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Reliability and combination rule in the theory of belief functions

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Abstract − *This paper presents a point of view to address an application with the theory of belief functions from a global approach. Indeed, in a belief application, the definition of the basic belief assignments and the tasks of reduction of focal elements number, discounting, combination and decision, must be thought at the same time. Moreover these tasks can be seen as a general process of belief transfer.*

The second aspect of this paper involves the introduction of the reliability in the combination rule directly and not before. Indeed, in general, the discounting process is made with a discounting factor that is a reliability factor of the sources. Here we propose to include in the combination rule an estimation of the reliability based on a local conflict estimation.

Keywords: Belief function, reliability, conflict, combination rule, Dempster-Shafer theory.

1 Introduction

The theory of belief functions, also called theory of evidence or Dempster-Shafer theory [3, 20], is based on the definition of basic belief assignments, we can see as an extension of the probability measures. Indeed basic belief assignments are defined on the power set 2^Θ, where Θ is a frame of discernment, instead of Θ in the probability theory. Therefore, the basic belief assignment is not an additive measure. From these basic belief assignments, we can define more belief functions and combination operators; allow information fusion, data association, decision tasks, *etc.*

With this framework of belief functions, we can see the combination of basic belief assignments as a transfer of belief on a subset of the power set [17]. Indeed, during the combination of basic belief assignments different combination rules propose to share the belief by different ways, especially to manage the conflict between the sources [23, 9, 11, 19, 13]. Another common mean to manage the conflict is the use of a discounting procedure. This process consist in decreasing the basic beliefs on precise focal elements and increase the basic beliefs on imprecise focal elements such as the total ignorance Θ.

The task of combination can be seen as a mapping from s basic belief assignments defined on 2^Θ, onto 2^Θ. Therefore the complexity of this task can be very important. This is the reason why we can reduce the number of focal elements (*i.e.* elements with a non null basic belief) [22, 21, 2, 5].

Finally, the belief functions are generally used to take a decision. In the most of applications, the decision is taken on one element of Θ and not 2^Θ, by the credibility, plausibility functions or by the pignistic probability [4, 18]. However, the decision can also be taken on all 2^Θ [1, 14]. Hence, the decision process is a mapping from 2^Θ onto Θ or 2^Θ.

In a belief application, the definition of the basic belief assignments and all these previous tasks (reduction of focal elements number, discounting, combination and decision) must be thought in the same time. Moreover these tasks can be seen as a general process of belief transfer. In the first part of this paper we present how we can formalize this belief transfer.

The second aspect of this paper involves the introduction of the reliability in the combination rule. Indeed, in general, the discounting process is made with a discounting factor that is a reliability factor of the sources. We have seen in [15] that a reliability can be estimated from a conflict measure. These two notions can be really near in a fusion process. Discounting procedure and combination rules are two means to manage the conflict and can also be seen as two belief transfer functions. Thus, we propose in this paper a formulation for a combination rule including the reliability.

Thereby, the rest of the paper is organized as follow: in the following section 2 we recall the theoretical background on the theory of the belief functions and we propose a general formulation for the belief transfer. In section 3 we present a formulation in order to address the discounting process and the combination in

the same time. Therefore, the section 4 proposes a combination rule including reliability: we first present the two experts case, then we extend the rule for s experts.

2 Theory of the belief functions

2.1 Theoretical background

Let Θ be a frame of discernment. A *basic belief assignment* (bba) m is the mapping from elements of the power set 2^Θ onto $[0,1]$ such that:

$$\sum_{X\in 2^\Theta} m(X) = 1. \tag{1}$$

A focal element X is an element of 2^Θ such that $m(X) \neq 0$.

2.1.1 Discounting procedure

When we can quantify the reliability of each expert, we can weaken the basic belief assignment before the combination by the discounting procedure:

$$\begin{cases} m_j^\alpha(X) = \alpha_j m_j(X), \forall X \in 2^\Theta \setminus \{\Theta\} \\ m_j^\alpha(\Theta) = 1 - \alpha_j(1 - m_j(\Theta)). \end{cases} \tag{2}$$

m_j and $\alpha_j \in [0,1]$ are respectively the bba and the discounting factor of the expert j. α_j is, in this case, the reliability of the expert j, eventually as a function of $X \in 2^\Theta$. This procedure has an impact on the level of conflict appearing during the combination step and on the belief interval given by both credibility and plausibility functions. Note also that α_j can depend on the focal element X.

