matrix layout and with other routines working with matrix data (msContour, msQuiver, msStreamline). If this orientation is not appropriate, use the coordinates (mfArrays X and Y), as described below.

The optional data_centering argument specifies how the data stored in the matrix are graphically displayed; it is a string which must be equal to "vertex" (default value - $(i, j)$ points to the vertex of a cell) or "cell" ( $(i, j)$ points to the center of a cell). In the cell centered model, the shading of the color is only flat, whatever the value specified by the use of msShading. This is summarized in the following figure:


The optional view argument is a character string specifying the type of view for displaying the data in the matrix Z : the default view is the usual top view, noted "ji" or "XY", according to the presence of the arguments $X, Y$. When these latter arguments are not present, view may be equal to "iZf", "iZb", "jZf" or "jZf", specifying a side view as noted by the letters i, j or Z (always the vertical axis); the third letter is a modifier, indicating a forward or a backward view. In the other case, i.e. when the arguments $\mathrm{X}, \mathrm{Y}$ are presents, the corresponding strings are "YZf", "YZb", "XZf" and "XZf". The different side views are summarized below:


Note that, in projection mode, only the default data centering (i.e. data_centering="vertex") is accepted.

A thin line may be drawn to show the boundary of each quadrilateral cell by using the optional argument grid equal to TRUE. The default is to not show these boundaries. The color of this thin line may be specified in the optional argument grid_color, with the same convention as in the msPlot, routine; by default, it is the foreground color. The optional argument grid_step allows to plot only some lines of the grid, indicating the step to be used along the X or the Y coordinate; it must be a vector of two integers. This is particularly useful when the size of the data is large.

The optional argument lighting (boolean variable) changes the use of colors: instead of selecting shades from the $Z$ values, they come from the lighting of the surface. In this case, it is recommended to select the grey colormap from the msColormap routine. Note also that this option is applicable only when X and X coordinates are provided.

When arguments X and Y are also provided, then their values are used as coordinates for the data points in Z :

- usually, X and Y are matrices and their shape must be the same as that of the data matrix Z . Use of these coordinates is required when, e.g., you want to swap the indices $i$ and $j$, or apply a transformation (linear or not) between the $(i, j)$ indices and the $(x, y)$ coordinates; these coordinate matrices may be created by the msMeshGrid routine.
- to economize memory, it is possible to pass only vectors: in this case, X and Y , provided as rank1 mf Arrays, must obey to the same constraint as edicted for the generator vectors (see again msMeshGrid).

Be aware that X and Y are not used to defined the axes of the plot: to do so, you must use the msAxis routine with appropriate argument(s).

The color axis may be set explicitly by the user via the msCAxis routine; otherwise it will be chosen automatically by the routine itself. In the color axis manual mode, data should be theoretically ranged entirely inside the color axis; on the contrary, the resulting behavior depends on the shading value (cf. msShading). When shading is equal to "flat", a quiet behavior is obtained and the colormap overflow can be monitored by use of the msSetColorOverflowPolicy routine; when shading is equal to "interp", a warning is emitted by the library because strange colors can be drawn. This latter case should be avoided if possible.

Note that a colormap must have been defined before the call, (via msColormap); otherwise, a warning is emitted and strange or unexpected results can occur.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks.

- The drawing is always clipped at the viewport.
- The function mfPColor has the same argument list; it returns the (integer) handle of the created graphic object.
- For making a pseudo color plot on a triangle (or a triangular patch), use the msTriPColor routine.
- NaN values are not (yet) allowed: an error is returned by the routine.

See also: msPatch, msContour, msImage, msQuiver

## mf/msContour

## data contouring

## Calling syntax:

```
call msContour( [ mfOut(C), X, Y,] Z
&
[, nb_levels, levels, order, linespec, linewidth,
&
    labels, labelscolor, labelsize ] )
```


## Description:

Draws a contour plot of the data contained in the mfArray Z. The plot is done in the current figure. As opposed as msPColor routine, msContour accepts only vertex-centered data.

It doesn't fill the regions between the contours with colors; for such a feature, see the $\mathrm{mf} / \mathrm{ms}$ ContourF routine.
$(i, j)$ indices of Z are mapped to the axis $(-Y, X)$, so data visualisation is coherent with the matrix layout and with other routines working with matrix data (msPColor, msQuiver, msStreamline). If this orientation is not appropriate, use the coordinates (mfArrays X and Y), as described below.

When arguments X and Y are also provided, then their values are used as coordinates for the data points in Z :

- usually, X and Y are matrices and their shape must be the same as that of the data matrix Z . Use of these coordinates is required when, e.g., you want to swap the indices $i$ and $j$, or apply a transformation (linear or not) between the $(i, j)$ indices and the $(x, y)$ coordinates; these coordinate matrices may be created by the msMeshGrid routine.
- to economize memory, it is possible to pass only vectors: in this case, X and Y , provided as rank1 mfArrays, must obey to the same constraint as edicted for the generator vectors (see again msMeshGrid).

If the optional $C$ mfArray is given in front of ( $\mathrm{X}, \mathrm{Y}$ ) coordinate matrices, then it will contain on output a two-row matrix storing the contour lines. Each contiguous drawing segment contains the value of the contour, the number of $(x, y)$ drawing pairs, and the pairs themselves. The segment are appended end-to-end as

```
C = [ level_1, x_1, x_2, ..., level_2, x_1, x_2, ...;
    pairs_1, y_1, y_2, ..., pairs_2, y_1, y_2, ...]
```

Be aware that these point coordinates must be interpreted in different ways, according the value of order (see below).

The optional (integer) argument nb_levels specifies the number of contours to be drawned; it cannot be used together with levels.

The optional argument levels, which must be a rank-1 mfArray, is used to specify the level values; it cannot be used together with nb_levels.

By default, the number of level curves is equal to 9 , and the level values are equally spaced within the min and max values of the data, or within thoses of the Color Axis if it has been set before (see msCAxis); this can be changed by using the optional arguments nb_levels and levels (see above). A warning is emitted if a level is outside the Color Axis, excepted when the color is prescribed in linespec.

By default, the level curves are labelled (use labels=.false. to remove the labels) and the contour curves are colored using the current colormap (use a color inside linespec to specify the color of all curves). Labels are displayed in the foreground color (e.g. black on white) by default; the user may choose another color via the labelscolor optional argument. Moreover the label size may be changed by using the optional labelsize real argument, which lets the user to specify a percentage of the standard size (default is labelsize=1.0d0).

The optional argument order is an integer (1 or 2) specifying the quality of the approximation of the contour lines. Default is order $=1$ : in this case, segments are straight lines. If order is set to 2 then a bilinear interpolation is used inside the cells, and segments are constituted by quadratic Bezier arcs. Note that in all cases, the approximation provides only continuity at the cell boundaries, therefore the tangent lines may appear as broken. Note also that the default case (order $=1$ ) is well adapted for most situations; only for situations where you have a small number of cells, or if you want to strongly zoom inside a contour map, you would choose order $=2$.

Optional arguments linespec (character string) and linewidth (real value) allow the user to change the appearance of the contour curves. Possible values for these arguments are described in msSetGrObj, under the "color", "linestyle" and "linewidth" entries. If a color has been specified, then all contour lines will be drawn with the same color. Be aware, however, that the choice of markers is disabled.

Note that a colormap (see msColormap) must have been defined before the call if a specific color is not present in linespec; otherwise, a warning is emitted and strange or unexpected results can occur.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- The drawing is always clipped at the viewport.
- The function mfContour has the same argument list; it returns the (integer) handle of the created graphic object.
- For making contours on a triangle (or a triangular patch), use the msTriContour routine.
- NaN values are not (yet) allowed: an error is returned by the routine.

See also: mf/msContourF, mf/msImage, mf/msQuiver

## mf/msContourF

## data contouring with filled regions

Calling syntax:

```
call msContourF( [ mfOut(C), X, Y,] Z
&
[, nb_levels, levels, linewidth,
&
labels, labelscolor, labelsize ] )
```

Description:
Similar to the msContour routine, except that the regions between the contours are filled with appropriate colors.

The whole region whose values are greater than the level is filled with the corresponding color. As a consequence, the region less than all specified levels is left in white. To avoid white regions, it is sufficient to add a level less than or equal the smaller value in the Z array.

Be aware however that some arguments of msContour are not available here.
See also: mf/msTriContourF

## Calling syntax:

```
call msQuiver( [ X, Y,] u, v &
    [, data_centering] [, color] [, arrow_length] [, step] &
    [, linewidth] [, arrow_head] )
```

Draws the vector field whose 2D components are stored in the $u$ and $v$ mfArrays, located at position ( $\mathrm{X}, \mathrm{Y}$ ) or whows location is deduced from the layout of u and v when X and Y are not present. The plot is done in the current figure.
$(i, j)$ indices are mapped to the axis $(-Y, X)$, so data visualisation is coherent with the matrix layout and with other routines working with matrix data (msContour, msPColor, msStreamline). If this orientation is not appropriate, use the coordinates (mfArrays X and Y ), as described below.

