Theory of belief functions

Arnaud Martin
Arnaud.Martin@ensieta.fr

ENSIETA - E3I2 EA3876
Brest, France

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The theory of belief functions

- Discernment space $\Theta = \{\theta_1, \dots, \theta_n\}$, where θ_i are the classes exclusive and exhaustive
- Mass functions defined onto $2^\Theta = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \{\theta_1 \cup \theta_2\}, \dots, \Theta\}$ with values on $[0, 1]$. Θ: ignorance
- $\sum_{X \in 2^\Theta} m(X) = 1$
- $m(\emptyset) = 0$
- Discounting by the reliability: $m^\alpha(X) = \alpha m(X)$, $m^\alpha(\Theta) = 1 - \alpha(1 - m(\Theta))$
- Dempster's combination:

$$m_{\mathrm{D}}(X) = \frac{1}{1-k} \sum_{Y_1 \cap Y_2 = X} m_1(Y_1) m_2(Y_2)$$

  where $k = m_{\mathrm{Conj}}(\emptyset)$
- Decision: credibility $\leq$ pignistic probability $\leq$ plausibility

An example of model

Discernment spaces:

- Is individual A dangerous? $\Theta_1 = \{Y_1, N_1\}$
- Is the suspect vehicle near the building B? $\Theta_2 = \{Y_2, N_2\}$

4 sources:

- S_0 Individual A is under surveillance due to previous unstable behavior
- S_1 Analyst 1 (who has 10 years of experience): it is probable that individual A is near building B
- S_2 ANPR: 30% probability that the vehicle is Individual A's white Toyota
- S_3 Analyst 2 (who is new in post): it is improbable that individual A is near building B

An example of model

Model

- S_0 Individual A is under surveillance due to previous unstable behavior
  Only on Θ_1: $m_0(Y_1) = \beta_0$ $m_0(Y_1 \cup N1) = 1 - \beta_0$ with $\beta_0 > 0.5$

An example of model

Model

- S_0 on Θ_1: $m_0(Y_1) = \beta_0$ $m_0(Y_1 \cup N_1) = 1 - \beta_0$ with $\beta_0 > 0.5$
- S_1 Analyst 1 (10 years of experience): it is probable that individual A is near building B
  Only on Θ_2: $m_1(Y_2) = \beta_1$ $m_1(N_2) = 1 - \beta_1$ with $\beta_1 > 0.5$
  Reliability: $\alpha_1 > 0.5$

An example of model

Model

- S_0 on Θ_1: $m_0(Y_1) = \beta_0$ $m_0(Y_1 \cup N_1) = 1 - \beta_0$ with $\beta_0 > 0.5$
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  Reliability: $\alpha_1 > 0.5$
  With the discounting: $m_1(Y_2) = \alpha_1\beta_1$, $m_1(N_2) = \alpha_1(1 - \beta_1)$, $m_1(Y_2 \cup N_2) = 1 - \alpha_1$

An example of model

Model

- S_0 on Θ_1: $m_0(Y_1) = \beta_0$ $m_0(Y_1 \cup N_1) = 1 - \beta_0$ with $\beta_0 > 0.5$
- S_1 on Θ_2: $m_1(Y_2) = \alpha_1\beta_1$, $m_1(N_2) = \alpha_1(1 - \beta_1)$, $m_1(Y_2 \cup N_2) = 1 - \alpha_1$
- S_2 ANPR: 30% probability that the vehicle is Individual A's white Toyota Only on Θ_2: $m_2(Y_2) = 0.3$ $m_2(N_2) = 0.7$

An example of model

Model

- S_0 on Θ_1: $m_0(Y_1) = \beta_0$ $m_0(Y_1 \cup N1) = 1 - \beta_0$ with $\beta_0 > 0.5$
- S_1 on Θ_2: $m_1(Y_2) = \alpha_1\beta_1$, $m_1(N_2) = \alpha_1(1 - \beta_1)$,
  $m_1(Y_2 \cup N_2) = 1 - \alpha_1$
- S_2 on Θ_2: $m_2(Y_2) = 0.3$ $m_2(N_2) = 0.7$
- S_3 Analyst 2 (who is new in post): it is improbable that individual A is near building B
  Only on Θ_2: $m_3(Y_2) = \beta_3$ $m_3(N_2) = 1 - \beta_3$ with $\beta_3 < 0.5$
  Reliability: $\alpha_3 < 0.5$
  With the discounting: $m_3(Y_2) = \alpha_3\beta_3$, $m_3(N_2) = \alpha_3(1 - \beta_3)$,
  $m_3(Y_2 \cup N_2) = 1 - \alpha_3$

An example of model

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Model on
$\Theta_1 \times \Theta_2 = \{(Y_1, Y_2), (Y_1, N_2), (N_1, Y_2), (N_1, N_2)\} = \{\theta_1, \theta_2, \theta_3, \theta_4\}$

- S_0 on Θ_1: $m_0(\theta_1 \cup \theta_2) = \beta_0$ $m_0(\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_4) = 1 - \beta_0$ with $\beta_0 > 0.5$
- S_1 on Θ_2: $m_1(\theta_1 \cup \theta_3) = \alpha_1\beta_1$, $m_1(\theta_2 \cup \theta_4) = \alpha_1(1 - \beta_1)$, $m_1(\theta_1 \cup \theta_2 \cup \theta_3 \cup \theta_4) = 1 - \alpha_1$
- S_2 on Θ_2: $m_2(\theta_1 \cup \theta_3) = 0.3$ $m_2(\theta_2 \cup \theta_4) = 0.7$
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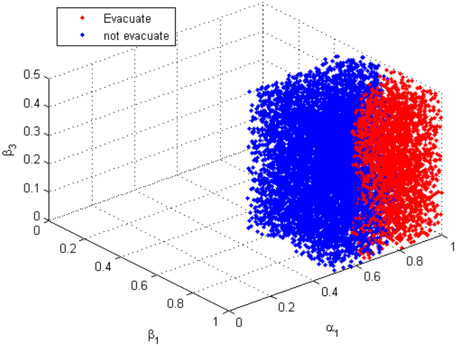
An example of model: Results

1000 generated $(\beta_0, \beta_1, \alpha_1, \beta_3, \alpha_3)$ Same decision with pignistic probability, credibility and plausibility

Decision according to the chosen values of β_1 and α_1



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