2.1.2 Reduction of focal elements number

Several approaches for the reduction of the focal elements number have been proposed in order to decrease the complexity of the combination rule [22, 21, 2, 5]. All these approaches are quite different. We do not present their mechanism because this is not the topic of this paper. However, some of them have *ad hoc* procedure, or are based on optimization principles. Moreover, some approaches are very depending on the combination rule. Generally the wanted focal elements number is fixed. Therefore this problem can be express as: how to transfer the belief given by a belief function to some elements of the power set in order to define another belief function with less focal elements.

2.1.3 Combination rules

Today there is a lot of combination rules in the belief functions framework. Most of them are based on the conjunction of the focal elements in order to increase the belief on the most precise elements of the power set. The conjunctive rule is given for two basic belief assignments m_1 and m_2 and for all $A \in 2^\Theta$ by:

$$m_c(A) = \sum_{B\cap C=A} m_1(B)m_2(C). \tag{3}$$

$m_c(\emptyset)$ can be interpreted as a non-expected solution and is generally called the global conflict of the combination or the inconsistence of the combination. The interpretation of $m_c(\emptyset)$ and the transfer of this belief on other elements of the power set gave birth to several combination rules [23, 9, 11, 19, 13]. The equation (3) can also be seen as a transfer of belief [17].

2.1.4 Decision process

From basic belief assignments, other belief functions can be defined such as credibility and plausibility. The credibility represents the intensity that the information given by one expert supports an element of 2^Θ, this is a minimal belief function given from a bba for all $X \in 2^\Theta$ by:

$$\mathrm{bel}(X) = \sum_{Y\subseteq X, Y\neq\emptyset} m(Y). \tag{4}$$

The plausibility represents the intensity with which there is no doubt on one element. This function is given from a bba for all $X \in 2^\Theta$ by:

$$\mathrm{pl}(X) = \sum_{Y\in 2^\Theta, Y\cap X\neq\emptyset} m(Y). \tag{5}$$

If the credibility function provides a pessimistic decision, the plausibility function is often too optimistic. The pignistic probability [16] is generally considered as a compromise. It is calculated from a basic belief assignment m for all $X \in 2^\Theta$, with $X \neq \emptyset$ by:

$$\mathrm{betP}(X) = \sum_{Y\in 2^\Theta, Y\neq\emptyset} \frac{|X\cap Y|}{|Y|} \frac{m(Y)}{1-m(\emptyset)}, \tag{6}$$

where $|X|$ is the cardinality of X.

In the most of applications, the decision is taken on Θ and not 2^Θ, thus the decided element is given by:

$$A = \arg\max_{X\in\Theta} f_d(X), \tag{7}$$

where f_d is the decision function (credibility, plausibility, pignistic probability, *etc.*). The decision function f_d can also be interpreted as a transfer of belief.

2.2 Formulation of belief transfer

We have seen that the belief functions framework proposes a lot of methods to discount the belief, to reduce the number of focal elements, to combine the basic belief assignments and to take a decision. The question of "which one we must choice?" has no general response and depends on the application. By analogy to the decision theory or to the pattern recognition approaches, we could propose a no free lunch theorem showing that any combination rule is still better [6, 10].

In the theory of the belief functions, we can see all the tasks of discounting, reducing the number of focal

elements, combination and decision making as a transfer of belief. A transfer of belief can be formalized as an operator F_T given by the mapping from the basic belief assignments space onto the same space: $m_T = F_T(m)$. The set of focal elements of m_T can be different of those of m and reduced. However, generally we add information; by example for the discounting process given by the equation (2) can be written by $m_T = F_T(m, \alpha)$. The focal elements are Θ and the same than with m. For the reduction of the number of focal elements, we can add the expected number of focal elements. In this case the set of focal elements can be really different from the initial set of m. The decision process includes generally a belief transfer, with in parameter the set $\mathcal{T}$ of elements of 2^Θ on which we want take the decision.