When arguments X and Y are also provided, then their values are used as coordinates for the data points in $u$ and $v$ :

- usually, X and Y are matrices and their shape must be the same as those of the data matrices $u$ and v . Use of these coordinates is required when, e.g., you want to swap the indices $i$ and $j$, or apply a transformation (linear or not) between the $(i, j)$ indices and the $(x, y)$ coordinates; these coordinate matrices may be created by the msMeshGrid routine.
- to economize memory, it is possible to pass only vectors: in this case, X and Y , provided as rank1 mf Arrays, must obey to the same constraint as edicted for the generator vectors (see again msMeshGrid).

Be aware that X and Y are not used to defined the axes of the plot: to do so, you must use the msAxis routine with appropriate argument(s).
u and v must be rank-2 (i.e. matrices) mfArrays of same shape. They represent respectively the $X$ and $Y$ components of the vector field to be drawn. If the orientation or the scaling is not appropriate, use the second form of call described below, by adding coordinates.

The optional data_centering argument specifies how the data are stored in the matrix; it is a string which must be equal to "vertex" (default value) or "cell".

The optional color argument specifies the color used for drawing the arrows: it is a one-letter color symbol or an escaped sequence, as described in the msSetGrObj routine.

The optional arrow_length argument (real) specifies the maximum length of the arrow field as a multiplicative factor applied to the mesh size; by default its value is 1 so that each arrow is inside the rectangular cell where its center is located. A value much less that 1 is useless, whereas a big value is not recommended.

The optional step argument (integer) the step in the loops over the $(i, j)$ indices to display the arrow; the default value is 1 . It is useful to avoid a too big number of arrows for high resolution meshes.

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1 . The line width is constant and is the same for all arrows, whatever their length is.

The optional arrow_head argument (real) specifies the size of the arrow head, based on the character height; by default its value is 1 .

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- The drawing is always clipped at the viewport.
- An arrow is not drawn if any component of the corresponding vector (u, v) is equal to $N a N$.
- The function mfQuiver has the same argument list; it returns the (integer) handle of the created graphic object.
- For drawing a 2D vector field on a triangular mesh, use the msTriQuiver routine.
- When specifying a color, the routine msColormap should be used before for definition of the colormap.

See also: mf/msContour, mf/msPColor
mf/msStreamline

## Calling syntax:

```
call msStreamline( [ X, Y,] u, v, start [, direction, n_arrow]
&
[, color] [, linewidth] [, arrow_head] [, npt_max] &
[, curv_tol] )
```

Based on a velocity vector field whose $x$ - and $y$-components are respectively u and v (mfArrays), draws streamlines in the current figure. There is no restriction about the velocity field: it may have a non-zero divergence, or a non-zero curl.

The streamlines are specified by the 2D-point coordinates stored in the start mfArray, which may have any number of lines, but exactly two columns. The integration always begins at the starting point, in two possible direction: forward and/or backward (see below the explanation about the direction optional argument). Usually, the end-points of a streamline are located at the boundary of the domain, but in some rare cases (velocity field having a non zero divergence) they can end inside the domain, at a stationnary point.
$(i, j)$ indices are mapped to the axis $(-Y, X)$, so data visualisation is coherent with the matrix layout and with other routines working with matrix data (msContour, msPColor, msQuiver). If this orientation is not appropriate, use the coordinates (mfArrays X and Y ), as described below.

When arguments X and Y are also provided, then their values are used as coordinates for the data points in $u$ and $v$ :

- usually, X and Y are matrices and their shape must be the same as those of the data matrices $u$ and v . Use of these coordinates is required when, e.g., you want to swap the indices $i$ and $j$, or apply a transformation (linear or not) between the $(i, j)$ indices and the $(x, y)$ coordinates; these coordinate matrices may be created by the msMeshGrid routine.
- to economize memory, it is possible to pass only vectors: in this case, X and Y , provided as rank1 mf Arrays, must obey to the same constraint as edicted for the generator vectors (see again msMeshGrid).

Be aware that X and Y are not used to defined the axes of the plot: to do so, you must use the msAxis routine with appropriate argument(s).
u and v must be rank-2 (i.e. matrices) mfArrays of same shape. They represent respectively the $X$ and $Y$ components of the vector field to be drawn. If the orientation or the scaling is not appropriate, use the second form of call described below, by adding coordinates.

The optional argument direction (character string) specifies whether the integration must be done following the flow direction ("forward"), against the flow direction ("backward") or in both direction ("both"); this latter case is the default.

The optional n_arrow argument specifies the number of arrow heads to be drawn at equally curvilinear abscissa along each streamline. It must be an integer greater or equal one (default value is 3 ).

The optional color argument specifies the color used for drawing the streamlines; it has the same meaning as in the routine msPlot.

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1 .

The optional arrow_head argument (real) specifies the size of the arrow head; by default its value is 1 .

The optional npt_max is the maximum number of points for each half-streamline (indeed, you can ask for a forward integration, a backward one or both - see above); its default value is 1000 .

The optional curv_tol argument concerns the integration process and more specifically the tolerance for high curvature detection; it is relative to the mesh size; its default value is 0.5 .

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- The drawing is always clipped at the viewport.
- The function mfStreamline has the same argument list; it returns the vector of the (integer) handles of the created graphic objects. Be aware that even for a single streamline, the returned handle is a vector (of size 1 ).
- For drawing 2D streamlines on a triangular mesh, use the msTriStreamline routine.
- NaN values are not allowed: an error is returned by the routine.

See also: mf/msQuiver

First interface:

```
call msPatch( x, y, c [, opacity] [, grid, grid_color] )
```


## Description:

This routine draws one colored polygonal shape from the mfArrays $x$, $y$ and $c$. These vectors contain respectively the coordinates of each vertex and the attached color. The polygonal shape doesn't need to be closed, e.g. a triangle may be defined only using three points.

The drawing is clipped at the viewport.
Before use, the color axis must be explicitly set by the user via the msCAxis routine. Theoretically, data should be ranged entirely inside the color axis; on the contrary, the resulting behavior depends on the shading value (cf. msShading). When shading is equal to "flat", a quiet behavior is obtained and the colormap overflow can be monitored by use of the msSetColorOverflowPolicy routine; when shading is equal to "interp", a warning is emitted by the library because strange colors can be drawn. This latter case should be avoided if possible.

If the optional argument opacity is present, transparency can be used: opacity must be a real number between 0 (full transparency, invisible object) and 1 (full opacity).

A thin line may be drawn to show the boundary of the polygonal cell by using the optional argument grid equal to TRUE. The default is to not show the boundary. The color of this thin line may be specified in the optional argument grid_color; by default, it is the foreground color.

A colormap must have been defined before the call, (via msColormap); otherwise, a warning is emitted and strange or unexpected results can occur.

## Second interface:

```
call msPatch( x, y [, color] [, opacity] [, grid, grid_color] [, clipping] )
```


## Description:

This second form fills the shape with a flat color (if specified), avoiding the use of an array for specifying the color. The color argument has the same meaning as in the routine msPlot.
grid and grid_color are described above for the first interface.
The optional argument clipping is a boolean (default value is $T R U E$ ). When set to $F A L S E$, it allows the drawing of a polygonal colored shape (only using flat color) outside the viewport. This facility is especially useful when the user wants to update a text string outside the viewport: due to the use of antialiased text drawing, he must erase before the ancient text with a rectangular patch of appropriate color.

## Remarks:

- the function mfPatch has the same argument list; it returns the (integer) handle of the created graphic object.
- this routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects of the figure, the Hold property must be set to "on" via the msHold routine.

See also: mf/msPColor

## data histogram

Calling syntax:

```
handle = mfPlotHist( x, x_min, x_max, n_bin
            [, color] [, filled] )
```

returns the (integer) handle of the newly created graphic object.
The vector mfArray $x$ contains the data. The histogram is computed for data ranged from x_min to x _max using a number of bins equal to n _bin.

If present, color specifies the color for the bar plot. It may be a character string containing the usual color code (see msPlot), a real triplet containing the RGB values or a string containing a color name from the RGB database. If the color is left unspecified, then it is taken from the current colortable (for more information about colortables - or color schemes - see the Muesli User's Guide).

If present, the logical argument filled specifies that bars are filled with the color used (default is FALSE).

The subroutine form:

```
call msPlotHist( mfOut( num [, x_bin] ), x, x_min, x_max, n_bin
    [, color] [, filled] )
```

allows the user to keep some internal data.
The optional arguments color (\& Co.) and filled have the same meaning as above.
The output mfArray num is the vector of the bins values.
If present, the mfArray x_bin is the vector containing the abscissas of the bins.

## Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- This routine may be called many times in the same figure with different data: histograms will be superposed one above the other, showing cumulative statistics. When not specified, colors will be recycled from the current colortable. Note that if your data is stored inside a two-dimensional array, then it may be easier to call the msCumulHist routine instead.