For the combination, the transfer of belief is a mapping from the space of s basic belief assignments onto the basic belief assignments space: $m_T = F_T(m_1, \ldots, m_s)$.

Generally, the decision is made after the combination whereas the reduction of the number of focal elements and the discounting process are conducted before the combination. However, these tasks are considered separately. All these tasks can be considered in a global approach of transfer of belief.

Of course this is not easy to address a particular application with a global approach. In the rest of the paper we focus on one approach to address the discounting process and the combination in the same time introducing the reliability in the combination rule.

3 A formulation of the combination including the reliability

General formulations have been proposed in order to represent the combination rules [1, 19, 13]. A general formulation of a combination including reliability can by done for all $X \in 2^\Theta$ by:

$$m(X) = \sum_{\mathbf{Y} \in (2^\Theta)^s} \prod_{j=1}^{s} m_j(Y_j) w(X, \mathbf{m}(\mathbf{Y}), \mathcal{T}(\mathbf{Y}), \boldsymbol{\alpha}(\mathbf{Y})), \quad (8)$$

where $\mathbf{Y} = (Y_1, \ldots, Y_s)$ is the responses of the s experts and $m_j(Y_j)$ the associated mass $(\mathbf{m}(\mathbf{Y}) = (m_1(Y_1), \ldots, m_j(Y_s))$, $\boldsymbol{\alpha}$ is a matrix of terms α_{ij} of the reliability of the expert j for the element i of 2^Θ, and $\mathcal{T}(\mathbf{Y})$ is the set of elements of 2^Θ on which we can transfer the masses $m_j(Y_j)$ for the given $\mathbf{Y}$ vector.

3.1 Reliability estimation based on local conflict

The goal to include the reliability in the combination rule is motivated by a local estimation of the reliability. In [15] an estimation of the reliability is proposed from the conflict between experts. This conflict is given by a distance between the basic belief assignments of the experts. With the same idea, we can propose an estimation of the reliability from a local measure of conflict. In [13], we introduced a local conflict function based on the number of experts in conflict for each response $Y_j \in 2^\Theta$ of all the experts $j = 1, \ldots, s$, given by the number of responses of the other experts in conflict with j. The local conflict function f_j is defined by the mapping of $(2^\Theta)^s$ onto $\left[0, \frac{1}{s}\right]$ with:

$$f_j(Y_1, ..., Y_s) = \frac{\sum_{i=1}^{s} \mathbb{1}_{\{Y_i \cap Y_j = \emptyset\}}}{s(s-1)}. \quad (9)$$

Therefore, we can propose a local estimation of the reliability $\alpha_j(Y_1, ..., Y_M)$ given by the mapping of $(2^\Theta)^s$ onto $\left[0, \frac{1}{s}\right]$ with:

$$\begin{aligned} \alpha_j(Y_1, ..., Y_s) &= \frac{1}{s} - f_j(Y_1, ..., Y_s) \\ &= \frac{\sum_{i=1, i \neq j}^{s} \mathbb{1}_{\{Y_i \cap Y_j \neq \emptyset\}}}{s(s-1)}. \end{aligned} \quad (10)$$

The $DPCR$ rule introduced in [13], based on this factor α_j, is a combination rule including the reliability.

Other local reliability estimations can be proposed according to the application. The local reliability is not necessary based on a local conflict in the rest of the paper.

3.2 Examples

We can rewrite all the combination rules with this formulation given by the equation (8). We present here some examples.

The conjunctive rule of the equation (3), is given by:

$$w(X, \mathbf{m}(\mathbf{Y}), \mathcal{T}(\mathbf{Y}), \boldsymbol{\alpha}(\mathbf{Y})) = 1 \text{ if } \bigcap_{j=1}^{s} Y_j = X \quad (11)$$

where $\mathcal{T}(\mathbf{Y}) = \bigcap_{j=1}^{s} Y_j$ and we do not consider $\boldsymbol{\alpha}(\mathbf{Y})$.

The disjunctive rule [8] is given by:

$$w(X, \mathbf{m}(\mathbf{Y}), \mathcal{T}(\mathbf{Y}), \boldsymbol{\alpha}(\mathbf{Y})) = 1 \text{ if } \bigcup_{j=1}^{s} Y_j = X \quad (12)$$

where $\mathcal{T}(\mathbf{Y}) = \bigcup_{j=1}^{s} Y_j$ and we do not consider $\boldsymbol{\alpha}(\mathbf{Y})$.

The Dubois & Prade rule proposed in [9] is given by:

$$w(X, \mathbf{m}(\mathbf{Y}), \mathcal{T}(\mathbf{Y}), \boldsymbol{\alpha}(\mathbf{Y})) = \begin{cases} 1 \text{ if } \bigcap_{j=1}^{s} Y_j = X; X \neq \emptyset \\ 1 \text{ if } \bigcup_{j=1}^{s} Y_j = X; X = \emptyset \end{cases} \quad (13)$$

where $\mathcal{T}(\mathbf{Y}) = \{\bigcap_{j=1}^{s} Y_j, \bigcup_{j=1}^{s} Y_j\} \setminus \emptyset$ and we do not consider $\boldsymbol{\alpha}(\mathbf{Y})$.

For the PCR6 rule introduced in [12], the weight $w(X, \mathbf{m}(\mathbf{Y}), \mathcal{T}(\mathbf{Y}), \boldsymbol{\alpha}(\mathbf{Y}))$ is given by:

- 1 if $\bigcap_{j=1}^{s} Y_j = X$ and $X \neq \emptyset$
- $\frac{\sum_{i=1}^{M} m_i(X)}{M-1}$, if $Y_i = X, i = 1, \ldots, s$, $m_i(X) + \sum_{j=1}^{} m_{\sigma_i(j)}(Y_{\sigma_i(j)})$ and if $\bigcap_{j=1}^{s} Y_j = \emptyset$ where:

$$\begin{cases} \sigma_i(j) = j, & \text{if } j < i, \\ \sigma_i(j) = j+1, & \text{if } j \geq i, \end{cases} \quad (14)$$

where $\mathcal{T}(\mathbf{Y}) = \{\bigcap_{j=1}^{s} Y_j, Y_1, \ldots, Y_s\} \setminus \emptyset$ and we do not consider $\boldsymbol{\alpha}(\mathbf{Y})$.

The rules introduced in [23, 11, 19, 13] can also be written with the equation (8).

4 A combination rule including reliability

Recently in [24, 25] some new combination rules are proposed to redistribute the belief to subsets or to the complementaries of the considered elements. Some of these rules are build quite similarly to the proportional conflict redistribution rules or to a mixed rule [7]. The principle is based on the hypothesis that the belief on the common part is strong and the belief on the other part is weak. Consequently, the partial conflict between focal elements is interpreted as total ignorance, and when there is no partial conflict a part of the belief is transferred onto the complementaries of the focal elements.

The proposed idea in this paper is a combination rule that transfer the basic belief on $2^T \setminus \{\emptyset\}$, with $T = \{Y_1, \ldots, Y_s, \overline{Y_1}, \ldots, \overline{Y_s}\}$ (where $\overline{Y_j}$ is the complementary of Y_j), according to their basic belief and reliability α_{ij}, $i = 1, \ldots, |2^\Theta|$ an arbitrary order on 2^Θ and $j = 1, \ldots, s$. Hence with the previous notations, $\mathcal{T}(\mathbf{Y}) = 2^T \setminus \{\emptyset\}$.

4.1 Two experts case

We first explain the idea for two experts given a basic belief assignment respectively on X and Y in 2^Θ.