See also: msHist, mfOut

## msCumulHist

## Interface:

```
subroutine msCumulHist( X, x_min, x_max, n_bin, x_nb )
type(mfArray), intent(in) :: X
real(kind=MF_DOUBLE), intent(in) :: x_min, x_max
integer, intent(in) :: n_bin
type(mfArray), optional :: x_nb
```


## Description:

Cumulative Histogram. Works on columns of the mfArray X.
If X has NCOL columns, draws first an histogram (via msHist) from the data of all columns, then draw a second histogram from the data of the NCOL-1 first columns, and so on. Each histogram is drawn in a different color, via the colors of the current colortable (for more information about colortables - or color schemes - see the Muesli User's Guide); if NCOL is greater than the maximum, following colors are cycled in the same set.

Columns of X may have been initialized from data which contain different number of element. In such a case, the optional argument $\mathrm{x} \_\mathrm{nb}$ is a vector which contains the useful number of elements in each column.

## Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

See also: msHist, mf/msPlotHist

## Interface:

```
call msImRead( mfOut(A,cmap), filename [, fmt, indexed] )
```


## Description:

Reads the image file filename: the colormap is extracted from the image and stored in the mfArray cmap and the pixels values are stored in the mfArray A.

The image can be displayed by using first msColormap and then msImage.
The optional argument fmt, if present, is a string which must contain the image format. Common accepted values for fmt are: "TIFF", "JPEG", "PNG", "XPM", ... . Actually, in the current version of MUESLI (but this should change in future), the file image is always converted in the XPM format via the 'convert' command of 'ImageMagick'. So, a great number of image formats is supported and, moreover, the argument fmt is not yet used for reading.

There are two options to fill the mfArray A, i.e. to give numerical values to pixels (the optional logical argument indexed is by default false):

1. real valued pixels - each pixel of the array contains a numerical real value (ranged in [0.,1.]), and a linear mapping is used between the whole range of these values and the index range of the colormap. This first way is rather oriented to the numerical processing of the pixels value, and is well adapted to image obtained via the pseudo colors from any numerical matrix.
For example, you may apply some filters or compute the gradient of the pixels values and you can visualize the image in different ways by changing the colormap, which is always a continuum of colors.
2. indexed pixels - each pixel of the array contains the index of the colormap.

This second way is more adapted to digital photos, or to color bitmaps which contain transparency (as for some PNG images - see below).
You should not modify the whole colormap, else you may obtain some strange results when you visualize the image. In contrast, you can easily remove or add some colors in the colormap.

## Remarks about transparent pixels:

If the original image contains transparent pixels ("NONE" color in an XPM file, or transparency in PNG files), then it cannot be opened using the default indexing scheme: the current routine will give an error. You can open it only via the "indexed pixels" scheme. Moreover, these transparent pixels will be colored in medium gray by the msImage routine.

## Limitations:

Currently, these limitations are linked to the XPM format:

- original XPM images having more than two characters per color are not supported (to check yourself, open the XPM file with any text editor: the fourth integer number of the third line must be equal to 1 or 2 ). Moreover, they must have less than 8281 colors.
- for original XPM images, they must have maximum 12000 pixels in width if colors are stored in one char (i.e. when the number of colors is small, less than 91 ), or 6000 pixels in width if colors are stored in two chars (i.e. when the number of colors is greater than 91).
- for images converted in XPM by the 'convert' tool of 'ImageMagick', the number of colors is always less than 256. So, the quality of true-color images is pretty degraded and, at least up to the version 6.9.3 of 'ImageMagick', the picture is darker.

Other interface:
call msImRead ( mfOut(infos), filename [, fmt], only_infos=.true. )
allows the user to obtain some information about the image (currently, only the width and the height in pixels unit), which are returned in the mfArray infos.

See also: msImWrite, mfOut

## msImWrite

## Interface:

```
call msImWrite( A, cmap, filename [, fmt, indexed] )
```


## Description:

Writes the mfArray A to the image file filename, using the colormap cmap.
The optional argument fmt, if present, is a string which must contain the image format. Common accepted values for fmt are: "TIFF", "JPEG", "PNG", "XPM", ... . Actually, in the current version of MUESLI (but this should change in future), the file image is always converted in the XPM format via the 'convert' command of 'ImageMagick'. So, a great number of image formats is supported.

The optional logical argument indexed (its default value is false) has the same meaning as those explained in the msImRead routine. It concerns obviously the mfArray A, not the image stored in the image file filename.

## Remarks about transparency:

- if the mfArray image contains transparent pixels, consider using an appropriate image format (XPM or PNG) with supports this transparency.
- transparency can be set in your image by two ways:

1. for the indexed pixels indexing scheme, set a color (in cmap) to the triplet ( $N a N, N a N, N a N$ ).
2. for the real valued pixels indexing scheme, set an element of the image mfArray A to $N a N$.

## Limitations:

- XPM images must have less than 8281 colors in their colormap.
- XPM images must have maximum 12000 pixels in width if colors are stored in one char (i.e. when the number of colors is small, less than 91 ), or 6000 pixels in width if colors are stored in two chars (i.e. when the number of colors is greater than 91 ).

See also: msImRead, mf/msImage

## $\mathrm{mf} / \mathrm{ms}$ Image

## image display

## Interface:

```
call msImage( image [, angle] [, flip] [, indexed] )
```


## Description:

Displays in the current figure the mfArray image, associated with the appropriate colormap previously set by msColormap.
$(i, j)$ indices of the image are mapped to the axis $(-Y, X)$, so data visualisation is coherent with Contour and PColor.

The optional real argument angle is used to define the orientation in degrees of the image (default is 0 degree, acceptable values are any multiple of 90: $90,180,270$, but also negative values such as -90 ).

The optional argument flip is a string can take the following values: "NONE", "HORIZ" (or mirror, left/right swap), "VERT" (or upside/downside swap); default value is "NONE".

The optional logical argument indexed (its default value is $F A L S E$ ) has the same meaning as those explained in the msImRead routine. It tells the current routine that the image mfArray contains integer indices of colors and not real values.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- The drawing is always clipped at the viewport.
- The function mfImage has the same argument list; it returns the (integer) handle of the created graphic object.

See also: mf/msContour, mf/msPColor

## Description:

This routine plots a PSLG domain and, optionally, shows the numbering of nodes, edges and holes.

Calling syntax:

```
    call msPlotPSLG( PSLG_domain [, color] [, linewidth] &
    [, nod_num] [, edg_num] [, hol_num] )
```

PSLG_domain, of type mfPSLG, represents the 2D domain definition. It must be built by the user.
The optional color argument specifies the color used for drawing the cells: it may be a character string or a vector of type real and length 3 (see $\mathrm{mf} / \mathrm{msPlot}$ ).

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1 .

The three last optional arguments nod_num, edg_num and hol_num, all booleans, are used to display respectively nodes', edges' and holes' numbering of the PSLG. By default, numbering is not displayed.

Be awre that only nodes involved in segments are drawn; unused nodes are therefore ignored. To see all registered nodes, use msPrintPSLG.

## Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- The function mfPlotPSLG has the same argument list; it returns the (integer) handle of the created graphic object.


## Description:

This routine plots a triangular mesh and, optionally, shows the numbering of triangles, nodes and faces.

First calling syntax:

```
call msTriMesh( x, y, tri [, color] [, linewidth] [, height]
&
[, tri_num] [, nod_num] [, boundary_only] )
```

The $m f A r r a y s \mathrm{x}$ and y are vectors of same length which describe the position $(x, y)$ of the nodes. The mfArray tri contains the triangulation (i.e. the triangles' indices, as returned by, e.g., the routine mfDelaunay).

The optional color argument specifies the color used for drawing the cells: it has the same meaning as in other plotting routine involving colors (see the msSetGrObj routine).

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1 .

The optional height argument (real) specifies, for the numbering of triangles, nodes and faces, the character height, as a relative factor; by default its value is 1 .

The two last optional arguments tri_num and nod_num, both booleans, are used to display triangles' and nodes' numbering of the triangular mesh. By default, numbering is not displayed.

When the optional boolean argument boundary_only is used with the value TRUE then only the boundary of the mesh is drawn. In this case, the triangles' numbers are never displayed, whatever the optional argument tri_num is.

## Second calling syntax:

```
call msTriMesh( tri_connect, ... [, fac_num] )
```

The first argument is tri_connect of type mfTriConnect, the mesh connectivity (see msBuildTriConnect); it replaces the three first mfArrays in the first calling syntax.

Same optional arguments as in the first calling syntax may be added. Note however that an additional optional argument is possible in this second case: the boolean fac_num concerns the numbering of the triangle faces.

This second use of msTriMesh uses a fast algorithm to draw the mesh; the printed files (both EPS and PDF) are also smaller in size.

## Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- The function mfTriMesh has the same argument list; it returns the (integer) handle of the created graphic object.

See also: msPrintTriConnect

```
mf/msTriPColor
```


## pseudo-color on a triangulation

Calling syntax:

```
call msTriPColor( x, y, z, tri )
```


## Description:

This routine is equivalent to the routine msPColor, but for data which are spread on a triangulation.
The mfArrays $\mathbf{x}, \mathrm{y}$ and $\mathbf{z}$ are vectors of same length which describes position $(x, y)$ of the vertices and the corresponding function value.

The mfArray tri contains the triangulation (i.e. the triangle's indices, as returned by, e.g. the routine mfDelaunay).

Remarks:

- The drawing is always clipped at the viewport.
- Use msTriMesh to show the grid or the boundary of the triangular mesh.
- The function mfTriPColor has the same argument list; it returns the (integer) handle of the created graphic object.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- NaN values are not (yet) allowed: an error is returned by the routine.

See also: mf/msTriContour, mf/msTriFill, msPatch

## mf/msTriFill

## coloring on a triangulation

Calling syntax:

```
call msTriFill( x, y, val, tri )
```


## Description:

This routine is equivalent to the routine msPatch (when using flat colors), but for data which are spread on a triangulation.

The $m f$ Arrays x and y are vectors of same length which describes position $(x, y)$ of the nodes.
The value to be colored is stored inside the vector mfArray val. This latter vector must have a length equal to the number of triangles. The mfArray val may contain $N a N$ values: in such a case the corresponding triangles are not drawn.

The mfArray tri contains the triangulation (i.e. the triangle's indices, as returned by, e.g. the routine mfDelaunay).

## Remarks:

- The drawing is always clipped at the viewport.
- Use msTriMesh to show the grid or the boundary of the triangular mesh.
- The function mfTriFill has the same argument list; it returns the (integer) handle of the created graphic object.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

See also: mf/msTriContour, mf/msTriPColor

## Description:

This routine is equivalent to the routine msContour, but for data which are spread on a triangular grid.
It doesn't fill the regions between the contours with colors; for such a feature, see the $\mathrm{mf} / \mathrm{msTriContourF}$ routine.

First calling syntax:

```
call msTriContour( [ mfOut(C),] X, Y, Z, tri &
    [, nb_levels, levels, linespec, linewidth, &
        labels, labelscolor, labelsize ] )
```

The mfArrays $\mathrm{X}, \mathrm{Y}$ and Z are vectors of same length which describes position $(x, y)$ of the nodes and the corresponding function value $z$.

The mfArray tri is the triangulation (e.g. returned by the routine mfDelaunay).
If the optional C mfArray is given in front of ( $\mathrm{X}, \mathrm{Y}$ ) coordinate matrices, then it will contain on output a two-row matrix storing the contour lines. Each contiguous drawing segment contains the value of the contour, the number of $(x, y)$ drawing pairs, and the pairs themselves. The segment are appended end-to-end as

```
C = [ level_1, x_1, x_2, ..., level_2, x_1, x_2, ...;
    pairs_1, y_1, y_2, ..., pairs_2, y_1, y_2, ...]
```

By default, the number of level curves is equal to 9 , and the level values are equally spaced within the min and max values of the Color Axis (see msCAxis); this can be changed by using the optional arguments nb_levels and levels (see below).

The optional (integer) argument nb_levels specifies the number of contours to be drawned; it cannot be used together with levels.

The optional argument levels, which must be a rank-1 mfArray, is used to specify the level values.
The optional arguments linespec (character string) and linewidth allow the user to change the appearance of the contour curves. The possible values for these arguments are described in msSetGrObj, under the "color", "linestyle" and "linewidth" entries.

By default, the level curves are labelled (use labels=.false. to remove the labels) and the contour curves are colored using the current colormap (use a color inside linespec to specify the color of all curves). Labels are displayed in the foreground color (e.g. black on white) by default; the user may choose another color via the labelscolor optional argument. Moreover the label size may be changed by using the optional labelsize real argument, which lets the user to specify a percentage of the standard size (default is labelsize=1.0d0).

## Second calling syntax:

```
call msTriContour( [ mfOut(C),] Z, tri_connect, [ ... ] )
```

Here, tri_connect (of type mfTriConnect) is the mesh connectivity (see msBuildTriConnect); it replaces the three mfArrays X, Y and tri in the first calling syntax. The other optional arguments are the same.

## Remarks:

- The triangular mesh used doesn't need to be convex, and may also have holes. See mfPSLG to create triangulation with holes.
- The drawing is always clipped at the viewport.
- Use msTriMesh to show the grid or the boundary of the triangular mesh.
- The function mfTriContour has the same argument list; it returns the (integer) handle of the created graphic object.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- $N a N$ values are allowed: all triangles involved by these special non finite values are simply ignored.

See also: mf/msContourF, mf/msTriPColor, mf/msTriFill

## mf/msTriContourF

## data contouring on triangles with filled regions

First calling syntax:

```
call msTriContourF( [ mfOut(C),] X, Y, Z, tri
&
[, nb_levels, levels, linewidth,
&
labels, labelscolor, labelsize ] )
```

Second calling syntax:

```
call msTriContourF( [ mfOut(C),] Z, tri_connect, [ ... ] )
```


## Description:

Similar to the msTriContour routine, except that the regions between the contours are filled with appropriate colors.

The whole region whose values are greater than a specified level is filled with the corresponding color. As a consequence, the region less than all specified levels is left in white. To avoid white regions, it is sufficient to add a level less than (or equal to) the smaller value in the Z array.

Be aware however that some arguments of msTriContour are not available here.
Lastly, be warned that the function version (i.e. mfTriContourF) returns a vector of handle(s), even if the number of handles is one. Indeed, under some circumstances, the whole triangular mesh may be splitted in many independant zones when the number of NaN values is great enough.

See also: mf/msContourF

Calling syntax:

```
call msTriQuiver ( x, y, u, v \&
    [, color ] [, arrow_length ] [, arrow_head ] )
```

This routine is equivalent to the routine msQuiver, but for data which are spread on a triangulation.
$\mathrm{x}, \mathrm{y}, \mathrm{u}$ and v must be rank-1 (i.e. vectors) mfArrays having the same number of rows. They represent respectively the $X$ and $Y$ coordinates of the points where the components $(u, v)$ of the vector field have to be drawn.

The optional color argument specifies the color used for drawing the arrows: it is a one-letter color symbol or an escaped sequence, as described in the msSetGrObj routine.

The optional arrow_length argument (real) specifies the length of the arrow as a multiplicative factor; by default its value is 1 so that the biggest arrow has approximately a length equal to $1 / 10$ th the full axis range.

The optional arrow_head argument (real) specifies the size of the arrow head; by default its value is 1 .

## Remarks:

- The drawing is always clipped at the viewport.
- An arrow is not drawn if any component of the corresponding vector (u, v) is equal to $N a N$.
- The function mfTriQuiver has the same argument list; it returns the (integer) handle of the created graphic object.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

See also: mf/msTriPColor, mf/msTriStreamline

Calling syntax:

```
call msTriStreamline( x, y, u, v, start [, tri | tri_connect]
[, direction, n_arrow] [, color]
    [, linewidth, arrow_head] [, npt_max] [, curv_tol] )
```

This routine is similar to the routine msStreamline, but for data which are spread on a triangulation. In particular, the arguments direction, n_arrow, color, linewidth, arrow_head, npt_max and curv_tol have exactly the same meanings.

The optional arguments tri and tri_connect (see msBuildTriConnect) describe respectively the triangulation of the mesh and its connectivity. If they are both absent, then the triangulation and its connectivity are done by the routine itself. If the user already knows the connectivity, it is preferable to pass it to the current routine, to avoid a redundant computation. On the other hand, if the user knows only the triangulation, it should pass it as argument, and the current routine will compute only the connectivity.

Remarks:

- The drawing is always clipped at the viewport.
- The function mfTriStreamline has the same argument list; it returns the vector of the (integer) handles of the created graphic objects. Be aware that even for a single streamline, the returned handle is a vector (of size 1).
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- NaN values are not allowed: an error is returned by the routine.

See also: mf/msTriQuiver

## Calling syntax:

```
call msSpy( A [, symbolspec, bitmap, continuous,
color_scale, show_nnz, nz_threshold ] )
```

draws non-zero elements of the mfArray A (usually sparse).
If the optional argument symbolspec (string) is present, it must contain the specification for a symbol and/or a color, as described in msSetGrObj at marker and color entries. By default, a star ("*") is used, but note that it is well adapted only for small size matrices: for big matrices, a dot (".") may be prefered.

If the optional argument continuous (logical) is present, the matrix A is drawn using a color/grey scale for the absolute value of each element, as in msPColor. If you intend to spy a dense matrix, the continuous argument must be used. Default value is .false.
In the continuous case:

- if the optional argument color_scale (string) is present, it must be equal to "lin" or "log". The latter case specifies the use of a logarithmic color/grey scale in the mapping between the value of the matrix elements and the color/grey index (default mapping is linear);
- if the optional argument nz_threshold (real) is present, it specifies the threshold value under which an element is considered as zero and, thus, not displayed. The default value is zero for the linear scale and $\max \left(\left|A_{i, j}\right|\right) \times \epsilon$ for the logarithmic scale. The threshold for small values is used for defining the color map range.
By setting nz_threshold to the special value -1 , the threshold will be set to the minimum value of the elements of $|A|$.