Hence $\mathcal{T}(X,Y) = 2^{\{X,Y,\overline{X},\overline{Y}\}} \setminus \{\emptyset\}$. We note that $X \cup \overline{X} = Y \cup \overline{Y} = \Theta$, and $\overline{X} \cap \overline{Y} = \overline{X \cup Y}$ and if $X \cap Y = \emptyset$: $X \cap \overline{Y} = X$, $Y \cap \overline{X} = Y$ and $\overline{X} \cup \overline{Y} = \Theta$. Hence:

$$\begin{aligned} \mathcal{T}(X,Y) \quad = \quad & \{X, Y, X \cap Y, X \cup Y, \overline{X}, \\ & \overline{X} \cap Y, \overline{X} \cup Y, \overline{Y}, \overline{Y} \cap X, \\ & \overline{Y} \cup X, \overline{X} \cup \overline{Y}, \overline{X} \cap \overline{Y}, \Theta\} \end{aligned}$$

If $X \cap Y \neq \emptyset$, and if the reliability $\alpha_{1X} = \alpha_{2Y} = 1$ and if $m_1(X) = m_2(Y) = 1$ then all the belief must be transferred onto $X \cap Y$. If the reliability $\alpha_{1X} = \alpha_{2Y} = 1$ but $m_1(X) \neq 1$ and $m_2(Y) \neq 1$, then the experts are not sure and a part of the mass $m_1(X).m_2(Y)$ can also be transferred onto $X \cup Y$. If for example $\alpha_{1X} = 0$ then we should also transfer mass onto $\overline{X}$.

If $X \cap Y = \emptyset$, we have a partial conflict between the experts. If the experts are reliable then, we can transfer the mass onto X, Y or $X \cup Y$, such as the $DPCR$ [13]. If the experts are not sure then a part of the mass can also be transferred onto the complementary of X and Y. Therefore, we propose the function w given by the following table if $X \cap Y = \emptyset$:

$X \cap Y = \emptyset$	
element	weight $.N$
X	$\alpha_{1X} m_1(X)$
Y	$\alpha_{2Y} m_2(Y)$
$\overline{X}$	$(1-\alpha_{1X})(1-m_1(X))$
$\overline{Y}$	$(1-\alpha_{2Y})(1-m_2(Y))$
$X \cup Y$	$(1-\alpha_{1X}\alpha_{2Y})(1-m_1(X)m_2(Y))$
$\overline{X} \cap \overline{Y} \neq \emptyset$	$(1-\alpha_{1X})(1-\alpha_{2Y})$ $(1-m_1(X))(1-m_2(Y))$
$\overline{X} \cup \overline{Y} = \Theta$	$(1-(1-\alpha_{1X})(1-\alpha_{2Y}))$ $(1-(1-m_1(X))(1-m_2(Y)))$

The given weights have to be normalized by a factor noted N in order to verify the equation (1).

A simple form of the function w could be given by the following table if $X \cap Y \neq \emptyset$:

$X \cap Y \neq \emptyset$	
element	weight $.N$
$X \cap Y$	$\alpha_{1X}\alpha_{2Y} m_1(X) m_2(Y)$
$X \cup Y$	$(1-\alpha_{1X}\alpha_{2Y})(1-m_1(X)m_2(Y))$

In this form, if for example, the expert 1 is not reliable, we do not transfer on $\overline{X}$. This is the reason why if $X \cap Y \neq \emptyset$, we propose another function w given by the table 1.

In this case the rule will have a behavior nearer the average than the conjunctive rule because the weights on X and Y are higher than the weight on $X \cap Y$. In order to avoid that, we can propose the function w given by the table 2.

With these proposed tables we could also transfer basic belief when one of the basic belief is null or when one of both considered element is empty. This is not interesting for two reasons: to consider null basic belief assignment means we are sure (or with a degree of reliability) that the belief is null. Therefore, if we cannot

element	weight $.N$
$X \cap Y$	$\alpha_{1X}\alpha_{2Y}m_1(X)m_2(Y)$
$X \cup Y$	$(1-\alpha_{1X}\alpha_{2Y})(1-m_1(X)m_2(Y))$
X	$\alpha_{1X}m_1(X)$
Y	$\alpha_{2Y}m_2(Y)$
$\overline{X} \neq \emptyset$	$(1-\alpha_{1X})(1-m_1(X))$
$\overline{Y} \neq \emptyset$	$(1-\alpha_{2Y})(1-m_2(Y))$
$\overline{X} \cap \overline{Y} \neq \emptyset$	$(1-\alpha_{1X})(1-\alpha_{2Y})(1-m_1(X))(1-m_2(Y))$
$\overline{X} \cup \overline{Y}$	$(1-(1-\alpha_{1X})(1-\alpha_{2Y}))(1-(1-m_1(X))(1-m_2(Y)))$
$X \cup \overline{Y}$	$(1-\alpha_{1X}(1-\alpha_{2Y}))(1-m_1(X)(1-m_2(Y)))$
$\overline{X} \cup Y$	$(1-(1-\alpha_{1X})\alpha_{2Y})(1-(1-m_1(X))m_2(Y))$
$X \cap \overline{Y} \neq \emptyset$	$\alpha_{1X}(1-\alpha_{2Y})m_1(X)(1-m_2(Y))$
$\overline{X} \cap Y \neq \emptyset$	$(1-\alpha_{1X})\alpha_{2Y}(1-m_1(X))m_2(Y)$

Table 1: The weight function when $X \cap Y \neq \emptyset$.

element	weight $.N$
$X \cap Y$	$\alpha_{1X}\alpha_{2Y}m_1(X)m_2(Y)$
$X \cup Y$	$(1-\alpha_{1X}\alpha_{2Y})(1-m_1(X)m_2(Y))$
X	$(\alpha_{1X}m_1(X))^2$
Y	$(\alpha_{2Y}m_2(Y))^2$
$\overline{X} \neq \emptyset$	$((1-\alpha_{1X})(1-m_1(X)))^2$
$\overline{Y} \neq \emptyset$	$((1-\alpha_{2Y})(1-m_2(Y)))^2$
$\overline{X} \cap \overline{Y} \neq \emptyset$	$(1-\alpha_{1X})(1-\alpha_{2Y})(1-m_1(X))(1-m_2(Y))$
$\overline{X} \cup \overline{Y}$	$(1-(1-\alpha_{1X})(1-\alpha_{2Y}))(1-(1-m_1(X))(1-m_2(Y)))$
$X \cup \overline{Y}$	$(1-\alpha_{1X}(1-\alpha_{2Y}))(1-m_1(X)(1-m_2(Y)))$
$\overline{X} \cup Y$	$(1-(1-\alpha_{1X})\alpha_{2Y})(1-(1-m_1(X))m_2(Y))$
$X \cap \overline{Y} \neq \emptyset$	$\alpha_{1X}(1-\alpha_{2Y})m_1(X)(1-m_2(Y))$
$\overline{X} \cap Y \neq \emptyset$	$(1-\alpha_{1X})\alpha_{2Y}(1-m_1(X))m_2(Y)$

Table 2: The weight function when $X \cap Y \neq \emptyset$, with a conjunctive kind of behavior.

give belief it becomes a problem, and we lost the interest of the theory of belief functions in this case. If we take into account empty complementary of elements, we transfer more belief on the other element, because $X \cup \emptyset = X$ or $\overline{X} \cup \emptyset = \overline{X}$. Moreover, more elements are considered and more complex is the combination rule.

As a result, we do not transfer basic belief if one of the considered element is empty or has a null basic belief assignment.

4.2 Some examples

In order to understand the principle of this rule, we illustrate it with simple examples.

- if $X \cap Y = \emptyset$ and $\alpha_{1X} = \alpha_{2Y} = 1$, then the only weights are $m_1(X)$ and $m_2(Y)$ respectively on X and Y.
- if $X \cap Y = \emptyset$, with $\alpha_{1X} = 1$ and $\alpha_{2Y} = 0$, then the only weights are $m_1(X)$, $(1 - m_2(Y))$, $m_1(X)(1 - m_2(Y))$ and $1 - m_1(X)m_2(Y)$ respectively on X, $\overline{Y}$, $X \cap \overline{Y}$ and $X \cup Y$.
- if $X \cap Y \neq \emptyset$ and $\alpha_{1X} = \alpha_{2Y} = 1$, then the only weights are $m_1(X)m_2(Y)$, $m_1(X)$ (or $m_1(X)^2$) and $m_2(Y)$ (or $m_2(Y)^2$) respectively on $X \cap Y$, X and Y.
- if $X \cap Y \neq \emptyset$, $\alpha_{1X} = 1$ and $\alpha_{2Y} = 0$, then the only weights are $m_1(X)$, $(1 - m_2(Y))$, $m_1(X)(1 - m_2(Y))$ and $1 - m_1(X)m_2(Y)$ respectively on X, $\overline{Y}$, $X \cap \overline{Y}$ and $X \cup Y$.
- if $X \cap Y = \emptyset$ and $m_1(X)m_2(Y) = 1$, then the only weights are α_{1X} and α_{2Y} respectively on X and Y.
- if $X \cap Y \neq \emptyset$ and $m_1(X)m_2(Y) = 1$, then the only weights are $\alpha_{1X}\alpha_{2Y}$, α_{1X} and α_{2Y} respectively on $X \cap Y$, X and Y.