If the optional argument show_nnz (logical) is present, the effective number of non-zero element is shown in the X -label. It's default value is '.true.'.

If the optional argument bitmap (character string) is present, it must contains the bitmap size (in pixels) which will be used for printing. The string reads under the form "WIDTHxHEIGHT" (e.g. "3000x3000"). This option is useful only for large, sparse matrices.

## Remarks:

- The drawing is always clipped at the viewport.
- The function mfSpy has the same argument list; it returns the (integer) handle of the created graphic object.
- The continuous and symbolspec options are exclusive.
- The grey scale mentionned above is the default colormap when none was selected by the user. Please use the msColormap routine to select another one, prior to the msSpy call.


### 2.6 Interactive routines

Caution: the routines available in this chapter require the selection of the X11 driver (which is usually selected by default). In batch mode, where the user selects the $N U L L$ driver (see msSetX11Device), these routines will not be available, and an error will occur if the user tries to call any of them.

A figure must be also opened (see mf/msFigure).
If you have multiple workspaces, switching from one workspace to another may breaks the interaction of the mouse with your Muesli program; therefore, it is recommended to stay in the same workspace during all the duration of the interactive process.

Some routines (msPan, msZoom and msPanAndZoom) accept the use of an additional keyboard key (usually the CTRL one) to modify the action. In such a case, it is required that the figure's window have the focus; on the contrary, the use of the key do nothing.

| mfGinput, msGinput | graphic input |
| :--- | :--- |
| mfGinputCustom |  |
| mfGinputRect |  |
| mfGetModKeys | rectangular graphic selection <br> modifier keys state |
| msPan | manual scrolling |
| msZoom |  |
| msPanAndZoom | figure zoom <br> combine scrolling and zooming |
| msMoveLegend | adjust legends frame in axes <br> msMoveGrObj graphic object |
| msAnimation | screen animation mode <br> msShowNow <br> msCla |
| screen update during an animation <br> clear current axes |  |
| msSetGBuffer | graphic buffer policy <br> msBBuf, msEBuf <br> msDefineCustomCursors | | begin and end of graphic buffering |
| :--- |
| define custom cursors for use with mfGinputCustom |

See also:
Global graphic settings
Window's and figure's management
Figure properties
Figure annotation - Low level graphic object's manipulation
High level plotting routines
mfGinput

## Interface:

```
function mfGinput( event, key ) result ( out )
    logical, intent(in), optional :: event, key
    type(mfArray) :: out
```


## Description:

This function is an interactive facility to get the pointer coordinates when it is inside the current figure.
Usually, this function is called without the optional event and key argument (by default, event is $T R U E)$ : the library waits for a mouse click or a key pressed, and the cursor show a cross shape in the current active window.

The returned mfArray contains the coordinates $(x, y)$ of the selected point. The point can be chosen by any mouse button (which can be also emulated via the L, M or R key). Be aware that the current routine implements a two-event procedure and therefore the location returned is that of the pointer for the second event! (mouse up in the usual case).

For other key pressed, the routine mfGinput terminates and leaves the returned mfArray empty.

## Remarks:

- this routine is usually employed for graphic interaction. If the user doesn't have a mouse, the pointer can be moved via the keyboard's arrows (the shift key accelerates the move by a factor of 10); moreover each mouse button can be emulated by using an equivalent sequence ( $\mathrm{L}, \mathrm{M}$ or R followed by any other key).
- see the msGinput routine for more features.

When called with the event argument equal to $F A L S E$, the library doesn't wait for a user event: it returns immediately the pointer coordinates in the mfArray out. A third element is added to out, indicating whether the pointer is inside the window (value 1 ) or outside (value 0 ); the user must use this flag to take any decision in its own program.

Moreover, if the key is present and equal to $T R U E$, then a fourth element is returned in out: the ASCII code of the key pressed recently.

See also: mfGinputRect
msGinput
graphic input

Calling syntax:

```
call msGinput( mfOut( coords, keycode [, color] )
[, whole_keyboard, rect_size, rect_inside_axes] )
```

\&

## Description:

This function is an interactive facility to get the pointer position inside the current figure. See mfGinput for a simpler use.

In case of multiple opened graphic windows, the cursor has a cross shape in the current active one instead of the classical pointer.

The returned mfArray coords contains the coordinates $(x, y)$ of the selected point. The point can be chosen either by the mouse (default behavior), or by a key pressed (see below).

The returned mfArray keycode contains an equivalent code for the event detected:

- If the optional argument whole_keyboard is FALSE (default behavior), then keycode contains the button number ( $\mathrm{left}=1$, middle $=2$, right $=3$ ), even if the equivalent keys ( $\mathrm{L}, \mathrm{M}$ or R ) have been used; Note that mfArray coords and keycode will be empty if any key different from L, M or R is pressed.
- On the contrary, keycode contains the value of the key pressed (actually ichar(key_pressed), where ichar is the standard Fortran function which returns the ASCII code of the letter). coords will be empty only if the ESCAPE key is pressed. Only when using keyboard, the mouse wheel can be detected. The values returned are ichar (8)=56 for wheel up, and ichar (2)=50 for wheel down (think that on a numeric keypad, 8 is for arrow up, and 2 is for arrow down).

If present, the returned mfArray color contains the RGB triple of the colored pixel just under the pointer.

Usually (i.e. when the optional argument rect_size is not present), the cursor shown on the screen has the shape of a big crosshair, whose length is that of the selected window; on the contrary, if the mfArray rect_size is present and not empty, it must contain the size (both width and height) of a rectangle which follows the pointer during its move. This rectangle is always centered around the pointer position. Moreover, if the logical optional argument rect_inside_axes is present and equal to TRUE, the rectangle drawn is constrained to be entirely inside the axes.

See also: mfGinputRect

## Description:

The first Calling syntax has no argument:

```
out = mfGinput( )
```

Returns in the mfArray out the coordinates of the pointer when the mouse has been clicked, and the case used (1 or 2):

1. without the Control key pressed, displaying the first custom cursor;
2. with the Control key pressed, displaying the second custom cursor.

The other Calling syntax implements a magnetic grid and has one argument:

```
out = mfGinput( magnetic_grid_rule )
```

The magnetic_grid_rule subroutine, provided by the user, must have the following interface:

```
subroutine magnetic_grid_rule( icase, x1, y1, x2, y2, valid )
    integer, intent(in) :: icase
    real(kind=MF_DOUBLE), intent(in) :: x1, y1
    real(kind=MF_DOUBLE), intent(out) :: x2, y2
    logical, intent(out) :: valid
end subroutine
```

where $\mathrm{x} 1, \mathrm{y} 1, \mathrm{x} 2, \mathrm{y} 2$ are world-coordinates. The $(\mathrm{x} 1, \mathrm{y} 1)$ is the pointer position in the figure, and the ( $\mathrm{x} 2, \mathrm{y} 2$ ), computed by the user, is where the user-defined cursor is displayed (icase is 1 or 2 , according the status of the Control Key, pressed or not, as mentioned above). If, for some reasons, no ( $\mathrm{x} 2, \mathrm{y} 2$ ) point can be computed, then the routine must set the last argument (valid) to $F A L S E$ and in this case the cursor will show its standard shape (i.e. the left arrow).

In this second calling syntax, four elements are returned in the mfArray out:

1. $x$-coordinate of the position clicked.
2. $y$-coordinate of the position clicked.
3. an integer indicating whether the Control Key has been pressed or not.
4. an integer giving the validity of the position; if the validity is zero, the position should be ignored.

Note: Before use, the msDefineCustomCursors routine must be called to define the two cursor shapes.

## mfGinputRect

## rectangular graphic selection

Interface:

```
function mfGinputRect() result ( out )
    type(mfArray) :: out
```

Description:
This function is an interactive facility to get the size and position of a rectangle drawn inside the current figure.

The returned mfArray contains the bounding box of the rectangle. This rectangle must be selected by the left mouse button only; for other key pressed, the routine mfGinputRect terminates and leaves the returned mfArray empty.

In case of multiple opened graphic windows, the cursor has a cross shape in the current active one, instead of the classical pointer.

See also: mfGinput

## mfGetModKeys

Interface:

```
function mfGetModKeys( ) result ( out )
    logical :: out(2)
```

Description:
This function checks the state of the keyboard for the Shift and the Control keys.
It returns an array of two booleans: the first one (resp. the second one) tells if one of the Shift keys (resp. the Control keys) was down when this routine was called.

## msPan

## manual scrolling

Calling syntax:

```
call msPan( )
```

Description:
Activates the 'pan' mode for scrolling in the current figure. It ends when the [ESCAPE] key is pressed.
Scrolling is monitored via the mouse (or the equivalent keyboard key):

- click-and-drag with the left button ('L' key) scrolls through the figure to a new view;
- middle-click does nothing;
- right-click ('R' key) resets the view to the initial one; if the [CTRL] key is pressed during the click (or the ' 6 ' key is pressed) then it centers the view with respect to the bounding box of all graphic objects.


## Remark:

Scrolling in the current figure should be used before putting annotations on the figure, otherwise, parts of graphic objects drawn outside the axes will not be moved with this routine; however, a call to msRedrawFigure will fix this problem.

See also: msZoom, msPanAndZoom

## msZoom

figure zoom

Calling syntax:
call msZoom( )

Description:
Activates the 'zoom' mode in the current figure. It ends when the [ESCAPE] key is pressed.
The zoom is monitored via the mouse:

- the left button is used to "zoom in" at the pointer location:
* a single click is used to enlarge the current view with a factor of 1.41 (or 1.19 if the [CTRL] key is pressed during the click);
* a click-and-drag zooms to the new selected area;
- a middle-click is used to "zoom out" at the pointer location (by a factor of 1.41 - or 1.19 if the [CTRL] key is pressed during the click);
- a right-click resets the view to the initial one; if the [CTRL] key is pressed during the click, then it sets the view to an area which shows all the graphic objects.

See also: msPan, msPanAndZoom

Calling syntax:
call msPanAndZoom( )

Description:
Activates both the 'pan' mode for scrolling and the 'zoom' mode for zooming in the current figure. It ends when the [ESCAPE] key is pressed.

Scrolling is monitored via the three mouse buttons:

- click-and-drag with the left button scrolls the figure to a new view;
- middle-click does nothing;
- right-click resets the view to the initial one; if the [CTRL] key is pressed during the click, then it sets the view to an area which shows all the graphic objects.

Zooming is monitored via the mouse wheel by a factor of 1.41 (or 1.19 if the [CTRL] key is pressed during the click). Note that the figure remains centered during this zooming process.

## Remark:

Scrolling in the current figure should be used before putting annotations on the figure, otherwise, parts of graphic objects drawn outside the axes will not be moved with this routine; however, a call to msRedrawFigure will fix this problem.

See also: msPan, msZoom

## msMoveLegend

## adjust legends frame in axes

Calling syntax:

```
call msMoveLegend( [ handle ] )
```


## Description:

This interactive routine allows the user to move the legend frame inside the figure via the mouse.
When a unique legends frame has been created by msLegend, the routine has no argument.
Use the optional argument to move a legends frame created by mfLegend and referenced by its handle.
Note that the legend frame may be located outside the axes; this may be useful in the case where a great number of curves are plotted in the figure. During the interactive move on the screen, the legend frame may appear truncated by the window boundary under X11; however, printing in EPS or PDF should give correct results.

The use of this routine could follow msPan, msZoom or msPanAndZoom in order to optimize its position.

## Interface:

```
subroutine msMoveGrObj( handle, prompt )
    integer, intent(in) :: handle
    character(len=*), intent(in), optional :: prompt
```


## Description:

Allows the user to move some types of graphic object, by interaction with the mouse (left button click only). It ends when the [ESCAPE] key is pressed. Supported graphic object types are: text (generated by mfText), arrow (generated by mfArrow) and polygon (generated by mfPatch using a flat color).

When this routine is called, it waits for a mouse down event (first part of a click) while the pointer have the shape of an opened hand. To grab a text or polygon object, you can click elsewhere inside its bounding box (displayed using a gray animated dashed line); for an arrow, you have three hotspots (sensitive area): one is the arrow base, the second is the middle and the third is the arrow head.

When a mouse down event is detected, the shape of the pointer becomes a closed hand and the graphic object (referenced by its handle) can be moved in the figure. Note that a text or polygon object can be only translated, without changing its orientation; however, for an arrow and according to the selected hotspot, you can change both its direction (when you have clicked on its base or its head) and its location (when you have clicked in its middle).
When a mouse up event is detected (end of a click), the pointer shape changes back to an opened hand.
The prompt argument lets you chosing an appropriate sentence on the screen. By default, the prompt is adapted to the type of graphic object:

```
text: -> Grab and move the text string surrounded by a dashed line...
arrow: -> Grab and move the arrow wearing three hotspots...
polygon: -> Grab and move the polygon surrounded by a dashed line...
```


## Remarks:

- Note that the graphic objects may be clipped at the axes framebox. If you move a graphic object out of the axes, it may no longer be visible; however, the dashed line(s) around it will be always visible and will be an appreciable help.
- An example of use of the msMoveGrObj routine can be found in the test program named MoveGrObj_test inside the tests/fgl folder.

See also: msPan, msZoom, msMoveLegend

Calling syntax:

```
call msAnimation( "on" | "off" )
```


## Description:

The aim of this routine is to avoid the flickering between frames when you want to show quickly (usually in a do loop) a succession of graphic objects.

Note that, at the end of the animation, you will not be able to print the figure in EPS or PDF files (graphic objects are not saved into memory), unless you run yourself the same graphic commands out of the animation.

Before setting animation to "on", a window must be opened (with msFigure). Moreover axis must be defined as constant, by calling the msAxis routine before.

After drawing each frame, the only way to show the display of the graphic object in the figure is to call the msShowNow routine. The MUESLI User Guide presents a detailed example of a typical sequence of such an animation.

At the end of the animation sequence, the user should set animation to "off".

## Remarks:

During an animation:

- the "hold" property cannot be changed by the user. Therefore, the msHold routine shouldn't be used.
- all graphic objects are volatile; as a consequence the handle returned by all plotting functions beginning by mf is equal to zero. Therefore, you cannot use any routine requiring a valid handle.
- don't use msClf (which does a complete erasing of the figure) between frames: in such a case, this will annihilate a large part of the benefit of the current routine. Use msCla instead.
- a varying text string may be displayed outside the axes, and erased between frames with the use of a white rectangle over it (see $\mathrm{mf} / \mathrm{msPatch}$ ).
- if the msPColor routine is used, then the optional arguments X and Y must be used.


## msShowNow

screen update during an animation

Calling syntax:
call msShowNow( )

Description:
This routine may be called only during an animation (see msAnimation). It forces the update of the screen to show the new display of the current frame.
msCla

Calling syntax:

```
call msCla()
```


## Description:

Clears the current axes, i.e. fills the rectangle inside the axes using the background color.
A typical usage concerns the animation of any graphic objects (curve, symbol, text, ...): at the beginning of a loop, the axes should be erased with msCla (quick way) and then graphic objects are drawn. Avoid to use msClf (heavy way) because this latter routine do a complete reset of the figure (among other things, the axes range is lost).

See also: msFigure, msClose

## msSetGBuffer

Interface:
subroutine msSetGBuffer ( flag )
character(len=*), intent(in) :: flag
flag may be equal to "on" or "off".

Description:
Activates the graphic buffer (default is "on"). Usually increases the display speed.
The graphic buffer may be turned "off" for debugging purpose, for example.
See also: msBBuf, msEBuf

## msBBuf

begin of graphic buffering

Calling syntax:
call msBBuf ( )

Description:
Begins to draw in the graphic buffer, instead of directly on the screen.

## Remarks:

- this routine is active only if the graphic buffer is not turned off by the msSetGBuffer routine;
- actually, many internal low-level graphic routines are already buffered.

See also: msSetGBuffer, msEBuf
msEBuf
end of graphic buffering

Calling syntax:

```
call msEBuf( )
```

Description:
Ends to draw in the graphics buffer.
msBBuf and msEBuf calls should always be paired.
See also: msSetGBuffer, msBBuf

## msDefineCustomCursors

define custom cursors for use with mfGinputCustom

## Interface:

```
subroutine msDefineCustomCursors( cursor_1, cursor_1_mask, color_1, &
    cursor_2, cursor_2_mask, color_2 )
    character(len=*), intent(in) :: cursor_1, cursor_1_mask, color_1, &
    cursor_2, cursor_2_mask, color_2
```

Description:
Register two customized cursors, provided by the user in the XBM format. The arguments cursor_* are the filenames of these images. Each cursor is defined by two bitmap images: one for the cursor shape itself and the other for the mask. A color must be also specified for each cursor.