Lets consider a more complicated example, with $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and the bbas and associated reliability given by:

bba/reliability	θ_1	$\theta_1 \cup \theta_2$	Θ
m_1/α_1	0.6 / 0.9		0.4 / 0.3
m_2/α_2		0.3 / 0.9	0.7 / 0.5

With the transfer based on the table 2 for the $X \cap Y \neq \emptyset$ case, we detail hereafter the transfer and give the resulting bba:

	θ_1	θ_2	$\theta_1 \cup \theta_2$	θ_3	$\theta_1 \cup \theta_3$	$\theta_2 \cup \theta_3$	Θ
θ_1,$\theta_1 \cup \theta_2$	0.44	0.01	0.23	0.01	0.53	0.71	0.80
θ_1, Θ	0.48					0.02	1.53
Θ, $\theta_1 \cup \theta_2$			0.11	0.01			1.36
Θ, Θ							0.79
m	0.14	0.002	0.05	0.003	0.08	0.11	0.62

Therefore this rule has the capability to take into account locally the conflict and reliability.

4.3 For s experts

We can propose many extensions to the two experts case. We note $Y_1, \ldots, Y_s$ the responses of the experts, with $Y_j \in 2^\Theta$.

The function w of the equation (8) can be done by the table 3, if $\bigcap_{j=1}^{s} Y_j = \emptyset$. In this case we transfer the basic beliefs onto all the focal elements Y_j, onto all the complementaries of the focal elements $\overline{Y_j}$, onto the union of the focal elements, onto all the combinations of unions and intersections of focal elements and complementaries of focal elements. The transferred mass depends on the initial bba and reliability of the focal element α. The principle is based on:

- For the complementaries of one focal element X, we consider 1-α for the reliability with α the reliability of X, and we consider 1-$m(X)$ for the bba.
- For the intersections: we do a product between reliabilities, and between bbas.
- For the unions: we take a dual form with one minus the product between reliabilities, and one minus the product between bbas.

If $\bigcap_{j=1}^{s} Y_j \neq \emptyset$, we propose the function w given by the table 4. The transfer of basic beliefs rests the same than in the $\bigcap_{j=1}^{s} Y_j = \emptyset$ case.

Note that with this extension, $\mathcal{T}(\mathbf{Y}) \neq 2^{\{\mathbf{Y},\overline{\mathbf{Y}}\}} \setminus \{\emptyset\}$, but $\mathcal{T}(\mathbf{Y}) \subset 2^{\{\mathbf{Y},\overline{\mathbf{Y}}\}} \setminus \{\emptyset\}$.

5 Conclusion

This paper formulates the idea that all the tasks for an application with the theory of belief functions must be define in the same time. Therefore, all the approaches to discount a basic belief assignment, to reduce the number of focal elements, to combine the belief and to take a decision have to be chosen jointly according to the application. Moreover, all these tasks can be seen as a function of belief transfer that we formalize.

A general approach for a given application is not an easy work. Discounting procedure and combination rules are two means to manage the conflict and can also be seen as two belief transfer functions. The second part of this paper presents how to consider the combination including the reliability often used in the discounting process. Such combination rules have the advantage to consider a local reliability. A local reliability can be obtained from an estimation of local conflict as we show before. We justify these kinds of combination rules with simple illustrative examples. However more experiments on generated bbas or on real data with precise applications are left for a future work.

element	weight .N
Y_j	$\alpha_{jY_j} m_j(Y_j)$
$\overline{Y_j}$	$(1-\alpha_{jY_j})(1-m_j(Y_j))$
$\bigcup_{j_1=1}^{n_1} Y_{j_1} \bigcup \bigcup_{j_2=1}^{n_2} \overline{Y_{j_2}}$ with $n_1+n_2=s$	$\left(1-\prod_{j_1=1}^{n_1}\alpha_{j_1Y_{j_1}}\prod_{j_2=1}^{n_2}(1-\alpha_{j_2Y_{j_2}})\right)\left(1-\prod_{j_1=1}^{n_1} m_{j_1}(Y_{j_1})\prod_{j_2=1}^{n_2}(1-m_{j_2}(Y_{j_2}))\right)$
$\bigcup_{j=1}^{s} Y_j$	$(1-\prod_{j=1}^{s}\alpha_{jY_j})(1-\prod_{j=1}^{s} m_j(Y_j))$
$\bigcap_{j_1=1}^{n_1} Y_{j_1} \bigcap \bigcap_{j_2=1}^{n_2} \overline{Y_{j_2}}$ if $\neq \emptyset$, with $n_1+n_2=s$	$\prod_{j_1=1}^{n_1}\alpha_{j_1Y_{j_1}} m_{j_1}(Y_{j_1})\prod_{j_2=1}^{n_2}(1-\alpha_{j_2Y_{j_2}})(1-m_{j_2}(Y_{j_2}))$
$\bigcap_{j=1}^{s} \overline{Y_j}$, if $\neq \emptyset$	$\prod_{j=1}^{ns}(1-\alpha_{jY_j})(1-m_j(Y_j))$
$\bigcup_{j=1}^{s} \overline{Y_j}$	$\left(1-\prod_{j=1}^{s}(1-\alpha_{jY_j})\right)\left(1-\prod_{j=1}^{s}(1-m_j(Y_j))\right)$

Table 3: The weight function given in the case of s experts when $\bigcap_{j=1}^{s} Y_j = \emptyset$.

element	weight .N
$\bigcap_{j=1}^{s} Y_j$	$\prod_{j=1}^{s}\alpha_{jY_j} m_j(Y_j))$
$\bigcup_{j=1}^{s} Y_j$	$(1-\prod_{j=1}^{s}\alpha_{jY_j})(1-\prod_{j=1}^{s} m_j(Y_j))$
Y_j	$\alpha_{jY_j} m_j(Y_j)$
$\overline{Y_j}$ if $\neq \emptyset$	$(1-\alpha_{jY_j})(1-m_j(Y_j))$
$\bigcup_{j_1=1}^{n_1} Y_{j_1} \bigcup \bigcup_{j_2=1}^{n_2} \overline{Y_{j_2}}$ if $\neq \emptyset$, with $n_1+n_2=s$	$\left(1-\prod_{j_1=1}^{n_1}\alpha_{j_1Y_{j_1}}\prod_{j_2=1}^{n_2}(1-\alpha_{j_2Y_{j_2}})\right)\left(1-\prod_{j_1=1}^{n_1} m_{j_1}(Y_{j_1})\prod_{j_2=1}^{n_2}(1-m_{j_2}(Y_{j_2}))\right)$
$\bigcap_{j_1=1}^{n_1} Y_{j_1} \bigcap \bigcap_{j_2=1}^{n_2} \overline{Y_{j_2}}$ if $\neq \emptyset$, with $n_1+n_2=s$	$\prod_{j_1=1}^{n_1}\alpha_{j_1Y_{j_1}} m_{j_1}(Y_{j_1})\prod_{j_2=1}^{n_2}(1-\alpha_{j_2Y_{j_2}})(1-m_{j_2}(Y_{j_2}))$
$\bigcap_{j=1}^{s} \overline{Y_j}$, if $\neq \emptyset$	$\prod_{j=1}^{ns}(1-\alpha_{jY_j})(1-m_j(Y_j))$
$\bigcup_{j=1}^{s} \overline{Y_j}$	$\left(1-\prod_{j=1}^{s}(1-\alpha_{jY_j})\right)\left(1-\prod_{j=1}^{s}(1-m_j(Y_j))\right)$

Table 4: The weight function given in the case of s experts when $\bigcap_{j=1}^{s} Y_j \neq \emptyset$.

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