A size of $16 \times 16$ pixels is recommended. The cursor main image must define the Hot Spot position, for example

```
#define _x_hot 8
#define _y_hot 8
```

Last, each color is a colorname from the RGB X11 database.
See also: mfGinputCustom

## Index

1 - Derived Typesmf_DE_Options ..... 434
mf_Int_List ..... 27
mf_NL_Options ..... 432
mf_Out ..... 22
mf_Real_List ..... 28
mfArray ..... 8
mfMatFactor ..... 458
mfPSLG ..... 383
mfTetraConnect ..... 398
mfTriConnect ..... 386
mfUnit ..... 812 - Parameters and Global Variables
MF_ALL ..... 4
MF_BESSEL_J0_ROOTS ..... 4
MF_BESSEL_J1_ROOTS ..... 4
MF_COLON ..... 4
MF_COMPILATION_CONFIG ..... 99
MF_COMPILER_VERSION ..... 98
MF_DOUBLE ..... 4
MF_E ..... 4
MF EMPTY ..... 4
MF_END ..... 4
MF_EPS ..... 4
MFI ..... 4
MF_INF ..... 4
MF LAPACK_VERSION ..... 362
MF_MUESLI_VERSION ..... 100
MF_NAN .....
MF_NUMERICAL_CHECK ..... 80
MF PI ..... 4
MF_REALMAX ..... 4
MF_REALMIN ..... 4
STDERR ..... 5
STDIN ..... 5
STDOUT ..... 5
3 - Operators

* ..... 162
** ..... 164
$+$ ..... 157
158
and ..... 12, 14, 186
but. ..... 12, 14
by. ..... 12, 14
.eqv ..... 188
h. ..... 166
.hc. ..... 168
.i. ..... 331
ix. ..... 340
neqv. ..... 189
not. ..... 185
or. ..... 187
step. ..... 12, 14
.t. ..... 165
.to. ..... 12, 14
.vc. ..... 167
x. ..... 159
.xi. ..... 342
/ ..... 163
$1=$ ..... 180
$=$ ..... 10
$=$ ..... 179
$>$ ..... 176
$>=$ ..... 175
$<$ ..... 178
<= ..... 177
4 - FML Routines
All ..... 44
Any ..... 45
isfinite ..... 307
isinf ..... 308
isnan ..... 309
mf .....  9
mfAbs ..... 234
mfACos ..... 197
mfACosh ..... 198
mfACot ..... 199
mfACoth ..... 200
mfACsc ..... 201
mfACsch ..... 202
mfAiry ..... 262
mfAll ..... 181
mfAngle ..... 235
mfAny ..... 182
mfASec ..... 203
mfASech ..... 204
mfASin ..... 205
mfASinh ..... 206
mfATan ..... 207
mfATan2 ..... 208
mfATanh ..... 209
mfBalance ..... 329
mfBesselI ..... 260
mfBesselJ ..... 258
mfBesselK ..... 261
mfBesselY ..... 259
mfBlkDiag ..... 289
mfCeil ..... 242
mfCheckPerm ..... 306
mfChol ..... 326
mfCmplx ..... 50
mfColon ..... 183
mfColPerm ..... 170
mfColScale ..... 172
mfCompan ..... 301
mfComplex ..... 236
mfCond ..... 322
mfCondEst ..... 355
mfConj ..... 237
mfCos ..... 210
mfCosh ..... 211
mfCot ..... 212
mfCoth ..... 213
mfCount ..... 15
mfCross ..... 160
mfCsc ..... 214
mfCsch ..... 215
mfCshift ..... 286
mfCSign ..... 248
mfDaeSolve ..... 426
mfDble ..... 49
mfDblQuad ..... 418
mfDelaunay ..... 385
mfDelaunay3D ..... 396
mfDet ..... 318
mfDiag ..... 288
mfDiff ..... 134
mfDisplayColumns ..... 17
mfEig ..... 337
mfEigs ..... 351
mfEoshift ..... 287
mfErf ..... 250
mfErfC ..... 252
mfErfCInv ..... 253
mfErfCScaled ..... 254
mfErfInv ..... 251
mfExp ..... 223
mfExpInt ..... 255
mfExpm ..... 346
mfExpm1 ..... 224
mfExtrema ..... 127
mfEye ..... 271
mfFactor ..... 265
mfFFT ..... 146
mfFFT2 ..... 148
mfFind ..... 290
mfFix ..... 243
mfFlipLR ..... 295
mfFlipUD ..... 296
mfFloor ..... 244
mfFlops ..... 72
mfFourierCos ..... 150
mfFourierLeg ..... 154
mfFourierSin ..... 152
mfFSolve ..... 407
mfFull ..... 440
mfFun ..... 231
mfFun2 ..... 232
mfFunFit ..... 371
mfFunm ..... 350
mfFZero ..... 406
mfGamma ..... 256
mfGammaLn ..... 257
mfGet ..... 14
mfGetAutoComplex ..... 75
mfGetAutoFilling ..... 65
mfGetMsgLevel ..... 51
mfGetRoundingMode ..... 78
mfGetTermWidth ..... 63
mfGetTrbLevel ..... 53
mfGradient ..... 135
mfGridData ..... 395
mfGridData3D ..... 404
mfGridFun ..... 233
mfHankel ..... 302
mfHasNoPhysDim ..... 85
mfHaveSamePhysDim ..... 86
mfHess ..... 338
mfHilb ..... 298
mfHypot ..... 240
mfImag ..... 238
mfInt ..... 48
mfInterp1 ..... 380
mfInterp2 ..... 382
mfIntersect ..... 191
mfInv ..... 331
mfInvFFT ..... 147
mfInvFFT2 ..... 149
mfInvFourierCos ..... 151
mfInvFourierLeg ..... 155
mfInvFourierSin ..... 153
mfInvHilb ..... 299
mfInvPerm ..... 174
mfIsColumn ..... 42
mfIsComplex ..... 34
mfIsDense ..... 36
mfIsDiag ..... 313
mfIsDiagDomCol ..... 359
mfIsEmpty ..... 29
mfIsEqual ..... 30
mfIsFinite ..... 310
mfIsFlopsOk ..... 73
mfIsFullRank ..... 361
mfIsInf ..... 311
mfIsLogical ..... 32
mfIsMatrix ..... 40
mfIsMember ..... 190
mfIsNaN ..... 312
mfIsNotEqual ..... 31
mfIsNumeric ..... 35
mfIsPerm ..... 43
mfIsPosDef ..... 358
mfIsPrime ..... 264
mfIsReal ..... 33
mfIsRow ..... 41
mfIsRowSorted ..... 454
mfIsScalar ..... 38
mfIsSorted ..... 132
mfIsSparse ..... 37
mfIsStrictDiagDomCol ..... 360
mfIsSymm ..... 356
mfIsTempoArray ..... 70
mfIsTril ..... 314
mfIsTriu ..... 315
mfIsVector ..... 39
mfIsVersion ..... 96
mfKron ..... 161
mfLDiv ..... 340
mfLegendre ..... 377
mfLinSpace ..... 272
mfLoad ..... 115
mfLoadAscii ..... 110
mfLoadHDF5 ..... 118
mfLoadSparse ..... 112
mfLoadTriConnect ..... 116
mfLog ..... 225
mfLog10 ..... 227
mfLog1p ..... 226
mfLog2 ..... 228
mfLogm ..... 347
mfLogSpace ..... 273
mfLsqNonLin ..... 410
mfMagic ..... 274
mfMax ..... 121
mfMean ..... 138
mfMedian ..... 139
mfMerge ..... 283
mfMin ..... 124
mfMod ..... 245
mfMoments ..... 144
mfMul ..... 159
mfNbPointers ..... 25
mfNcolMax ..... 445
mfNnz ..... 443
mfNodeSearch ..... 394
mfNodeSearch3D ..... 403
mfNonZeros ..... 291
mfNorm ..... 320
mfNormEst ..... 354
mfNormEst1 ..... 354
mfNull ..... 344
mfNzMax ..... 444
mfOdeSolve ..... 420
mfOnes ..... 277
mfOrth ..... 345
mfOut ..... 21
mfPack ..... 284
mfPerm ..... 304
mfPoly ..... 374
mfPolyFit ..... 369
mfPolyVal ..... 368
mfPow10 ..... 229
mfPow2 ..... 230
mfPowm ..... 349
mfPPDer ..... 379
mfPPVal ..... 378
mfProd ..... 129
mfPseudoInv ..... 332
mfQleft ..... 335
mfQR ..... 333
mfQright ..... 336
mfQuad ..... 416
mfRand ..... 278
mfRandN ..... 280
mfRandPerm ..... 305
mfRandPoiss ..... 281
mfRank ..... 321
mfRCond ..... 323
mfRDiv ..... 342
mfReadLine ..... 102
mfReal ..... 239
mfRem ..... 246
mfRepMat ..... 282
mfReshape ..... 292
mfRMS ..... 142
mfRoots ..... 373
mfRot90 ..... 297
mfRound ..... 241
mfRowPerm ..... 171
mfRowScale ..... 173
mfSchur ..... 339
mfSec ..... 216
mfSech ..... 217
mfShape ..... 46
mfSign ..... 247
mfSimpson ..... 415
mfSin ..... 218
mfSinh ..... 219
mfSize ..... 47
mfSmooth ..... 145
mfSort ..... 130
mfSortRows ..... 133
mfSpAlloc ..... 441
mfSparse ..... 439
mfSpCut ..... 451
mfSpDiags ..... 450
mfSpEye ..... 446
mfSpImport ..... 452
mfSpline ..... 375
mfSpOnes ..... 447
mfSpRand ..... 448
mfSpRandN ..... 449
mfSqrt ..... 222
mfSqrtm ..... 348
mfStd ..... 141
mfSum ..... 128
mfSVD ..... 330
mfSVDS ..... 353
mfTan ..... 220
mfTanh ..... 221
mfTetraSearch ..... 402
mfToeplitz ..... 303
mfTolForSymm ..... 357
mfToLower ..... 93
mfToUpper ..... 94
mfTrace ..... 319
mfTrapz ..... 414
mfTriL ..... 293
mfTriSearch ..... 393
mfTriU ..... 294
mfUnion ..... 192
mfUnique ..... 193
mfUnpack ..... 285
mfVander ..... 300
mfVar ..... 140
mfZeros ..... 276
msAddEntryInHistory ..... 106
msAssign ..... 11
msAutoRelease ..... 20
msBalance ..... 329
msBuildTetraConnect ..... 399
msBuildTriConnect ..... 387
msCheckDomainConvexity ..... 390
msChol ..... 326
msCholSpNum ..... 328
msCholSpSymb ..... 327
msCleanPrimeNumbers ..... 266
msClearHistory ..... 105
msColPerm ..... 170
msColScale ..... 172
msDaeSolve ..... 430
msDblQuad ..... 419
msDel3DNodeNeighbors ..... 401
msDiag ..... 288
msDisableFPE ..... 77
msDisplay ..... 16
msEig ..... 337
msEigs ..... 351
msEllipKE ..... 263
msEnableFPE ..... 76
msEndDelaunay3D ..... 397
msEquiv ..... 26
msExtractTriConnect ..... 389
msFind ..... 290
msFindIOUnit ..... 95
msFlops ..... 71
msFlush ..... 60
msFormat ..... 18
msFreeArgs ..... 67
msFreeMatFactor ..... 459
msFreePointer ..... 24
msFSolve ..... 409
msFunFit ..... 372
msFunm ..... 350
msGetArpackInfo ..... 365
msGetAutoRowSorted ..... 456
msGetBlasLib ..... 363
msGetLapackLib ..... 364
msGetStdIO ..... 58
msGetSuiteSparseLib ..... 366
msGradient ..... 136
msHess ..... 338
msHist ..... 143
msHorizConcat ..... 169
msInitArgs ..... 66
msLDLT ..... 325
msLoadSparse ..... 113
msLog2 ..... 228
msLsqNonLin ..... 412
msLU ..... 324
msMax ..... 123
msMedit ..... 119
msMeshGrid ..... 275
msMin ..... 126
msMuesliTrace ..... 101
msOdeSolve ..... 424
msPause ..... 61
msPointer ..... 23
msPolyFit ..... 370
msPostHashes ..... 89
msPostProgress ..... 92
msPrepHashes ..... 87
msPrepProgress ..... 90
msPrintColoredMsg ..... 56
msPrintHashes ..... 88
msPrintProgress ..... 91
msPrintPSLG ..... 384
msPrintTetraConnect ..... 400
msPrintTriConnect ..... 391
msQR ..... 334
msQuad ..... 417
msRand ..... 279
msRat ..... 267
msReadHistoryFile ..... 103
msRelease ..... 19
msRemoveLastEntryInHistory ..... 107
msRequMuesliVer ..... 97
msReshape ..... 292
msReturnArray ..... 69
msRowPerm ..... 171
msRowScale ..... 173
msRowSort ..... 455
msRref ..... 343
msSave ..... 114
msSaveAscii ..... 109
msSaveHDF5 ..... 117
msSaveSparse ..... 111
msSchur ..... 339
msSet ..... 12
msSetAsParameter ..... 68
msSetAutoComplex ..... 74
msSetAutoFilling ..... 64
msSetAutoRowSorted ..... 457
msSetColoredMsg ..... 55
msSetMsgLevel ..... 52
msSetPhysDim ..... 83
msSetPhysUnitAbbrev ..... 84
msSetRoundingMode ..... 79
msSetStdIO ..... 59
msSetTermColor ..... 57
msSetTermWidth ..... 62
msSetTrbLevel ..... 54
msSort ..... 131
msSpExport ..... 453
msSpline ..... 376
msSpReAlloc ..... 442
msSVD ..... 330
msSVDS ..... 353
msTriNodeNeighbors ..... 392
msUpdateTriConnect ..... 388
msUsePhysUnits ..... 82
msWriteHistoryFile ..... 104
Shape ..... 46
Size ..... 47
5 - FGL Routines
mfArrow ..... 515
mfAxis ..... 486
mfBar ..... 529
mfCAxis ..... 490
mfColormap ..... 493
mfColormapSize ..... 494
mfContour ..... 532
mfContourF ..... 534
mfErrorBar ..... 525
mfFigure ..... 476
mfGetAllGrObj ..... 517
mfGetCharEncoding ..... 473
mfGetColorOverflowPolicy ..... 463
mfGetColorScheme ..... 480
mfGetDefaultCapStyle ..... 469
mfGetDefaultJoinStyle ..... 471
mfGetModKeys ..... 561
mfGetTypeGrObj ..... 518
mfGetWinId ..... 478
mfGetX11ColorDepth ..... 467
mfGetX11Device ..... 465
mfGinput ..... 557
mfGinputCustom ..... 559
mfGinputRect ..... 560
mfImage ..... 545
mfLegend ..... 513
mfPatch ..... 539
mfPColor ..... 530
mfPlot ..... 524
mfPlotCubicBezier ..... 527
mfPlotCubicSpline ..... 528
mfPlotHist ..... 540
mfPlotPSLG ..... 546
mfPlotQuadrBezier ..... 526
mfQuiver ..... 535
mfSelectTypeGrObj ..... 519
mfSpy ..... 555
mfStreamline ..... 537
mfText ..... 516
mfTitle ..... 504
mfTriContour ..... 550
mfTriContourF ..... 552
mfTriFill ..... 549
mfTriMesh ..... 547
mfTriPColor ..... 548
mfTriQuiver ..... 553
mfTriStreamline ..... 554
mfXLabel ..... 506
mfYLabel ..... 507
msAnimation ..... 567
msArrow ..... 515
msAxis ..... 486
msAxisFontSize ..... 487
msAxisLabelFormat ..... 509
msAxisLineWidth ..... 488
msBar ..... 529
msBBuf ..... 571
msCAxis ..... 490
msCharInPixels ..... 489
msCla ..... 569
msClf ..... 491
msClose ..... 479
msColorbar ..... 492
msColormap ..... 493
msColormapSize ..... 494
msContour ..... 532
msContourF ..... 534
msCumulHist ..... 541
msDefineCustomCursors ..... 573
msDrawBox ..... 495
msDrawGrid ..... 496
msEBuf ..... 572
msErrorBar ..... 525
msExitFgl ..... 468
msFigure ..... 476
msGetX11Pixmap ..... 498
msGinput ..... 558
msGrid ..... 499
msHold ..... 500
msImage ..... 545
msImRead ..... 542
msImWrite ..... 544
msLabelFontSize ..... 508
msLegend ..... 511
msMoveGrObj ..... 566
msMoveLegend ..... 565
msPan ..... 562
msPanAndZoom ..... 564
msPatch ..... 539
msPColor ..... 530
msPlot ..... 524
msPlotCubicBezier ..... 527
msPlotCubicSpline ..... 528
msPlotHist ..... 540
msPlotPSLG ..... 546
msPlotQuadrBezier ..... 526
msPrint ..... 482
msQuiver ..... 535
msRedrawFigure ..... 497
msRemoveClipBox ..... 501
msRemoveGrObj ..... 522
msResizeWindow ..... 477
msSetBackgroundColor ..... 462
msSetCharEncoding ..... 474
msSetClipBox ..... 502
msSetColorOverflowPolicy ..... 464
msSetColorScheme ..... 481
msSetDefaultCapStyle ..... 470
msSetDefaultJoinStyle ..... 472
msSetGBuffer ..... 570
msSetGrObj ..... 520
msSetPdfOC ..... 483
msSetWinProp ..... 510
msSetX11Device ..... 466
msShading ..... 503
msShowNow ..... 568
msSpy ..... 555
msStreamline ..... 537
msText ..... 516
msTitle ..... 504
msTitleFontSize ..... 505
msTriContour ..... 550
msTriContourF ..... 552
msTriFill ..... 549
msTriMesh ..... 547
msTriPColor ..... 548
msTriQuiver ..... 553
msTriStreamline ..... 554
msXLabel ..... 506
msYLabel ..... 507
msZoom ..... 563


[^0]:    *IPR/CNRS, Rennes, France

[^1]:    ${ }^{1}$ in order to keep adjacent the complex conjugate pairs

[^2]:    ${ }^{1}$ For implicit systems, the mfDaeSolve routine should be used instead.
    ${ }^{2}$ Currently, only the $B D F$ integrator has this feature.

[^3]:    ${ }^{3}$ Not just linearly implicit, which would have a mass matrix. For explicit systems, the mfOdeSolve routine should be used instead.
    ${ }^{4}$ For higher-index problems, the current algorithm may or may not work!

[^4]:    ${ }^{5}$ It is so called because it is the linear combination of two jacobian. Actually, it is close to the iteration matrix of the non-linear solver. See also this remark.

[^5]:    ${ }^{6}$ For the ODE solver, the jacobian provided by the user may be singular, it doesn't matter for the integration process. See the remark at the end of this entry.

